CBSE Test Paper 01

CH-10 Straight Lines

1. The lines x + 2y – 3 = 0, 2x + y – 3 = 0 and the line l are concurrent . If the line I passes through the origin, then its equation is
   a. x – y = 0
   b. x + y + 0
   c. x + 2y = 0
   d. none of these
2. Projection (the foot of perpendicular) from (x , y) on the x – axis is
   a. (- x , 0)
   b. (0 , y)
   c. (x , 0)
   d. (0 , - y)
3. The distance of the point (α, β) from X axis is
   a. $|\beta|$
   b. $|\alpha|$
   c. none of these.
   d. α
4. A line is drawn through the points (3 , 4) and (5 , 6) . If the is extended to a point whose ordinate is – 1, then the abscissa of that point is
   a. none of these
   b. 1
   c. 0
   d. - 2
5. The line which is parallel to X axis and crosses the curve y = $\sqrt{x}$ at an angle of 45^0 is:
   a. y = 1/2
   b. y = 1
   c. none of these
   d. y = $\frac{1}{4}$
6. Fill in the blanks:

   If a, b, c are in A.P., then the straight line ax + by + c = 0 will always pass through

________.

7. Fill in the blanks:

   The slope of the line, whose inclination is 150^{o} is ________.

8. Prove that the line through the point (x_1, y_1) and parallel to the line Ax + By + C = 0 is $A(x - x_1) + B(y - y_1) = 0$.

9. Prove that the points A (1, 4), B (3, -2) and C (4, - 5) are collinear.

10. Find the equation of the line, where length of the perpendicular segment from the origin to the line is 4 and the inclination of the perpendicular segment with the positive direction of X - axis is 30^{o}.

11. If a, b, c are variables such that 3a + 2b + 4c = 0, then show that the family of lines given by ax + by + c = 0 pass through a fixed point. Also, find the point.

12. Show that the lines 4x + y - 9 = 0, x - 2y + 3 = 0, 5x - y - 6 = 0 make equal intercepts on any line of gradient 2.

13. A line forms a triangle in the first quadrant with the coordinate axes. If the area of the triangle is $54\sqrt{3}$ sq units and perpendicular drawn from the origin to the line makes an angle 60^{o} with X-axis, then find the equation of the line.

14. Find the equation of the straight lines which passes through the origin and trisect the intercept of line 3x + 4y = 12 between the axes.

15. A rectangle has two opposite vertices at the points (1, 2) and (5,5). If the other vertices lie on the line x = 3, find the equations of the sides of the rectangle.

Solution

1. (a) x – y = 0

   Explanation: Equation of a line passing through the intersection of two lies is given by $ax_1 + by_1 + c_1 + k(ax_2 + by_2 + c_2) = 0$

   Hence x+2y-3 + k(2x+y-3) = 0

   Since it passes through (0,0)

   -3 -3k = 0

   This implies k = -1

   Sustituting for k we get,

   x+2y-3 +(-1)(2x+y-3) = 0

   -x +y =0 or x - y = 0

2. (c) (x , 0)

   Explanation: Let L be the foot of the perpendicular from the X axis. Therefore its y coocrdinate is zero

   Therefore the coordiantes of the point L is (x,0)

   Hence option 1 is the correct answer

3. (a) $|\beta|$

   Explanation:
   The distance of a point from X axis is its y coordinate.
   Hence the distance of the point (α, β) from X axis is $|\beta|$

4. (d) - 2

   Explanation:

The slope of the given line is $\frac{y_2-y_1}{x_2-x_1}$ = $\frac{6-4}{5-3}$ = 1

Therefore $\frac{4-(-1)}{3-x}$ = 1

That is 4+1 = 3 - x

Therefore x = -2

5. (a) y = 1/2

   Explanation: The equation of the line which is a tangent to the curve y = $\sqrt{x}$ is

   y = mx + a/m

   Since it makes and angle of 45^0, m = 1

   y^2 = x implies a = 1/4

   Hence the equation of the tangent is y = x+

   That is the y intercept is $\sqrt{\frac{1}{4}}$ = 1/2

   Hence the equation of the line is y = 1/2

6. (1, -2)

7. $-\frac{1}{\sqrt{3}}$

8. Equation of the line parallel to line Ax + By + C = 0 is Ax + By + K = 0 . . . (i)

   Since line (i) passes through (x_1, y_1)

   $Ax_1 + By_1 + K = 0$...(ii)

   Subtracting (ii) from (i), we have

   $A(x - x_1) + B(y - y_1) = 0$

9. Given points are A (1, 4), B (3, -2) and C(4, -5)

   From the condition for collinearity of points A, B and C, we have

   The slope of AB = Slope of BC.

   $\therefore \quad \frac{-2-4}{3-1} = \frac{-5+2}{4-3} \left[\because \text{ slope } = \frac{y_2-y_1}{x_2-x_1}\right]$

   $\Rightarrow \frac{-6}{2} = \frac{-3}{1} \Rightarrow -3 = -3,$ which is true.

   Hence, points A, B and C are collinear.

10. The normal form of the equation of the line is x cos ω + y sinω = p

Given, p = 4, ω= 30^0. Therefore, the equation of the line is x cos 30^0 + y sin 30^0 = 4.

$\Rightarrow x\frac{\sqrt{3}}{2}+y\frac{1}{2}=4 \Rightarrow \sqrt{3}x$+ y = 8

11. We have, 3a + 2b + 4c = 0

$\Rightarrow \quad c=-\frac{3}{4}a-\frac{1}{2}b$

On substituting this value of c in ax + by + c = 0, we get

$ax+by-\frac{3}{4}a-\frac{1}{2}b=0$

$\Rightarrow \quad a\left(x-\frac{3}{4}\right)+b\left(y-\frac{1}{2}\right)=0$

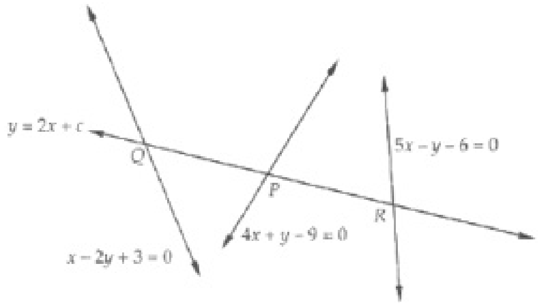
$\Rightarrow \quad \left(x-\frac{3}{4}\right)+\lambda\left(y-\frac{1}{2}\right)=0, \text{ where } \lambda=\frac{b}{a}.$

This equation is of the form $L_1+\lambda L_2=0$, which represents a straight line through the intersection of the Line L_1 = 0 and L_2 = 0 i.e., $x-\frac{3}{4}=0$ and $y-\frac{1}{2}=0$.

On solving these two equations, we get the point $\left(\frac{3}{4},\frac{1}{2}\right)$ which is a fixed point.

12. The equation of any line of gradient 2 is

y = 2x + c ...(i)



The equations of given lines are

4x + y - 9 = 0 ...(ii)

x - 2y + 3 = 0 ...(iii)

5x - y - 6 = 0 ...(iv)

Solving (i) with (ii), (iii) and (iv) respectively, we obtain the coordinates of P, Q and R as

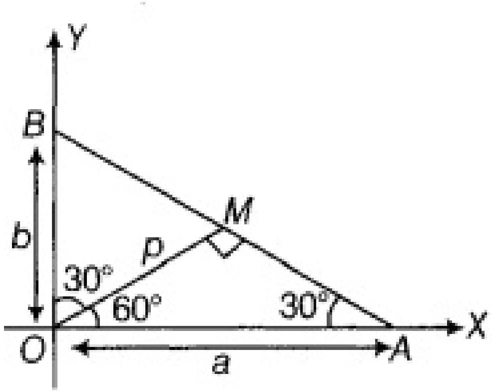
P $\left(\frac{3}{2}-\frac{c}{6},3+\frac{2c}{3}\right)$, Q $\left(1-\frac{2c}{3},2-\frac{c}{3}\right)$ and R $\left(2+\frac{c}{3},4+\frac{5c}{3}\right)$

Clearly, P is the mid-point of QR. Therefore PQ = PR.

Hence, lines (ii), (iii) and (iv) make equal intercepts on any line of gradient 2.

13. Since, OM is the perpendicular line on AB.

Here, $\angle MOB=30^{\circ},\angle MOA=60^{\circ}$



Let OM = p, OA = a, OB = b

In ΔOMA,

$\frac{p}{a} = \cos 60^\circ = \frac{1}{2} \Rightarrow a = 2p$

In ΔOMB,

$\frac{p}{b} = \cos 30^\circ = \frac{\sqrt{3}}{2} \Rightarrow b = \frac{2p}{\sqrt{3}}$

$\therefore$ Area of $\Delta OAB = \frac{1}{2} \times ab = 54\sqrt{3}$ [given]

$\Rightarrow \quad \frac{1}{2}(2p) \times \frac{(2p)}{\sqrt{3}} = 54\sqrt{3}$

$\Rightarrow \quad 4p^2 = 54\sqrt{3} \times 2\sqrt{3}$

$\Rightarrow p^2 = 81 \Rightarrow p = 9$ [$\because$ distance is always positive, so we take positive sign]

Using normal form of the equation of line AB is

x cosα + y sinα = p

$\therefore$ x cos 60^0 + y sin60^0 = 9

$\Rightarrow \quad x\left(\frac{1}{2}\right) + y\left(\frac{\sqrt{3}}{2}\right) = 9$

$\Rightarrow \quad x + \sqrt{3}y - 18 = 0$

14. The given line is 3x + 4y = 12 $\Rightarrow \frac{x}{4} + \frac{y}{3} = 1$ ...(i)

Let the line (i) cuts X and Y-axes at A and B, respectively.

Then, A = (4, 0) and B = (0, 3).

Let the line AB be trisected at P and Q, then

AP : PB = 1 : 2

$\therefore \quad P = \left(\frac{1 \cdot 0 + 2 \cdot 4}{1+2}, \frac{1 \cdot 3 + 2 \cdot 0}{1+2}\right)$

$\Rightarrow \quad P = \left(\frac{8}{3}, 1\right)$

and AQ : QB = 2:1

Also, $Q = \left(\frac{2 \cdot 0 + 1 \cdot 4}{1+2}, \frac{2 \cdot 3 + 1 \cdot 0}{2+1}\right) = \left(\frac{4}{3}, 2\right)$

Now, equation of line OP passing through (0, 0) and $\left(\frac{8}{3}, 1\right)$ is

$y - 0 = \frac{1-0}{\frac{8}{3}-0}(x - 0) \Rightarrow y = \frac{3}{8}x$

$\Rightarrow$ 3x - 8y = 0

And equation of the line OQ passing through (0,0) and $\left(\frac{4}{3}, 2\right)$ is

$y - 0 = \frac{2-0}{\frac{4}{3}-0}(x - 0) \Rightarrow$ 2y = 3x $\Rightarrow$ 3x - 2y = 0

15. Let ABCD be a rectangle whose two opposite vertices are A (1, 2) and C (5,5).

    Let the coordinates of the other two vertices B and D of rectangle ABCD be B (3, y_1) and D (3, y_2). Since diagonals, AC and BD bisect each other. Therefore, the mid-points of AC and BD are the same.

    $\therefore \frac{y_1+y_2}{2} = \frac{2+5}{2} \Rightarrow y_1 + y_2 = 7$ ...(i)

    Since ABCD is a rectangle.

    $\therefore$ AC = BD

    $\Rightarrow AC^2 = BD^2$

    $\Rightarrow (1 - 5)^2 + (2 - 5)^2 = (3 - 3)^2 + (y_1 - y_2)^2$

    $\Rightarrow 16 + 9 = (y_1 - y_2)^2$

    $\Rightarrow y_1 - y_2 + \pm 5$ ....(ii)

    Solving (i) and (ii), we get

    $y_1 = 6$ and $y_2 = 1$ or, $y_1 = 1$ and $y_2 = 6$

    Thus, the coordinates of B and D are B (3,1) and D (3, 6).

    The equation of side AB is

    y - 2 = $\frac{1-2}{3-1}$ (x - 1) or, y - 2 = -$\frac{1}{2}$ (x - 1) or, x + 2y - 5 = 0

    The equation of side BC is

    y - 1 = $\frac{5-1}{5-3}$ (x - 3) or, y - 1 = 2(x - 3) or, 2x - y - 5 = 0

    The equation of side CD is

    y - 5 = $\frac{6-5}{3-5}$ (x - 5) or, y - 5 = - $\frac{1}{2}$ (x - 5) or, x + 2y -15 = 0

    The equation of side AD is

    y - 2 = $\frac{6-2}{3-1}$ (x - 1) or, y - 2 = 2 (x - 1) or, 2x - y = 0