# CBSE Test Paper 04
## CH-13 Limits and Derivatives

1. $\underset{h \to 0}{Lt} \frac{\sin^2(x+h) - \sin^2 x}{h}$ is equal to
   a. sin 2x
   b. 2 sin x
   c. sin x cos x
   d. $\cos^2 x$
2. $\frac{d}{dx}(\sin^{-1}(1-x))$ is equal to
   a. $\frac{-1}{\sqrt{2x-x^2}}$
   b. $\frac{1}{\sqrt{x^2-2x}}$
   c. $\frac{1-x}{(2x-x^2)^{\frac{3}{2}}}$
   d. $\frac{1}{\sqrt{2x-x^2}}$
3. If $y = \sin^{-1}$ x and z = $\cos^{-1}\sqrt{1-x^2}$, then $\frac{dy}{dz} =$
   a. - 1
   b. $\tan^{-1}\frac{x}{\sqrt{1-x^2}}$
   c. 1
   d. 0
4. If y = $\sqrt{x + \sqrt{x + \sqrt{x + \ldots + to\ \infty}}}$ then $\frac{dy}{dx} =$
   a. $\frac{1}{2y+1}$

b. $\frac{1}{2y-1}$

c. $\frac{x}{y+1}$

d. $\sqrt{\frac{x}{y+1}}$

5. $\underset{x\to 0}{Lt} \frac{\tan x - x}{x^2 \tan x}$ is equal to

   a. 1

   b. $\frac{1}{2}$

   c. 0

   d. $\frac{1}{3}$

6. Fill in the blanks: The value of the given limit $\lim_{x\to 0} x\sec x$ is ________.
7. Fill in the blanks: The value of the limit $\lim_{x\to -1} \frac{x^{10}+x^5+1}{x-1}$ is ________.
8. Evaluate $\lim_{\theta\to 0} \frac{\sin a\theta}{\sin b\theta}$.
9. Evaluate $\lim_{\theta\to 0} \theta cosec\theta$.
10. Evaluate $\lim_{x\to 0} x\sec x$
11. Evaluate: $\lim_{x\to 1} \frac{x^3-1}{x-1}$.
12. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\sin^n x$
13. Find the derivative of $x^n + ax^{n-1} + a^2x^{n-2} + ...+ a^{n-1} x + a^n$ for some fixed real number a.
14. Find the derivative of f(x) = tan (ax + b), by first principle.
15. Find the derivative of x sinx from first principle.

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### Solution

1. (a) sin 2x

   **Explanation:** $\underset{h \to 0}{Lt} \frac{[\sin(x+h)-\sin x][\sin(x+h)+\sin x]}{h}$

   $\Rightarrow \underset{h \to 0}{\mathrm{Lt}} \frac{2\sin\left(\frac{2\pi+h}{2}\right)\cos\left(\frac{h}{2}\right)2\cos\left(\frac{2\pi+h}{2}\right)\sin\left(\frac{h}{2}\right)}{h}$

   $\Rightarrow \sin 2x$

2. (c) $\frac{1-x}{(2x-x^2)^{\frac{3}{2}}}$

   **Explanation:** $y = \sin^{-1}(1-x)$

   $\Rightarrow \sin y = 1 - x$

   $\Rightarrow (\cos y)y' = -1$

   $\Rightarrow y'' = (y')^2(\tan y) = \frac{(\sin y)}{(\cos y)^3}$

   $\Rightarrow \frac{1-x}{(2x-x^2)^{\frac{3}{2}}}$

3. (c) 1

   **Explanation:** We have;

   siny = x and cos z = $\sqrt{1-x^2} \Rightarrow$ cos z = cos y

   $\Rightarrow z = y$

   $\Rightarrow \frac{dy}{dz} = 1$

4. (b) $\frac{1}{2y-1}$

   **Explanation:**

   $y = \sqrt{}(x+y)$

   $y^2 = x + y$

   $2yy' = 1 + y'$

5. (d) $\frac{1}{3}$

   **Explanation:** The following limit is in the form of 0/0

   $\Rightarrow$ using L'Hospital rule;

   $\Rightarrow \underset{x \to 0}{Lt} \frac{\tan^2 x}{(x^2)+(x^2\tan^2 x)+(\tan x)(2x)}$

$$= \underset{x \to 0}{Lt} \left( \frac{(\tan x)^2}{x^2} \right) \cdot \left( \frac{1}{1+\tan^2 x + \left( \frac{2 \tan x}{x} \right)} \right)$$
$$\Rightarrow 1 \cdot \left( \frac{1}{1+0+2} \right)$$
$$\Rightarrow \frac{1}{3}$$

6. 0

7. $-\frac{1}{2}$

8. We have,
$$\lim_{\theta \to 0} \frac{\sin a\theta}{\sin b\theta} = \lim_{\theta \to 0} \frac{\sin a\theta}{\theta} \div \lim_{\theta \to 0} \frac{\sin b\theta}{\theta}$$
$$= a \lim_{\theta \to 0} \frac{\sin a\theta}{a\theta} \div b \lim_{\theta \to 0} \frac{\sin b\theta}{b\theta}$$
$$= a(1) \div b(1) = \frac{a}{b}$$

9. We have,
$$\lim_{\theta \to 0} \theta \; cosec\theta = \lim_{\theta \to 0} \frac{\theta}{\sin \theta} = \lim_{\theta \to 0} \frac{1}{\frac{\sin \theta}{\theta}} = \frac{1}{1} = 1$$

10. Here $\lim\limits_{x \to 0} x \sec x$
$$= \lim_{x \to 0} x \times \frac{1}{\cos x} \to \lim_{x \to 0} \frac{x}{\cos x} = \frac{0}{1} = 0$$

11. When x = 1 the expression $\frac{x^3-1}{x-1}$ assumes the indeterminate form $\frac{0}{0}$. Therefore, (x -1 ) is a common factor in numerator and denominator. Factorising the numerator and denominator, we obtain
$$\lim_{x \to 1} \frac{x^3-1}{x-1} \left( \text{form } \frac{0}{0} \right)$$
$= \lim\limits_{x \to 1} \frac{(x-1)(x^2+x+1)}{(x-1)} = \lim\limits_{x \to 1}$ $x^2 + x + 1 = 1^2 + 1 + 1 = 3$.

12. Here f (x) = $\sin^n x$
$$\therefore \; f'(x) = \frac{d}{dx}(\sin^n x)$$
$$= n\sin^{n-1} x . \frac{d}{dx}(\sin x)$$
$= n\sin^{n-1}x \cos x$

13. Let $f(x) = x^n + ax^{n-1} + a^2x^{n-2} + .... + a^{n-1} x + a^n$

On differentiating both sides, we get

f'(x) = $nx^{n-1} + a(n - 1)x^{n-2} + a^2(n - 2)x^{n-3} + .... + a^{n-1}.1 + 0$

On putting x = a both sides, we get

f'(a) = $na^{n-1} + a(n - 1)a^{n-2} + a^2 (n - 2)a^{n-3} +...+ a^{n-1}$

= $n a^{n-1} + (n - 1) a^{n-1} + (n - 2) a^{n-1} + ... + a^{n-1}$

= $a^{n-1} [n + (n - 1) + (n - 2) + .... + 1]$

[$\because$ sum of n natural numbers $= \frac{n(n+1)}{2}$]

f'(a)$= \frac{n(n+1)}{2} a^{n-1}$

14. we have, f(x) = tan(ax + b)

By first principle of derivative, we have

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\tan[a(x+h)+b]-\tan(ax+b)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{\sin(ax+ah+b)}{\cos(ax+ah+b)} - \frac{\sin(ax+b)}{\cos(ax+b)}}{h}$$

$$= \lim_{h\to 0} \frac{[\sin(ax+ah+b)\cos(ax+b)-\sin(ax+b)\cos(ax+ah+b)]}{h\cos(ax+b)\cos(ax+ah+b)}$$

$$= \lim_{h\to 0} \frac{a\sin(ah)}{a\cdot h\cos(ax+b)\cos(ax+ah+b)}$$

$$= \lim_{h\to 0} \frac{a}{\cos(ax+b)\cos(ax+ah+b)} \lim_{ah\to 0} \frac{\sin ah}{ah}$$

[as h $\to$ 0, then ah $\to$ 0]

$$= \frac{a}{\cos^2(ax+b)} \times 1\left[\because \lim_{x\to 0} \frac{\sin x}{x} = 1\right]$$

= $a \sec^2(ax + b)$

15. We have, f(x) = x sinx

By using first principle of derivative,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)\sin(x+h)-x\sin x}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)[\sin x\cdot\cos h+\cos x\cdot\sin h]-x\sin x}{h}$$ [$\because$ sin(x + y) = sinx cosy + cosx siny]

$$= \lim_{h\to 0} \frac{[x\sin x\cdot\cos h+x\cdot\cos x\cdot\sin h+h\sin x\cdot\cos h+h\cos x\cdot\sin h-x\sin x]}{h}$$

$$= \lim_{h\to 0} \frac{[x\sin x(\cos h-1)+x\cdot\cos x\cdot\sin h+h(sinx\cdot cosh+cosx\cdot\sin h)]}{h}$$

$$= \lim_{h\to 0} \frac{x\sin x(\cos h - 1)}{h} + \lim_{h\to 0} x\cdot\cos x\cdot\frac{\sin h}{h} + \lim_{h\to 0}\frac{h(\sin x\cdot\cos h + \cos x\cdot\sin h)}{h}$$

$= x\sin x \lim_{h\to 0}\left[\frac{-(1-\cos h)}{h}\right]$ + x cos x + sin x

$= -2x\sin x\cdot \lim_{\frac{h}{2}\to 0}\frac{\sin^2\frac{h}{2}}{h\times\frac{h}{4}}\times\frac{h}{4}$ + xcos x + sin x

$= -x\cdot\sin x\cdot\frac{2}{4}\lim_{\frac{h}{2}\to 0}\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)^2\times$ h + xcos x + sin x

$= -x\sin x\cdot\frac{1}{2}(1)\times$ 0 + x cos x + sin x$\left[\because \lim_{x\to 0}\frac{\sin x}{x} = 1\right]$

= xcos x + sin x