34. Magnetic Field

Short Answer

1. Question

Suppose a charged particle moves with a velocity v near a wire carrying an electric current. A magnetic force, therefore, acts on it. If the same particle is seen from a frame moving with velocity v in the same direction, the charge will be found at rest. Will the magnetic force become zero in this frame? Will the magnetic field become zero in this frame?

Answer

Formula used:

The magnetic force is given by

$F = qvBsin\theta$

Where F is magnetic force

q is electric charge $=1.6 \times 10^{-19}C$

v is velocity

Angle between magnetic force and velocity component

A Charge particle is moving near a conducting wire. And also a frame is moving in the same direction which the charge is moving. Seeing from the frame it is known that the charge is at rest. So velocity of the charge particles will be zero. Thus magnetic force will be zero.

When the charge is at rest the current will be zero. We don't know either it is a positive or negative charge. Let us consider it is a negative charge. Seeing from the frame negative charge is at rest. But there is a positive charge flow. Due to this magnetic field will exists. So the magnetic field should not be zero.

2. Question

Can a charged particle be accelerated by a magnetic field? Can its speed be increased?

Answer

Formula used:

$F = ma$

Where F is magnetic force

m is mass

a is acceleration

Here the force (magnetic force) which is acting on the charged particle will try to accelerate it. If the charged particle is in the direction perpendicular to the magnetic field, then the particle will move in a circular path. In that circular path the direction of the charged particle will change.

Formula used: Magnetic force is given by

$F = qvBsin\Theta$

Where F is magnetic force

q is electric charge =$1.6 \times 10^{-19}C$

v is velocity

Angle between magnetic force and velocity component

Force is always act perpendicular to the direction of velocity of a charged particle. So it can't change the magnitude of the velocity of the charged particle. Velocity, magnetic field and force are perpendicular to each other. So it can change the direction of the velocity of a charged particle. So our answer is Yes, a charged particle can be accelerated by a magnetic filed but its speed can't be increase.

3. Question

Will a current loop placed in a magnetic field always experience a zero force?

Answer

Effect of the magnetic force on a current loop is totally based on the uniformity of the magnetic field. If the field is non uniform, then current loop will not experience a force. So magnetic force on the loop will be zero. If the field is uniform, then the force experienced on the current loop either zero or non zero.

4. Question

The free electrons in a conducting wire are in constant thermal motion. If such a wire, carrying no current, is placed in a magnetic field, is there a magnetic force on each free electron? Is there a magnetic force on the wire?

Answer

Formula used: The magnetic force is given by

$F = ILBsin\theta$

Where F is magnetic force

I is current

L is length of the wire

B is magnetic field

If the electron is in motion definitely there will be a magnetic force on it. Electrons are present in conducting wire. But current flowing in the wire is zero. When this wire is placed in a magnetic field the electrons will come to motion but the motion will be random. So we can neglect the effect of force on each electron. So total magnetic force acting on the wire will be zero.

5. Question

Assume that the magnetic field is uniform in a cubical region and is zero outside. Can you project a charged particle from outside into the field so that the particle describes a complete circle in the field?

Answer

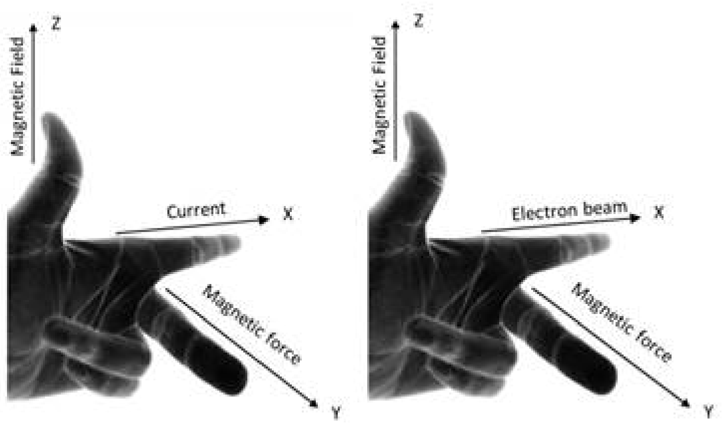
Magnetic field is uniform(assumption). If the charged particle is perpendicular to the magnetic field the particle will exhibits a circular trajectory. Let us consider that the charged particle is coming into the field from outside along the positive y-axis. And Let us take the field is in x-axis. X-axis is perpendicular to the y-axis. So magnetic field will be perpendicular to the changed particle. Then the particle will make a circular path.

6. Question

An electron beam projected along the positive x-axis deflects along the positive y-axis. If this deflection is caused by a magnetic field, what is the direction of the field? Can we conclude that the field is parallel to the z-axis?

Answer

If the force is acting in the y-axis then the electron beam will be deflected from positive x-axis along y-axis. The electrons motion is in positive x-axis and then current will be along negative x-axis. And the force is acting along the y-axis. According to Fleming's left hand rule the magnetic filed will be in z-axis. So we can conclude that the field is parallel to the z-axis.



7. Question

Is it possible for a current loop to stay without rotating in a uniform magnetic field? If yes, what should be the orientation of the loop?

Answer

Yes

Formula used: The net torque of a loop is given by

$\tau = iABsin\Theta$

Where τ is torque

i is current

A is area vector

B is magnetic field

θ is the angle between the area vector and magnetic field

When θ=0, $\tau = iABsin(0)\ \sin(0) = 0$

$\tau = 0$

When θ=180,

$\tau = iABsin(180)\ \sin(180) = 0$

$\tau = 0$

If θ is zero or 180° (integral multiple of π) then torque will be zero. So coil will stop its rotation. θ=nπ represents that the magnetic field is in parallel with area vector. So the loop can stay in a uniform magnetic field without rotation.

8. Question

The net charge in a current-carrying wire is zero. Then, why does a magnetic field exert a force on it?

Answer

Mostly positive charge on the wire is because of protons which are containing nucleus. When the protons are not in motion the force acting on them will be zero. So charge will be zero. But there is a negative charge too. Negative charge is due to electrons.

Formula used: $F = qvBsin\Theta$

Where F is magnetic force

q is electric charge =$1.6 \times 10^{-19}C$

v is velocity

B is magnetic field

Θ is angle between magnetic field and a charge

So when the electrons are in motion the wire will carry current. Magnetic force will act on the wire. That's why magnetic field exerts a force on the wire.

9. Question

The torque on a current loop is zero if the angel between the positive normal and the magnetic field is either θ = 0 or θ = 180°. In which of the two orientations, the equilibrium is stable?

Answer

Formula used: Potential energy is given by

$U = -mBcos\theta$

Where m is magnetic moment

B is magnetic field

Let is take B=0.1T, m = 7.85×10^{-11} A m �

If θ is zero, then

$U = -7.85 \times 10^{-11} \times 0.1 \times \cos(0)$

$= -7.85 \times 10^{-11} \times 0.1 \times 1$

$U = -7.85 \times 10^{-12} J$

If θ=180 then

$U = -7.85 \times 10^{-11} \times 0.1 \times \cos(180)$

$= -7.85 \times 10^{-11} \times 0.1 \times (-1)$

$U = +7.85 \times 10^{-12} J$

If energy is minimum, then system will be more stable. If energy is increase, then the system will loose it's stability. For θ=0 the energy is minimum and the equilibrium will stable. For θ=180 the energy is maximum and equilibrium will unstable.

10. Question

Verify that the unit's weber and volt second are the same.

Answer

Formula used: Magnetic force is by

$F = qvB$

Where F is magnetic force

q is electric charge =$1.6 \times 10^{-19} C$

v is velocity

B is magnetic field

$$B = \frac{F}{qv}$$

One of the unit that magnetic force measured is Weber/m �.

N m² is newton meter²

C is Coulomb

m/s is meter per second

V/m is volt per meter

$$\frac{weber}{m^2} = \frac{N\,m^2}{C\,\frac{m}{s}}$$

$$weber = \frac{N}{C}\,m\,s$$

$$= \frac{V}{m}\,m\,s$$

(We know that $\frac{N}{C} = \frac{V}{m}$)

$$weber = V\,s$$

$$1\,weber = 1 volt\ second$$

Thus weber and the volt second are same.

Objective I

1. Question

A positively charged particle projected towards east is deflected towards north by a magnetic field. The field may be

A. towards west

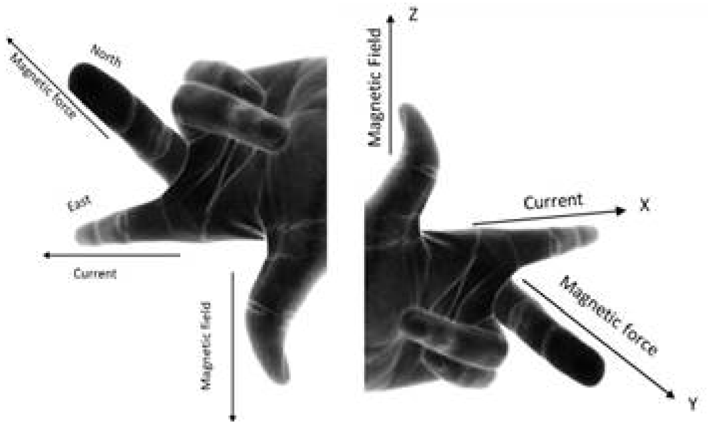
B. towards south

C. upward

D. downward.

Answer

The positive charge is moving towards east. So the current is in the direction of east. It is deflected towards north. So the magnetic force is acting in the north direction. Motion is in east and force is acting in north. Magnetic field is always perpendicular to the both force and velocity of a particle. According to left hand Fleming's rule the magnetic force will be in downward direction.



2. Question

A charged particle is whirled in a horizontal circle on a frictionless table by attaching it to a string fixed at one point. If a magnetic field is switched on in the vertical direction, the tension in the string

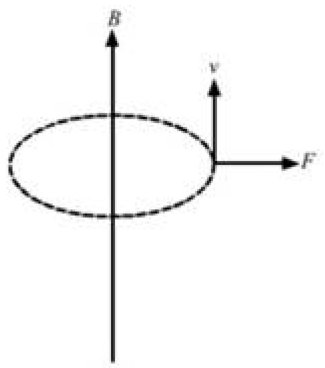
A. will increase

B. will decrease

C. will remain the same

D. may increase or decrease

Answer



Force is depending upon the direction of the whirl and also charge of a particle. Which type of charge particle it is not given. And We don't know which direction the whirl is rotating. Based on these things we can say that force(tension) may increase or decrease. The magnetic field is in vertical direction and also we don't know it is upward or downward. According to Flemings left hand rule the force will act either inward or outward. So option D is correct.

3. Question

Which of the following particles will experience maximum magnetic force (magnitude) when projected with the same velocity perpendicular to a magnetic field?

A. Electron

B. Proton

C. He^+

D. Li^{++}

Answer

Here the particle is experiencing a maximum magnetic filed and projected with a same velocity. There are no variations in the both. So both magnetic field and velocity are constant over here.

Formula used: The magnetic force is given by

$F = qvBsin\Theta$

Where F is magnetic force

q is electric charge $=1.6 \times 10^{-19}C$

v is velocity

Θ is angle between magnetic field and a charge

In the above equation magnetic field and velocity are constant. So force will depend on the charge. In the option given Li^{++} has the highest charge. After Li^{++}, He^+ has highest charge. So Li^{++} will experience a maximum magnetic field.

4. Question

Which of the following particles will describe the smallest circle when projected with the same velocity perpendicular to a magnetic field?

A. Electron

B. Proton

C. He^+

D. Li^+

Answer

In the options given election has a lowest charge.

Electron < Proton < He^+ <Li^{++}

Formula used: The magnetic force is given by

$F = qvB$

Where F is magnetic force

q is electric charge $=1.6 \times 10^{-19}C$

v is velocity

$$F = \frac{mv^2}{r}$$

m is mass of the charge particle

v is velocity

r is radius of the projectile motion

$$\frac{mv^2}{r} = qvB$$

$$mv^2 = rqvB$$

$$mv = rqB$$

$$r = \frac{mv}{qB}$$

Here v and B are constants and q is same for all the charge particles. r is proportional to m. If radius is small radius will become small. Electron exhibits light weight among the all other charge particles. Because of light weight it will creates a circle with a small radius when it is projected. So option A is correct.

5. Question

Which of the following particles will have minimum frequency of revolution when projected with the same velocity perpendicular to a magnetic field?

A. Electron

B. Proton

C. He^+

D. Li^+

Answer

Formula used: Time period of revolution of a particle is given by

$$T = \frac{2\pi m}{qB}$$

$$f = \frac{qB}{2\pi m}$$

In the above options Electron is light weight particle and Li^+ has highest weight compared to other particles in the options given. Frequency f is inversely proportional to mass m. So Li^+ will have minimum frequency of revolution. So option D is correct.

6. Question

A circular loop of area 1 cm^2, carrying a current of 10 A, is placed in a magnetic field of 0.1 T perpendicular to the plane of the loop. The torque on the loop due to the magnetic field is

A. zero

B. 10^{-4} Nm

C. 10^{-2} Nm

D. 1 N m

Answer

Given a circular loop area =1cm �

Current =10A

Magnetic field strength =0.1 T

Given loop is circular. On each and every point of this circular loop there exists two forces. It will happen in case of circular loop only. The forces are equal but their magnitudes are opposite. Because of the opposite magnitudes they cancel each other. Then net force will be equal to zero. If we place any loop in magnetic field it definitely conducts current. In case of circular loop, the torque due to magnetic field on loop will be zero.

7. Question

A beam consisting of protons and electrons moving at the same speed goes through a thin region in which there is a magnetic field perpendicular to the beam. The protons and the electrons

A. will go undeviated

B. will be deviated by the same angle and will not separate

C. will be deviated by different angels and hence separate

D. will be deviated by the same angle but will separate.

Answer

Formula used: Magnetic force is given by

$F = qvB$

Where F is magnetic force

q is electric charge =$1.6 \times 10^{-19} C$

v is velocity

Force on the electron is $F = -qvB$

Force on a the proton is $F = +qvB$

The magnitudes of force on proton and electron are different but magnetic force acting on the both are same. The force acting on the proton is in positive in direction and the force acting on the electron is in negative direction. Even same force is acting on them but because of different directions they got deviated by different angles and then they will separate. So option C is correct.

8. Question

A charged particle moves in a uniform magnetic field. The velocity of the particle at some instant makes an acute angel with the magnetic field. The path of the particle will be

A. a straight line

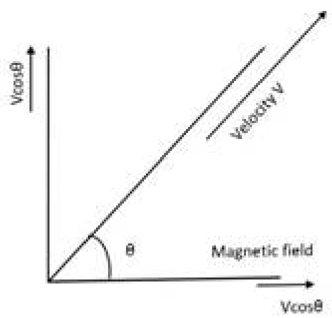
B. a circle

C. a helix with uniform pitch

D. a helix with non-uniform pitch

Answer

Given that velocity of a particle is making an acute angle with the magnetic field. Velocity of a particle is resolves into two components. Magnetic field always change the direction of the velocity. One component is perpendicular to the magnetic field that is Vsinθ and another component is parallel to the magnetic field that is Vcosθ. Vertical component of a velocity is perpendicular to the magnetic field. Then force will act on the charged particle. Because of the vertical force(Vsinθ), the particle will make a circular path. Because of the two components of the velocity the particle will move in a helical path. This path will maintain the uniformity. So option C is correct.



9. Question

A particle moves in a region having a uniform magnetic field and a parallel, uniform electric field. At some instant, the velocity of the particle is perpendicular to the field direction. The path of the particle will be

A. a straight line

B. a circle

C. a helix with uniform pitch

D. a helix with non-uniform pitch

Answer

The region has a uniform magnetic field and an electric field.

Formula used: Lorentz's force is given by

$F = qE + qvB$

Where F is Lorentz's force

q is electric charge $= 1.6 \times 10^{-19}C$

E is electric field strength

B is magnetic field strength

V is velocity

The electric field will accelerate the particle to move in the direction of the magnetic field. Magnetic field will always change the direction of velocity of a particle. Because of these the particle will move in a helix path with non-uniformity. So option D is correct.

10. Question

An electric current i enters and leaves a uniform circular wire of radius a through diametrically opposite points. A charge particle q moving along the axis of the circular wire passes tough its center at speed v. The magnetic force acting on the particle when it passes through the center has a magnitude.

A. $qv\frac{\mu_0 i}{2\alpha}$

B. $qv\frac{\mu_0 i}{2\pi\alpha}$

C. $qv\frac{\mu_0 i}{\alpha}$

D. zero

Answer

Current is entering into the circular wire. According to right hand thumb rule the magnetic field will be in axis of the circular wire. It is given that the charge particle is moving along the axis of the wire.

Formula used: Magnetic force is given by the formula

$F = qvBsin\Theta$

Where F is magnetic force

q is electric charge =$1.6 \times 10^{-19}C$

v is velocity

B is magnetic filed

Θ is angle between magnetic field and a charge

θ will be zero because the angle between the magnetic field and charge particle is 0° or 180°.

$F = qvBsin(0°)$

$F = 0$ [$\sin(0°) = 0$]

When the particle is moving along the axis of a wire the magnetic force acting on it will be zero. So option D is correct.

Objective II

1. Question

If a charged particle at rest experiences no electromagnetic force,

A. the electric field must be zero.

B. the magnetic field must be zero

C. the electric field may or may not be zero

D. the magnetic field may or may not be zero

Answer

Formula used: The magnetic force is given by

$F = qvBsin\Theta$

Where F is magnetic force

q is electric charge =$1.6 \times 10^{-19}C$

v is velocity

B is magnetic filed

Θ is angle between magnetic field and charge

Here the particle is rest so velocity is zero. Then magnetic force is zero. But we don't know whether the magnetic field is zero or not as the force exerted on a particle under the influence of magnetic field depends on the orientation of the particle with respect to the magnetic field. And the electromagnetic force on particle is also not present. Magnetic field may not be zero. So electric field must be zero. So option A and D are correct.

2. Question

If a charged particle kept at rest experiences an electromagnetic force,

A. the electric field must not be zero

B. the magnetic field must not be zero

C. the electric field may or may not be zero

D. The magnetic field may or may not be zero.

Answer

The electromagnetic force is acting on the charge particle.

Formula used: 1. Electric force is given by

$F = qE$

Where F is electric force

q is electric charge $= 1.6 \times 10^{-19} C$

E is electric field

From the above formula we can say that electric field should not be zero. Because electric force is acting on the particle. And the particle is rest, so velocity of the charged particle is zero.

2. The magnetic force is given by

$F = qvBsin\Theta$

Where F is magnetic force

q is electric charge =$1.6 \times 10^{-19} C$

v is velocity

B is magnetic filed

Θ is angle between magnetic field and charge

From the above equation we can say that magnetic fore will become zero. But we don't know whether the magnetic filed is zero or not. We cannot make the statement about magnetic field. So magnetic field may or may not be zero. Option A and D are correct.

3. Question

If a charged particle projected in a gravity-free room deflects,

A. there must be an electric field

B. there must be a magnetic filed

C. both fields cannot be zero.

D. both fields can be non zero.

Answer

The particle is projected in gravity-free room. If no force is acting on the charge particle means magnetic and electric force is not acting on the particle, then the particle will not deflect. If any one the force is acting on the charged particle, then it gets deflects. And also if both the force is acting on it then also the particle will deflect. So both electric and magnetic field cannot be zero. And also they can be non zero. So options C an D are correct.

4. Question

A charged particle moves in a gravity-free space without change in velocity. Which of the following is/are possible?

A. $E = 0, B = 0$

B. $E = 0, B \neq 0$

C. $E \neq 0, B = 0$

D. $E \neq 0, B \neq 0$

Answer

The particle is in a gravity free space. If no force is acting on the particle, the velocity will be constant. If both the force electric and magnetic forces acting on the particle is same then both forces get canceled and force acting on the particle will be zero. And if electric force is zero and magnetic force is non zero, in this case if the magnetic field is in the direction of velocity of a particle then the magnetic force acting on it will become zero. Option A, B and D are correct.

5. Question

A charged particle moves along a circle under the action of possible constant electric and magnetic fields. Which of the following are possible?

A. $E = 0, B = 0$

B. $E = 0, B \neq 0$

C. $E \neq 0, B = 0$

D. $E \neq 0, B \neq 0$

Answer

charged particle is moving in a circle. If Electric force is acting on the particle, the speed of the particle will increase. Because of this the particle cannot be move in a circle. In case of magnetic field, it will only change the direction of the velocity of a particle. It can't change speed of the charged particle. So magnetic field cannot be zero. So option b is correct.

6. Question

A charged particle goes undeflected in a region containing electric and magnetic field. It is possible that

A. $\vec{E} \parallel \vec{B}, \vec{v} \parallel \vec{E}$

B. $\vec{E}$ is not parallel to $\vec{B}$

C. $\vec{v} \parallel \vec{B}$ but $\vec{E}$ is not parallel to $\vec{B}$

D. $\vec{E} \parallel \vec{B}$ but $\vec{v}$ is not parallel to $\vec{E}$.

Answer

Let us consider the electric field is parallel to the magnetic field. In this case the charges particle will accelerate and also it will move in same direction. Then it cannot deflect. If electric field is not parallel to magnetic field means that may be in perpendicular to each other. In this case magnetic field, electric field and velocity of a particle will be in perpendicular to each other. If both forces are equal, then force acting on the particle will be zero. Then particle will not deflect. So option A and B are correct. Option C and D are not valid conditions.

7. Question

If a charged particle goes unaccelerated in a region containing electric and magnetic fields.

A. $\vec{E}$ must be perpendicular to $\vec{B}$

B. $\vec{v}$ must be perpendicular to $\vec{E}$

C. $\vec{v}$ must be perpendicular to $\vec{B}$

D. E must be equal to vB.

Answer

The charged particle must not accelerate in the region. If both the field are counter balance, then the particle won't accelerate. If Electric force is acting along the direction of magnetic force and both are same then the particle will not effect by these fields. So particle won't accelerate. And if the electric field is parallel to the velocity of a particle then it will definitely accelerate. So electric field must be perpendicular to the direction of the velocity. So options A and B are correct.

8. Question

Two ions have equal masses but one is singly ionized and the other is doubly ionized. They are projected from the same place in a uniform magnetic field with the same velocity perpendicular to the field.

A. Both ions will go along circles of equal radii.

B. The circle described by the singly ionized charge will have a radius double that of the other circle.

C. The two circles do not touch each other.

D. The two circles touch each other.

Answer

Formula used: Radius of the orbit of a particle is given by

$$r = \frac{mv}{qB}$$

Where r is radius of the orbit

m is mass

v is velocity

q is electric charge

B is electric field

Radius r is inversely proportional to the charge. So singly ionized charge will have a double radius than doubly ionized charge. And both the charges are projected from the same place. So they will touch each other at the beginning. So options B and D are correct.

9. Question

An electron is moving along the positive x-axis. You want to apply a magnetic field for a short time so that the electron may reverse its direction and move parallel to the negative x-axis. This can be done by applying the magnetic field along.

A. y-axis

B. z-axis

C. y-axis only

D. z-axis only

Answer

Electron is moving along the positive x-axis now. If the particle is moving in the direction of magnetic field, then the field does not change the direction of velocity of the electron. If we apply the magnetic field in the y-axis it can reverse the direction of velocity of the electron. And also field in z-axis also can reverse the direction of the electron. So option A and B are correct.

10. Question

Let $\vec{E}$ and $\vec{B}$ denote electric and magnetic fields in a frame S. $\vec{E}'$ and $\vec{B}'$ in another frame S' moving with respect to S at a velocity $\vec{v}$. Two of the following equations are wrong. Identify them.

A. $B_y' = B_y + \frac{vE_z}{c^2}$

B. $E_y' = E_y - \frac{vB_z}{c^2}$

C. $B_y' = B_y + vE_z$

D. $E_y' = E_y + vB_z$

Answer

Formula used:

Electric force is given by $F = qE$

Where F is electric force

q is electric charge $= 1.6 \times 10^{-19} C$

E is electric field

Magnetic force is given by $F = qv$

Where F is magnetic force

q is electric charge $= 1.6 \times 10^{-19} C$

v is velocity

B is magnetic force

From the above formulas $E = vB$ and $B = \frac{E}{v}$

From the dimensional analysis we can write equations as

i. $B_y' = B_y + \frac{vE_z}{c^2}$

ii. $E'_y = E_y + vB_z$

So only options A and D are correct.

Exercises

1. Question

An alpha particle is projected vertically upward with a speed of 3.0×10^4 kg s^{-1} in a region where a magnetic field of magnitude 1.0 T exists in the direction south to north. Find the magnetic force that acts on the α-particle.

Answer

Given -

Speed of the alpha particle in upward direction,

$v = 3 \times 10^4$ km/s

since 1Km = 1000m

$\Rightarrow v = 3 \times 10^7$ m/s

Magnetic field, $B = 1.0$ T

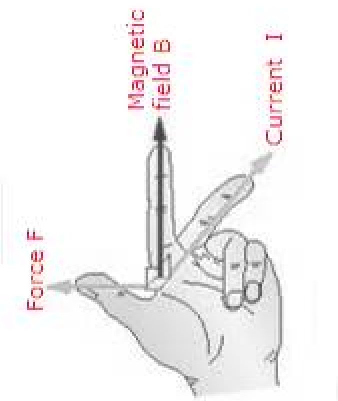
Charge on the alpha particle is given as ,

$$q = 2 \times e$$

where e is the charge of an electron.

$\Rightarrow q = 2 \times 1.6 \times 10^{-19}$ C,

We need to calculate the magnetic force that acts on the α-particl



The direction of magnetic force can be found using Fleming's left-hand rule.

The direction of the magnetic field is from south to north.

We know, Lorentz force F is given by -

$$F = q \times v \times B \sin\theta,$$

where,

q = charge

v = velocity of the charge

B=magnetic field

and

θ= angle between V and B

Now, magnetic force acting on the α -particle,

$F = qvB\ sin\theta,$

Substituting the values

$F = qvB\ sin90°$

$= 2 \times 1.6 \times 10^{-19} \times 3 \times 10^{7} \times 1$

$= 9.6 \times 10^{-12}$ N

2. Question

An electron is projected horizontally with a kinetic energy of 10 keV. A magnetic filed of strength 1.0×10^{-7} T exists in the vertically upward direction.

(a) Will the electron deflect toward right or towards left of its motion?

(b) Calculate the sideways deflection of the electron in travelling through 1m. Make appropriate approximations.

Answer

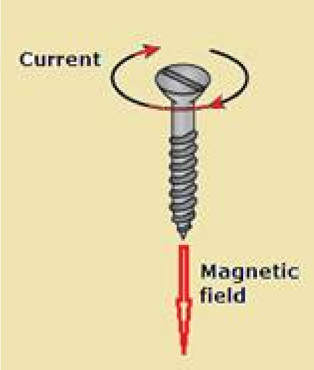
Given-The kinetic energy of the electron when projected towards horizontal direction,

$K.E$ = 10 keV = 1.6×10^{-15} J

Magnetic field, $B = 1 \times 10^{-7}$ T

Charge on an electron =1.60×10^{-19}

mass of an electron = 9.1×10^{-31} kilograms



(a) The direction electron deflection can be found by the right-hand screw rule.

Given the direction of magnetic field is vertically upward.

So, the electron will be deflected towards left.

(b) Kinetic energy-

$$K.E. = \frac{1}{2}mv^2$$

Where

m is the mass of the electron

v is the velocity of the electron

Solving for v, we get

$$\Rightarrow \mathbf{v} = \sqrt{\frac{K.E. \times 2}{m}} \ (1)$$

We know, Lorentz force F is given by -

$$F = qvB \sin\theta \ (2)$$

where,

q = charge

v = velocity of the charge

B=magnetic field

and

θ= angle between V and B

And Newton's second law of motion

$$F = m \times a \ (3)$$

where m = mass

a= acceleration

we know

$$velocity = \frac{distance}{time}$$

⇒

$$time = \frac{distance}{velocity}$$

From (1) and (2) -

$$a = \frac{q \times v \times B}{m} \ (4)$$

Applying 2nd equation of motion

$$s = ut + \frac{1}{2}at^2 \ (5)$$

where,

a= acceleration

u = initial velocity

t= Time taken to cross the magnetic field

Since, the initial velocity is zero,

from (1),(2) and (3)

$$s = \frac{1}{2}at^2$$

$$s = \frac{1}{2} \times \frac{q \times v \times B}{m} \times \left(\frac{distance}{velocity}\right)^2$$

substituting the values -

$$\Rightarrow s = \frac{1}{2} \times \frac{q \times v \times B}{m} \times t^2$$

$$= \frac{1}{2} \times \frac{q \times v \times B}{m} \times \frac{x^{\ 2}}{v^2}$$

$$= \frac{1}{2} \times \frac{q \times B}{m} \times \frac{x^2}{v}$$

From (1)

$$\Rightarrow s = \frac{1}{2} \times \frac{q \times B}{m} \times \frac{x^2}{\sqrt{\frac{K.E. \times 2}{m}}}$$

substituting the values-

$$\Rightarrow s = \frac{1}{2} \times \frac{1.6 \times 10^{-19} \times 1 \times 10^{-7}}{9.1 \times 10^{-31}} \times \frac{1^2}{\sqrt{\frac{1.6 \times 10^{-15} . \times 2}{9.1 \times 10^{-31}}}}$$

$$\Rightarrow s = 1.5 \times 10^{-2}\ cm$$

3. Question

A magnetic field of (4.0 x 10^{-3} vector k)T exerts a force of (4.0 vector i + 3.0 vector j) x 10^{-10}N on a particle having a charge of 1.0 x 10^{-9}C and going in the X-Y plane. Find the velocity of the particle.

Answer

Given -

$$Force, F = (4.0\,\hat{\imath} + 3.0\,\hat{\jmath}) \times 10 - 10N$$

Magnetic field, $B = 4.0 \times 10^{-3}\,\hat{k}$ T

Electric charge on the particle, $q = 1 \times 10^{-9}$ C

Also given that the charge is going in the X-Y plane,

Therefore, the x-component of force, $F_x = 4 \times 10^{-10}$ N

and the y-component of force, $F_y = 3 \times 10^{-10}$ N

We know, Lorentz force F is given by -

$$F = qvB\ sin\theta$$

where,

q = charge

v = velocity of the charge

B=magnetic field

and

θ= angle between V and B

$$\Rightarrow v = \frac{F}{q \times B\ sin\theta}\ (1)$$

On putting the respective values, we get

Let's take motion along x-axis-

From (1)

$$v_x = \frac{4 \times 10^{-10}}{1 \times 10^{-9} \times 4.0 \times 10^{-3}}$$

$$= 100\, m/s$$

Similarly, motion along y-axis

from (1)

$$v_y = \frac{3 \times 10^{-10}}{1 \times 10^{-9} \times 4.0 \times 10^{-3}}$$

$$= 75\, m/s$$

Hence. velocity of the particle, $= -75i + 100j$ m/s

4. Question

An experimenter's diary reads as follow: "a charged particle is projected in a magnetic field of $\left(7.0\vec{i} - 3.0\vec{j}\right) \times 10^{-3}$ T. The acceleration of the particle is found to be $\left(3.0\vec{i} + 7.0\vec{j}\right) \times 10^{-6}$ ms^{-2n}. The number to the left of $\vec{i}$ in in the last expression was not readable. What can this number be?

Answer

Given-

Magnetic field, $B = (7.0\hat{i} - 3.0\hat{j}) \times 10^{-3}\, T$

Acceleration of the particle, $a = (\alpha\hat{i} + 7\hat{j}) \times 10^{-6}\, m/s^2$

Let denoted the unidentified number as α

Since, magnetic force always acts perpendicular to the motion of

the particle, so, B and a are perpendicular to each other.

So, the dot product of the two quantities should be zero.

That is,

$$B.a = 0$$

$$\Rightarrow (\alpha\hat{i} + 7\hat{j}) \times 10^{-6} . (7.0\hat{i} - 3.0\hat{j}) \times 10^{-3} = 0$$

$$\Rightarrow 7\alpha \times 10^{-3} \times 10^{-6} - 3 \times 10^{-3} \times 7 \times 10^{-6} = 0$$

$\Rightarrow 7\alpha - 21 = 0$

$$\alpha = \frac{21}{7} = 3$$

Hence acceleration of the particle is $(3\hat{i} + 7\hat{j}) \times 10^{-6}$ m/s^2.

5. Question

A 10 g bullet having a charge of 4.00 μC is fired at a speed of 270 ms^{-1} in a horizontal direction. A vertical magnetic field of 500 μT exists in the space. Find the deflection of the bullet due to the magnetic field as it travels through 100 m. Make appropriate approximations.

Answer

Given-

Mass of the bullet, m = 10g = 10^{-3} Kg

Charge of the bullet, q = 4.00 μC =10^{-6} C

Speed of the bullet in horizontal direction, v = 270 m/s

Vertical magnetic field, B = 500 μT = 500× 10^{-6} T

Distance travelled by the bullet, d = 100 m

Magnetic force,

We know, Lorentz force F is given by -

$F = qvB\ sin\theta\ (1)$

where,

q = charge

v = velocity of the charge

B=magnetic field

and

θ= angle between V and B

Also,

And Newton's second law of motion

$F = m \times a\ (3)$

where m = mass

a= acceleration

Using equation (1) –

$$m \times a = qvB\ sin\theta$$

$$\Rightarrow a = \frac{qvB}{m}$$

$$= \frac{4.00 \times 10^{-6} \times 270 \times 500 \times 10^{-6} 10}{10^{-3}}$$

we know

$$velocity = \frac{distance}{time}$$

$\Rightarrow$

$$time = \frac{distance}{velocity}$$

Substituting the values, time taken by the bullet to travel 100 m horizontally,

$$t = \frac{d}{v} = \frac{100}{270}\ s$$

Applying 2nd equation of motion

$$s = ut + \tfrac{1}{2}at^2\ (5)$$

where,

a= acceleration

u = initial velocity

t= Time taken to cross the magnetic field

Since, the initial velocity is zero,

from (1),(2) and (3)

$$s = \frac{1}{2}at^2$$

Now, the deflection caused by the magnetic field in this time

interval,

$$s = \frac{1}{2}at^2$$

$$= \frac{1}{2} \times 4.00 \times 10^{-6} \times 270 \times 500 \times 10^{-6} 10 \times 10 - 3 \times (\frac{100}{270})^2$$

$$= 3.7 \times 10^{-6}\ m.$$

6. Question

When a proton is released from rest in a room, it starts with an initial acceleration a_0 towards west. When it is projected towards north with a sped v_0, it moves with an initial acceleration $3a_0$ towards west. Find the electric field and the minimum possible magnetic field in the room.

Answer

Given-

Proton is released from rest in a room, it starts with an initial acceleration a_0 towards west

Now, we know coulomb's force F given by –

$$F = qE$$

where

q = charge

E= electric field

Also,

And Newton's second law of motion

$$F = m \times a \ (1)$$

Where

m = mass

a= acceleration

$$\Rightarrow F = ma_0 \ (2)$$

From (1) and (2)

$$qE = ma_0$$

Electric field,

$$\Rightarrow E = \frac{ma_0}{q}$$

which acts towards west.

We know, Lorentz force F is given by -

$$F = qvB\ sin\theta\ (1)$$

where,

q = charge

v = velocity of the charge

B=magnetic field

and

and θ = the angle between B and l

Now, given that when the proton is projected towards north with a speed v_0, it moves with an initial acceleration $3a_0$ towards west.

$$F = q \times v_0 \times B\ sin\theta$$

$$\Rightarrow B = \frac{F}{q \times v_0}$$

An electric force will act on the proton in the west direction, which produces an acceleration a_0 on the proton.

Initially the proton was moving with velocity v, so a magnetic force was also acting on the proton.

So, magnetic force acting will be the only cause behind the change in acceleration of the proton

Change in acceleration towards west due to the magnetic force acting on it is -

$\Delta a = 3a_0 - a_0 = 2a_0$

So from (2), the force will become -

$$F = m \times 2 \times a_0$$

Hence, required magnetic field

$$B = 2m \times a_0 \times q \times v_0$$

7. Question

Consider a 10-cm long portion of a straight wire carrying a current of 10 A placed in a magnetic field of 0.1 T making an angle of 53° with the wire. What magnetic force does the wire experience?

Answer

Given-Length of wire, l = 10 cm

Electric current passing through the wire, I = 10 A

Magnetic field, B = 0.1 T

Angle between the wire and magnetic field, θ = 53°

Magnetic Force on a Current carrying wire is given by

$$F = BIL\sin\theta$$

where,

B= magnetic field

I = current

L = length of the wire

and θ = the angle between B and l

hence, magnetic force,

$$F = BIL\ sin\ \theta$$

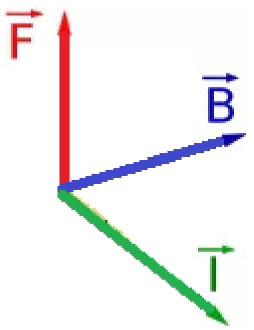
$$F = BIL\ sin 53°$$

$$\Rightarrow F = 10 \times 10 \times 10^{-2} \times 0.1 \times 0.798$$

$$\Rightarrow F = 0.0798 \approx 0.08\ N$$

The direction of force can be found using Fleming's left-hand rule –

which states that –

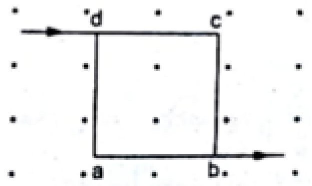


whenever a current carrying conductor is placed inside a magnetic field, a force acts on the conductor, in a direction perpendicular to both the directions of the current and the magnetic field

Therefore, the direction of magnetic force is perpendicular to the wire as well as the magnetic field.

8. Question

A current of 2A enters at the corner d of a square frame abcd of side 20 cm and leaves at the opposite corner b. A magnetic field B = 0.1 T exists in the space in a direction perpendicular to the plane of the frame as shown in figure. Find the magnitude and direction of the magnetic force on the four sides of the frame.



Answer

Given-

square of side, l = 20 cm

Electric current passing through the wire, I = 2 A

Magnetic field, B = 0.1 T

The direction of magnetic field is perpendicular to the plane of the frame, coming out of the plane.

Given in the question, that current enters at the corner *d* of the square frame and leaves at the opposite corner *b.*

Hence, angle between the frame and magnetic field, θ = 90°

Now, we know-

Magnetic Force on a Current carrying wire is given by

$$F = BIL\sin\theta$$

where,

B= magnetic field

I = current

L = length of the wire

and θ = the angle between B and l

Hence, magnetic force,

$$F = BIL\sin\theta$$

For wire along the sides da and cb,

$$F = BIL\sin\theta$$

$$F = B \times I \times L\ sin\ 90°$$

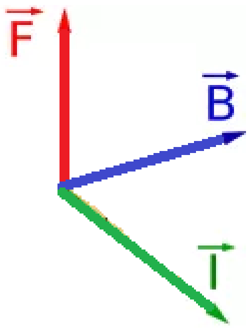
$= 22 \times 20 \times 10^{-2} \times 0.1$

$= 0.02\,N$

The direction of force can be found using Fleming's left-hand

rule.

which states that –



whenever a current carrying conductor is placed inside a magnetic field, a force acts on the conductor, in a direction perpendicular to both the directions of the current and the magnetic field

Thus, the direction of magnetic force will be towards the left.

now, for wires along sides, *dc* and *ab*,

$F = \hat{B}I\hat{l}\sin\theta$

$F = B \times I \times L\,sin\,90°$

$= 22 \times 20 \times 10^{-2} \times 0.1$

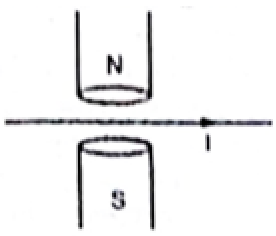
$= 0.02\,N$

Again, here the direction of force can be found using Fleming's left-hand rule.

Thus, the direction of magnetic force will be downwards.

9. Question

A magnetic field of strength 1.0 T is produced by a strong electromagnet in a cylindrical region of radius 4.0 cm as shown in figure. A wire, carrying a current of 2.0 A, is placed perpendicular to and intersecting the axis of the cylindrical region. Find the magnitude of the force acting on the wire.



Answer

Given-

Magnetic field, (B) = 1 T

Radius of the cylindrical region, r = 4.0 cm

Electric current through the wire, I = 2 A

The wire is placed perpendicular the direction of magnetic field

So, angle between wire and magnetic field, θ = 90

Magnetic Force on a Current carrying wire is given by

$$\boldsymbol{Force, F = BIL \sin\theta}$$

where,

B= magnetic field

I = current

l = length of the wire

and θ = the angle between B and l

Magnetic force,

$$\boldsymbol{Force, F = BIL \sin\theta}$$

$$\boldsymbol{F = B \times I \times L \; sin90^\circ}$$

Here, length l which is under the base of the cylinder is of radius 2r

$$\boldsymbol{l = 2r = 8 \times 10^{-2} m}$$

$$\boldsymbol{F = B \times 2r \times I \times \; sin90^\circ}$$

$$\boldsymbol{= 2.0\, A \times 8 \times 10^{-2} m \times 1.0 T \times 1}$$

$$\boldsymbol{= 0.16 N}$$

10. Question

A wire of length ℓ carries a current i along the x-axis. A magnetic field exits which is given as $\vec{B} = B_0\left(\vec{i} + \vec{j} + \vec{k}\right)T$. Find the magnitude of the magnetic force acting on the wire.

Answer

Given-

length of wire l cm

Electric current through the wire = $I\,\hat{\imath}$ A

Magnetic field in vector form is given as –

$$B = B_0(\hat{\imath} + \hat{\jmath} + \hat{k})T.$$

Given in the question, that the current is passing along the *X*-axis.

Magnetic Force on a Current carrying wire is given by

$$F = BIL\sin\theta$$

where,

B= magnetic field

I = current

l = length of the wire

and θ = the angle between *B* and *l*

Magnetic force-

$$F = \hat{B}I\hat{l}\sin\theta$$

Substituting the values –

$$F = B_0\left((\hat{\imath} + \hat{\jmath} + \hat{k})\right) \times I\hat{\imath} \times l\,sin\,\theta$$

since current I is along x- axis, from vector laws, cross product of same vectors is zero –

$$ie\,,\hat{\imath} \times \hat{\imath} = 0$$

$$\Rightarrow F = B_0\left(\hat{\imath} + \hat{\jmath} + \hat{k}\right) \times I\hat{\imath} \times l$$

$$\Rightarrow F = B_0\left(\hat{\imath} + \hat{\jmath} + \hat{k}\right) \times I\hat{\imath} \times l$$

$$= B_0Il\hat{k} - B_0Il\hat{\jmath}$$

The magnitude of the magnetic force,

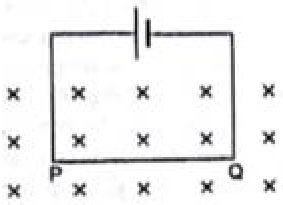
$$|\vec{F}| = \sqrt{(B_0Il)^2 + (B_0Il)^2}$$

$$F = \sqrt{2}\,B_0Il$$

11. Question

A current of 5.0 A exists in the circuit shown in figure. The wire PQ has a length of 50 cm and the magnetic field in which it is immersed has a magnitude of 0.20 T.

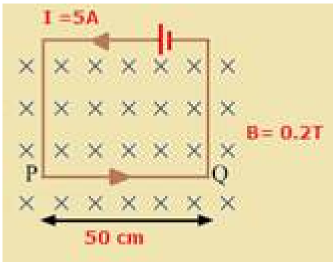
Find the magnetic force acting on the wire PQ.



Answer

Given-Length of the wire PQ inside the magnetic field, *l* = 50 cmElectric current , *I* = 5 AMagnetic field, *B* = 0.2 T

From fig since magnetic field is displayed as "cross", we can say that the direction of magnetic field is perpendicular to the plane and it is going into the plane.



Angle between the plane of the wire and the magnetic field,

$\theta = 90^0$

$$F = BIL\sin\theta$$

where,

B= magnetic field

I = current

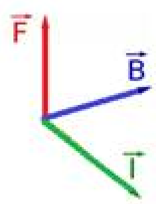
L = length of the wire

and θ = the angle between *B* and *L*

$$F = B \times I \times L\ sin\ 90^0$$

$$= 5 \times 50 \times 10^{-2} \times 0.2 \times 1$$

$$= 0.50\ N$$



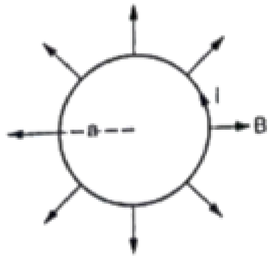
The direction of force can be found using Fleming's left-hand rule.

whenever a current carrying conductor is placed inside a magnetic field, a force acts on the conductor, in a direction perpendicular to both the directions of the current and the magnetic field

Thus, the direction of magnetic force is upwards in the plane of the paper.

12. Question

A circular loop of radius a, carrying a current i, is placed in a two-dimensional magnetic field. The centre of the loop coincides with the centre of the field figure. The strength of the magnetic field at the periphery of the loop is B. Find the magnetic force on the wire.



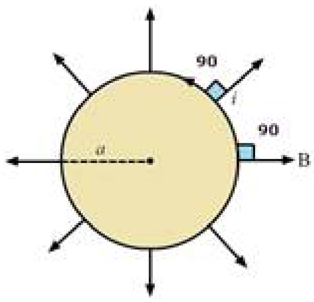
Answer

Given –

circular loop of radius = *a*

So, the length of the loop is its circumference ,

$$l = 2 \times \pi \times a \ (1)$$



Electric current through the loop = *i*

Given that the loop is placed in a two-dimensional magnetic field.

Also the centre of the loop coincides with the centre of the field.

The strength of the magnetic field at the periphery of the loop is *B*

Hence, the direction of the magnetic is radially outwards.

Here, the angle between the length of the loop and the magnetic field, $\theta = 90^0$

$$F = BIL \sin\theta$$

where,

B= magnetic field

I = current

l = length of the wire

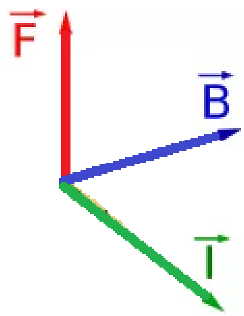
and θ = the angle between *B* and *l*

$$F = BIL\sin\theta$$

$$F = B \times I \times (2 \times \pi \times a)\ sin\ 90^0$$

$$F = B \times I \times (2 \times \pi \times a)$$

The direction of force can be found using Fleming's left-hand rule -



whenever a current carrying conductor is placed inside a magnetic field, a force acts on the conductor, in a direction perpendicular to both the directions of the current and the magnetic field

Thus, the direction of magnetic force lies perpendicular to the plane pointing inwards.

13. Question

A hypothetical magnetic field existing in a region is given by $\vec{B} = B_0\vec{e}_r$, where $\vec{e}_r$ denotes the unit vector along the radial direction. A circular loop of radius a, carrying a current i, is placed with its plane parallel to the x-y plane and the centre at (0, 0, d). Find the magnitude of the magnetic force acting on the loop.

Answer

Given-hypothetical magnetic field exists in a region,

$$\hat{B} = B_0\hat{e}_r$$

Where

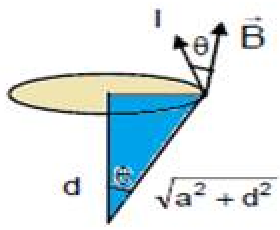
$\hat{e}_r$, is a unit vector along the radial direction a.

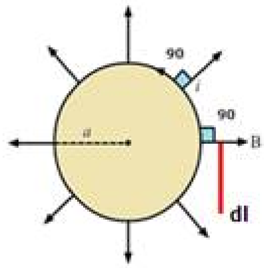
It is a circular loop of radius *a*

So, the length of the loop, $l = 2\pi a$

Electric current through loop = *i*

Also, the loop is placed with its plane parallel to the *X*−*Y* plane and its centre lies at (0,0, *d*).





The angle between the length of the loop l and the magnetic field B is θ

Magnetic force is given by

$$F = BIL\sin\theta$$

where,

B= magnetic field

I = current

l = length of the wire

and θ = the angle between B and l

Substituting the values -

$$F = B \times I \times 2\pi a \sin\theta$$

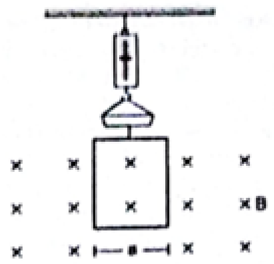
Looking into the fig, $B\sin\theta = B_0 \frac{a}{\sqrt{a^2+d^2}}$

$$\Rightarrow F = I \times 2\pi a \times B_0 \frac{a}{\sqrt{a^2+d^2}}$$

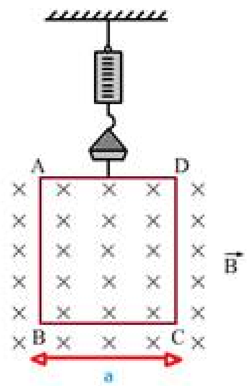
$$\Rightarrow F = I \times 2\pi \times B_0 \frac{a^2}{\sqrt{a^2+d^2}}$$

14. Question

A rectangular wire-loop of width a is suspended from the insulated pan of a spring balance as shown in figure. A current i exists in the anticlockwise direction in the loop. A magnetic field B exists in the lower region. Find the change in the tension of the spring if the current in the loop is reversed.



Answer



Given-

width of wire loop = *a*

Electric current through the loop = *i*

Direction of the current is anti-clockwise.

Strength of the magnetic field in the lower region = *B*From fig, we can say that direction of the magnetic field is into the plane of the loop.

Here, angle between the length of the loop and magnetic field, $\theta = 90^0$

Magnetic force is given by

$$F = BIL\sin\theta$$

where,

B= magnetic field

I = current

l = length of the wire

and θ = the angle between *B* and *l*

The magnetic force will act only on side AD and BC.

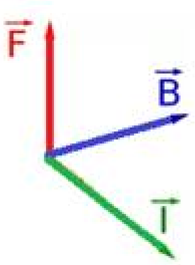
As side AD is outside the magnetic field, so *F* = 0Magnetic force on side BC is

$$F = \hat{B}Ia\sin\theta$$

$$\hat{B}Ia\sin 90^0$$

$= \hat{B}Ia$

Direction of force can be found using Fleming's left-hand rule.



whenever a current carrying conductor is placed inside a magnetic field, a force acts on the conductor, in a direction perpendicular to both the directions of the current and the magnetic field

Thus, the direction of the magnetic force will be upward.

Similarly altering the direction of current to clockwise, the force

along BC-

$F = \hat{B}I\hat{l}\sin\theta$

$= -\hat{B} \times I \times a$

Thus, the change in force is equal to the change in tension

$F_{net} = \hat{B} \times I \times a - (-\hat{B} \times I \times a)$

$= 2\hat{B} \times I \times a$

15. Question

A current loop of arbitrary shape lies in a uniform magnetic field B. Show that the net magnetic force acting on the loop is zero.

Answer

Let's

assume a square magnetic loop

Let,Uniform magnetic field existing in the region of the arbitrary loop = *B*current flowing through the loop be *i*.

Length of each side of the loop be *l*.

Assuming the direction of the current clockwise.

Direction of the magnetic field is going towards the plane of the loop.

Magnetic force is given by

$$F = B \times I \times l \sin\theta$$

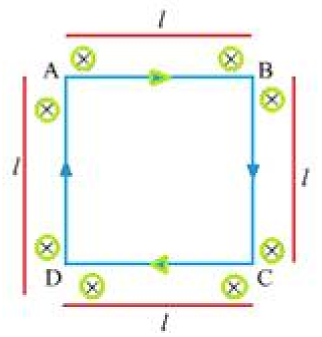
where,

B= magnetic field

I = current

l = length of the wire

and θ = the angle between *B* and *l*



$$F = B \times I \times L \sin\theta$$

Here, $\theta = 90^0$

Direction of force can be found using Fleming's left-hand rule.

Force F_1 acting on side AB is

= $B \times I \times L$ directed upwards

Force F_2 acting on side DC = $B \times I \times L$

directed downwards

Since, F_1 and F_2 are equal and in opposite direction, they will cancel each other.

Similarly,

Force F_3 acting on AD = $B \times I \times L$

directed outwards pointing

And

Force F_4 acting on BC = $B \times I \times L$ directed outwards

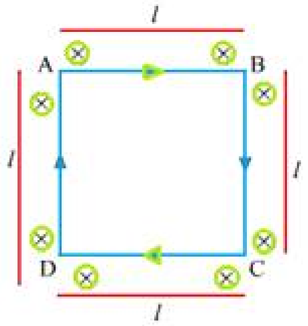
Since, F_3 and F_4 are equal and in opposite direction, they will cancel each other

Hence, the net force acting on the arbitrary loop is 0.

16. Question

Prove that the force acting on a current carrying wire, joining two fixed points a and b in a uniform magnetic field, is independent of the shape of the wire.

Answer



Magnetic force acting on a current carrying wire in an uniform magnetic field is given by

$$F = BIL\sin\theta$$

where,

B= magnetic field

I = current

l = length of the wire

and θ = the angle between *B* and *l*

Since

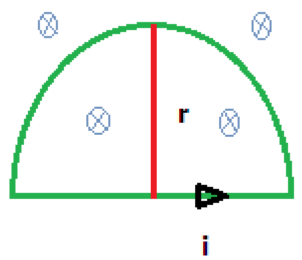
$F = BIl\sin\theta$ is vector

Also, the length of the wire is fixed from A to B, so force is independent of the shape of the wire.

17. Question

A semicircular wire of radius 5.0 cm carries a current of 5.0 A. A magnetic field B of magnitude 0.50 T exists along the perpendicular to the plane of the wire. Find the magnitude of the magnetic force acting on the wire.

Answer



Given-

Radius of semicircular wire, r = 5.0 cm

Thus, the length of the wire = $2r = 10\ cm$

Electric current flowing through wire = 5.0 A

Magnetic field, B = 0.50 T

Direction of magnetic field is perpendicular to the plane of the.

Hence angle between length of the wire and magnetic field, $\theta = 90^0$

As we know the magnetic force is given by

$$\hat{F} = I\hat{B} \times \hat{l}$$

$$\hat{F} = I\hat{B}\hat{l} \sin 90^0$$

$$= 5 \times 2 \times 0.05 \times 0.5$$

$$= 0.25\ N$$

18. Question

A wire, carrying a current i, is kept in the x-y plane along the curve $y = A \sin\left(\frac{2\pi}{\lambda}x\right)$.A magnetic field B exists in the z-direction. Find the magnitude of the magnetic force on the portion of the wire between x = 0 and x = λ.

Answer

Given-

Current passing through the wire = I A

The wire is kept in the x–y plane along the curve,

$$y = A \sin\{\frac{2\pi}{\lambda}x\}$$

Also given that the magnetic field exists in the z direction.

To find the magnetic force on the portion of the wire between $x = 0$ and $x = \lambda$.

We know ,magnetic force acting on a current carrying wire in an uniform magnetic field is given by

$$F = B \times I \times L$$

where,

B= magnetic field

I = current

l = length of the wire

and θ = the angle between *B* and *l*

For a differential length *dl*,

$$F = IB \times dI \sin\theta$$

The effective force on the whole wire due to force acting on the wire of length λ placed along the x axis.

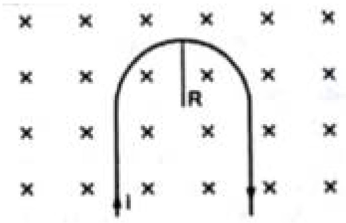
So,

$$F = iB\int_0^{\lambda} dl$$

$$\Rightarrow F = i\lambda B$$

19. Question

A rigid wire consists of a semicircular portion of radius R and two straight sections figure. The wire is partially immersed in a perpendicular magnetic field B as shown in the figure. Find the magnetic force on the wire if it carries a current i.



Answer

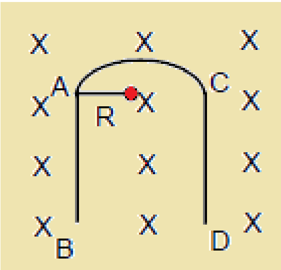
Given-

Radius of the semi-circular portion = *R*

Perpendicular Magnetic field = *B*

Electric current flowing through the wire = *I* A

Given in the question, the wire is partially immersed in a perpendicular magnetic field.



As AB and CD are straight wires of length *l* each and strength of the magnetic field is also same on both the wires, the force acting on these wires will be equal in magnitude

Direction of force can be found out using Fleming's left hand rule.

So, their directions will be opposite to each other.

So, the magnetic force on the wire AB and the force on the wire CD are equal and opposite to each other. Both the forces cancel out each other.

Therefore, only the semicircular loop RC will experience a net magnetic force.

Here, angle between the length of the wire and magnetic field, $\theta = 90^0$

We know ,magnetic force acting on a current carrying wire in an uniform magnetic field is given by

$$\hat{F} = \hat{B} \times I \times \hat{l}$$

where,

B= magnetic field

I = current

l = length of the wire

and θ = the angle between B and l

Here, length $l = 2R$

$$\hat{F} = \hat{B} \times I \times 2R$$

$$\hat{F} = I\hat{B} \times 2R\ sin90^0$$

$$= 2\ iRB$$

20. Question

A straight horizontal wire of mass 10 mg and length 1.0 m carries a current of 2.0 A. What minimum magnetic field B should be applied in the region so that the magnetic force on the wire may balance its weight?

Answer

Given-

Mass of the wire, M = 10 mg = 10^{-5} Kg

Length of the wire, l = 1.0 m

Electric current flowing through wire, I = 2.0 A

It is said in the question, the weight of the wire should be balanced by the magnetic force acting on the wire.

Also angle between the length of the wire and magnetic field is

90°.

We k now weight of an object is given by

$$w = m \times g$$

where,

m = mass

g = acceleration due to gravity =9.8 m/s^2

We know,magnetic force acting on a current carrying wire in an uniform magnetic field is given by

$$F = B \times I \times L$$

where,

B= magnetic field

I = current

l = length of the wire

and θ = the angle between *B* and *l*

Thus,

$$Mg = IBl\, sin90^0$$

$$\Rightarrow B = \frac{Mg}{Il}$$

$$= 10^{-5} \times 9.82 \times 1$$

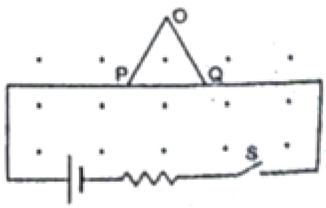
$$= 4.9 \times 10^{-5}\, T$$

21. Question

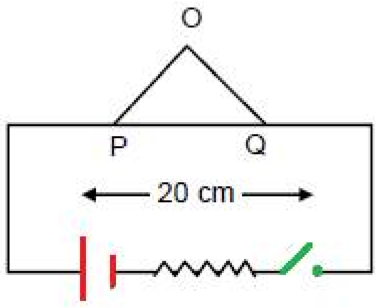
Figure shows a rod PQ of length 20.0 cm and mass 200 g suspended through a fixed point O by two threads of lengths 20.0 cm each. A magnetic field of strength 0.500 T exists in the vicinity of the wire PQ as shown in the figure. The wires connecting PQ with the battery are loose and exert no force on PQ.

(a) Find the tension in the threads when the switch S is open.

(b) A current of 2.0 A is established when the switch S is closed. Find the tension in the threads now.



Answer



Given-

Length of the rod PQ = 20.0 cm

Mass of the rod, *M* = 200 g

Length of the two threads, *l* = 20.0 cm

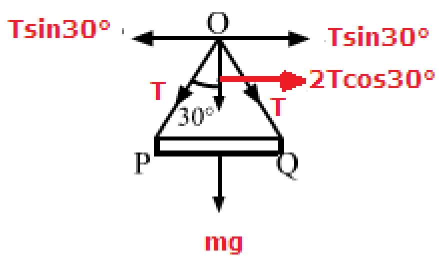
Magnetic field applied, *B* = 0.500 T

(a) When the switch is open-

The weight of the rod is balanced by the tension in the rod.

So,

$$2Tcos30^\circ = Mg$$



$$T = Mg2cos30^\circ$$

$$= 0.2 \times 9.8232$$

$$= 1.13\ N$$

(b)When switch is closed and current flowing through the circuit = 2 A

Then, there exists a magnetic force due to presence of current given by –

$$F = B \times I \times L$$

where,

B= magnetic field

I = current

l = length of the wire

and θ = the angle between B and l

Hence, $\Rightarrow 2T\cos 30° = Mg + ilB$

substituting the values

$\Rightarrow 2T\cos 30° = (0.200 \times 9.8) + (2 \times 0.2 \times 0.5)$

$= 1.95 + 0.2$

$= 2.16$

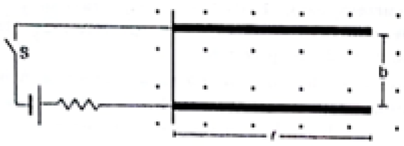
$\Rightarrow 2T = 2.16 \times 23$

$\Rightarrow T = 1.245$

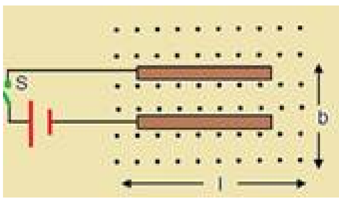
$= 1.25\ N$

22. Question

Two metal strips, each of length ℓ, are clamped parallel to each other on a horizontal floor with a separation b between them, A wire of mass m lies on them perpendicularly as shown in figure. A vertically upward magnetic field of strength B exists in the space. The metal strips are smooth but the coefficient of friction between the wire and the floor is μ. A current i is established when the switch S is closed at the instant t = 0. Discuss the motion of the wire after the switch is closed. How far away from the strips will the wire reach?



Answer



Given-

Length of the two metallic strips = l

Distance between the strips = b

Mass of the wire = m

Strength magnetic field = B

Coefficient of friction between the wire and the floor = μ

Let the wire moved by a distance *x.*

The magnetic field present,will act on the wire towards the right.

As coefficient of friction is zero as the space between the wire and strip is smooth .

Due to the influence of magnetic force, the wire firstly will travel a distance equal to the length of the strips.

After this, it travels a distance x and then ,a frictional force will act opposite to its direction of motion on the wire.

So work done by the magnetic force and the frictional

force will be equal.

$$F_f = \mu W$$

where

μ is the coefficient of friction for the two surfaces

W is the weight of the object

= mass × acceleration due to gravity

= mg

Magnetic force due to presence of current given by –

$$F = B \times I \times L$$

where,

B= magnetic field

I = current

l = length of the wire

and θ = the angle between *B* and *l*

Thus,

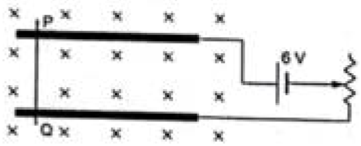
$$F \times l = \mu mg \times x,$$

$$\Rightarrow ibBl = \mu mgx$$

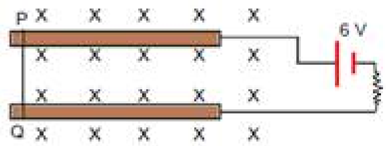
$$\Rightarrow x = \frac{iblB}{\mu mg}$$

23. Question

A metal wire PQ of mass 10 g lies at rest on two horizontal metal rails separated by 4.90 cm figure. A vertically downward magnetic field of magnitude 0.800 T exists in the space. The resistance of the circuit is slowly decreased and it is found that when the resistance goes below 20.0 Ω, the wire PQ starts sliding on the rails. Find the coefficient of friction.



Answer



Given-

Mass of the metal wire, M = 10 g

Distance between the two metallic strips, l = 4.90 cm

Magnetic field acting vertically-downward, B = 0.800 T

Given in the question that when the resistance of the circuit is slowly decreased below 20.0Ω the wire end PQ starts sliding on the metallic rails.

At that time, current I from ohm's law becomes

$$i = \frac{v}{R}$$

where,

v= applied voltage and

R = resistance of the circuit

$$i = 620A$$

From Fleming's left-hand rule, we can confer that the magnetic force will act towards the right.

This magnetic force will make the wire glide on the rails.

The frictional force present at the surface of the metallic rails will try to oppose this motion of the wire.

When the wire starts sliding on the rails, the frictional force acting on the wire present between the wire and the metallic rail will just balance the magnetic force acting on the wire due to current flowing through it.

Thus,

$\mu R = F,$

where

μ is the coefficient of friction

R is the normal reaction force and

F is the magnetic force

frictional force will be equal.

$F_f = \mu W$

where

μ is the coefficient of friction for the two surfaces

W is the weight of the object

= mass × acceleration due to gravity

= mg

Also,

Magnetic force due to presence of current given by –

$\hat{F} = \hat{B} \times I \times \hat{l}$

where,

B= magnetic field

I = current

l = length of the wire

and θ = the angle between *B* and *l*

Hence,

$\Rightarrow \mu \times M \times g = ilB$

substituting values

$\mu \times 10 \times 10^{-3} \times 9.8 = 620 \times 4.9 \times 10^{-2} \times 0.8$

$$\mu = \frac{620 \times 4.9 \times 10^{-2} \times 0.8}{10 \times 10^{-3} \times 9.8}$$

$\Rightarrow \mu = 0.12$

24. Question

A straight wire of length ℓ can slide on two parallel plastic rails kept in a horizontal plane with a separation d. The coefficient of friction between the wire and the rails is μ. If the wire carries a current i, what minimum magnetic field should exist in the space in order to slide the wire on the rails.

Answer

Given-Length of the wire = *l*

Distance between the plastic rails = *d*

The coefficient of friction between the wire and the rails = μ

Electric current flowing through the wire = *i*

The magnetic force will make the wire glide on the rails.

The frictional force present at the surface of the metallic rails will try to oppose the motion of the wire.

The minimum magnetic field required in the space, in order to slide the wire on the rails, will be such that this magnetic force acting on the wire should be able to balance the frictional force on the wire.

Thus,

$$\mu \boldsymbol{R} = \boldsymbol{F}_f,$$

where

μ is the coefficient of friction

R is the normal reaction force and

F is the magnetic force

Frictional force will be equal.

$$\boldsymbol{F}_f = \boldsymbol{\mu W}$$

where

μ is the coefficient of friction for the two surfaces

W is the weight of the object

= mass × acceleration due to gravity

= mg

Also,

Magnetic force due to presence of current given by –

$$\widehat{\boldsymbol{F}} = \widehat{\boldsymbol{B}} \times \boldsymbol{I} \times \hat{\boldsymbol{l}}$$

where,

B= magnetic field

I = current

l = length of the wire

and θ = the angle between *B* and *l*

Hence,

$$\Rightarrow \mu \times M \times g = ilB$$

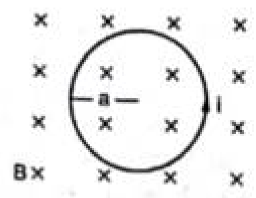
$$\Rightarrow B = \frac{\mu \times M \times g}{il}$$

25. Question

Figure shows a circular wire-loop of radius a, carrying a current i, placed in a perpendicular magnetic field B.

(a) Consider a small part dl of the wire. Find the force on this part of the wire exerted by the magnetic field.

(b) Find the force of compression in the wire.



Answer

Given-

Radius of the circular wire = *a*

Electric current passing through the loop = *i*

Magnetic field Perpendicular to the plane = *B*

(a) Magnetic force due to presence of current on a small differential length dl given by –

$$\hat{F} = \hat{B} \times I \times \widehat{dl}$$

where,

B= magnetic field

I = current

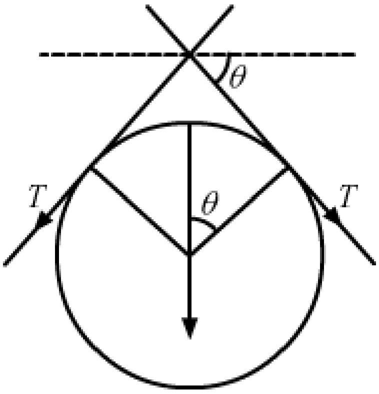
dl =differential length of the wire

and θ = the angle between *B* and *dl*

The direction of magnetic force,using Fleming's left-hand rule is towards the centre for any differential length *dl* of the wire.

Also, d*l* and B are perpendicular to each other

(b) Suppose a part of loop subtends a small angle 2θ at the centre of a circular loop as shown in fig.



Then, looking into the fig. we can say

$$2Tsin\theta = \hat{B} \times I \times \widehat{dl}$$

We know length *l* of an arc –

$$l = r\theta$$

where ,

r is radius of the circle and θ the angle subtended by the arc at the center

here, the arc is subtending an angle 2θ

$$\Rightarrow 2Tsin\theta = \hat{B} \times I \times 2\theta$$

Since is small, sinθ will become negligible

$$\Rightarrow sin\theta = \theta$$

$$\Rightarrow 2T \times \theta = \hat{B} \times I \times 2\theta$$

$$2T\theta = I.2\theta.B$$

$$\Rightarrow T = i.B$$

26. Question

Suppose that the radius of cross section of the wire used in the previous problem is r. Find the increase in the radius of the loop if the magnetic field is switched off. The Young modulus of the material of the wire is Y.

Answer

Given-

Radius of cross section of the wire used in the previous

problem = *r*

Young's modulus of the metallic wire = *Y*.

It said in the question that, when the applied magnetic field is switched off, the tension in the wire increased and so iyd length is increased.

We know Young's modulus,

$$\gamma = \frac{Stress}{Strain}$$

we know, stress is, s is –

$$s = \frac{Force\ or\ Tension}{unit\ Area} = \frac{T}{A}$$

and strain

$$\delta = \frac{length\ of\ strech}{original\ stretch} = \frac{\Delta l}{l}$$

Young's modulus,

$$\gamma = \frac{\frac{T}{A}}{\frac{\Delta l}{l}} = \frac{T}{A} \times \frac{l}{\Delta l}$$

$$\Delta l = \frac{T}{A} \times \frac{l}{\gamma}\ (1)$$

Area of circle

$$A = \pi r^2$$

Let, a be the radius of loop

$$l = 2\ \pi\ \Delta a$$

from (1)

$$\Delta l = \frac{T}{\pi r^2} \times \frac{2\ \pi\ \Delta r}{\gamma}$$

Now the tensile force will be the magnetic force acting on a conductor of length *l* given by –

$$\hat{F} = \hat{B} \times I \times a$$

where,

B= magnetic field

I = current

a =differential length of the wire

and θ = the angle between B and a

$$\Rightarrow \ \Delta l \ = \frac{\hat{B} \times I \times a}{\pi r^2} \times \frac{2\,\pi\,\Delta a}{\gamma}$$

$$\Rightarrow \ \Delta l \ = \frac{B \times I a^2}{\pi r^2\,\gamma}$$

27. Question

The magnetic field existing in a region is given by

$$\vec{B} = B_0\left(1+\frac{x}{\ell}\right)\vec{k}.$$

A square loop edge ℓ and carrying a current i, is placed with its edges parallel to the x-y axes. Find the magnitude of the net magnetic force experienced by the loop.

Answer

Given-

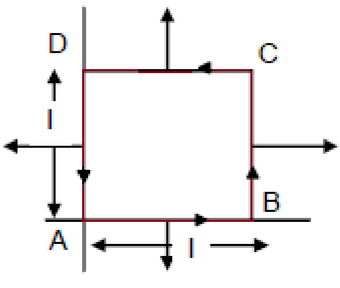
A square loop with-

Magnetic field, $\hat{B} = B_0(1+\frac{x}{l})\vec{k}$

Length of the edge of a square loop = l

Electric current flowing through it = i

Given in the question that the loop is lies with its edges parallel to the X–Y axes.



In the fig, arrow shows the direction of force on different sides of the square.

Magnetic force acting due to presence of current on a small length l given by –

$$\vec{F} \ = \ \vec{B} \times I \times l$$

where,

$\vec{F}$ = magnetic field

I = current

$\vec{l}$ = length of the wire

Now,

Force on side AB -

Consider a differential element of length dx at a distance x from the origin along line AB

Force on this small element,

$$\overrightarrow{dF} = i\,B_0\left(1+\frac{x}{l}\right)dx$$

Force on the full length of AB,

$$F_{AB} = i\,B_0\int_0^l\left(1dx+\frac{x}{l}\,dx\right)$$

$$= i\,B_0\,(l+\frac{l}{2})$$

Force on AB will be acting downwards.

Similarly, force on CD,

$$F_{CD} = i\,B_0\,(l+\frac{l}{2})$$

The net force acting vertically will be -

$$= F_{AB} - F_{CD}$$

$$= 0$$

Force on AD,

$$F_{AD} = i\,B_0\,(l+\frac{0}{2})$$

$$= iB_0 l$$

Force on BC

$$F_{BC} = i\,B_0\,(l+\frac{1}{2})$$

Then, the net horizontal force

$$\mathbf{F_{net}} = F_{AD} - F_{BC}$$

$= iB_0 l$

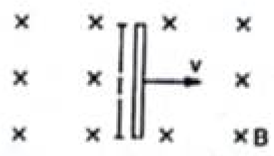
28. Question

A conducting wire of length ℓ, lying normal to a magnetic field B, moves with velocity v as shown in figure.

(a) Find the average magnetic force on a free electron of the wire.

(b) Due to this magnetic force, electrons concentrate at one end resulting in the electric field inside the wire. The redistribution stops when the electric force on the free electrons balances the magnetic force. Find the electric field developed inside the wire when the redistribution stops.

(c) What potential difference is developed between the ends of the wire?



Answer

Given-

Length of the conducting wire = *l*

Inward magnetic field = *B*

Velocity of the conducting wire = *v*

As the wire is moving with velocity *v,* we can take this as the motion of free electrons present inside the wire with velocity *v.*

(a) The average magnetic force on a free electron of the wire

We know, Lorentz force F is given by -

$$F = e(v \times B)$$

where,

e = charge on an electron

v = velocity of the electron

B=magnetic field

(b)The redistribution of electrons stops when the electric force is just balanced by the magnetic force.

Electric force coulomb's law,

$$F = eE$$

where

e = charge

E=electric field of the charge

and also magnetic force, we know, Lorentz force F is given by -

$$F = e(v \times B)$$

where,

e = charge on an electron

v = velocity of the electron

B=magnetic field

On equating these two forces, we get-

$$e(v \times B) = eE$$

$$\Rightarrow E = vB \ (1)$$

(c) The potential difference developed between the ends of the wire , V is -

$$V = lE$$

where l is length of wire and E is applied electric field

From (1)

$$V = lvB$$

29. Question

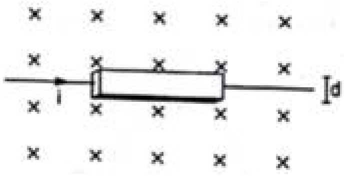
A current i is passed through a silver strip of width d and area of cross section A. The number of free electrons per unit volume is π.

(a) Find the drift velocity v of the electrons.

(b) If a magnetic field B exists in the region as shown in figure, what is the average magnetic force on the free electrons?

(c) Due to the magnetic force, the free electrons get accumulated on one side of the conductor along its length. This produces a transverse electric field in the conductor which opposes the magnetic force on the electrons. Find the magnitude of the electric field which will stop further accumulation of electrons.

(d) What will be the potential difference developed across the width of the conductor due to the electron-accumulation? The appearance of a transvers emf, when a current carrying wire is placed in a magnetic field, is called Hall effect.



Answer

Given-Width of the silver strip = *d*

Area of cross-section = *A*

Electric current flowing through the strip = *i*

The number of free electrons per unit volume = *n*

(a) We know the relation between the drift velocity of electrons and current through any wire,

$$i = v_d n A e$$

where

e is the charge of an electron

v_d is the drift velocity.

A is the area of the conductor

On solving for the drift velocity, we get

$$\Rightarrow v_d = \frac{i}{nAe}$$

(b)The magnetic field existing in the region is *B*

.Magnetic force acting due to presence of current on a small length l given by –

$$\vec{F} = \vec{B} \times I \times \vec{l}$$

where,

$\vec{F}$ = magnetic field

I = current

$\vec{l}$ = length of the wire

So, the force on a free electron

$$F = \frac{ilB}{nAl}$$

$$= \frac{iB}{nA} \; (1)$$

which acts towards upward direction

(c) Let us consider, electric field as *E*.

Now, the accumulation of electrons will stop when magnetic force just balances the electric force.

Electric force, coulomb's law,

$$F = eE\ (2)$$

where

e = charge

E=electric field of the charge

From (1) and (2)

$$eE = \frac{iB}{nA}$$

$$\Rightarrow E = \frac{iB \times e}{nA}$$

(d)The potential difference , V developed across the width d of

the conductor due to the electron-accumulation is given by-

$$V = Ed$$

where

d is length of wire

and E is applied electric field

$$\Rightarrow V = Ed = \frac{iB \times e}{nA}$$

30. Question

A particle having a charge of 2.0×10^{-8} C and a mass of 2.0×10^{-10} g is projected with a speed of 2.0×10^{3} ms^{-1} in a region having a uniform magnetic field of 0.10 T. The velocity is perpendicular to the field. Find the radius of the circle formed by the particle and also the time period.

Answer

Given-

Charge on the particle, $q = 2.0 \times 10^{-8}$ C

Mass of the particle, $m = 2.0 \times 10^{-10}$ g velocity of the particle when projected, $v = 2.0 \times 10^3$ m s^{-1}

Magnetic field, $B = 0.10$ T.

Given in the question that, the velocity is perpendicular to the field.

So, for the particle to move in a circle, the centrifugal force comes into acts which is provided by the magnetic force acting on it.

Also magnetic force, we know, Lorentz force F is given by -

$$F = e(v \times B)$$

where,

e = charge on an electron

v = velocity of the electron

B=magnetic field

Using the formula for centrifugal force

$$F_c = \frac{mv^2}{r}$$

where,

v= velocity of the particle

r= radius of circle form

m =mass of the electron

Equating the two forces, we will get-

$$qvB = \frac{mv^2}{r}$$

$$r = \frac{mv}{qB}$$

$$= \frac{2 \times 10^{-13} \times 2 \times 10}{32 \times 10^{-8} \times 0.10}$$

$$= 20\ cm$$

Now,

Time period,

$$T = \frac{2\pi m}{qB}$$

$$= \frac{2 \times 3.14 \times 2 \times 10^{-13}}{2 \times 10^{-8} \times 0.10}$$

$$= 6.28 \times 10^{-4}\ s$$

31. Question

A proton describes a circle of radius 1 cm is a magnetic field of strength 0.10 T. What would be the radius of the circle described by an α-particle moving with the same speed in the same magnetic field?

Answer

Given-Radius of the circle, r = 1 cm

Magnetic field , B= 0.10 T

We know that the charge on a proton is e and that of an alpha

particle is 2e.

Also, the mass of a proton is m

Mass of an alpha particle is 4m.

let assume that both the particles are moving with speed v.

So, for the particle to move in a circle, the centrifugal force comes into acts which is provided by the magnetic force acting on it.

Also magnetic force, we know, Lorentz force F is given by -

$$F = e(v \times B)$$

where,

e = charge on an electron

v = velocity of the electron

B=magnetic field

Using the formula for centrifugal force

$$F_C = \frac{mv^2}{r}$$

where,

v= velocity of the particle

r= radius of circle form

Equating the two forces, we will get-

$$qvB = \frac{mv^2}{r}$$

$$r = \frac{mv}{qB}$$

Then, we can confer from question that-

$$r_p = \frac{mv}{eB},$$

Where

r_p is the radius of the circle described by the proton which is 0.01

$$\Rightarrow 0.01 = \frac{mv}{e \times 0.1} \ (1)$$

For alpha particle, radius is given by –

$$r_\alpha = \frac{4mv}{2eB}$$

$$= \frac{4mv}{2e \times 0.1} \ (2)$$

On dividing equation (1) by (2), we get:

$$\frac{r_\alpha}{0.01} = \frac{4mv \times e \times 0.1}{2e \times 0.1 \times mv}$$

$$\Rightarrow r_\alpha = 0.02 \ m$$

$$= 2 \ cm$$

32. Question

An electron having a kinetic energy of 100 eV circulates in a path of radius 10 cm in a magnetic field. Find the magnetic field and the number of revolutions per second made by the electron.

Answer

Given-Kinetic energy of an electron = 100 eV

Radius of the circle = 10 cm

We now kinetic energy is given by

$$KE = \frac{1}{2}\text{mv}^2$$

Where

From question , we can confer that

$$\frac{1}{2}mv^2 = 100 \text{ eV}$$

Here,*m* is the mass of an electron

v is the velocity of an electron.

We know that 1 eV = 1.6 × 10^{-19} J

Thus,

$$\frac{1}{2} \times 9.1 \times 10^{-31} \times v^2 = 1.6 \times 10^{17}$$

$$\Rightarrow v^2 = 0.35 \times 10^{14}$$

$v = 0.591 \times 10^7 \; m/s$Now, radius r -

$$r = \frac{mv}{eB} \; (1)$$

$$\Rightarrow B = \frac{mv}{er}$$

$$= \frac{9.1 \times 10^{-31} \times 0.591 \times 10^7}{1.6 \times 10 - 19 \times 0.1}$$

$$= 3.3613 \times 10^{-4} \; T$$

Therefore, magnetic field applied is 3.4 × 10^{-4} T

Number of revolutions per second of the electron is nothing but the frequency given by ,

$$f = \frac{1}{T}$$

$$T = \frac{2\pi r}{v}$$

From (1)

$$v = \frac{reB}{m}$$

$$\Rightarrow T = \frac{2\pi m}{eB}$$

$$T = \frac{2\pi m}{eB}$$

$$f = \frac{Be}{2\pi m}$$

$$\Rightarrow f = \frac{3.4 \times 10^{-4} \times 1.6 \times 10^{-19}}{2 \times 3.14 \times 9.1 \times 10^{-31}}$$

$$= 0.094 \times 10^{8} = 9.4 \times 10^{6}$$

$$f = 9.4 \times 10^{6}\, Hz$$

33. Question

Protons having kinetic energy K emerge from an accelerator as a narrow beam. The beam is bent by a perpendicular magnetic field so that it just misses a plane target kept at a distance ℓ in front of the accelerator. Find the magnetic field.

Answer

Given-Kinetic energy of proton = K

Distance between the target and the accelerator = l

Therefore, radius of the circular orbit ≤ l

From question it is given that, the beam is bent by a

perpendicular magnetic field, let it be B.

We know-

Now, radius r -

$$r = \frac{mv}{eB}$$

where,

m is the mass of a proton

v= velocity of the particle

B = magnetic force

e= charge on the particle

For a proton, the above equation can be written as-

$$l = \frac{m_p v}{eB} \; (1)$$

where r = l

Kinetic energy, K $K = \frac{1}{2}\mathrm{mv}^2$

$$\Rightarrow v = \sqrt{\frac{2K}{m_p}}$$

Substituting the values of v in the equation (1)

$$l = \frac{2K\,m_p}{eB}$$

$$\Rightarrow B = \frac{2K\,m_p}{el}\ T$$

34. Question

A charged particle is accelerated through a potential difference of 12 kV and acquires a speed of $1.0 \times 10^8 \text{ms}^{-1}$. It is then injected perpendicularly into a magnetic filed of strength 0.2 T. Find the radius of the circle described by it.

Answer

Given-

Applied potential difference to the charged particle , $V = 12$ kV $= 12 \times 10^3$ V

Speed acquire by the charged particle, $v = 1.0 \times 10^6$ m s^{-1}

Perpendicular Magnetic field $B = 0.2$ T

We know

The kinetic energy acquired by the particle is produced due to

applied potential difference, hence

$$qV = \frac{1}{2}mv^2$$

$$\Rightarrow \frac{m}{q} = \frac{2V}{v^2}$$

$$= \frac{2 \times 12 \times 10^3}{(1 \times 10^6)^2}$$

$$\Rightarrow \frac{m}{q} = 24 \times 10^{-9}$$

and

$$r = \frac{mv}{qB}$$

where,

m is the mass of a proton

v= velocity of the particle

B = magnetic force

q= charge on the particle

$$\Rightarrow r = \frac{24 \times 10^{-9} \times 10^{6}}{0.2}$$

$$\Rightarrow r = 12 \times 10^{-2}\ m$$

$$= 12\ cm$$

35. Question

Doubly ionized helium ions are projected with a speed of 10 km s^{-1} in a direction perpendicular to a uniform magnetic field of magnitude 1.0 T. Find

(a) the force acting on an ion,

(b) the radius of the circle in which it circulates and

(c) the time taken by an ion to complete the circle.

Answer

Given-

Speed of the helium ions, v = 10 km s^{-1} = 10^4 m/s

Uniform magnetic field, B = 1.0 T

Charge on the helium ions = 2e

Mass of a helium ion, m = $4 \times 1.6 \times 10^{-27}$ Kg

(a) The force acting on an ion –

Magnetic force, we know, Lorentz force F is given by -

$$F = qvBsin\theta$$

where,

e = charge on an electron

v = velocity of the electron

B=magnetic field

θ= angle between B and v

$$F = qvBsin\theta$$

$= 2 \times 1.6 \times 10^{-19} \times 10^{4} \times 1.0$

$= 3.2 \times 10^{-15}\ N$

(b) The radius of the circle is given by,

$$r = \frac{mv}{qB}$$

where,

m is the mass of a proton

v= velocity of the particle

B = magnetic force

q= charge on the particle

$$r = \frac{mv}{qB}$$

$$= \frac{4 \times 1.6 \times 10^{-27} \times 10^{4}}{2 \times 1.6 \times 10^{-19} \times 1}$$

$2 \times 10^{-4}\ m$

(c) The time taken by an ion to complete the circle,

we know

$$speed = \frac{distance}{time}$$

$$\Rightarrow time = \frac{distance}{speed}$$

Since the total time taken for a complete cycle will be its circumference $2\pi r$ and the velocity is V -

$$\Rightarrow T = \frac{2\pi r}{v}$$

$$= \frac{6.28 \times 2.1 \times 10^{-4}}{10^{4}}$$

$= 1.31 \times 10^{-7}\ s$

36. Question

A proton is projected with a velocity of 3×10^{6} m^{-1}perpendicular to a uniform magnetic field of 0.6 T. Find the acceleration of the proton.

Answer

Given-

Velocity of the proton, $v = 3 \times 10^6$ m s^{-1}

Uniform magnetic field, B = 0.6 T

As per the question, the proton is projected perpendicular to a

uniform magnetic field.

We know, Newton's second law, Force F is given by –

$$F = ma$$

where

m is mass of the object

a=acceleration

For proton

$$\Rightarrow F = m_p a \ (1)$$

Magnetic force, we know, Lorentz force F is given by -

$$F = evBsin\theta \ (2)$$

where,

e = charge on an electron

v = velocity of the electron

B=magnetic field

θ= angle between B and v

Equating (1) and (2),

$$m_p a = evBsin\theta$$

As $\theta = 90^0$

$$\Rightarrow a = \frac{evB}{m}$$

$$= \frac{1.6 \times 10^{-19} \times 3 \times 10^6 \times 0.6}{1.67 \times 10^{-27}}$$

$$= 1.72 \times 10^{14} \ m/s^2$$

37. Question

(a) An electron moves along a circle of radius 1 m in a perpendicular magnetic field of strength 0.50 T. What would be its sped? Is it reasonable?

(b) If a proton moves along a circle of the same radius in the same magnetic field, what would be its speed?

Answer

Given-

(a) For electron

Radius of the circle = 1 m

Magnetic field strength = 0.50 T

Now,

The radius of the circle is given by,

$$r = \frac{mv}{qB}$$

where,

m is the mass of a electron

v= velocity of the particle

B = magnetic force

q= charge on the particle $= 1.6 \times 10{-}^{19}$ C

$$r = \frac{mv}{qB}$$

$$\Rightarrow v = \frac{reB}{m}$$

$$= \frac{1 \times 0.50 \times 1.6 \times 10{-}^{19}}{9.1 \times 10^{-31}}$$

$$\approx 8.8 \times 10^{10}\ m/s^2$$

Speed of light is 3×10^8 m/s^2

Here, the speed of the electron moving along the circle is greater than the speed of light, so it is not reasonable.

(b) For a proton,

The radius of the circle is given by,

$$r = \frac{mv}{qB}$$

where,

m is the mass of a proton

v= velocity of the particle

B = magnetic force

q= charge on the particle =$1.6 \times 10 - ^{19}$ C

$$r = \frac{mv}{qB}$$

$$\Rightarrow v = \frac{reB}{m}$$

$$= \frac{1 \times 0.50 \times 1.6 \times 10^{-19}}{1.6 \times 10^{-27}}$$

$$= 5 \times 10^{7}\ m/s$$

38. Question

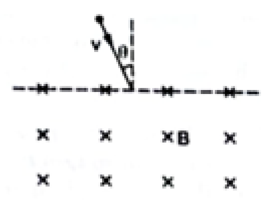
A particle of mass m and positive charge q, moving with a uniform velocity v, enters a magnetic field B as shown in figure.

(a) Find the radius of the circular arc it describes in the magnetic field.

(b) Find the angle subtended by the arc at the centre.

(c) How long does the particle stay inside the magnetic field?

(d) Solve the three parts on the above problem if the charge q on the particle is negative.



Answer

Given-Mass of the particle = *m*Positive charge on the particle = *q*velocity of the particle = *v*Magnetic field = *B*

(a)The radius of the circular arc described by the particle in the magnetic field-

We know,

The radius of the circle is given by,

$r = \frac{mv}{qB}$

where,

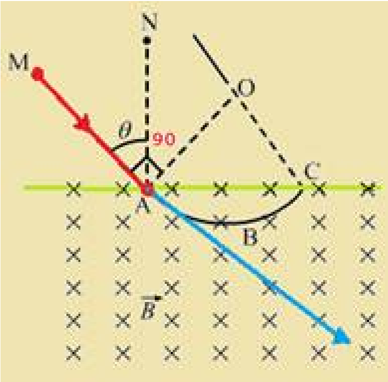
m is the mass of a proton

v= velocity of the particle

B = magnetic force

q= charge on the particle =$1.6 \times 10{-}^{19}$ C

(b) The angle subtended by the arc at the centre



Line MAB is the tangent to arc ABC,

When the particle enters into the magnetic field, it follows a path in the form of arc as shown in fig.

Now, the angle described by the charged particle is nothing but the angle ∠ MAO is ,

∠MAO = 90°

Also from fig , ∠NAC = 90°

∠OAC = ∠ OCA = θThen, by angle-sum property of a triangle, sum of all angles of a triangle is 180^0

∠AOC =180°− (θ + θ)

= $\pi - 2\theta$ (1)

(c)The time for which the particle stay inside the magnetic field is given by-

Distance covered by the particle inside the magnetic field,is the length of arc subtended by angle θ and the radius

l = *r*θ

from (1)

l= *r* $(\pi - 2\theta)$

time taken for a complete cycle will be its circumference $2\pi r$ and

the velocity is V -

$$\Rightarrow T = \frac{2\pi r}{v}$$

Also The radius of the circle is given by,

$$r = \frac{mv}{qB}$$

where,

m is the mass of a proton

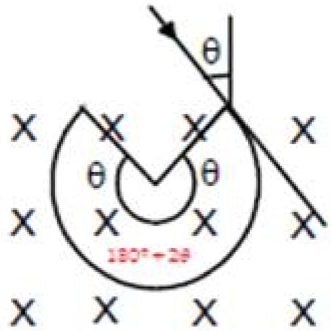
v= velocity of the particle

B = magnetic force

q= charge on the particle =$1.6 \times 10-^{19}$ C

$$\Rightarrow T = \frac{m}{qB}(\pi - 2\theta)$$

(d) If the charge *q* on the particle is negative, then



(i) Radius of circular arc is given by

$$r = \frac{mv}{qB}$$

(ii) The centre of the arc lies within the magnetic field

Therefore, the angle subtended by the arc = $\pi + 2\theta$

(iii) The time taken by the particle to cover the path inside the magnetic field

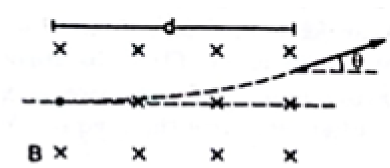
$$\Rightarrow T = \frac{m}{qB}(\pi + 2\theta)$$

39. Question

A particle of mass m and charge q is projected into a region having a perpendicular magnetic field B. Find the angle of deviation figure of the particle as it comes out of the magnetic field if the width d of the region is very slightly smaller than

(a) $\frac{mv}{qB}$ (b). $\frac{mv}{2qB}$

(c) $\frac{2mv}{qB}$



Answer

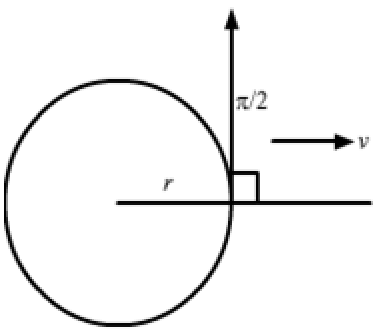
Given-

Mass of the particle = *m*

Charge of the particle = *q*

Perpendicular magnetic field = *B*

To find the angle of deviation figure of the particle as it comes out of the magnetic field when -



(a) When the width,

$$d = \frac{mv}{qB}$$

d is equal to the radius

θ is the angle between the radius and tangent drawn to the circle , which is equal to $\frac{\pi}{2}$.

(b) When the width,

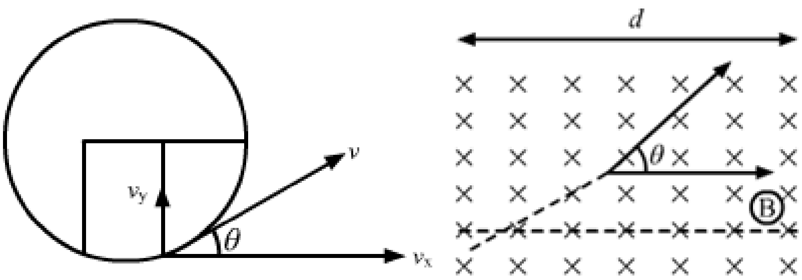
$$d = \frac{mv}{2qB}$$

Now, in this case, the width of the region in where magnetic field is applied is half of the radius of the circular path.

As the magnetic force is acting only along the y –axis,the velocity of the particle will remain constant along the x-axis.

So, if *d* distance is travelled along the x axis,

then,



$$d = v_x \times t$$

$$t = \frac{d}{t} (1)$$

where,

v=velocity

t=time

for constant velocity the acceleration along the x direction is zero.

Hence the force will act only along the y direction.

Using the 3rd equation of motion along the y axis-

$$V_y = u_y + a_y t$$

where

u = initial velocity

a = acceleration

t= time taken

since, initial velocity is 0

Also, as $\theta = 90^0$

$$\Rightarrow a = \frac{evB}{m}$$

$$\Rightarrow V_y = 0 + \frac{qu_x Bt}{m}$$

$$= \frac{qu_x Bt}{m}$$

From (1)

$$V_y == \frac{qu_x Bt\,d}{mv_x}$$

We know

$$tan\theta = \frac{v_y}{v_x}$$

From above fig.

$$\frac{qBd}{mv_x} = \frac{qBmv_x}{2qBmv_x}$$

$$= \frac{1}{2}$$

$$\Rightarrow \theta = \tan^{-1}\frac{1}{2}$$

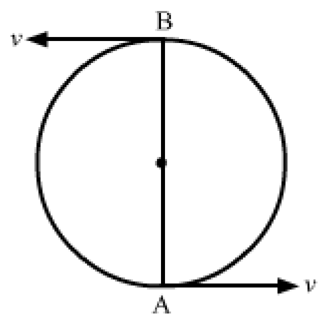
$$= 26.4$$

$$= 30°$$

$$= \frac{\pi}{6}$$

(c) When the width, *d* =

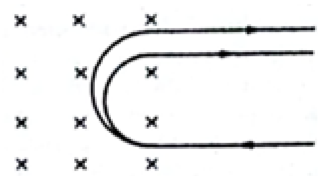
$$d = \frac{2mv}{qB}$$



From above fig., it can be concluded that the angle between the initial direction and final direction of velocity is π.

40. Question

A narrow beam of singly charged carbon ions, moving at a constant velocity of 6.0 × $10^4$$ms^{-1}$, is sent perpendicularly in a rectangular region having uniform magnetic field B = 0.5 T figure. It is found that two beams emerge from the field in the backward direction, the separations from the incident beam being 3.0 cm and 3.5 cm. Identify the isotopes present in the ion beam. Take the mass of an ion = A(1.6 × 10^{-27}) kg, where A is the mass number.



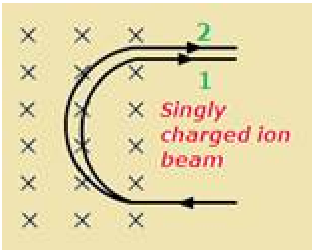
Answer

Given-

Velocity of a narrow beam of singly-charged carbon ions,

$v = 6.0 \times 10^4$ m s^{-1}

Strength of magnetic field $B = 0.5$ T



Separation between the two beams from the incident beam are 3.0 cm and 3.5 cm.

Mass of an ion = $A(1.6 \times 10^{-27})$ kg

The radius of the curved path taken by the beam 1 -

$$r_1 = \frac{3}{2} = 1.5\ cm\ (1)$$

The radius of the curved path taken by the beam 2-

$$r_2 = 3.52\ cm\ (2)$$

The radius of the circle is given by,

$$r = \frac{mv}{qB}$$

where,

m is the mass of a proton

v= velocity of the particle

B = magnetic force

q= charge on the particle =$1.6 \times 10-^{19}$ C

For the first beam

$$r_1 = \frac{m_1 v}{qB}$$

where

m_1 = mass of the first isotope

q = s the charge.

For the second beam

$$r_2 = \frac{m_2 v}{qB}$$

where

m_2 = mass of the second isotope

q = the charge.

$$\frac{r_1}{r_2} = \frac{m_1}{m_2}$$

from (1) and (2),

$$\frac{\frac{3}{2}}{3.52} = \frac{A_1 \times 1.6 \times 10^{-27}}{A_2 \times 1.6 \times 10^{-27}}$$

$$\frac{6}{7} = \frac{A_1}{A_2} \ (3)$$

Now, we know,

$$r_1 = \frac{m_1 v}{qB}$$

$$\Rightarrow m_1 = \frac{qBr_1}{v}$$

$$= \frac{1.6 \times 10^{-19} \times 0.5 \times 0.015}{6 \times 10^4}$$

$$= 20 \times 10^{-27}\ kg$$

$$= \frac{20 \times 10^{-27}}{1.6 \times 10^{-27}}\ u$$

$$= 12.5\ u$$

Also, from (3)

$$A_2 = \frac{7}{6} A1$$

$$= \frac{7}{6} \times 12.5$$

$$= 14.58\, u$$

Looking at the mass of the material obtained, we can infer that these are the two isotopes of carbon used are $^{12}C_6$ and $^{14}C_6$.

41. Question

Fe^+ions are acceleration through a potential difference of 500 V and are injected normally into a homogeneous magnetic field B of strength 20.0 mT. Find the radius of the circular paths followed by the isotopes with mass numbers 57 and 58. Take the mass of an ion = A(1.6×10^{-27}) kg where A is the mass number.

Answer

Given-Potential difference through which the Fe^+, V = 500 V

Strength of the homogeneous magnetic field

B = 20.0 mT= 20×10^{-3} T

Mass numbers of the two isotopes ,

m_1 =57 and m_2 =58.

Mass of an ion = A (1.6×10^{-27}) kg

The radius of the circular path described by a particle in a magnetic field,

$$r = \frac{mv}{qB}$$

where,

m is the mass of a proton

v= velocity of the particle

B = magnetic force

q= charge on the particle =$1.6 \times 10-^{19}$ C

For calculating the radius of isotope 1-

$$r_1 = \frac{m_1 v_1}{qB}$$

For isotope 2,

$$r_2 = \frac{m_2 v_2}{qB}$$

$$\frac{r_1}{r_2} = \frac{m_1 v_1}{m_2 v_2}$$

Since isotopes are accelerated from the same potential V, the Kinetic energy gained by the two particles will be same for both the particles.

We, know the force developed by potential difference –

$$F = qV$$

where

v = applied potential

q=charge

$$\Rightarrow qV = \frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_2 v_2^2 \ (1)$$

$$= \frac{v_1^2}{v_2^2}$$

$$\Rightarrow \frac{r_1}{r_2} = \left(\frac{m_1}{m_2}\right)^{\frac{3}{2}} \ (2)$$

Also,we have

$$r_1 = \frac{m_1 v_1}{qB}$$

From (1)

$$r_1 = \frac{m_1 \sqrt{2qVm_1}}{qB}$$

$$= \frac{1}{B} \frac{\sqrt{2Vm_1}}{q}$$

$$= \frac{\sqrt{1000 \times 57 \times 1.6 \times 10^{-27}}}{\sqrt{1.6 \times 10^{-19} \times 20 \times 10^{-3}}}$$

$$= 1.19 \times 10^{-2}\ m$$

$$= 119\ cm$$

For calculating the radius of 2 isotope-

From (2)

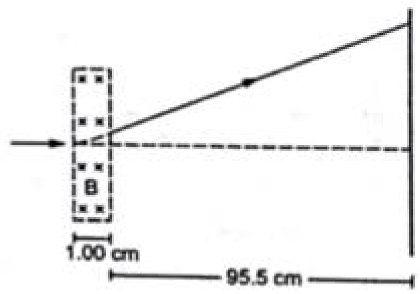
$$\frac{r_1}{\left(\frac{m_1}{m_2}\right)^{\frac{3}{2}}} = r_2$$

$$= \left(\frac{58}{57}\right)^{\frac{3}{2}} \times 119\ cm$$

$$= 120\ cm$$

42. Question

A narrow beam of singly charged potassium ions of kinetic energy 32 keV is injected into a region of width 1.00 cm having a magnetic field of strength 0.500 T as shown in figure. The ions are collected at a screen 95.5 cm away form the field region. If the beam contains isotopes of atomic weights 39 and 41, find the separation between the points where these isotopes strike the screen. Take the mass of a potassium ion = A(1.6 × 10^{-27}) kg where A is the mass number.



Answer

Given -

Kinetic energy of singly-charged potassium ions = 32 keV

Width of the magnetic region = 1.00 cm

Magnetic field's strength, *B* = 0.500 T

Distance between the screen and the region = 95.5 cm

Atomic weights of the two isotopes are m_1 =39 and m_2 =41

.Mass of a potassium ion = *A* (1.6 × 10^{-27}) kg

For a singly-charged potassium ion K-39, separation between the points can be calculated as follows –

Mass of K-39 = 39 × 1.6 × 10^{-27} kg

Charge, *q* = 1.6 × 10^{-19} C

Given in the question that the narrow beam of singly-charged potassium ions is injected into a region of magnetic field.

As, given that kinetic energy K.E is

K.E = 32 keV

$$\frac{1}{2}mv^2 = 32 \times 10^3 \times 1.6 \times 10^{-19}$$

$$12 \times 39 \times (1.6 \times 10^{-27}) \times v^2 = 32 \times 10^3 \times 1.6 \times 10^{-19}$$

$$v = 4.05 \times 10^5\ m/s$$

We know that throughout the motion, the horizontal velocity remains constant.

So, the time taken to cross the magnetic field,

$$t = dv$$

$$= 0.014.05 \times 105$$

$$= 24.7 \times 10^{-9}\ s$$

Now, the acceleration in the magnetic field region-

$$F = qvB = ma$$

$$a = \frac{qvB}{m}$$

$$= \frac{1.6 \times 10^{-19} \times 4.05 \times 10^5}{39 \times 1.6 \times 10^{-27}}$$

$$= 5192 \times 10^8\ m/s2$$

Now, velocity in the vertical direction can be found from Newton's 2nd law as

$$v_y = U_Y + at$$

Since, initial velocity is 0,

$$v_y = +at$$

Substituting the values-

$$v_y = 5193.53 \times 10^8 \times 24.7 \times 10^{-9}$$

$$= 12824.24\ m/s$$

Time taken by the ion to reach the screen-

$$time = \frac{distance}{velocity}$$

$$= \frac{0.955}{4.05 \times 10^5}$$

$$\Rightarrow t = 0.000002358\ s.$$

Now, distance moved by the ion vertically in this given time -

$$d = v_y \times t$$

$$= 12824.24 \times 2358 \times 10^{-9}$$

$$= 3023.95 \times 10^{-5}\ m$$

Vertical distance travelled by the particle inside magnetic field

can be calculated by using 3rd equations of motion –

$$v^2 - u^2 = 2aS$$

where,

v= final velocity

u=initial velocity

a= acceleration acting on the ion

S=distance travelled

Since initial velocity is zero,

$$\Rightarrow v^2 = 2aS$$

$$\Rightarrow (12824.24)2 = 2 \times 5192 \times 108 \times S \Rightarrow 15.83 \times 10^{-5} = S$$

Now, time taken t,

$$t = 15.83 \times 10^{-5} + 3023.95 \times 10^{-5}$$

$$= 3039.787 \times 10^{-5}\ m.$$

Now,for the potassium ion K-41

$$\Rightarrow \frac{1}{2}mv^2 = 32 \times 10^3 \times 1.6 \times 10^{-9}$$

$$\Rightarrow \frac{1}{2} \times 41 \times 1.6 \times 10^{-27}\, v^2 = 32 \times 10^3 \times 1.6 \times 10^{-9}$$

$$\Rightarrow v = 3.94 \times 10^5\ m/s$$

And, acceleration a is given by-

$$a = 4805 \times 10^8\ m/s2$$

t = time taken by the ion to exit the magnetic field is given by -

$= 25.4 \times 10^{-9}\ sec$

From newton's law velocity in vertical direction-

$v_{y1} = a\,t$

$= 4805 \times 10^{8} \times 25.4 \times 10^{-9}$

$= 12204.7 \times 10^{-9}\ m/s$

Time to reach the screen, t -

$t = 2423 \times 10 - 9\ s.$

Now distance moved in vertical direction is-

$$= \frac{12204.7 \times 10^{-9}}{2423 \times 10^{-9}}$$

$= 2957.1 \times 10^{-5}\ m\,(1)$

Now,Vertical distance travelled by the particle inside magnetic field can be found out by using 3rd equation of motion given by

$v^2 - u^2 = 2aS$

where,

v= final velocity

u=initial velocity

a= acceleration acting on the ion

S=distance travelled

Since initial velocity is zero,

$v^2 = 2aS$

$(12204.7)^2 = 2 \times 4805 \times 10^{8}\ S$

$\Rightarrow S = 15.49 \times 10^{-5}\ m$

Net distance travelled by K-41 potassium ion

$= 15.49 \times 10^{-5} + 2957.1 \times 10^{-5} = 2972.68 \times 10^{-5}\ m\,(2)$

Net gap between K-39 and K-41 potassium ions is given by-

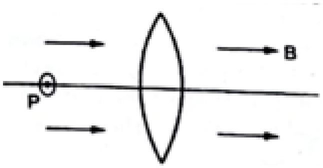
From (1) and (2)

$= 3039.787 \times 10^{-5} - 2972.68 \times 10^{-5}$

$= 67\ mm.$

43. Question

Figure shows a convex lens of focal length 12 cm lying in a uniform magnetic field B of magnitude 1.2 T parallel to tis principal axis. A particle having a charge 2.0 × 10^{-3} C and mass 2.0 × 10^{-6} kg is projected perpendicular to the plane of the diagram with a speed of 4.8 ms^{-1}. The particle moves along a circle with its centre on the principal axis at a distance of 18 cm from the lens. Show that the image of the particle goes along a circle and find the radius of that circle.



Answer

Given-Focal length of the convex lens = 12 cm

Uniform magnetic field, B = 1.2 T

Charge of the particle, $q = 2.0 \times 10^{-3}$ C

an mass, $m = 2.0 \times 10^{-5}$ kg

Speed of the particle, $v = 4.8$ m s^{-1}

Distance between the particle and the lens = 18 cm

Given in the question that the object is projected perpendicularly on the plane of the paper.

The radius of the circular path described by a particle in a magnetic field r,

$$r = \frac{mv}{qB}$$

where,

m is the mass of a proton

v= velocity of the particle

B = magnetic force

q= charge on the particle =$1.6 \times 10-^{19}$ C

$$r = \frac{2 \times 10^{-5} \times 4.8}{2 \times 10^{-3} \times 1.2}$$

$$r = 0.04\,m$$

$$= 4\,cm$$

Given that, the object distance, *u* = -18 cm

Using the lens formula –

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

where,

v=distance of image formed from lens

u=distance of the object from lens

f =focal length of the lens

substituting the values-

$$\frac{1}{v} - \frac{1}{8} = \frac{1}{12}$$

$$\Rightarrow v = 36\,cm$$

Let the radius of the circular path of image be r'.

Hence magnification -

$$\frac{v}{u} = \frac{r'}{r}$$

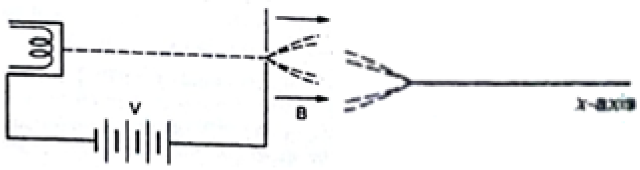
$$\Rightarrow r' = \frac{v}{u} \times r$$

$$= 8\,cm$$

Therefore, the radius of the circular path in which the image of the object formed from the lens moves is 8 cm.

44. Question

Electrons emitted with negligible speed from an electron gun are acceleration through a potential difference V



Along the x-axis. These electrons emerge form a narrow hole into a uniform magnetic field B directed along this axis. However, some of the electrons emerging from the hole make slightly divergent angles as shown in figure. Show that these paraxial electrons are refocused on the x-axis at a distance.

$$\sqrt{\frac{8\pi^2 mV}{eB^2}}.$$

Answer

Given-Electrons are accelerated by applying a potential difference = *V*

Let the mass of an electron = *m*

charge of an electron= *e*

Electric field,

$$E = \frac{V}{r}$$

Force experienced by the electron by coulomb's law is given by,

$$F = eE$$

Acceleration a, of the electron is given by,

$$a = \frac{eV}{rm} \ (1)$$

where,

e= electronic charge

V= applied potential difference

r= radius of the curve

m= mass of the object

Using the 3rd equation of motion

$$v^2 - u^2 = 2aS$$

where,

v= final velocity

u=initial velocity

a= acceleration acting on the ion

S=distance travelled

Since initial velocity is zero,

$$v^2 = 2 \times a \times s$$

Here, $s = r$ which is the radius of the curve

From (1)

$$\mathbf{v} = \sqrt{\frac{2eV}{rm}r}$$

$$= \sqrt{\frac{2eV}{m}}$$

We know that time taken by electron to cover the curved path is given as,

$$T = \frac{2\pi m}{eB}$$

As the acceleration of the electron is along the *y* axis only, it travels along the *x*axis with uniform velocity.

Velocity of the electron moving along the field remains *v*.

Therefore, the distance at which the beam is

d = velocity × Time

$$d = \sqrt{\frac{2eV}{m}} \times \frac{2\pi m}{eB}$$

$$= \sqrt{\frac{8\pi^2 mV}{eB^2}}$$

45. Question

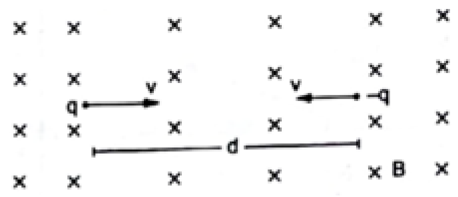
Two particles, each having a mass m are placed at a separation d in a uniform magnetic filed B as shown in figure. They have opposite charges of equal magnitude q. At time t = 0, the particles are projected towards each other, each with a speed v. Suppose the Coulomb force between the charges is switched off.

(a) Find the maximum value v_m of the projection speed so that the two particles do not collide.

(b) What would be the minimum and maximum separation between the particles if $v = v_m/2$?

(c) At what instant will a collision occur between the particles if $v = 2v_m$?

(d) Suppose $v = 2v_m$ and the collision between the particles is completely inelastic. Describe the motion after the collision.



Answer

Given-Mass of two particles = mDistance between the particles = dAlso magnitude of charges of both the particles are equal but opposite polarity equal = q.From the question we can confer that, both the particles are projected towards each other with equal speed v.

Assuming that Coulomb force between the charges is switched off.

(a) The maximum value v_m of the projection speed so that the two particles do not collide-

The particles will not collide with each other if

$d = r_1 + r_2$

where, $r_1 = r_2$ = radius of circular orbit followed by the charged particles

The radius of the circular path described by a particle in a magnetic field r,

$$r = \frac{mv}{qB}$$

where,

m is the mass of a proton

v= velocity of the particle

B = magnetic force

q= charge on the particle =$1.6 \times 10{-}^{19}$ C

$$\Rightarrow d = \frac{mv}{qB} + \frac{mv}{qB}$$

$$= 2\frac{mv}{qB}$$

So,

$$\Rightarrow v_m = \frac{qB}{d2m}\,(1)$$

(b) The minimum and maximum separation between the particles if $v = v_m/2$-

Let the radius of the curved path taken by the particles in the condition when $v_m/2$, be r

So, minimum separation between the particles will be

$$= (d - 2r)$$

$$\Rightarrow (d - 2r) = \frac{2m_v v}{qB} - \frac{2mv}{qB}$$

$$\Rightarrow (d - 2r) = \frac{m_v v}{qB}$$

$$\Rightarrow (d - 2r) = \frac{d}{2}$$

Now, the maximum distance of separation between the particles will be = $(d + 2r)$

$$\Rightarrow d + 2r = d + \frac{d}{2}$$

$$= \frac{3d}{2}$$

(c) The instant at which the collision occurs between the particles if $v=2v_m$ –

The particles along the horizontal direction will collide at a distance , $d/2$

Let they collide after time *t*.

Velocity of the particles before and after collision along the horizontal direction will remain the same.

Therefore,

$$t = \frac{\frac{d}{2}}{2v_m}$$

From (1)

$$\Rightarrow t = \frac{d}{4} \times \frac{2mq}{Bd}$$

$$\Rightarrow t = \frac{m}{2qB} \ (1)$$

(d) The motion of the two particles after collision when the collision is completely inelastic and $v = 2vm$ –

Let the P be t he point of particles collision.

And at point P, both the particles will have motion in upwards

As the collision is inelastic these particles will stick together.

Distance between centers = *d*

Velocity of the particles before and after collision along the horizontal direction will remain the same.

At point P, velocities along the horizontal direction are equal in magnitude and opposite in direction.

So, they will cancel out each other.

So, the velocity along the vertical upward direction will add up.

Magnetic force acting along the vertical direction,

Magnetic force, we know, Lorentz force F is given by -

$$F = qvBsin\theta$$

where,

q = charge on an electron

v = velocity of the electron

B=magnetic field

θ= angle between B and v

$$\Rightarrow F = q2v_mB$$

Now, from Newton's second law, the acceleration along the vertical direction,

$$a = \frac{F}{m}$$

$$= \frac{2qv_mB}{m}$$

So, from 1st equation of motions-

$$v = u + at$$

where

v= final velocity

u=initial velocity

a= acceleration due to gravity

t=time taken

since initital velocity u =0,

Velocity of the combined mass at point P is along the vertical

direction v'.

$$v' = a \times t$$

$$v' = \frac{2qv_mB}{m} \times \frac{m}{2qB}$$

$$\Rightarrow v' = v_m$$

Hence, the particles will behave as a combined mass and move with same velocity v_m.

46. Question

A uniform magnetic field of magnitude 0.20 T exists in space from east to west. With what speed should a particle of mass 0.010 g and having a charge 1.0×10^{-5} C projected from south to north so that it moves with a uniform velocity?

Answer

Given-

Magnetic field, B = 0.20 T

Mass of the particle, m = 0.010 g = 1×10^{-5} kg

Charge of the particle, q = 1.0×10^{-5} C

Given in the question that, if the particle has to move with

uniform velocity in the region of the applied field,

the gravitational force experienced by the particle must be

equal to the magnetic force experienced by the particle.

Gravitational force,

$$\mathbf{F} = mg$$

where

m is the massof theobject

g=acceleration due to gravity

And

Magnetic force, we know, Lorentz force F is given by -

$$F = qvB$$

where,

q = charge on an electron

v = velocity of the electron

B=magnetic field

θ= angle between B and v

So,

$$qvB = mg$$

$$\Rightarrow 1 \times 10^{-5} \times v \times 2 \times 10^{-1} = 1 \times 10^{-5} \times 9.8$$

$$\Rightarrow v = 4.9 \times 10$$

$$= 49\ m/s$$

47. Question

A particle moves in a circle of diameter 1.0 cm under the action of a magnetic field of 0.40 T. An electric field of 200 V m^{-1} makes the path straight. Find the charge/mass ratio of the particle.

Answer

Given-Diameter of the circle = 1.0 cm

Thus, radius of circle, r = = 0.5×10^{-2} m

Magnetic field, B = 0.40 T

Electric field, E = 200 V m^{-1}

From the question we can confer that, the particle is moving in a circle under the action of a magnetic field.

But when an electric field is applied on the particle, it moves in a straight line.

So, we can say that the electric field is balanced by the magnetic field ie,

$$F_e = F_m$$

Magnetic force, we know, Lorentz force F is given by -

$$F_m = qvB$$

where,

q = charge on an electron

v = velocity of the electron

B=magnetic field

θ= angle between B and v

$$qE = qvB$$

$$\Rightarrow v = \frac{E}{B}$$

$$= \frac{200}{0.4}$$

$$= 500\ m/s$$

The radius of the circular path described by a particle in a magnetic field r,

$$r = \frac{mv}{qB}$$

where,

m is the mass of a proton

v= velocity of the particle

B = magnetic force

q= charge on the particle =$1.6 \times 10{-}^{19}$ C

$$\frac{q}{m} = \frac{v}{rB}$$

$$r = \frac{500}{0.5 \times 10^{-2} \times 0.4}$$

$$= 2.5 \times 10^5\ C/kg$$

48. Question

A proton goes undeflected in a crossed electric and magnetic field (the fields are perpendicular to each other) at a speed of 2.0×10^6 ms^{-1}. The velocity is perpendicular to both the fields. When the electric field is switched off, the proton moves along a circle of radius 4.0 cm. Find the magnitudes of the electric and the magnetic fields. Take the mass of the proton = 1.6×10^{-27} kg.

Answer

Given-Mass of the proton, $m = 1.6 \times 10^{-27}$ kg

Speed of the proton inside the crossed electric and magnetic field, $v = 2.0 \times 10^5$ ms^{-1}

Given from question we can confer that , the proton is not deflected under the combined action of the electric and magnetic fields.

Thus, the forces applied by both the fields are equal and opposite to each other

Magnetic force, we know, Lorentz force F is given by -

$$F_m = qvB$$

where,

q = charge on an electron

v = velocity of the electron

B=magnetic field

θ= angle between B and v

Also,

We, know the Columb's force–

$$F = eE$$

where

E = applied electric field

e=charge

Now,

$$qE = qvB$$

$$\Rightarrow E = vB \ (1)$$

But when the electric field is switched off, the proton moves follows circular path due to the presence of force of the magnetic field.

The radius of the circular path described by a particle in a magnetic field r,

$$r = \frac{mv}{qB}$$

where,

m is the mass of a proton

v= velocity of the particle

B = magnetic force

q= charge on the particle =$1.6 \times 10{-}^{19}$ C

$$\Rightarrow B = \frac{mv}{qr}$$

$$= \frac{1.6 \times 10^{-27} \times 2 \times 10^{5}}{1.6 \times 10^{-19} \times 4 \times 10^{-2}}$$

$= 0.05\, T$

Substituting the value of B in equation (1), we get

$E = 2 \times 10^5 \times 0.05$

$= 1 \times 10^4\, N/c$

Hence magnitudes of the electric and the magnetic fields are

$= 0.05\, T\ and\ 1 \times 10^4\, N/c$ respectively

49. Question

A particle having a charge of 5.0 μC and a mass of 5.0×10^{-12} kg is projected with a speed of 1.0 km s^{-1} in a magnetic field of magnitude 5.0 mT. The angle between the magnetic field and the velocity is $\sin^{-1}(0.90)$. Show that the path of the particle will be a helix. Find the diameter of the helix and its pitch.

Answer

Given-Charge on the particle, $q = 5$ μC $= 5 \times 10^{-6}$ C

Magnetic field, $B = 5 \times 10^{-3}$ T

Mass of the particle, $m = 5 \times 10^{-12}$ kg

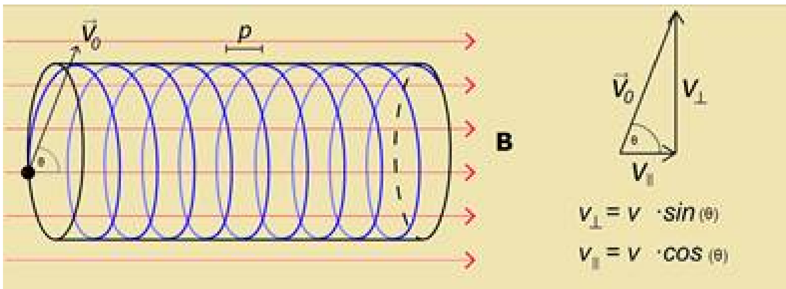
Velocity of projection, $v = 1$ Km/s $= 10^3$ m/s

Angle between the magnetic field and velocity,

$\theta = \sin^{-1}(0.9)$

Since there are no forces in the horizontal direction ie there is no force in the direction of magnetic field, so, the particle moves with uniform velocity in horizontal direction.

Thus, it moves in a helical form.In helical motion we have two components of velocity.



Component of velocity which is perpendicular to the magnetic field is given by -

$v_\perp = v\sin\theta$

Similarly component of velocity in the direction of magnetic field will be the parallel component given by -

$$v_{\parallel} = v\cos\theta$$

The velocity has a vertical component along which it accelerates with an acceleration a and moves in a circular cross-section.

We know motion in helical direction which is centripetal force and is given by –

$$F_c = \frac{mv_{\perp}^2}{r}$$

Now, this force is balanced by Lorentz force acting due to presence of magnetic field ,

$$\frac{mv_{\perp}^2}{r} = qv_{\perp}B$$

$$\Rightarrow r = \frac{mv\sin\theta}{qB}$$

$$= \frac{5 \times 10^{-12} \times 10^3 \times 0.90}{5 \times 10^{-6} \times 5 \times 10^{-3}}$$

$$= 0.18\ m$$

Hence, diameter of the helix can be calculated as ,

$$2r = 0.36\ m = 36\ cm$$

And the Pitch of the helix is ,

$$P = \frac{2\pi r}{v\sin\theta} \times v\cos\theta$$

$$= \frac{2 \times 3.14 \times 0.18}{0.90} \times \sqrt{1 - 0.81}$$

$$= 0.55\ m$$

$$= 55\ cm$$

50. Question

A proton projected in a magnetic field of 0.020 T travels along a helical path of radius 5.0 cm and pitch 20 cm. Find the components of the velocity of the proton along and perpendicular to the magnetic field. Take the mass of the proton = 1.6 × 10^{-27} kg.

Answer

Given

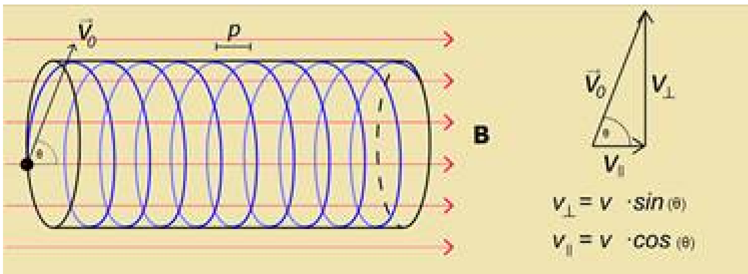
Mass of the proton, $m_p = 1.6 \times 10^{-27}$ kg

Magnetic field intensity, $B = 0.02$ T

Radius of the helical path, $r = 5$ cm $= 5 \times 10^{-2}$ m

Pitch of the helical path, $p = 20$cm $= 2 \times 10^{-1}$ m

In helical motion we have two components of velocity.



Component of velocity which is perpendicular to the magnetic field is given by -

$$v_\perp = vsin\theta$$

Similarly component of velocity in the direction of magnetic field will be the parallel component given by -

$$v_\parallel = vcos\theta$$

Now,

Now, this force is balanced by Lorentz force acting due to presence of magnetic field ,

$$\frac{mv_\perp^2}{r} = qv_\perp B$$

$$\Rightarrow r = \frac{mvsin\theta}{qB}$$

$$\Rightarrow 5 \times 10^{-2} = \frac{1.6 \times 10^{-27} \times v_\perp}{1.6 \times 10^{-19} \times 0.02}$$

$$\Rightarrow v_\perp = 1 \times 10^5\ m/s$$

Now, pitch of the helix is calculated as –

$$Pitch = \frac{v_\parallel 2\pi r}{v_\perp}$$

$$v_\parallel = \frac{v_\perp P}{2\pi r}$$

$$= \frac{10^5 \times 0.2}{2 \times 3.14 \times 5 \times 10^{-2}}$$

$= 0.6369 \times 10^5$

$= 6.4 \times 10^4 \ m/s$

51. Question

A particle having mass m and charge q is release from the origin in a region in which electric field and magnetic field are given by

$\vec{B} = B_0 \vec{j}$ and $\vec{E} = E_0 \vec{k}$.

Find the speed of the particle as a function of its z-coordinate.

Answer

Given-

Mass of the particle = *m*

Charge of the particle = *q*

Electric field and magnetic field are given by

$\vec{B} = -B_0 \vec{J}$

$\vec{E} = E_0 \vec{k}$

Velocity,

$v = v_x \hat{\imath} + v_y \hat{\jmath} + v_z \hat{k}$

Magnetic force, we know, Lorentz force F is given by -

$F_m = qvB$

where,

q = charge on an electron

v = velocity of the electron

B=magnetic field

θ= angle between B and v

Also, coulomb's force experienced by the electron is given by,

$F = eE$

where e= charge on the electron and

E= electric field applied

So, total force on the particle,

$F = q(E + v \times B)$

$=q(E_0\overrightarrow{k} + v_x\hat{\imath} + v_y\hat{\jmath} + v_z\widehat{k} \times -\overrightarrow{B_0 J})$

$= q(E_0\overrightarrow{k} + B_0 v_x\hat{\imath} + B_0 v_z\widehat{k})$

Now, since

$vx = 0,$

$\Rightarrow F_z = qE_0$

So, acceleration is given by

$$a_z = \frac{qE_0}{m}$$

From 3rd equation for motion

$v^2 = u^2 + 2as$

where

u = initial velocity

v= final velocity

s=distance travelled

and a = acceleration of the particle

$$v^2 = \frac{2qE_0 z}{m}$$

So,

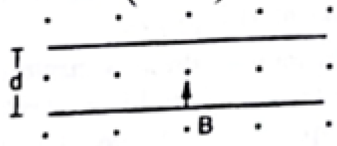
$$v = \sqrt{\frac{2qE_0 z}{m}}$$

Here, *z* is the distance along the *z*-direction.

52. Question

An electron is emitted with negligible speed from the negative plate of a parallel plate capacitor charged to a potential difference V. The separation between the plates is d and a magnetic field B exists in the space as shown in figure. Show that the electron will fail to strike the upper plate if.

$$d > \left(\frac{2m_e V}{eB_0^2}\right)^{\frac{1}{2}}.$$



Answer

Given-Potential difference applied across the plates of the capacitor = *V*Separation between the plates = *d*Magnetic field intensity = *B*

The electric field applied across the plates of a capacitor

,

$$E = \frac{V}{d}$$

Also, coulomb's force experienced by the electron is given by,

$$F = eE$$

where e= charge on the electron and

E= electric field applied

Hence, the force experienced by the electron due to this electric field,

$$F = e\frac{V}{d}$$

Now, acceleration a is given by-

$$acceleration = force \times mass$$

$$\Rightarrow a = \frac{F}{m} = \frac{eV}{m_e \times d'}$$

where

e = charge of the electron

m_e = mass of the electron

From 3rd equation for motion

$$v^2 = u^2 + 2as$$

where

u = initial velocity

v= final velocity

s=distance travelled

and a = acceleration of the particle

substituting the value of *a*-

$$v^2 = 2 \times \frac{eV}{m_e \times d} \times d$$

$$\Rightarrow v = \sqrt{\frac{2\,e\,V}{m_e}}$$

The electron will move in a circular path due to the presence of the magnetic field.

The radius of the circular path described by a particle in a magnetic field r,

$$r = \frac{mv}{qB}$$

where,

m is the mass of a proton

v= velocity of the particle

B = magnetic force

q= charge on the particle =$1.6 \times 10{-}^{19}$ C

Radius of the circular path followed by the electron is ,

$$r = \frac{m_e v}{eB}$$

And the electron will fail to strike the upper plate of the

capacitor if and only if the radius of the circular path will be less

than *d*,

i.e. d>r

$$\Rightarrow d > \frac{m_e}{eB} \times \frac{m_e v}{eB}$$

$$\Rightarrow d > \sqrt{\frac{2m_e v}{eB^2}}$$

Thus, the electron will fail to strike the upper plate if

$$d > \sqrt{\frac{2m_e v}{eB^2}}$$

53. Question

A rectangular coil of 100 turns has length 5 cm and width 4 cm. It is placed with its plane parallel to a uniform magnetic field and a current of 2A is sent through the coil. Find the magnitude of the magnetic field B, if the torque acting on the coil is 0.2 N m^{-1}.

Answer

Given-No. of turns in the coil, n = 100

Area of the coil, $A = 5 \times 4\ cm^2 = 20 \times 10^{-4}\ m^2$

Magnitude of current = 2 A

Torque acting on the coil, $\tau = 0.2\ N\ m^{-1}$

We know that torque acting on a rectangular coil having n turns is given by

$$Torque, \tau = niA \times B$$

Where

B= applied magnetic field

A= area of rectangular loop

I = current flowing through coil

n = number of turns

$$\tau = niA \times B$$

$$\Rightarrow \tau = niBA \sin 90°$$

$$\Rightarrow 0.2 = 100 \times 2 \times 20 \times 10 - 4 \times B$$

$$\Rightarrow B = 0.5\,T$$

54. Question

A 50 turn circular coil of radius 2.0 cm carrying a current of 5.0 A is rotated in a magnetic field of strength 0.20 T.

(a) What is the maximum torque that acts on the coil?

(b) In a particular position of the coil, the torque acting on it is half of this maximum. What is the angle between the magnetic field and the plane of the coil?

Answer

Given-No. of turns of the coil, n = 50

Magnetic field intensity, $B = 0.20$ T = 2×10^{-1} T

Radius of the coil, $r = 0.02$ m = 2×10^{-2} m

Magnitude of current =5 A

We know that torque acting on a rectangular coil having n turns is given by

$$\tau = niA\,B\sin\theta$$

Where

B= applied magnetic field

A= area of rectangular loop

I = current flowing through coil

θ = angle between the area vector and magnetic field

Torque is maximum if

$\theta = 90°$.

$$\tau_{max} = niAB\sin 90°$$

$$= 50 \times 5 \times 3.14 \times 4 \times 10^{-4} \times 2 \times 10^{-1}$$

$$= 6.28 \times 10^{-2}\ N-m$$

Given that ,

$$\tau = \frac{1}{2}\tau_{max}$$

$$\Rightarrow \sin\theta = \frac{1}{2}$$

$$\Rightarrow \theta = 30°$$

So, the angle between the magnetic field and the plane of coil is given by -

$$90° - 30° = 60°$$

55. Question

A rectangular loop of sides 20 cm and 10 cm carries a current of 5.0 A. A uniform magnetic field of magnitude 0.20 T exists parallel to the longer side of the loop.

(a) What is the force acting on the loop?

(b) What is the torque acting on the loop?

Answer

Given-No. of turns of the coil, n = 50

Magnetic field, $B = 0.20\text{ T} = 2 \times 10^{-1}\text{ T}$

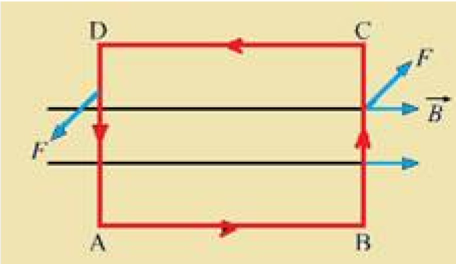
Magnitude of current, I = 5 A

Length of the loop, $l = 20\text{ cm} = 20 \times 10^{-2}\text{ m}$,

Breadth of the loop, $w = 10\text{ cm} = 10 \times 10^{-2}\text{ m}$,

So, area of the loop, A = length ×width = 0.02 m^2,

Let ABCD be the rectangular loop.



(a)There is no force acts along the the sides *ab* and *cd*, as they are parallel to the magnetic field.

$$\Rightarrow \tau = niBA \sin 0^\circ$$

$$= 0$$

But the force acting on the sides *ad* and *bc* are equal in magnitude but opposite

So, they cancel out each other

.Hence, the net force on the loop is zero.

(b) Torque acting on the coil,

$$\Rightarrow \tau = niBA \sin \theta$$

Where

B= applied magnetic field

A= area of rectangular loop

I = current flowing through coil

θ = angle between the area vector and magnetic field

$$\Rightarrow \tau = niBA \sin 90^0$$

$$= 1 \times 5 \times 0.02 \times 0.2$$

$= 0.02\ Nm$

So, the torque acting on the loop is 0.02 Nm and is parallel to the shorter side

56. Question

A circular coil of radius 2.0 cm ahs 500 turns in it and carries a current of 1.0 A. Its axis makes an angle of 30° with the uniform magnetic field of magnitude 0.40 T that exist in the space. Find the torque acting on the coil.

Answer

Given-

No. of turns of the coil, $n = 500$

Magnetic field intensity, $B = 0.40\text{ T} = 4 \times 10^{-1}\text{ T}$

Radius of the coil, $r = 2\text{ cm} = 2 \times 10^{-2}\text{ m}$

Magnitude of current, $i = 1\text{ A}$

Angle between the area vector and magnetic field, $\theta = 30°$

Torque acting on the coil,

$\Rightarrow \tau = niBA \sin\theta$

Where

B= applied magnetic field

A= area of rectangular loop

I = current flowing through coil

θ = angle between the area vector and magnetic field

$\tau = 500 \times 1 \times 3.14 \times 4 \times 10^{-4} \times 4 \times 10^{-1} \times 12$

$= 12.56 \times 10^{-2}$

$= 0.1256$

$= 0.13\ Nm$

57. Question

A circular loop carrying a current i has wire of total length L. A uniform magnetic field B exists parallel to the plane of the loop.

(a) Find the torque on the loop.

(b) If the same length of the wire is used to form a square loop, what would be the torque? Which is larger?

Answer

Given-Magnetic field intensity = *B*

Magnitude of current = *i*

Circumference, $L = 2\pi r$,

where

r is the radius of the coil

So, area of the coil,

$$A = \frac{L^2}{4\pi}$$

Torque acting on the coil,

$$\Rightarrow \tau = niBA \sin\theta$$

Where

B= applied magnetic field

A= area of rectangular loop

I = current flowing through coil

θ = angle between the area vector and magnetic field

$$\Rightarrow \tau = niBA$$

$$= \frac{iL^2B}{4\pi}$$

(b) Let *s* be the length of the square loop.

Then given in the question that

$$4s = L$$

$$\Rightarrow s = \frac{L}{4}$$

$$A = \left(\frac{L}{4}\right)^2$$

$$= \frac{L^2}{16}$$

$$\Rightarrow \tau = niBA = \frac{iL^2B}{16}$$

So, the torque in case of the circular loop is larger.

58. Question

A square coil of edge ℓ having n turns carries a current i. It is kept on a smooth horizontal plate. A uniform magnetic field B exists in a direction parallel to an edge. The total mass of the coil is M. What should be the minimum value of B for which the coil will start tipping over?

Answer

Given-Number of turns in the coil = n

Edge of the square loop = l

Magnetic field intensity = B

Magnitude of current = i

Angle between area vector and magnetic field, θ = 90°

Torque acting on the coil,

$$\Rightarrow \tau = niBA \sin\theta$$

Where

B= applied magnetic field

A= area of rectangular loop

I = current flowing through coil

θ = angle between the area vector and magnetic field

$$\tau = nil2Bsin90^\circ$$

$$= nil^2B \ (1)$$

Torque produced due to weight is

given by,

$$T_{weight} = \frac{mgl}{2} \ (2)$$

where m is the mass

g is the acceleration due to gravity

For the coil to start tipping towards downwards,

$T \geq T_{weight}$

For minimum value of *B,*

$T = T_{weight}$

From (1) and(2)

$$\Rightarrow nil^2B = \frac{mgl}{2}$$

$$\Rightarrow B = \frac{mg}{2nil}$$

59. Question

Consider a non-conducting ring of radius r and mass m which has a total charge q distributed uniformly on it. The ring is rotated about its axis with an angular speed ω.

(a) Find the equivalent electric current in the ring.

(b) Find the magnetic moment μ of the ring.

(c) Show that $\mu = \frac{q}{2m}\ell$ where ℓ is the angular momentum of the ring about its axis of rotation.

Answer

Given-Radius of the ring = *r*

Mass of the ring = *m*

Total charge enclosed on the ring = *q*

(a) Angular speed-

We know that angular speed is given by-

$$\omega = 2\pi f$$

now frequency f

$$f = \frac{1}{T}$$

$$\omega = \frac{2\pi}{T}$$

$$\Rightarrow T = \frac{2\pi}{\omega}$$

Current in the ring,

$i = charge \times time$

$= qT$

$= q\frac{\omega}{2\pi}$ (1)

(b)For a ring of area *A* with current *i* flowing through it, magnetic moment,

$\mu = niA$

where,

A= area of cross section

i=current flowing through it

n = number of turns

for number of turns *n* = 1

$\mu = ia$

From (1)

$\mu = q\frac{\omega}{2\pi} \times \pi r^2 =$

$\mu = \frac{q\omega r^2}{2}$ (2)

(d) Angular momentum *l*,

$l = I\omega$

where

I is moment of inertia of the ring about its axis of rotation.

ω is angular velocity

$I = mr^2$

Where

m is the mass

r is the radius of gyration.

$I = mr^2\omega$

So,

$\Rightarrow \omega r^2 = \frac{l}{m}$

Putting this value in equation (2), we get-

$$\mu = \frac{q\omega r^2}{2} = \frac{ql}{2m}$$

60. Question

Consider a non-conducting plate of radius r and mass m which has a charge q distributed uniformly over it. The plate is rotated about its axis with an angular speed ω. Show that the magnetic moment μ and the angular momentum ℓ of the plate are related as $\mu = \frac{q}{2m}\ell$.

Answer

Given-Radius of the ring = *r*

Mass of the ring = *m*

Total charge of the ring = *q*

Angular speed, $\omega = 2\pi f$

Where

F is the frequency

Now frequency f

$$f = \frac{1}{T}$$

Where,

T is the time period

$$\Rightarrow \omega = \frac{2\pi}{T}$$

$$\Rightarrow T = \frac{2\pi}{\omega}$$

Magnetic moment, $\mu = niA$

where,

A= area of cross section

i=current flowing through it

n = number of turns

for number of turns $n = 1$

$\mu = ia$

Current in the ring,

$$current = charge \times time$$

$$i = qT$$

$$= q\frac{2\pi}{\omega}$$

For the ring of area *A* with current *i,* magnetic moment,m is given by

$$m = iA$$

Where A is the area of the loop

$$\Rightarrow m = q\frac{2\pi}{\omega} \times \pi r^2 = \frac{q\omega r^2}{2} \ (1)$$

Angular momentum,

$$l = I\omega \ (2)$$

where

I is the moment of inertia of the ring about its axis of rotation.

ω is the angular moment

Moment of inertia, *I* is given by-

$$I = mr^2$$

where,

m is the mass of the object

r is the radius of the circular object

So, substituting in (2)

$$l = mr^2\omega$$

$$\Rightarrow \omega r^2 = \frac{l}{m}$$

Putting this value ωr^2 in equation (1),

$$\mu = \frac{q\omega r^2}{2} = \frac{ql}{2m}$$

61. Question

Consider a solid sphere of radius r and mass m which has a charge q distributed uniformly over its volume. The sphere is rotated about a diameter with an angular speed ω. Show that the magnetic moment μ and the angular momentum ℓ of the sphere are related as $\mu = \frac{q}{2m}\ell$.

Answer

Given

radius of solid sphere = r

mass of sphere = m

charge on the sphere =q

angular speed of the sphere = ω

magnetic moment and the angular momentum ℓ of the sphere are related –

$$\mu = \frac{q}{2m} l$$

Lets consider a differential strip of width *dx* at a distance *x* from the centre of the sphere.

Area of the strip is given as,

$$da = 4\pi \times dx$$

now, current i is given by-

$$i = \frac{dq}{dt}$$

Angular speed,

$$\omega = 2\pi f$$

Now frequency f

$$f = \frac{1}{T}$$

$$\Rightarrow \omega = \frac{2\pi}{T}$$

$$\Rightarrow T = \frac{2\pi}{\omega}$$

current i becomes

$$i = q\frac{\omega}{2\pi}$$

Magnetic moment is given by-$\mu = niA$

where,

A= area of cross section

i=current flowing through it

n = number of turns

for number of turns $n = 1$

$$\mu = ia$$

Integrating over the sphere-

$$\mu = \frac{q\omega}{2\pi} 4\pi \int_0^r x\,dx$$

$$= q.\omega r^2$$

$$= \frac{q}{m} l$$