Ques (27) lecture 14

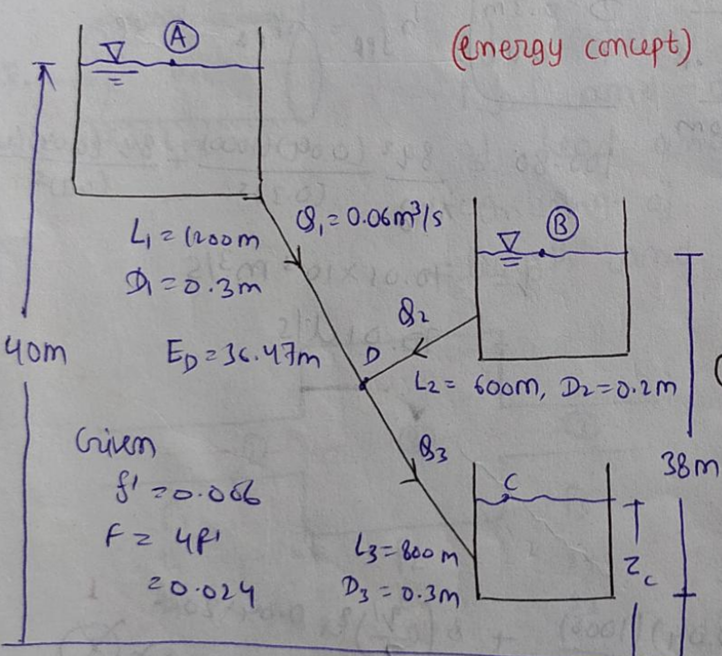
9/04/16

(energy concept)

Determine

$Q_2 = ??$

$Z_c = ??$



Given

$f' = 0.06$

$F = 4f'$

$= 0.024$

$$E_B = \frac{\cancel{P_B}}{\cancel{\rho g}} + \frac{\cancel{V_B^2}}{\cancel{2g}} + Z_B = 38$$

$$\boxed{E_B = 38m}$$

① Soln apply energy eqn b/w (A) and (D)

$E_A = E_D + h_{fAD}$

$$40 = E_D + \frac{8 Q_1^2}{\pi^2 g} \cdot \frac{f L_1}{D_1^5}$$

$$40 = E_D + \frac{8(0.06)^2}{\pi^2 g} \cdot \frac{(0.024)(1200)}{(0.3)^5}$$

$E_D = 36.47m$

{since $E_B > E_D$ so flow is from B to D}

① Apply energy eqn b/w (B) and (D)

$E_B = E_D + h_{fBD}$

$$38 = 36.47 + \frac{8 Q_2^2}{\pi^2 g} \cdot \frac{f \cdot L_2}{D_2^5}$$

$$= 36.47 + \frac{8 Q_2^2}{\pi^2 g} \frac{(0.024)(600)}{(0.2)^5}$$

$Q_2 = 20.28 \times 10^{-3} \ m^3/s$

$$Q_3 = Q_1 + Q_2$$
$$= 0.06 + 0.02028$$
$$= 0.08028\ m^3/s$$

(3) Apply energy eq^n b/w (D) and (C)

***
***

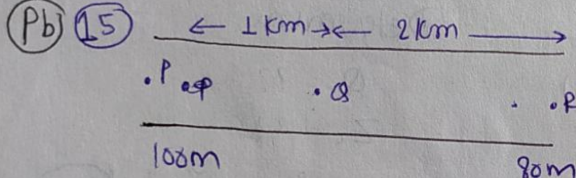
$$E_D = E_C + h_{fDC}$$

$$36.47 = Z_C + \frac{8Q_3^2 \cdot f L_3}{\pi^2 g \cdot D_3^5}$$

$$36.47 = Z_C + \frac{8(0.08028)^2}{\pi^2 g} \cdot \frac{(0.024)(800)}{(0.3)^5}$$

$$\boxed{Z_C = 32.26\ m}$$

(Pb) (15)



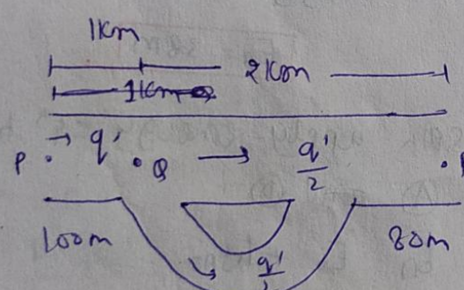
D = 0.3 m
f = 0.04

(q = same but hf different)

$$h_{fPR} = h_{fPQ} + h_{fQR}$$

$$100 - 80 = \frac{8q^2}{\pi^2 g} \frac{(0.04)(1000)}{(0.3)^5} + \frac{8q^2 (0.04)(2000)}{(0.3)^5}$$

$$q = 70.01 \times 10^{-3}\ m^3/s$$
$$= 70.01\ l/s$$



$$h_{fPR} = h_{fPQ} + h_{fQR}$$

$$100 - 80 = \frac{8q'^2}{\pi^2 g} \frac{(0.04)(1000)}{(0.3)^5} + \frac{8\left(\frac{q'}{2}\right)^2 \times 0.04 \times 2000}{\pi^2 g (0.3)^5} \times \cancel{2}$$

(h will be equal)

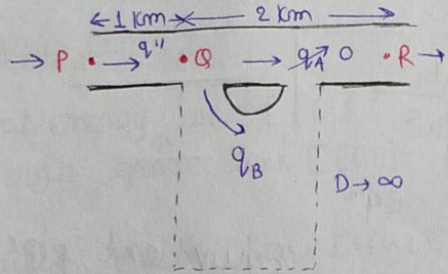
$$q' = 99.01 \times 10^{-3}\ m^3/s$$

$$\boxed{q' = 99.01\ l/s}$$

$$\%\ \text{increase} = \frac{99.01 - 70.01}{70.01} \times 100$$

$$= 41.42\ \text{Ans}$$

Pb - (16)



$h_{fA} = h_{fB}$

$$\frac{8 q_A^2}{\pi^2 g}\frac{f.L_A}{D_A^5} = \left[\frac{8 q_B^2 \, f.L_B}{\pi^2 g \, D_B^5 \to \infty}\right] \to \infty$$

Non zero

$\left(\frac{Q_2 - Q_1}{Q_1}\right) \times 100$

$h_{fPR} = h_{fPQ} + \cancel{h_{fQR}} \to 0$

$$100 - 80 = \frac{8 q'^2}{\pi^2 g}\frac{(0.04)(1000)}{(0.3)^5}$$

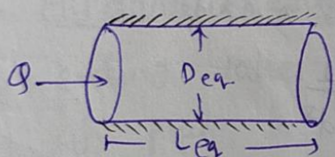
$q' = 121.26 \times 10^{-3}\ m^3/s$

$= 121.26\ l/s$

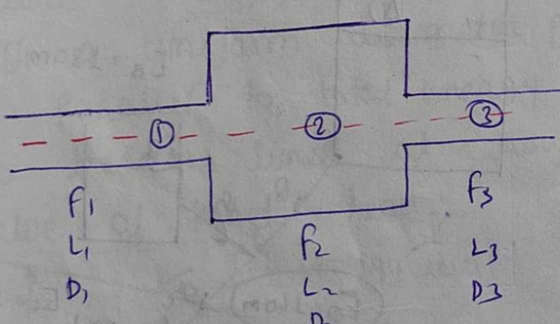
% increase $= \frac{121.26 - 70.01}{70.01} \times 100$

$= 73.20$

① Equivalent pipe →



★★★★ (G+E) It is a pipe of uniform diameter of certain length having the loss of head and discharge equal to the loss of head and discharge of compound pipe consisting of several pipe of different length and diameters

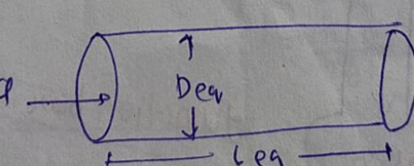


$h_f = h_{f_1} + h_{f_2} + h_{f_3} + \cdots$

$$= \frac{8Q^2}{\pi^2 g}\left[\frac{f_1 . L_1}{D_1^5} + \frac{f_2 L_2}{D_2^5} + \cdots\right] \text{ -- ①}$$

equivalent pipe

$$h_f = \frac{8Q^2}{\pi^2 g} . \frac{f L_{eq}}{D_{eq}^5} \text{ -- ②}$$



By eq ① and eq②

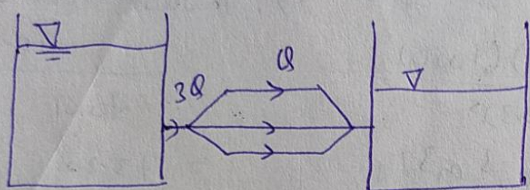
$$\frac{8Q^2}{\pi^2 g} \times \frac{f L_{eq}}{D_{eq}^5} = \frac{8Q^2}{\pi^2 g}\left[\frac{f_1 L_1}{D_1^5} + \frac{f_2 L_2}{D_2^5} + \cdots\right]$$

If $f_1 = f_2 = f_3 = \cdots$

$$\boxed{\frac{L_{eq}}{D_{eq}^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \cdots}$$

Dupuit's eqⁿ

Pb-⑦ Parallel connection



Dia = d

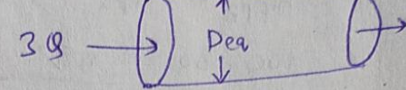
length = L

$$h_f = \frac{8Q^2 \cdot fL}{\pi^2 g\, d^5} \times \cancel{8} \quad \text{---} ①$$

2009 ESE

$$\boxed{D_{eq} = (3)^{\frac{2}{5}} d}$$

Equivalent pipe



$$h_f = \frac{8(3Q)^2 \cdot f \cdot L}{\pi^2 g\, D_{eq}^5} = ②$$

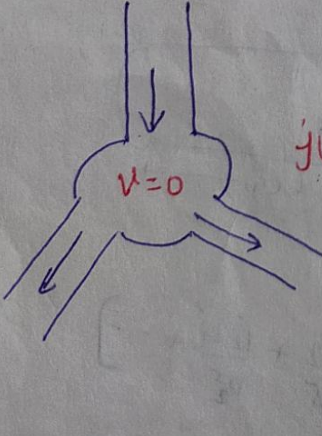
By eq ① and ②

$$\frac{\cancel{8}(3Q)^2 \cancel{fL}}{\cancel{\pi^2 g} D_{eq}^5} = \frac{\cancel{8}Q^2 \times \cancel{fL}}{\cancel{\pi^2 g} \times d^5}$$

$$\frac{3^2}{D_{eq}^5} = \frac{1}{d^5}$$

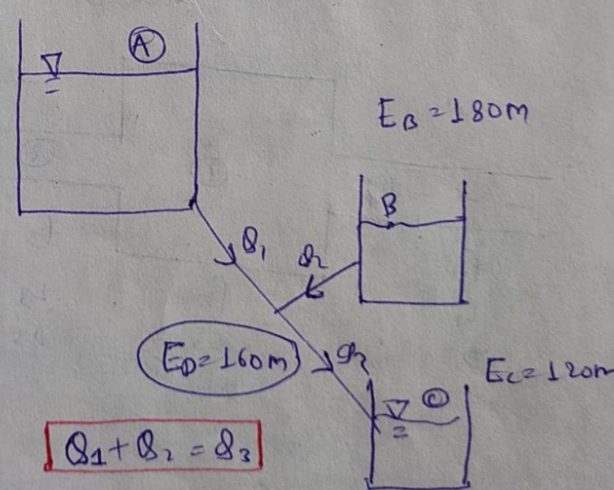
Theory of junction:

velocity of flow at the junⁿ is (0) the flow will be only due to high pressure therefore the total head is piezo-metric head



junction box

Pb-⑬



$$\boxed{Q_1 + Q_2 = Q_3}$$

Energy lines

→ total energy lines [TEL] (or) Energy grade line [EGL] $\left[\frac{p}{\rho g} + \frac{V^2}{2g} + z\right]$

→ Hydraulic grade line (HGL) $\left[\frac{p}{\rho g} + z\right]$

[total EL] or [EGL] ⇒ It is the line joining the points representing the value of total head at the various cross-section of the pipe in a pipe flow

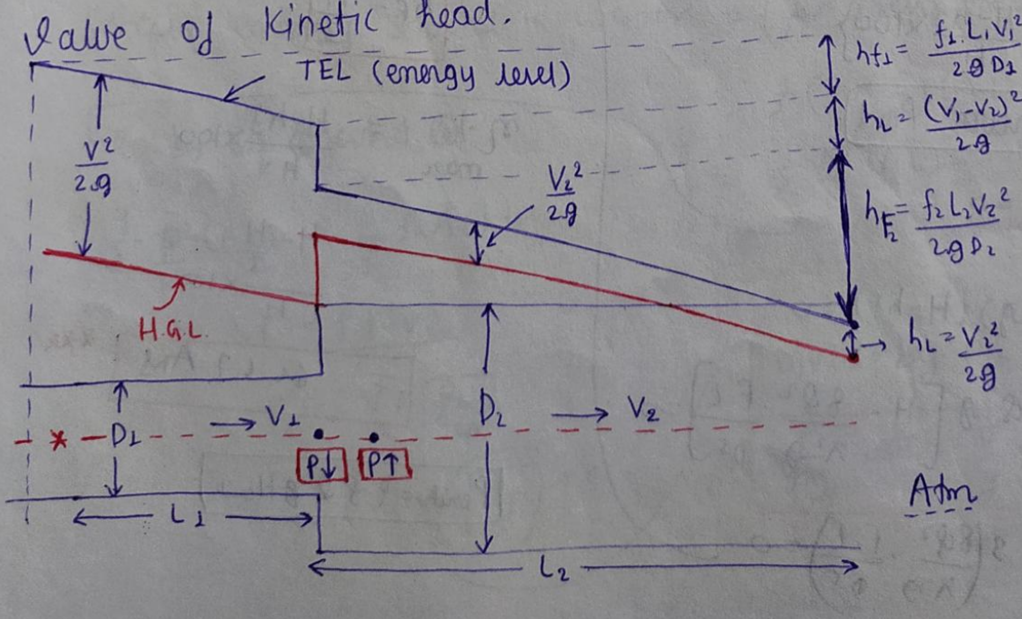
② this line always goes down in the dirⁿ of flow untill or unless some energy supplies externally

Hydraulic grade line [H.G.L] line joining the points representing values of piezometric head at various cross-section of pipe in the pipe flow

② this line may go up or down in the fluid flow

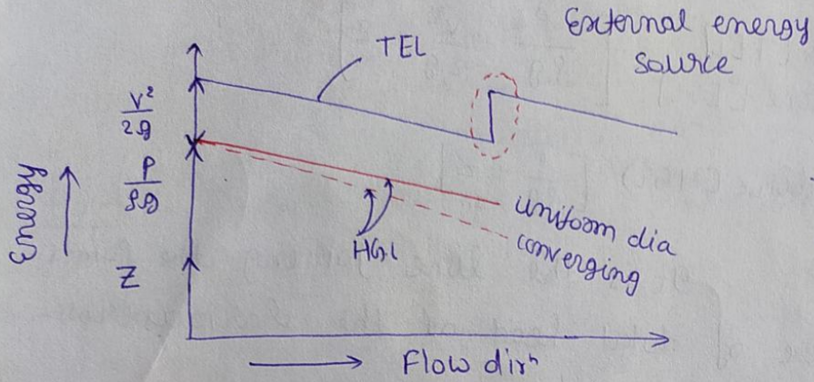
③ this line will always lies below to total energy line in the pipe flow.

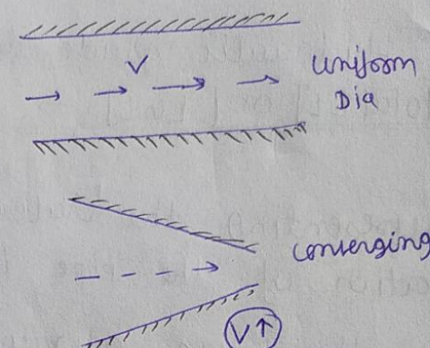
④ In a uniform diameter pipe, this line will always be parallel to total energy line and the vertical gap b/w these 2 lines at any section of pipe represent the value of kinetic head.



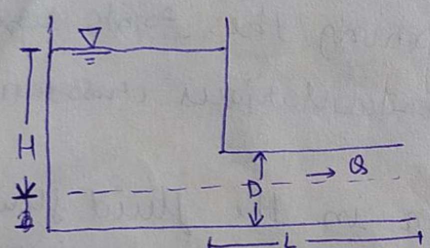
[Vertical gap b/w HGL and TEL is Kinetic head]

In general:





Power transmission through pipe



Energy $= m.gH$ (Joule)

Power $= \dot{m}.gH$ (Watt)

$= \rho Q.g.H$

$P_{in} = \dot{m}.g.H$

$P_{out} = \dot{m}g(H-h_f)$

(Neglect minor losses)

$$\boxed{h_f = \frac{8Q^2}{\pi^2 g}.\frac{fL}{D^5}}$$

$$\eta\% = \frac{P_{out}}{P_{in}}\times 100$$

$$= \frac{\dot{m}g(H-h_f)}{\dot{m}g(H)}\times 100$$

$$= \left\{\frac{H-h_f}{H}\times 100\right\}$$

For maximum P_{out}

$$\frac{dP_{out}}{dQ} = 0$$

$$\frac{d}{dQ}(\dot{m}g)(H-h_f) = 0$$

$$\frac{d}{dQ}\rho.Q.g\left[H - \frac{8Q^2}{\pi^2 g}.\frac{f.L}{D^5}\right] = 0$$

$$H - 3\left(\frac{8Q^2}{\pi^2 g}.\frac{f.L}{D^5}\right) = 0$$

$$H - 3h_f = 0$$

$$\boxed{h_f = \frac{H}{3}}$$ ***

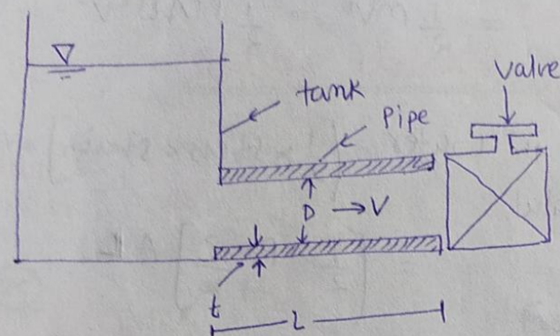
$$\eta\%_{max} = \frac{H-h_f}{H}\times 100$$

$$= \frac{H-\frac{H}{3}}{H}\times 100$$

$$\boxed{= 66.67 \text{ Ans}}$$ ***

$$\boxed{P_{exit} = \rho Q.g H_{exit}}$$

Water Hammer effect:



C/S Area of pipe → A

thickness of pipe material $= (t)$

time taken by the Pressure waves to travel from valve to tank and tank to valve

$$= \frac{L+L}{c} = \frac{2L}{c}$$

$$\boxed{T = \frac{2L}{c}}$$

C → velocity of pressure wave

T → time taken to close the valve

TYPE OF CLOSURE

* Gradual closure $T > \frac{2L}{c}$
* Sudden closure $T < \frac{2L}{c}$

① Gradual closure :-

Newton 2nd law

$$F = ma$$

$$PA = \frac{m(V-0)}{T}$$

$$P\cancel{A} = \frac{\rho \cancel{A}[A.L].V}{T}$$

$$P = \rho . V . \frac{L}{T}$$

divide by ρg

$$\boxed{\frac{P}{\rho g} = \frac{V}{g} . \frac{L}{T}}$$

[No compressible effect]

② Sudden closure :- ① Rigid pipe ② elastic pipe

① Rigid Pipe

K.E. of water (Before closure of valve) $= \frac{1}{2} mV^2 = \frac{1}{2} \rho (A.L). V^2$ --- ①

Strain energy stored in water $= \left(\frac{1}{2} \times \text{Stress} \times \text{Strain}\right) \times \text{volume}$

$$= \frac{1}{2}\left(P \times \frac{P}{K}\right)(A.L)$$

$$= \frac{1}{2} \frac{P^2}{K} (A.L) \text{ --- ②}$$

by eq ① and ②

$$\frac{1}{2} \frac{P^2}{k} (A.L) = \frac{1}{2} \rho (A.L). V^2$$

$$P^2 = \rho . K . V^2$$

$$P = V . \sqrt{\frac{\rho^2 . K}{\rho}}$$

$$P = \rho . V \sqrt{\frac{K}{\rho}} \rightarrow C$$

Divide by ρg

$$\boxed{\frac{P}{\rho g} \cancel{\quad} = \frac{V}{g} . \sqrt{\frac{K}{\rho}}}$$

1st iteration loop BACB :-

S.NO.	Pipe	r	Q_0	rQ_0^2	$\lvert 2rQ_0 \rvert$
1	BA	4	15	-900	120
2	AC	2	45	$+4050$	180
3	CB	1	25	$+625$	50

$\Sigma rQ_0^2 = 3775$, $\Sigma \lvert 2rQ_0 \rvert = 350$

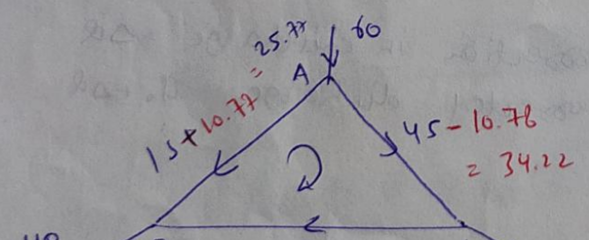
2nd iteration loop BACB :-

SNO	Pipe	r	Q_0	rQ_0^2	$\lvert 2rQ_0 \rvert$
1	BA	4	25.78	-2658.43	206.43
2	AC	2	34.22	$+2342.02$	136.88
3	CB	1	14.22	$+202.2$	28.44

$\Sigma rQ_0^2 = 114.2$ $\lvert 2rQ_0 \rvert = 371.56$

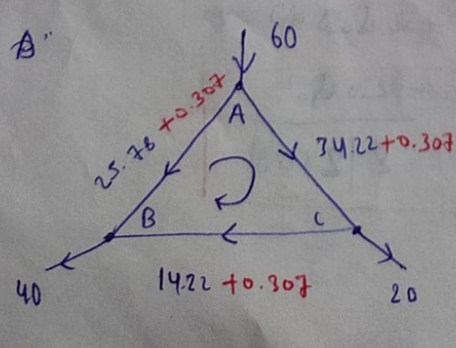
$$\Delta Q = \frac{\Sigma rQ_0^2}{\Sigma \lvert 2rQ_0 \rvert} = \frac{3775}{350} = 10.78$$

[means 10.78 discharge in anticlockwise dirn]

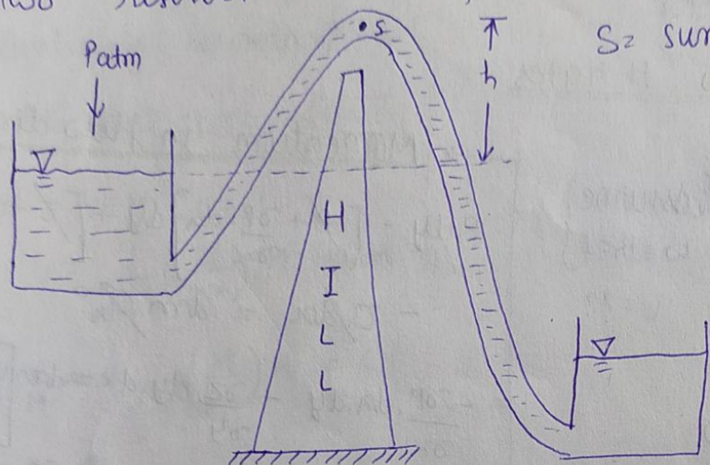


$$\Delta Q = -\frac{\Sigma rQ_0^2}{\Sigma \lvert 2rQ_0 \rvert} = -\frac{(-114.2)}{371.56} = 0.307$$

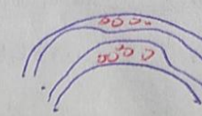
means 0.307 discharge in clockwise dirn



Flow through syphon = it is long bend pipe which is used to transfer liquid from a resorvoir at a higher elevation to another reservoir at a lower level. when a two reservoir are separated by a hill



S = summit



$$\frac{P_{atm}}{\rho g} = 10.3\text{ m of } H_2O \text{ column}$$

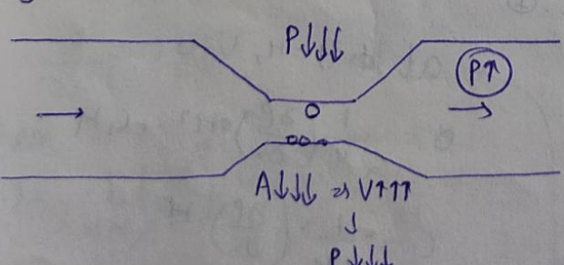
$P_{v(sep)}$ (vapour pressure head separation)

$$\frac{P_{v(sep)}}{\rho g} = 2.5 - 2.7\text{ m of } H_2O \text{ column}$$

(the Point at the height of the syphon is called summit)

Cavitation: In a fluid flow system if any where press. of fluid falls below the vapour pressure, the dissolve gases inside the liquid will start coming out in the form of bubble when these bubble are carried out by liquid in a different vapour zone where the pressure is above the vapour pressure the bubble start to burst and form cavity on the surface is so known as Cavitation

In general,

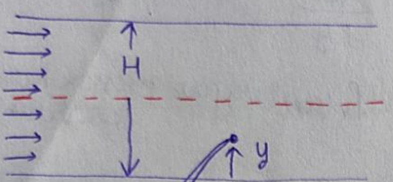


*** (Cavitation may (Possibility) be form in venturimeter) due to very very low Pressure)

Laminar FLOW

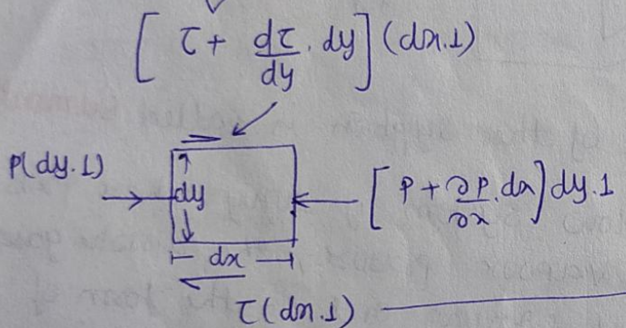
- laminar Flow b/w || Plate
- laminar Flow through pipe
- correction Factor.

→ Laminar Flow b/w || Plates.



{Assume width=1}

U=??



→ Momentum in flow dirn

$$P.\cancel{dy}-\left[\cancel{P}+\frac{\partial P}{\partial x}.dx\right]dy+\left[\cancel{\tau}+\frac{\partial\tau}{\partial y}.dy\right]dx$$
$$-\tau/dx = dm\,\cancel{a_x}^0$$

$$= -\frac{\partial P}{\partial x}.dx.dy+\frac{\partial\tau}{\partial y}.dy.dx = dm\left[\cancel{u\frac{\partial u}{\partial x}}+\cancel{v\frac{\partial u}{\partial y}}+\cancel{w\frac{\partial u}{\partial z}}+\cancel{\frac{\partial u}{\partial t}}\right]$$

★★★

$$\boxed{\frac{\partial\tau}{\partial y}=\frac{\partial P}{\partial x}}$$ IES Objective

Assumption: For Newtonian fluid

$$\tau=\mu\frac{du}{dy}$$

$$\mu\frac{\partial^2 U}{\partial y^2}=\frac{\partial P}{\partial x}$$
$$\frac{\partial^2 U}{\partial y^2}=\frac{1}{\mu}\left(\frac{\partial P}{\partial x}\right)$$

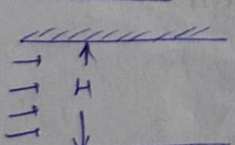
Int. it

$$\frac{\partial U}{\partial y}=\frac{1}{\mu}\left(\frac{\partial P}{\partial x}\right).y+C_1$$

Int. it

$$U=\frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right).y^2+C_1y+C_2 \quad \text{--- (1)}$$

$\boxed{\text{Case (1)}}$ Fixed Plates



Applying boundary cond

At $y=0$, $U=0$, $\boxed{C_2=0}$

at $y=H$, $U=0$

$$0=\frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right).H^2+C_1H$$

$$C_1=-\frac{1}{2\mu}\times\left(\frac{\partial P}{\partial x}\right).H$$

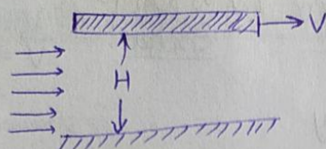
By eq (1)

$$U=\frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right).y^2-\frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right).H.y$$

Imp ★★★

$$\boxed{U=-\frac{1}{2\mu}\times\left(\frac{\partial P}{\partial x}\right)(Hy-y^2)}$$

Case - ②



by eqⁿ ①

Applying boundary condᵈ

At $y=0,\ U=0,\ \boxed{C_2=0}$

At $y=H,\ U=V$

$$V = \frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right).H^2 + C_1 H$$

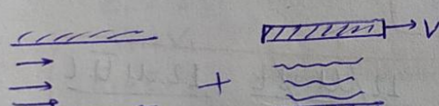
$$\frac{V}{H} - \frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right).H = C_1$$

By eq. ①

$$U = \frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right).y^2 + \left[\frac{V}{H} - \frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right).H\right].y$$

$$\boxed{U = -\frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right)(Hy-y^2) + \frac{V}{H}.y}$$

In general,



★★★★

Simple or Plain Couette Flow

$$\frac{\partial P}{\partial x} = 0$$

then,

$$U = -\frac{1}{2\mu}\cancel{\left(\frac{\partial P}{\partial x}\right)}^{0}(Hy-y^2) + \frac{V}{H}.y$$

$$\boxed{U = \frac{V}{H}.y}$$

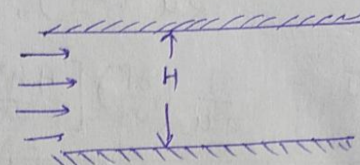
Shear distribution

$$\tau = \mu.\frac{du}{dy}$$

$$= \mu.\frac{d}{dy}\left(\frac{V}{H}.y\right) = \mu.\frac{V}{H} \qquad \boxed{\tau = \frac{V}{H}\mu}$$

Case - ① Fixed Plates

maximum velocity (U_{max})



$$U = -\frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right)(Hy-y^2)$$

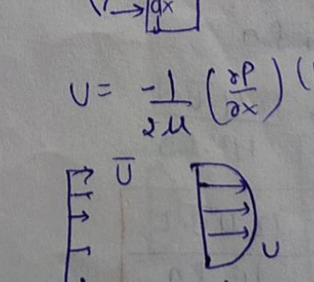
At $y = \frac{H}{2}$, $U = U_{max}$

$$U_{max} = -\frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right)\left[H.\frac{H}{2} - \frac{H^2}{4}\right]$$

$$\boxed{U_{max} = -\frac{1}{8\mu}\left(\frac{\partial P}{\partial x}\right).H^2}$$

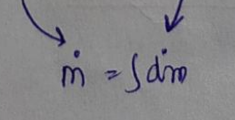
Mean velocity ($\bar{V}$ or V)

Constant velocity at any section show that the mass flow rate remain same as in original



Assume (width=1)

$$U = -\frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right)(Hy-y^2)$$



$$\dot{m} = \int d\dot{m}$$

$$\rho (H.1)\bar{U} = \int_0^H \rho (dy.1) U$$

(max no of question)

Extra: $\vec{V} = \frac{Q}{(H.1)}$

$$H.\bar{U} = \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)\int_0^H (Hy-y^2).dy$$

$$\frac{P_1-P_2}{\rho g} = \frac{12\mu\left(\frac{Q}{H}\right).L}{\rho g H^2}$$

$$H.\bar{U} = -\frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)\left[\frac{H.H^2}{2} - \frac{H^3}{3}\right]$$

***

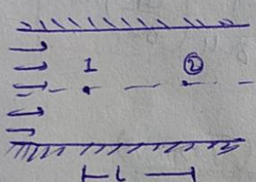
$$\boxed{\bar{U} = -\frac{1}{12\mu}\left(\frac{\partial p}{\partial x}\right)H^2}$$

$$\boxed{\frac{P_1-P_2}{\rho g} = \frac{12.\mu.Q.L}{\rho g.H^3}}$$

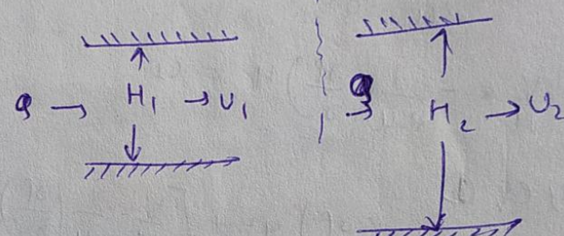
Relationship →

$U_{max} = 1.5\,\bar{U}$

Head loss



In general,



$$\bar{U} = -\frac{1}{12\mu}\left(\frac{\partial p}{\partial x}\right)H^2$$

$$\frac{12\mu.\bar{U}}{H^2}.\partial x = -\partial p$$

Int it.

$$-\int_1^2 \partial p = \frac{12\mu.\bar{U}}{H^2}\int_0^L \partial x$$

$$P_1-P_2 = \frac{12.\mu.\bar{U}.L}{H^2}$$

Divide by ρg

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = \frac{12\mu.\bar{U}.L}{H^2.\rho g}$$

$$\boxed{\frac{P_1-P_2}{\rho g} = \frac{12\mu\bar{U}.L}{H^2.\rho g}}$$

$$\boxed{U = -\frac{1}{2\mu}\frac{\partial p}{\partial x}(Hy-y^2)}$$

$$\boxed{U_{max} = 1.5\,\bar{U}}$$

$$\boxed{\frac{P_1-P_2}{\rho g} = \frac{12\mu.Q.L}{\rho g H^3} \quad \frac{12.\mu\bar{U}.L}{\rho g H^2}}$$

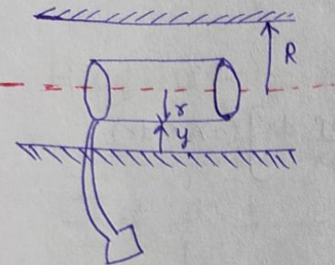
Shear distribution.

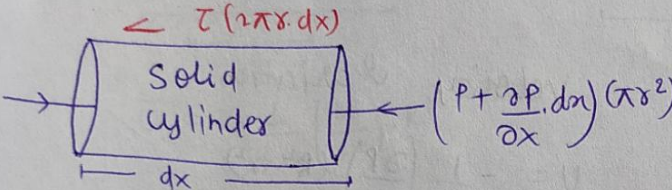
$$\tau = \mu.\frac{du}{dy}$$

$$\tau = \mu.\frac{d}{dy}\left(-\frac{1}{2\mu}.\frac{\partial p}{\partial x}.(Hy-y^2)\right)$$

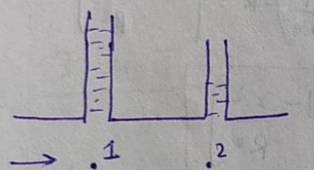
$$\boxed{\tau = -\frac{1}{2}\frac{\partial p}{\partial x}.[H-2y]}$$

laminar Flow through pipe:





In general



$$\boxed{\frac{\partial P}{\partial x} = (-ve)}$$

$Z_1 = Z_2$

$V_1 = V_2$

$E_1 > E_2$

For Newtonian fluid

$$\mu \frac{du}{dy} = -\frac{r}{2}\left(\frac{\partial P}{\partial x}\right) \quad \left\{\tau = \mu . \frac{du}{dy}\right\}$$

$$-\mu . \frac{du}{dr} = -\frac{r}{2}\left(\frac{\partial P}{\partial x}\right) \quad \{ dy = -dr \}$$

$$\frac{du}{dr} = \frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right) . r$$

Int. w

$$\boxed{u = \frac{1}{4\mu}\left(\frac{\partial P}{\partial x}\right) r^2 + C} \quad \text{---} ①$$

Momentum eqn in flow dirn

$$\vec{F} = m\cancel{a}^{\,0}$$

$$P(\cancel{\pi r^2}) - \left[\cancel{P} + \frac{\partial P}{\partial x}.dx\right](\pi r^2) - \tau(2\pi r dx) = 0$$

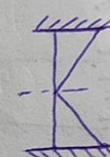
$$-\frac{\partial P}{\partial x}.\cancel{dx}(\cancel{\pi} r \cancel{r}) - \tau(2\cancel{\pi}\cancel{r}\cancel{dx}) = 0$$

$$-\frac{\partial P}{\partial x}.r - \tau(2) = 0$$

$$2\tau = -\frac{\partial P}{\partial x}.r$$

★★★★★

$$\boxed{\tau = -\frac{r}{2}\frac{\partial P}{\partial x}}$$



At $r = R$, $u = 0$

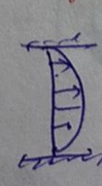
$$C = \frac{1}{4\mu}\left(\frac{\partial P}{\partial x}\right).R^2$$

By eq ①

$$\cancel{u = \frac{1}{4\mu}\left(\frac{\partial P}{\partial x}\right)(R^2 - r^2)}$$

$$u = \frac{1}{4\mu}\left(\frac{\partial P}{\partial x}\right)r^2 - \frac{1}{4\mu}\left(\frac{\partial P}{\partial x}\right)^2 . R^2$$

$$\underbrace{u = -\frac{1}{4\mu}\underbrace{\left(\frac{\partial P}{\partial x}\right)}_{(-ve)}\underbrace{(R^2 - r^2)}_{(+ve)}}_{\text{Parabolic}}$$



$y + r = R$

diff w

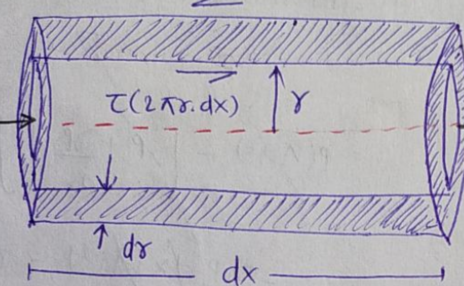
$dy + dr = 0$

$dy = -dr$

In general

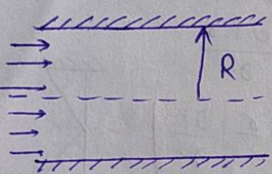
$$\left(\tau + \frac{\partial \tau}{\partial r}.dr\right)(2\pi(r+dr)dx]$$

$P(2\pi r.dr) \rightarrow$



$\leftarrow \left[P + \frac{\partial P}{\partial x}dx\right](2\pi r.dr)$

- Maximum velocity (U_{max})



$$U = -\frac{1}{4\mu}\left(\frac{\partial P}{\partial x}\right)(R^2 - r^2)$$

At, $r = 0$, $U = U_{max}$

$$\boxed{U_{max} = -\frac{1}{4\mu}\left(\frac{\partial P}{\partial x}\right)(R^2)}$$

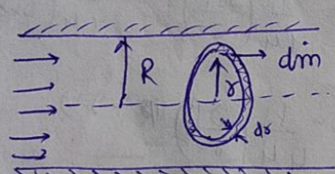
velocity variation

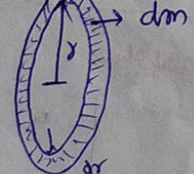
$$U = -\frac{1}{4\mu}\left(\frac{\partial P}{\partial x}\right)(R^2 - r^2)$$

$$U = -\frac{1}{4\mu}\left(\frac{\partial P}{\partial x}\right).R^2\left[1 - \frac{r^2}{R^2}\right]$$

$$\boxed{U = U_{max}\left[1 - \frac{r^2}{R^2}\right]}$$

Mean velocity - ($\bar{U}$ or V)





$$\dot{m} = \int d\dot{m}$$

$$\rho(\pi R^2).\bar{U} = \int_0^R \rho(2\pi.dr)\,U$$

$$U = U_{max}\left(1 - \frac{r^2}{R^2}\right)$$

$$R^2\bar{U} = 2\int_0^R U_{max}\left(1 - \frac{r^2}{R^2}\right)r.dr$$

$$R^2.\bar{U} = 2U_{max}\int_0^R\left[r - \frac{r^3}{R^2}\right]dr$$

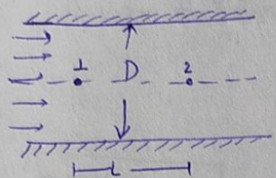
$$R^2.\bar{U} = 2U_{max}\left(\frac{R^2}{2} - \frac{R^4}{4R^2}\right)$$

$$\cancel{R^2}.\bar{U} = 2U_{max}\frac{\cancel{R^2}}{4}$$

$$\boxed{U_{max} = 2\bar{U}}$$

$$\boxed{\bar{U} = \frac{-1}{8\mu}\left(\frac{\partial P}{\partial x}\right)R^2}$$

Head loss →



$$\bar{U} = -\frac{1}{8\mu}\left(\frac{\partial P}{\partial x}\right).R^2$$

$$\bar{U} = -\frac{1}{8\mu}\left(\frac{\partial P}{\partial x}\right)\frac{.D^2}{4}$$

$$\frac{32\mu.\bar{U}.\partial x}{D^2} = -\partial P$$

Int. it

$$-\int_1^2 \partial P = \frac{32\mu.\bar{U}}{D^2}\int_0^L dx$$

$$P_1 - P_2 = \frac{32\mu.\bar{U}.L}{D^2}$$

Divide by ρg

$$\frac{P_1 - P_2}{\rho g} = \frac{32\mu.\bar{U}.L}{\rho.g.D^2}$$

(Hagen Poiseuille eqn)

$$\bar{U} = \frac{Q}{\frac{\pi}{4}D^2}$$

$$\frac{P_1 - P_2}{\rho g} = \frac{32\mu\left(\frac{Q}{\frac{\pi}{4}.D^2}\right).L}{\rho.g.D^2}$$

$$\boxed{\frac{P_1 - P_2}{\rho g} = \frac{128.\mu}{\pi}\frac{Q.L}{\rho g.D^4}}$$

Comparison of Result

$$h_f = \frac{32.\mu.V.L}{\rho.g\,D^2} \quad (\vec{U} \text{ or } V)$$

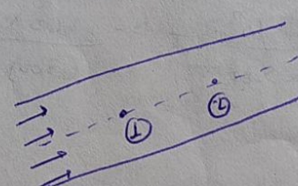
$$\frac{f.\cancel{L}.V^{\cancel{2}}}{2\cancel{g}D} = \frac{32\mu.\cancel{V}.\cancel{L}}{\rho.\cancel{g}.D^2}$$

$$\frac{fV}{2} = \frac{32.\mu}{\rho.D}$$

$$f = \frac{64.\mu}{\rho.V.D}$$

$$\boxed{f = \frac{64}{Re}}$$

Inclined
(laminar flow through pipe)



Apply energy eqn b/w 1 and 2

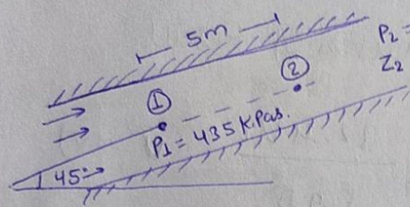
$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + \frac{f.L.V^2}{2gD}$$

$$\left(f = \frac{64}{Re}\right)$$

$$\boxed{\left(\frac{P_1}{\rho g} + Z_1\right) - \left(\frac{P_2}{\rho g} + Z_2\right) = \frac{32.\mu V.L}{\rho.g.D^2}}$$

$$\boxed{\left(\frac{P_1}{\rho g} + Z_1\right) - \left(\frac{P_2}{\rho g} + Z_2\right) = \frac{128}{\pi}\frac{\mu.Q.L}{\rho g\,D^4}}$$

Pb-(11) laminar flow through pipe



$P_2 = 200$ KPascal

$Z_2 = 5\sin 45$

$Q = ?$

$D = 0.1m$

$\rho_{oil} = 800 kg/m^3$

$\mu_{oil} = \frac{0.8 kg}{m\text{-}s}$

① $\frac{P_1 - P_2}{\rho g} = \frac{128}{\pi} \cdot \frac{\mu \cdot Q \cdot L}{\rho \cdot g D^4}$

$\boxed{Q = 0.144 m^3/s}$

$\frac{(435-200)\times 10^3}{800\times 9.81} = \frac{128}{\pi}\left(\frac{0.8\times Q.5}{800\times 9.81\times(0.1)^4}\right)$

② $\left[\frac{P_1}{\rho g} + \cancel{Z_1}^{0}\right] - \left[\frac{P_2}{\rho g} + Z_2\right] = \frac{128}{\pi} \cdot \frac{\mu \cdot Q \cdot L}{\rho \cdot g \cdot D^4}$

$\frac{435\times 10^3}{800\times 9.81} + 0 - \frac{200\times 10^3}{800\times 9.81} + 5\sin 45 = \frac{128}{\pi}\times\frac{0.8\times(Q.)5}{800\times(9.81)(0.1)^4}$

$Q = 0.161 m^3/s$

③ $\frac{435\times 10^3}{800\times 9.81} + 0 - \frac{200\times 10^3}{(800)(9.81)} - 5\sin 45 = \frac{128}{\pi}\frac{(0.8)Q.5}{800\times 9.81\times(0.1)^4}$

$\boxed{Q = 0.127 m^3/s}$