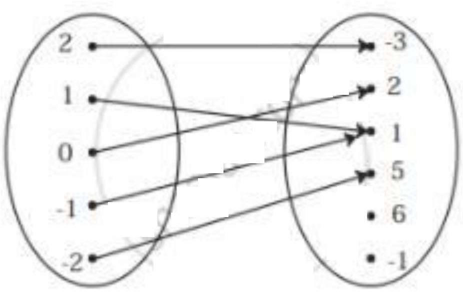
Chapter 2: Functions

EXERCISE 2.1 [PAGES 30 - 31]

Exercise 2.1 | Q 1.1 | Page 30

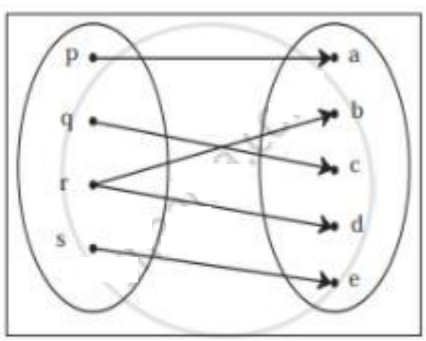
Check if the following relation is function.



SOLUTION

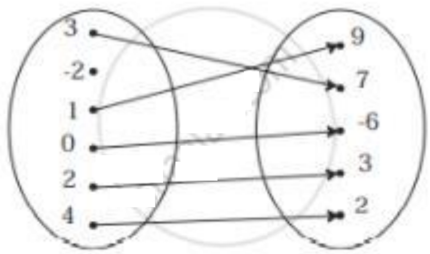
Yes
Reason:
Every element of set A has been assigned a unique element in set B.

Exercise 2.1 | Q 1.2 | Page 31
Check if the following relation is function.



No.
Reason:
An element of set A has been assigned more than one element from set B.

Exercise 2.1 | Q 1.3 | Page 31
Check if the following relation is function.



SOLUTION

No.
Reason:
Not every element of set A has been assigned an image from set B.

Exercise 2.1 | Q 2.1 | Page 31
Which sets of ordered pairs represent functions from
A = {1,2,3,4} to B = {−1,0,1,2,3}? Justify
{(1,0), (3,3), (2,−1), (4,1), (2,2)}

SOLUTION

{(1, 0), (3, 3), (2, –1), (4, 1), (2, 2)} does not represent a function.
Reason:
(2, –1) and (2, 2) show that element 2∈ A has been assigned two images – 1 and 2 from set B.

Exercise 2.1 | Q 2.2 | Page 31
Which sets of ordered pairs represent functions from
A = {1,2,3,4} to B = {−1,0,1,2,3}? Justify
{(1,2), (2,−1), (3,1), (4,3)}

SOLUTION

{(1, 2), (2, –1), (3, 1), (4, 3)} represents a function. **Reason:**
Every element of set A has a unique image in set B.

Exercise 2.1 | Q 2.3 | Page 31
Which set of ordered pair represent function from
A = {1,2,3,4}to B = {−1,0,1,2,3}? Justify.
{(1,3), (4,1), (2,2)}

SOLUTION

{(1, 3), (4, 1), (2, 2)} does not represent a function.
Reason:
3∈ A does not have an image in set B.

Exercise 2.1 | Q 2.4 | Page 31

Which set of ordered pair represent function from
A = {1,2,3,4}to B = {−1,0,1,2,3}? Justify
{(1,1), (2,1), (3,1), (4,1)}

SOLUTION

{(1, 1), (2, 1), (3, 1), (4, 1)} represents a function **Reason:**
Every element of set A has been assigned a unique image in set B.

Exercise 2.1 | Q 3.1 | Page 31
If f(m) = $m^2 - 3m + 1$, find f(0)

SOLUTION

f(m) = $m^2 - 3m + 1$
f(0) = $0^2 - 3(0) + 1 = 1$

Exercise 2.1 | Q 3.2 | Page 31
If f(m) = $m^2 - 3m + 1$, find f(−3)

SOLUTION

f(–3) = $(-3)^2 - 3(-3) + 1$
= 9 + 9 + 1 = 19

Exercise 2.1 | Q 3.3 | Page 31

If f(m) = $m^2 - 3m + 1$, find $f\left(\frac{1}{2}\right)$

SOLUTION

$$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 = 3\left(\frac{1}{2}\right) + 1 = \frac{1}{4} = \frac{3}{2} + 1$$
$$= \frac{1-6+4}{4} = -\frac{1}{4}$$

Exercise 2.1 | Q 3.4 | Page 31
If f(m) = $m^2 - 3m + 1$, find f(x + 1)

SOLUTION

$f(x + 1) = (x + 1)^2 - 3(x + 1) + 1$
$= x^2 + 2x + 1 - 3x - 3 + 1$
$= x^2 - x - 1$

Exercise 2.1 | Q 3.5 | Page 31
If f(m) = m2 − 3m + 1, find f(−x)

SOLUTION

$f(-x) = (-x)^2 - 3(-x) + 1$
$= x^2 + 3x + 1$

Exercise 2.1 | Q 4.1 | Page 31

Find x, if g(x) = 0 where

$g(x) = \frac{5x - 6}{7}$

SOLUTION

$g(x) = \frac{5x - 6}{7}$

$g(x) = 0$

$\therefore \frac{5x - 6}{7} = 0$

$\therefore 5x - 6 = 0$

$\therefore x = \frac{6}{5}$

Exercise 2.1 | Q 4.2 | Page 31

Find x, if g(x) = 0 where

$g(x) = \frac{18 - 2x^2}{7}$

SOLUTION

$g(x) = \frac{18 - 2x^2}{7}$

$g(x) = 0$

$\therefore \frac{18 - 2x^2}{7} = 0$

$\therefore 18 - 2x^2 = 0$

$\therefore x^2 = \frac{18}{2} = 9$

$\therefore x = \pm 3$

Exercise 2.1 | Q 4.3 | Page 31
Find x, if g(x) = 0 where
$g(x) = 6x^2 + x - 2$

SOLUTION

$g(x) = 6x^2 + x - 2$

$g(x) = 0$

$\therefore 6x^2 + x - 2 = 0$

$\therefore 6x^2 + 4x - 3x - 2 = 0$

$\therefore 2x(3x + 2) - 1(3x + 2) = 0$

$\therefore (2x - 1)(3x + 2) = 0$

$\therefore 2x - 1 = 0$ or $3x + 2 = 0$

$\therefore x = \frac{1}{2}$ or $x = -\frac{2}{3}$

Exercise 2.1 | Q 5 | Page 31
Find x, if $f(x) = g(x)$ where
$f(x) = x^4 + 2x^2$, $g(x) = 11x^2$

SOLUTION

$f(x) = x^4 + 2x^2$, $g(x) = 11x^2$

$f(x) = g(x)$

$\therefore x^4 + 2x^2 = 11x^2$

$\therefore x^4 - 9x^2 = 0$

$\therefore x^2(x^2 - 9) = 0$

$\therefore x = 0$ or $x^2 - 9 = 0$

$\therefore x = 0$ or $x^2 = 9$

$\therefore x = 0$ or $x = \pm 3$

Exercise 2.1 | Q 6 | Page 31
If (x) ={$x^2 + 3$, $x \le 2$, $5x + 7$, $x > 2$, then find
f(3)

f(2)
f(0)

SOLUTION

$x^2 + 3$, x≤2, 5x+ 7, x>2

i. f(3) = 5(3) + 7 = 15 + 7 = 22

ii. f(2) = 2^2 + 3 = 4 + 3 = 7

iii. f(0) = 0^2 + 3 = 3

Exercise 2.1 | Q 7 | Page 31
If ƒ(x) ={ 4x – 2,
x ≤ – 3 5,
–3 < x < 3,
x^2,
x ≥3 then find ƒ(–4), ƒ(–3),ƒ(1), ƒ(5)

SOLUTION

ƒ(x) =4x – 2,
x ≤ – 3 5,
–3 < x < 3,
x^2,
x ≥3

i. f(– 4) = 4(– 4) – 2 = – 16 – 2 = –18

ii. f(–3) = 4(–3) – 2 = – 12 – 2 = –14

iii. f(1) = 5

iv. f(5) = 5^2 = 25

Exercise 2.1 | Q 8.1 | Page 31
If ƒ(x) = 3x + 5, g(x) = 6x − 1, then find (ƒ+g) (x)

SOLUTION

ƒ(x) = 3x + 5, g(x) = 6x – 1

(ƒ+ g) x = ƒ(x) + g(x)

= 3x + 5 + 6x – 1 = 9x + 4

Exercise 2.1 | Q 8.2 | Page 31
If ƒ(x) = 3x + 5, g(x) = 6x – 1, then find (ƒ – g) (2)

SOLUTION

(ƒ – g) (2) = ƒ(2) – g(2)

= [3 (2) + 5] – [6 (2) – 1]

= 6 + 5 – 12 + 1= 0

Exercise 2.1 | Q 8.3 | Page 31

If $f(x) = 3x + 5$, $g(x) = 6x - 1$, then find (f g) (3)

SOLUTION

$(f\ g)(3) = f(3)\ g(3)$

$= [3(3) + 5]\ [6(3) - 1]$

$= (14)(17) = 238$

Exercise 2.1 | Q 8.4 | Page 31

If f(x) = 3x + 5, g(x) = 6x – 1, then find $\left(\frac{f}{g}\right)(x)$ and its domain

SOLUTION

$$\left(\frac{f}{g}\right)x = \frac{f(x)}{g(x)} = \frac{3x+5}{6x-1}, x \neq \frac{1}{6}$$

Domain = R - $\left\{\frac{1}{6}\right\}$

Exercise 2.1 | Q 9.1 | Page 31

If $f(x) = 2x^2 + 3$, $g(x) = 5x - 2$, then find $f \circ g$

SOLUTION

$f(x) = 2x^2 + 3$, $g(x) = 5x - 2$

$(fog)(x) = f(g(x)) = f(5x - 2)$

$= 2(5x - 2)^2 + 3$

$= 2(25x^2 - 20x + 4) + 3$

$= 50x^2 - 40x + 8 + 3$

$= 50x^2 - 40x + 11$

Exercise 2.1 | Q 9.2 | Page 31

If $f(x) = 2x^2 + 3$, $g(x) = 5x - 2$, then find g of

SOLUTION

$(g\ of)(x) = g(f(x)) = g(2x^2 + 3)$

$= 5(2x^2 + 3) – 2$

$= 10 x^2 + 15 – 2$

$= 10 x^2 + 13$

NOTES

$(gof)(x) = g(f(x)) = g(2x^2 + 3)$

$= 5(2x^2 + 3) – 2$

= 10 x 2 + 15 – 2

$= 10 x^2 + 13$

Exercise 2.1 | Q 9.3 | Page 31
If $f(x) = 2x^2 + 3$, $g(x) = 5x – 2$, then find fof

SOLUTION

$(fof)(x) = f(f(x)) = f(2x^2 + 3)$

$= 2(2x^2 + 3)^2 + 3$

$= 2(4x^4 + 12x^2 + 9) + 3$

$= 8x^4 + 24x^2 + 18 + 3$

$= 8x^4 + 24x^2 + 21$

Exercise 2.1 | Q 9.4 | Page 31
If $f(x) = 2x^2 + 3$, $g(x) = 5x – 2$, then find gog

SOLUTION

$(gog)(x) = g(g(x)) = g(5x – 2)$

$= 5(5x – 2) – 2$

$= 25x – 10 – 2$

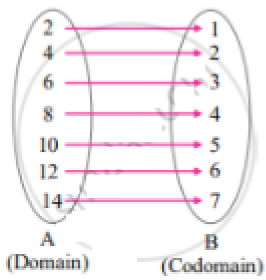
$= 25x – 12$

MISCELLANEOUS EXERCISE 2 [PAGE 32]

Miscellaneous Exercise 2 | Q 1.1 | Page 32
Which of the following relations are functions? If it is a function determine its domain and range.
{(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)}

SOLUTION

{(2, 1), (4, 2), (6, 3), (8, 4), (10, 5) (12, 6), (14, 7)}



Every element of set A has been assigned a unique element in set B.

∴ Given relation is a function.
Domain = {2, 4, 6, 8, 10, 12, 14},
Range = {1, 2, 3, 4, 5, 6, 7}

Miscellaneous Exercise 2 | Q 1.2 | Page 32
Which of the following relations are functions? If it is a function determine its domain and range.
{(0, 0), (1, 1), (1, −1), (4, 2), (4, −2), (9, 3), (9, −3), (16, 4), (16, -4)}

SOLUTION

{(0, 0), (1, 1), (1, –1), (4, 2), (4, –2), (9, 3), (9, –3), (16, 4), (16, – 4)}

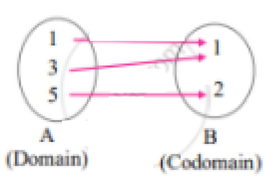
∵ (1, 1), (1, –1) ∈ the relation

∴ Given relation is not a function. As the element 1 of the domain has not been assigned a unique element of co-domain.

Miscellaneous Exercise 2 | Q 1.3 | Page 32
Which of the following relations are functions? If it is a function determine its domain and range.
{(1, 1), (3, 1), (5, 2)}

SOLUTION

{(1, 1), (3, 1), (5, 2)}



Every element of set A has been assigned a unique element in set B.

∴ Given relation is a function.
Domain = {1, 3, 5}, Range = {1, 2}

Miscellaneous Exercise 2 | Q 2 | Page 32

A function f: R→R defined by f(x) = $3\frac{x}{5} + 2,$ x∈R. Show that f is one-one and onto. Hence find f^{-1}

SOLUTION

f: R →R defined by f(x) = $\frac{3x}{5}$ +2

First we have to prove that f is one-one function for that we have to prove if $f(x_1) = f(x_2)$ then $x_1 = x_2$

Here f(x) = $\frac{3x}{5} + 2$

Let f(x1) = f(x2)

$$\therefore \frac{3x_1}{5} + 2 = \frac{3x_2}{5} + 2$$

$$\therefore \frac{3x_1}{5} = \frac{3x_2}{5}$$

$\therefore x_1 = x_2$

∴ f is a one-one function. Now, we have to prove that f is an onto function. Let y ∈ R be such that y = f(x)

$\therefore$ y = $\frac{3x}{5} + 2$

$\therefore$ y -2 = $\frac{3x}{5}$

$$\therefore x = \frac{5(y-2)}{3} \in R$$

∴ for any y ∈ co-domain R, there exist an element x = $\frac{5(y-2)}{3}$ domain R such that f(x) = y

∴ f is an onto function.

∴ f is one-one onto function.

∴ f^{-1} exists

∴ $f^{-1}(y) = \frac{5(y-2)}{3}$

∴ $f^{-1}(x) = \frac{5(x-2)}{3}$

Miscellaneous Exercise 2 | Q 3 | Page 32

A function f is defined as follows

f(x) = 4x + 5, for −4 ≤ x < 0.

Find the values of f(−1), f(−2), f(0), if they exist.

SOLUTION

f(x) = 4x + 5, – 4 ≤ x < 0

f(–1) = 4(–1) + 5 = –4 + 5 = 1

f(–2) = 4(–2) + 5 = –8 + 5 = –3

x = 0 ∉ domain of f

∴ f(0) does not exist.

Miscellaneous Exercise 2 | Q 4 | Page 32

A function f is defined as follows: f(x) = 5 − x for 0 ≤ x ≤ 4 Find the value of x such that f(x) = 3

SOLUTION

f(x) = 5 -x

f(x) = 3

∴ 5 – x = 3

∴ x = 5 – 3 = 2

Miscellaneous Exercise 2 | Q 5 | Page 32

If $f(x) = 3x^2 - 5x + 7$ find f(x − 1).

SOLUTION

$f(x) = 3x^2 - 5x + 7$

∴ $f(x - 1) = 3(x - 1)^2 - 5(x - 1) + 7$

$= 3(x^2 - 2x + 1) - 5(x - 1) + 7$

$= 3x^2 - 6x + 3 - 5x + 5 + 7$

$= 3x^2 - 11x + 15$

Miscellaneous Exercise 2 | Q 6 | Page 32
If f(x) = 3x + a and f(1) = 7 find a and f(4).

SOLUTION

f(x) = 3x + a

f(1) = 7

∴ 3(1) + a = 7

∴ a = 7 – 3 = 4

∴ f(x) = 3x + 4

∴ f(4) = 3(4) + 4 = 12 + 4 = 16

Miscellaneous Exercise 2 | Q 7 | Page 32
If f(x) = ax^2 + bx + 2 and f(1) = 3, f(4) = 42. find a and b.

SOLUTION

f(x) = ax^2 + bx + 2

f(1) = 3

∴ a$(1)^2$ + b(1) + 2 = 3

∴ a + b = 1 …(i)

∴ f(4) = 42

∴ a(4)2 + b(4) + 2 = 42

∴ 16a + 4b = 40

Dividing by 4, we get

4a + b = 10 …(ii)

Solving (i) and (ii), we get

a = 3, b = –2

Miscellaneous Exercise 2 | Q 8 | Page 32

If f(x) = $\frac{2x-1}{5x-2}, x \neq \frac{2}{5}$

Verify whether (fof) (x) = x

SOLUTION

(fof) (x) = f(f(x))

$$= f\left(\frac{2x-1}{5x-2}\right)$$

$$= \frac{2\frac{2x-1}{5x-2}-1}{5\frac{2x-1}{5x-2}-2}$$

$$= \frac{4x-2-5x+2}{10x-5-10x+4} = \frac{-x}{-1} = x$$

Miscellaneous Exercise 2 | Q 9 | Page 32

If f(x) = $\frac{x+3}{4x-5}$, g(x) = $\frac{3+5x}{4x-1}$ then verify that (fog) (x) = x.

SOLUTION

f(x) = $\frac{x+3}{4x-5}$, g(x) = $\frac{3+5x}{4x-1}$

(fog)(x) = f(g(x))

$$= f\left(\frac{3+5x}{4x-1}\right)$$

$$= \frac{\frac{3+5x}{4x-1}+3}{\left(4\frac{3+5x}{4x-1}\right)-5}$$

$$= \frac{3+5x+12x-3}{12+20x-20x+5} = \frac{17x}{17} = x$$