Limits

Exercise 27A

Q. 1. Evaluate

$$\lim_{x \to 2} (5 - x)$$

Answer : To evaluate:

$$\lim_{x \to 2} (5 - x)$$

Formula used:

We have,

$$\lim_{x \to a} f(x) = f(a)$$

As $x \to 2$, we have

$$\lim_{x \to 2} (5 - x) = 5 - 2$$

$$\lim_{x \to 2} (5 - x) = 3$$

Thus, the value of $\lim_{x \to 2} (5 - x)$ is 3.

Q. 2. Evaluate

$$\lim_{x \to 1} (6x^2 - 4x + 3)$$

Answer : To evaluate:

$$\lim_{x \to 1} (x^2 - 4x + 3)$$

Formula used: We have,

$$\lim_{x \to a} f(x) = f(a)$$

As $x \to 1$, we have

$$\lim_{x \to 1}(x^2 - 4x + 3) = 1^2 - 4(1) + 3$$

$$\lim_{x \to 1}(x^2 - 4x + 3) = 0$$

Thus, the value of $\lim_{x \to 1}(x^2 - 4x + 3)$ is 0.

Q. 3. Evaluate

$$\lim_{x \to 3}\left(\frac{x^2 + 9}{x + 3}\right)$$

Answer : To evaluate:

$$\lim_{x \to 3}\frac{x^2 + 9}{x + 3}$$

Formula used:

We have,

$$\lim_{x \to a} f(x) = f(a)$$

As $x \to 3$, we have

$$\lim_{x \to 3}\frac{x^2 + 9}{x + 3} = \frac{3^2 + 9}{3 + 3}$$

$$\lim_{x \to 3}\frac{x^2 + 9}{x + 3} = \frac{18}{6}$$

$$\lim_{x\to 3}\frac{x^2+9}{x+3}=3$$

Thus, the value of $\lim_{x\to 3}\frac{x^2+9}{x+3}$ is 3.

Q. 4. Evaluate

$$\lim_{x\to 3}\left(\frac{x^2-4x}{x-2}\right)$$

Answer : To evaluate:

$$\lim_{x\to 3}\frac{x^2-4x}{x-2}$$

Formula used: We have,

$$\lim_{x\to a} f(x) = f(a)$$

As $x \to 3$, we have

$$\lim_{x\to 3}\frac{x^2-4x}{x-2}=\frac{3^2-4(3)}{3-2}$$

$$\lim_{x\to 3}\frac{x^2-4x}{x-2}=\frac{3^2-4(3)}{3-2}$$

$$\lim_{x\to 3}\frac{x^2-4x}{x-2}=-3$$

Thus, the value of $\lim_{x\to 3}\frac{x^2-4x}{x-2}$ is -3.

Q. 5. Evaluate

$$\lim_{x\to 5}\left(\frac{x^2-25}{x-5}\right)$$

Answer : To evaluate:

$$\lim_{x\to 5}\frac{x^2-25}{x-5}$$

Formula used:

We have,

$$\lim_{x\to a} f(x) = f(a)$$

As $x \to 5$, we have

$$\lim_{x\to 5}\frac{x^2-25}{x-5} = \lim_{x\to 5}\frac{(x+5)(x-5)}{x-5}$$

$$\lim_{x\to 5}\frac{x^2-25}{x-5} = x+5$$

$$\lim_{x\to 5}\frac{x^2-25}{x-5} = 5+5$$

$$\lim_{x\to 5}\frac{x^2-25}{x-5} = 10$$

Thus, the value of $\lim_{x\to 5}\frac{x^2-25}{x-5}$ is -10.

Q. 6. Evaluate

$$\lim_{x\to 1}\left(\frac{x^3-1}{x-1}\right)$$

Answer : To evaluate:

$$\lim_{x\to 1}\frac{x^3-1}{x-1}$$

Formula used: We have,

$\lim_{x \to a} f(x) = f(a)$ and

$x^3 - y^3 = (x-y)(x^2+xy+y^2)$

As $x \to 1$, we have

$$\lim_{x\to 1}\frac{x^3-1}{x-1} = \lim_{x\to 1}\frac{(x-1)(x^2+x+1)}{x-1}$$

$$\lim_{x\to 1}\frac{x^3-1}{x-1} = \lim_{x\to 1}(x^2+x+1)$$

$$\lim_{x\to 1}\frac{x^3-1}{x-1} = 1+1+1$$

$$\lim_{x\to 1}\frac{x^3-1}{x-1} = 3$$

Thus, the value of $\lim_{x\to 1}\frac{x^3-1}{x-1}$ is 3.

Q. 7. Evaluate

$$\lim_{x\to -2}\left(\frac{x^3+8}{x+2}\right)$$

Answer :

To evaluate: $\lim_{x \to -2} \frac{x^3+8}{x+2}$

Formula used:

We have,

$\lim_{x \to a} f(x) = f(a)$ and

$x^3+y^3=(x+y)(x^2-xy+y^2)$

As $x \to -2$, we have

$$\lim_{x \to -2} \frac{x^3+8}{x+2} = \lim_{x \to -2} \frac{(x+2)(x^2-2x+4)}{x+2}$$

$$\lim_{x \to -2} \frac{x^3+8}{x+2} = \lim_{x \to -2} (x^2-2x+4)$$

$\lim_{x \to -2} \frac{x^3+8}{x+2} =4-4+4$

$\lim_{x \to -2} \frac{x^3+8}{x+2} =4$

Thus, the value of $\lim_{x \to -2} \frac{x^3+8}{x+2}$ is 4

Q. 8. Evaluate $\lim_{x \to 3}\left(\frac{x^4 - 81}{x - 3}\right)$

Answer :

Answer :

To evaluate: $\lim_{x \to 3} \frac{x^4 - 81}{x - 3}$

Formula used:

We have,

$\lim_{x \to a} f(x) = f(a)$ and

As $x \to 3$, we have

$$\lim_{x \to 3} \frac{x^4 - 81}{x - 3} = \lim_{x \to 3} \frac{(x^2 + 9)(x^2 - 9)}{x - 3}$$

$$\lim_{x \to 3} \frac{x^4 - 81}{x - 3} = \lim_{x \to 3} \frac{(x^2 + 9)(x + 3)(x - 3)}{x - 3}$$

$$\lim_{x \to 3} \frac{x^4 - 81}{x - 3} = \lim_{x \to 3}(x^2 + 9)(x + 3)$$

$$\lim_{x \to 3} \frac{x^4 - 81}{x - 3} = (9 + 9)(3 + 3)$$

$$\lim_{x\to 3}\frac{x^4-81}{x-3}=486$$

Thus, the value of $\lim_{x\to 3}\frac{x^4-81}{x-3}$ is 486.

Q. 9. Evaluate

$$\lim_{x\to 3}\left(\frac{x^2-4x+3}{x^2-2x-3}\right)$$

Answer : To evaluate:

$$\lim_{x\to 3}\frac{x^2-4x+3}{x^2-2x-3}$$

Formula used:

We have,

$\lim_{x\to a} f(x) = f(a)$ and

As $x \to 3$, we have

$$\lim_{x\to 3}\frac{x^2-4x+3}{x^2-2x-3}=\lim_{x\to 3}\frac{(x-3)(x-1)}{(x-3)(x+2)}$$

$$\lim_{x\to 3}\frac{x^2-4x+3}{x^2-2x-3}=\lim_{x\to 3}\frac{(x-1)}{(x+2)}$$

$$\lim_{x\to 3}\frac{x^2-4x+3}{x^2-2x-3}=\frac{2}{5}$$

Thus, the value of

$\lim\limits_{x \to 3} \frac{x^2-4x+3}{x^2-2x-3}$ is $\frac{2}{5}$.

Q. 10. Evaluate

$$\lim_{x \to \frac{1}{2}} \left(\frac{4x^2-1}{2x-1} \right)$$

Answer : To evaluate:

$$\lim_{x \to \frac{1}{2}} \frac{4x^2-1}{2x-1}$$

Formula used:

We have,

$$\lim_{x \to a} f(x) = f(a)$$

As $x \to \frac{1}{2}$, we have

$$\lim_{x \to \frac{1}{2}} \frac{4x^2-1}{2x-1} = \lim_{x \to \frac{1}{2}} \frac{(2x+1)(2x-1)}{2x-1}$$

$$\lim_{x \to \frac{1}{2}} \frac{4x^2-1}{2x-1} = \lim_{x \to \frac{1}{2}} (2x+1)$$

$$\lim_{x \to \frac{1}{2}} \frac{4x^2-1}{2x-1} = 2$$

Thus, the value of $\lim\limits_{x \to \frac{1}{2}} \frac{4x^2-1}{2x-1}$ is 2.

Q. 11. Evaluate

$$\lim_{x \to 4}\left(\frac{x^3 - 64}{x^2 - 16}\right)$$

Answer : To evaluate:

$$\lim_{x \to 4}\frac{x^3 - 64}{x^2 - 16}$$

Formula used: We have,

$$\lim_{x \to a} f(x) = f(a)$$

and

$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

As $x \to 4$, we have

$$\lim_{x \to 4}\frac{x^3 - 64}{x^2 - 16} = \lim_{x \to 4}\frac{(x - 4)(x^2 + 4x + 16)}{(x + 4)(x - 4)}$$

$$\lim_{x \to 4}\frac{x^3 - 64}{x^2 - 16} = \lim_{x \to 4}\frac{(x^2 + 4x + 16)}{(x + 4)}$$

$$\lim_{x \to 4}\frac{x^3 - 64}{x^2 - 16} = \frac{48}{8}$$

$$\lim_{x \to 4}\frac{x^3 - 64}{x^2 - 16} = 6$$

Thus, the value of

$\lim_{x \to 4}\frac{x^3 - 64}{x^2 - 16}$ is 6.

Q. 12. Evaluate

$$\lim_{x \to 2}\left(\frac{x^5 - 32}{x^3 - 8}\right)$$

Answer : To evaluate:

$$\lim_{x \to 2}\frac{x^5 - 32}{x^3 - 8}$$

Formula used: We have,

$$\frac{x^m - y^m}{x - y} = my^{m-1}$$

As $x \to 4$, we have

$$\lim_{x \to 2}\frac{x^5 - 32}{x^3 - 8} = \lim_{x \to 2}\frac{x^5 - 2^5}{x^3 - 2^3}$$

$$\lim_{x \to 2}\frac{x^5 - 32}{x^3 - 8} = \lim_{x \to 2}\frac{\frac{x^5 - 2^5}{x - 2}}{\frac{x^3 - 2^3}{x - 2}}$$

$$\lim_{x \to 2}\frac{x^5 - 32}{x^3 - 8} = \lim_{x \to 2}\frac{5(2)^4}{3(2)^2}$$

$$\lim_{x \to 2}\frac{x^5 - 32}{x^3 - 8} = \frac{20}{3}$$

Thus, the value of $\lim_{x \to 2}\frac{x^5 - 32}{x^3 - 8}$ is $\frac{20}{3}$

Q. 13. Evaluate

$$\lim_{x \to a}\left(\frac{x^{5/2} - a^{5/2}}{x - a}\right)$$

Answer : To evaluate:

$$\lim_{x \to a} \frac{x^{\frac{5}{2}} - a^{\frac{5}{2}}}{x - a}$$

Formula used: We have,

$$\frac{x^m - y^m}{x - y} = my^{m-1}$$

As $x \to a$, we have

$$\lim_{x \to a} \frac{x^{\frac{5}{2}} - a^{\frac{5}{2}}}{x - a} = \frac{5}{2} a^{\frac{5}{2}-1}$$

$$\lim_{x \to a} \frac{x^{\frac{5}{2}} - a^{\frac{5}{2}}}{x - a} = \frac{5}{2} a^{\frac{3}{2}}$$

Thus, the value of $\lim_{x \to a} \frac{x^{\frac{5}{2}} - a^{\frac{5}{2}}}{x - a}$ is $\frac{5}{2} a^{\frac{3}{2}}$

Q. 14. Evaluate

$$\lim_{x \to a} \left\{ \frac{(x+2)^{5/3} - (a+2)^{5/3}}{x - a} \right\}$$

Answer : To evaluate:

$$\lim_{x \to a} \left\{ \frac{(x+2)^{\frac{5}{3}} - (a+2)^{\frac{5}{3}}}{x - a} \right\}$$

Formula used: We have,

$\frac{x^m - y^m}{x-y} = my^{m-1}$

As $x \to a$, we have

$$\lim_{x\to a}\left\{\frac{(x+2)^{\frac{5}{3}} - (a+2)^{\frac{5}{3}}}{x-a}\right\} = \lim_{x\to a}\left\{\frac{(x+2)^{\frac{5}{3}} - (a+2)^{\frac{5}{3}}}{(x+2)-(a+2)}\right\}$$

$$\lim_{x\to a}\left\{\frac{(x+2)^{\frac{5}{3}} - (a+2)^{\frac{5}{3}}}{x-a}\right\} = \frac{5}{3}(a+2)^{\frac{5}{3}-1}$$

$$\lim_{x\to a}\left\{\frac{(x+2)^{\frac{5}{3}} - (a+2)^{\frac{5}{3}}}{x-a}\right\} = \frac{5}{3}(a+2)^{\frac{2}{3}}$$

Thus, the value of $\lim_{x\to a}\left\{\frac{(x+2)^{\frac{5}{3}} - (a+2)^{\frac{5}{3}}}{x-a}\right\}$ is $\frac{5}{3}(a+2)^{\frac{2}{3}}$

Q. 15. Evaluate

$$\lim_{x\to 1}\left(\frac{x^n - 1}{x-1}\right)$$

Answer : To evaluate:

$\lim_{x\to 1}\frac{x^n-1}{x-1}$

Formula used: We have,

$\frac{x^m - y^m}{x-y} = my^{m-1}$

As $x \to a$, we have

$\frac{x^m - y^m}{x-y} = my^{m-1}$

As $x \to 1$, we have

$$\lim_{x \to a} \frac{x^n - 1}{x - 1} = n$$

Thus, the value of $\lim_{x \to a} \frac{x^n - 1}{x - 1}$ is n.

Q. 16. Evaluate

$$\lim_{x \to a} \left(\frac{\sqrt{x} - \sqrt{a}}{x - a} \right)$$

Answer : To evaluate:

$\lim_{x \to a} \frac{\sqrt{x} - \sqrt{a}}{x - a}$

Formula used:

We have,

$\frac{x^m - y^m}{x-y} = my^{m-1}$

As $x \to a$, we have

$$\lim_{x\to a}\frac{x^{\frac{1}{2}}-a^{\frac{1}{2}}}{x-a}=\frac{1}{2}a^{\frac{1}{2}-1}$$

$$\lim_{x\to a}\frac{x^{\frac{1}{2}}-a^{\frac{1}{2}}}{x-a}=\frac{1}{2\sqrt{a}}$$

Thus, the value of $\lim_{x\to a}\frac{x^{\frac{1}{2}}-a^{\frac{1}{2}}}{x-a}$ is $\frac{1}{2\sqrt{a}}$

Q. 17. Evaluate

$$\lim_{h\to 0}\left(\frac{\sqrt{x+h}-\sqrt{x}}{h}\right)$$

Answer : To evaluate:

$$\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}$$

Formula used:

L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim_{x\to a} f(x)=\lim_{x\to a} g(x)=0 \text{ or } \pm\infty$ then

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$$

As $x\to 0$, we have

$$\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}=\frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}=\lim_{h\to 0}\frac{\frac{d}{dh}\left(\sqrt{x+h}-\sqrt{x}\right)}{\frac{d}{dh}(h)}$$

$$\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}=\lim_{h\to 0}\frac{\frac{1}{2\sqrt{x+h}}}{1}$$

$$\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}=\frac{1}{2\sqrt{x}}$$

Thus, the value of $\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}$ is $\frac{1}{2\sqrt{x}}$

Q. 18. Evaluate

$$\lim_{h\to 0}\frac{1}{h}\left\{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}\right\}$$

Answer : To evaluate:

$$\lim_{h\to 0}\frac{1}{h}\left\{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}\right\}$$

Formula used: L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim_{x\to a} f(x)=\lim_{x\to a} g(x)=0 \text{ or } \pm\infty$ then

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$$

As $x\to 0$, we have

$$\lim_{h\to 0}\frac{1}{h}\left\{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}\right\}=\frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{h\to 0}\frac{1}{h}\left\{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}\right\}=\lim_{h\to 0}\frac{\frac{d}{dh}\left(\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}\right)}{\frac{d}{dh}(h)}$$

$$\lim_{h\to 0}\frac{1}{h}\left\{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}\right\}=\lim_{h\to 0}\frac{\frac{-1}{2\sqrt{x+h}}+\frac{1}{2\sqrt{x}}}{1}$$

$$\lim_{h\to 0}\frac{1}{h}\left\{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}\right\}=0$$

Thus, the value of $\lim_{h\to 0}\frac{1}{h}\left\{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}\right\}$ is 0.

Q. 19. Evaluate

$$\lim_{x\to 0}\left(\frac{\sqrt{1+x}-1}{x}\right)$$

Answer : To evaluate:

$$\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}$$

Formula used: L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$$\lim_{x\to a}f(x)=\lim_{x\to a}g(x)=0 \text{ or } \pm\infty$$ then

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$$

As $x \to 0$, we have

$$\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}=\frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}=\lim_{x\to 0}\frac{\frac{d}{dx}\left(\sqrt{1+x}-1\right)}{\frac{d}{dx}(x)}$$

$$\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}=\lim_{x\to 0}\frac{\frac{1}{2\sqrt{x+1}}}{1}$$

$$\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}=\frac{1}{2}$$

Thus, the value of $\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}$ is $\frac{1}{2}$

Q. 20. Evaluate

$$\lim_{x\to 0}\left(\frac{\sqrt{2-x}-\sqrt{2+x}}{x}\right)$$

Answer : To evaluate:

$$\lim_{x\to 0}\frac{\sqrt{2-x}-\sqrt{2+x}}{x}$$

Formula used: L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim_{x\to a} f(x)=\lim_{x\to a} g(x)=0 \text{ or } \pm\infty$ then

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$$

As $x\to 0$, we have

$$\lim_{x\to 0}\frac{\sqrt{2-x}-\sqrt{2+x}}{x}=\frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x\to 0}\frac{\sqrt{2-x}-\sqrt{2+x}}{x}=\lim_{x\to 0}\frac{\frac{d}{dx}\left(\sqrt{2-x}-\sqrt{2+x}\right)}{\frac{d}{dx}(x)}$$

$$\lim_{x\to 0}\frac{\sqrt{2-x}-\sqrt{2+x}}{x}=\lim_{x\to 0}\frac{-\frac{1}{2\sqrt{2-x}}-\frac{1}{2\sqrt{2+x}}}{1}$$

$$\lim_{x\to 0}\frac{\sqrt{2-x}-\sqrt{2+x}}{x}=\frac{-2}{2\sqrt{2}}$$

$$\lim_{x\to 0}\frac{\sqrt{2-x}-\sqrt{2+x}}{x}=\frac{-1}{\sqrt{2}}$$

Thus, the value of $\lim_{x\to 0}\frac{\sqrt{2-x}-\sqrt{2+x}}{x}$ is $\frac{-1}{\sqrt{2}}$

Q. 21. Evaluate

$$\lim_{x\to 0}\left(\frac{\sqrt{1+x+x^2}-1}{x}\right)$$

Answer : To evaluate:

$$\lim_{x\to 0}\frac{\sqrt{1+x+x^2}-1}{x}$$

Formula used: L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim_{x \to a} f(x) = \lim_{x \to a} g(x) = 0 \text{ or } \pm\infty$ then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

As $x \to 0$, we have

$$\lim_{x \to 0} \frac{\sqrt{1+x+x^2}-1}{x} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x \to 0} \frac{\sqrt{1+x+x^2}-1}{x} = \lim_{x \to 0} \frac{\frac{d}{dx}\left(\sqrt{1+x+x^2}-1\right)}{\frac{d}{dx}(x)}$$

$$\lim_{x \to 0} \frac{\sqrt{1+x+x^2}-1}{x} = \lim_{x \to 0} \frac{\frac{1+2x}{2\sqrt{1+x+x^2}}}{1}$$

$$\lim_{x \to 0} \frac{\sqrt{1+x+x^2}-1}{x} = \frac{1}{2}$$

Thus, the value of $\lim_{x \to 0} \frac{\sqrt{1+x+x^2}-1}{x}$ is $\frac{1}{2}$

Q. 22. Evaluate

$$\lim_{x \to 0} \left(\frac{\sqrt{3-x}-1}{2-x} \right)$$

Answer : To evaluate:

$$\lim_{x \to 0} \frac{\sqrt{3-x}-1}{2-x}$$

Formula used: We have,

$$\lim_{x\to a} f(x) = f(a)$$

As $x \to 0$, we have

$$\lim_{x\to 0}\frac{\sqrt{3-x}-1}{2-x} = \frac{\sqrt{3}-1}{2}$$

Thus, the value of $\lim_{x\to 0}\frac{\sqrt{3-x}-1}{2-x}$ is $\frac{\sqrt{3}-1}{2}$

Q. 23. Evaluate

$$\lim_{x\to 0}\left(\frac{2x}{\sqrt{a+x}-\sqrt{a-x}}\right)$$

Answer : To evaluate:

$$\lim_{x\to 0}\frac{2x}{\sqrt{a+x}-\sqrt{a-x}}$$

Formula used:

Multiplying numerator and denominator by

$$\sqrt{a+x}+\sqrt{a-x}$$

$$\lim_{x\to 0}\frac{2x}{\sqrt{a+x}-\sqrt{a-x}} = \lim_{x\to 0}\frac{2x}{\sqrt{a+x}-\sqrt{a-x}}\left(\frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a+x}-\sqrt{a-x}}\right)$$

$$\lim_{x\to 0}\frac{2x}{\sqrt{a+x}-\sqrt{a-x}} = \lim_{x\to 0}\frac{2x\left(\sqrt{a+x}+\sqrt{a-x}\right)}{a+x-a+x}$$

$$\lim_{x\to 0}\frac{2x}{\sqrt{a+x}-\sqrt{a-x}} = \lim_{x\to 0}\frac{2x\left(\sqrt{a+x}+\sqrt{a-x}\right)}{2x}$$

$$\lim_{x\to 0}\frac{2x}{\sqrt{a+x}-\sqrt{a-x}} = \lim_{x\to 0}\sqrt{a+x}+\sqrt{a-x}$$

$$\lim_{x\to 0}\frac{2x}{\sqrt{a+x}-\sqrt{a-x}} = 2\sqrt{a}$$

Thus, the value of $\lim_{x\to 0}\frac{2x}{\sqrt{a+x}-\sqrt{a-x}}$ is $2\sqrt{a}$.

Q. 24. Evaluate

$$\lim_{x\to 1}\left(\frac{\sqrt{3+x}-\sqrt{5-x}}{x^2-1}\right)$$

Answer : To evaluate:

$$\lim_{x\to 1}\frac{\sqrt{3+x}-\sqrt{5-x}}{x^2-1}$$

Formula used:

L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0 \text{ or } \pm\infty$ then

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$$

As $x \to 0$, we have

$$\lim_{x\to 0}\frac{\sqrt{3+x}-\sqrt{5-x}}{x^2-1} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x\to 1}\frac{\sqrt{3+x}-\sqrt{5-x}}{x^2-1} = \lim_{x\to 1}\frac{\frac{d}{dx}\left(\sqrt{3+x}-\sqrt{5-x}\right)}{\frac{d}{dx}(x^2-1)}$$

$$\lim_{x\to 1}\frac{\sqrt{3+x}-\sqrt{5-x}}{x^2-1}=\lim_{x\to 1}\frac{\frac{1}{2\sqrt{3+x}}+\frac{1}{2\sqrt{5-x}}}{2x}$$

$$\lim_{x\to 1}\frac{\sqrt{3+x}-\sqrt{5-x}}{x^2-1}=\frac{1}{4}$$

Thus, the value of $\lim_{x\to 1}\frac{\sqrt{3+x}-\sqrt{5-x}}{x^2-1}$ is $\frac{1}{4}$

Q. 25. Evaluate

$$\lim_{x\to 2}\left(\frac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}}\right)$$

Answer : To evaluate:

$$\lim_{x\to 2}\frac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}}$$

Formula used: L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim_{x\to a}f(x)=\lim_{x\to a}g(x)=0 \text{ or } \pm\infty$ then

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$$

As $x\to 0$, we have

$$\lim_{x\to 2}\frac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}}=\frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x\to 2}\frac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}} = \lim_{x\to 2}\frac{\frac{d}{dx}(x^2-4)}{\frac{d}{dx}\left(\sqrt{x+2}-\sqrt{3x-2}\right)}$$

$$\lim_{x\to 2}\frac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}} = \lim_{x\to 2}\frac{2x}{\frac{1}{2\sqrt{x+2}}-\frac{3}{2\sqrt{3x-2}}}$$

$$\lim_{x\to 2}\frac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}} = \frac{4}{\frac{1}{2\sqrt{2+2}}-\frac{3}{2\sqrt{6-2}}}$$

$$\lim_{x\to 2}\frac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}} = \frac{8}{\frac{1}{2}-\frac{3}{2}}$$

$$\lim_{x\to 2}\frac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}} = -8$$

Thus, the value of $\lim_{x\to 2}\frac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}}$ is -8.

Q. 26. Evaluate

$$\lim_{x\to 4}\left(\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}\right)$$

Answer : To evaluate:

$$\lim_{x\to 4}\left(\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}\right)$$

Formula used:

Multiplying numerator and denominator with conjugates of numerator and denominator i.e

$$\left(1+\sqrt{5-x}\right)\left(3+\sqrt{5+x}\right)$$

$$\lim_{x\to 4}\left(\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}\right) = \lim_{x\to 4}\left(\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}\right)\left(\frac{1+\sqrt{5-x}}{1+\sqrt{5-x}}\right)\left(\frac{3+\sqrt{5+x}}{3+\sqrt{5+x}}\right)$$

$$\lim_{x\to4}\left(\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}\right) = \lim_{x\to4}\left(\frac{4-x}{x-4}\right)\left(\frac{1+\sqrt{5-x}}{3+\sqrt{5+x}}\right)$$

$$\lim_{x\to4}\left(\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}\right) = \lim_{x\to4} -\left(\frac{1+\sqrt{5-x}}{3+\sqrt{5+x}}\right)$$

$$\lim_{x\to4}\left(\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}\right) = -\frac{1}{3}$$

Thus, the value of $\lim_{x\to4}\left(\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}\right)$ is $-\frac{1}{3}$

Q. 27. Evaluate

$$\lim_{x\to0}\left(\frac{\sqrt{a+x}-\sqrt{a}}{x\sqrt{a(a+x)}}\right)$$

Answer : To evaluate:

$$\lim_{x\to0}\frac{\sqrt{a+x}-\sqrt{a}}{x\sqrt{a(a+x)}}$$

Formula used: L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0 \text{ or } \pm\infty$ then

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$$

As $x \to 0$, we have

$$\lim_{x\to0}\frac{\sqrt{a+x}-\sqrt{a}}{x\sqrt{a(a+x)}} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x\to 0}\frac{\sqrt{a+x}-\sqrt{a}}{x\sqrt{a(a+x)}} = \lim_{x\to 0}\frac{\frac{d}{dx}\left(\sqrt{a+x}-\sqrt{a}\right)}{\frac{d}{dx}\left(x\sqrt{a(a+x)}\right)}$$

$$\lim_{x\to 0}\frac{\sqrt{a+x}-\sqrt{a}}{x\sqrt{a(a+x)}} = \lim_{x\to 0}\frac{\frac{1}{2\sqrt{a+x}}}{x\left(\frac{a}{2\sqrt{a(a+x)}}\right)+\sqrt{a(a+x)}}$$

$$\lim_{x\to 0}\frac{\sqrt{a+x}-\sqrt{a}}{x\sqrt{a(a+x)}} = \frac{\frac{1}{2\sqrt{a}}}{a}$$

$$\lim_{x\to 0}\frac{\sqrt{a+x}-\sqrt{a}}{x\sqrt{a(a+x)}} = \frac{1}{2a\sqrt{a}}$$

Thus, the value of $\lim_{x\to 0}\frac{\sqrt{a+x}-\sqrt{a}}{x\sqrt{a(a+x)}}$ is $\frac{1}{2a\sqrt{a}}$

Thus, the value of $\lim_{x\to 0}\frac{\sqrt{a+x}-\sqrt{a}}{x\sqrt{a(a+x)}}$ is $\frac{1}{2a\sqrt{a}}$

Q. 28. Evaluate

$$\lim_{x\to 0}\left(\frac{\sqrt{1+x^2}-\sqrt{1+x}}{\sqrt{1+x^3}-\sqrt{1+x}}\right)$$

Answer : To evaluate:

$$\lim_{x\to 0}\frac{\sqrt{1+x^2}-\sqrt{1+x}}{\sqrt{1+x^3}-\sqrt{1+x}}$$

Formula used: L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0 \text{ or } \pm\infty$ then

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$$

As $x \to 0$, we have

$$\lim_{x\to 0}\frac{\sqrt{1+x^2}-\sqrt{1+x}}{\sqrt{1+x^3}-\sqrt{1+x}}=\frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x\to 0}\frac{\sqrt{1+x^2}-\sqrt{1+x}}{\sqrt{1+x^3}-\sqrt{1+x}}=\lim_{x\to 0}\frac{\frac{d}{dx}\left(\sqrt{1+x^2}-\sqrt{1+x}\right)}{\frac{d}{dx}\left(\sqrt{1+x^3}-\sqrt{1+x}\right)}$$

$$\lim_{x\to 0}\frac{\sqrt{1+x^2}-\sqrt{1+x}}{\sqrt{1+x^3}-\sqrt{1+x}}=\lim_{x\to 0}\frac{\frac{2x}{2\sqrt{1+x^2}}-\frac{1}{2\sqrt{1+x}}}{\frac{3x^2}{2\sqrt{1+x^3}}-\frac{1}{2\sqrt{1-x}}}$$

$$\lim_{x\to 0}\frac{\sqrt{1+x^2}-\sqrt{1+x}}{\sqrt{1+x^3}-\sqrt{1+x}}=\lim_{x\to 0}\frac{-\frac{1}{2}}{-\frac{1}{2}}$$

$$\lim_{x\to 0}\frac{\sqrt{1+x^2}-\sqrt{1+x}}{\sqrt{1+x^3}-\sqrt{1+x}}=-1$$

Thus, the value of $\lim_{x\to 0}\frac{\sqrt{1+x^2}-\sqrt{1+x}}{\sqrt{1+x^3}-\sqrt{1+x}}$ is -1.

Q. 29. Evaluate

$$\lim_{x\to 1}\left(\frac{x^4-3x^2+2}{x^3-5x^2+3x+1}\right)$$

Answer : To Evaluate:

$$\lim_{x\to 1}\left(\frac{x^4-3x^2+2}{x^3-5x^2+3x+1}\right)$$

L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$$\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0 \text{ or } \pm\infty$$ then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

As $x \to 0$, we have

$$\lim_{x\to 1}\left(\frac{x^4 - 3x^2 + 2}{x^3 - 5x^2 + 3x + 1}\right) = \frac{0}{0}$$

Therefore,

$$\lim_{x\to 1}\left(\frac{x^4 - 3x^2 + 2}{x^3 - 5x^2 + 3x + 1}\right) = \lim_{x\to 1} \frac{4x^3 - 6x}{3x^2 - 10x + 3} = \frac{4-6}{3-10+3} = -\frac{2}{-4} = -\frac{1}{2}$$

Hence,

$$\lim_{x\to 1}\left(\frac{x^4 - 3x^2 + 2}{x^3 - 5x^2 + 3x + 1}\right) = -\frac{1}{2}$$

Q. 30. Evaluate

$$\lim_{x\to 2}\left(\frac{3^x - 3^{3-x} - 12}{3^{3-x} - 3^{x/2}}\right)$$

Answer : To evaluate:

$$\lim_{x\to 2}\left(\frac{3^x - 3^{3-x} - 12}{3^{3-x} - 3^{\frac{x}{2}}}\right)$$

Formula used:

L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$$\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0 \text{ or } \pm\infty$$ then

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$$

As $x \to 0$, we have

$$\lim_{x\to 0}\frac{2^x - 1}{x} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x\to 2}\left(\frac{3^x - 3^{3-x} - 12}{3^{3-x} - 3^{\frac{x}{2}}}\right) = \lim_{x\to 2}\frac{\frac{d}{dx}(3^x - 3^{3-x} - 12)}{\frac{d}{dx}\left(3^{3-x} - 3^{\frac{x}{2}}\right)}$$

$$\lim_{x\to 2}\left(\frac{3^x - 3^{3-x} - 12}{3^{3-x} - 3^{\frac{x}{2}}}\right) = \lim_{x\to 0}\frac{3^x \ln 3 + 3^{3-x}\ln 3}{-3^{3-x}\ln 3 + 3^{\frac{x}{2}}\left(\frac{1}{2}\right)\ln 3}$$

$$\lim_{x\to 2}\left(\frac{3^x - 3^{3-x} - 12}{3^{3-x} - 3^{\frac{x}{2}}}\right) = \frac{\ln 3 + 27\ln 3}{-27\ln 3 + \left(\frac{1}{2}\right)\ln 3}$$

$$\lim_{x\to 2}\left(\frac{3^x - 3^{3-x} - 12}{3^{3-x} - 3^{\frac{x}{2}}}\right) = \frac{28\ln 3}{-26.5\ln 3}$$

Thus, the value of $\lim_{x\to 2}\left(\frac{3^x - 3^{3-x} - 12}{3^{3-x} - 3^{\frac{x}{2}}}\right)$ is $\frac{28\ln 3}{-26.5\ln 3}$

Q. 31. Evaluate

$$\lim_{x\to 0}\left(\frac{e^{4x} - 1}{x}\right)$$

Answer : To evaluate:

$$\lim_{x\to 0}\frac{e^{4x} - 1}{x}$$

Formula used:

L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim_{x \to a} f(x) = \lim_{x \to a} g(x) = 0 \text{ or } \pm\infty$ then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

As $x \to 0$, we have

$$\lim_{x \to 0} \frac{e^{4x} - 1}{x} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x \to 0} \frac{e^{4x} - 1}{x} = \lim_{x \to 0} \frac{\frac{d}{dx}(e^{4x} - 1)}{\frac{d}{dx}(x)}$$

$$\lim_{x \to 0} \frac{e^{4x} - 1}{x} = \lim_{x \to 0} \frac{4e^{4x}}{1}$$

$$\lim_{x \to 0} \frac{e^{4x} - 1}{x} = 4$$

Thus, the value of $\lim_{x \to 0} \frac{e^{4x} - 1}{x}$ is 4.

Q. 32. Evaluate

$$\lim_{x \to 0} \left(\frac{e^{2+x} - e^2}{x} \right)$$

Answer : To evaluate:

$$\lim_{x \to 0} \frac{e^{2+x} - e^2}{x}$$

Formula used: L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim_{x \to a} f(x) = \lim_{x \to a} g(x) = 0 \text{ or } \pm\infty$ then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

As $x \to 0$, we have

$$\lim_{x \to 0} \frac{e^{2+x} - e^2}{x} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x \to 0} \frac{e^{2+x} - e^2}{x} = \lim_{x \to 0} \frac{\frac{d}{dx}(e^{2+x} - e^2)}{\frac{d}{dx}(x)}$$

$$\lim_{x \to 0} \frac{e^{2+x} - e^2}{x} = \lim_{x \to 0} \frac{e^{2+x}}{1}$$

$$\lim_{x \to 0} \frac{e^{2+x} - e^2}{x} = e^2$$

Thus, the value of $\lim_{x \to 0} \frac{e^{2+x} - e^2}{x}$ is e^2.

Q. 33. Evaluate

$$\lim_{x \to 4} \left(\frac{e^x - e^4}{x - 4} \right)$$

Answer : To evaluate:

$$\lim_{x \to 4} \frac{e^x - e^4}{x - 4}$$

Formula used:

L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$$\lim_{x \to a} f(x) = \lim_{x \to a} g(x) = 0 \text{ or } \pm\infty$$ then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

As $x \to 0$, we have

$$\lim_{x \to 0} \frac{e^{2+x} - e^2}{x} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x \to 4} \frac{e^x - e^4}{x - 4} = \lim_{x \to 4} \frac{\frac{d}{dx}(e^x - e^4)}{\frac{d}{dx}(x - 4)}$$

$$\lim_{x \to 4} \frac{e^x - e^4}{x - 4} = \lim_{x \to 4} \frac{e^x}{1}$$

$$\lim_{x \to 4} \frac{e^x - e^4}{x - 4} = e^4$$

Thus, the value of $\lim_{x \to 4} \frac{e^x - e^4}{x - 4}$ is e^4.

Q. 34. Evaluate

$$\lim_{x \to 0} \left(\frac{e^{3x} - e^{2x}}{x} \right)$$

Answer : To evaluate:

$$\lim_{x\to 0}\frac{e^{3x}-e^{2x}}{x}$$

Formula used:

L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0 \text{ or } \pm\infty$ then

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$$

As $x \to 0$, we have

$$\lim_{x\to 0}\frac{e^{3x}-e^{2x}}{x} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x\to 0}\frac{e^{3x}-e^{2x}}{x} = \lim_{x\to 0}\frac{\frac{d}{dx}(e^{3x}-e^{2x})}{\frac{d}{dx}(x)}$$

$$\lim_{x\to 0}\frac{e^{3x}-e^{2x}}{x} = \lim_{x\to 0}\frac{3e^{3x}-2e^{2x}}{1}$$

$$\lim_{x\to 0}\frac{e^{3x}-e^{2x}}{x} = 3\text{-}2$$

$$\lim_{x\to 0}\frac{e^{3x}-e^{2x}}{x} = 1$$

Thus, the value of $\lim_{x\to 0}\frac{e^{3x}-e^{2x}}{x}$ is 1.

Q. 35. Evaluate

$$\lim_{x \to 0}\left(\frac{e^x - x - 1}{x}\right)$$

Answer : To evaluate:

$$\lim_{x\to 0}\frac{e^x - x - 1}{x}$$

Formula used:

L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0 \text{ or } \pm\infty$ then

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$$

As $x \to 0$, we have

$$\lim_{x\to 0}\frac{e^x - x - 1}{x} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x\to 0}\frac{e^x - x - 1}{x} = \lim_{x\to 0}\frac{\frac{d}{dx}(e^x - x - 1)}{\frac{d}{dx}(x)}$$

$$\lim_{x\to 0}\frac{e^x - x - 1}{x} = \lim_{x\to 0}\frac{e^x - 1}{1}$$

$$\lim_{x\to 0}\frac{e^{x}-x-1}{x}=1-1$$

$$\lim_{x\to 0}\frac{e^{x}-x-1}{x}=0$$

Thus, the value of $\lim\limits_{x\to 0}\lim\limits_{x\to 0}\frac{e^{x}-x-1}{x}$ is 0.

Q. 36. Evaluate

$$\lim_{x\to 0}\left(\frac{e^{bx}-e^{ax}}{x}\right),0<a<b$$

Answer : To evaluate:

$$\lim_{x\to 0}\frac{e^{bx}-e^{ax}}{x}$$

Formula used:

L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim\limits_{x\to a}f(x)=\lim\limits_{x\to a}g(x)=0 \text{ or } \pm\infty$ then

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$$

As $x\to 0$, we have

$$\lim_{x\to 0}\frac{e^{bx}-e^{ax}}{x}=\frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x\to 0}\frac{e^{bx}-e^{ax}}{x}=\lim_{x\to 0}\frac{\frac{d}{dx}\left(e^{bx}-e^{ax}\right)}{\frac{d}{dx}(x)}$$

$$\lim_{x\to 0}\frac{e^{bx}-e^{ax}}{x}=\lim_{x\to 0}\frac{be^{bx}-ae^{ax}}{1}$$

$$\lim_{x\to 0}\frac{e^{bx}-e^{ax}}{x}=\text{b-a}$$

Thus, the value of $\lim_{x\to 0}\frac{e^{bx}-e^{ax}}{x}$ is b-a.

Q. 37. Evaluate

$$\lim_{x\to 0}\left(\frac{a^x-b^x}{x}\right)$$

Answer : To evaluate:

$$\lim_{x\to 0}\frac{a^x-b^x}{x}$$

Formula used:

L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim_{x\to a} f(x)=\lim_{x\to a} g(x)=0 \text{ or } \pm\infty$ then

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$$

As $x\to 0$, we have

$$\lim_{x \to 0} \frac{a^x - b^x}{x} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x \to 0} \frac{a^x - b^x}{x} = \lim_{x \to 0} \frac{\frac{d}{dx}(a^x - b^x)}{\frac{d}{dx}(x)}$$

$$\lim_{x \to 0} \frac{a^x - b^x}{x} = \lim_{x \to 0} \frac{a^x \ln a - b^x \ln b}{1}$$

$$\lim_{x \to 0} \frac{a^x - b^x}{x} = \ln a - \ln b$$

$$\lim_{x \to 0} \frac{a^x - b^x}{x} = \ln \frac{a}{b}$$

Thus, the value of $\lim_{x \to 0} \frac{a^x - b^x}{x}$ is $\ln \frac{a}{b}$.

Q. 38. Evaluate

$$\lim_{x \to 0} \left(\frac{a^x - a^{-x}}{x} \right)$$

Answer : To evaluate:

$$\lim_{x \to 0} \frac{a^x - a^{-x}}{x}$$

Formula used:

L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim_{x \to a} f(x) = \lim_{x \to a} g(x) = 0 \text{ or } \pm \infty$ then

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$$

As $x \to 0$, we have

$$\lim_{x\to 0}\frac{a^x-a^{-x}}{x}=\frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x\to 0}\frac{a^x-a^{-x}}{x}=\lim_{x\to 0}\frac{\frac{d}{dx}(a^x-a^{-x})}{\frac{d}{dx}(x)}$$

$$\lim_{x\to 0}\frac{a^x-a^{-x}}{x}=\lim_{x\to 0}\frac{a^x\ln a+a^{-x}\ln a}{1}$$

$$\lim_{x\to 0}\frac{a^x-a^{-x}}{x}=2\ln a$$

Thus, the value of $\lim_{x\to 0}\frac{a^x-a^{-x}}{x}$ is $2\ln a$.

Q. 39. Evaluate

$$\lim_{x\to 0}\left(\frac{2^x-1}{x}\right)$$

Answer : To evaluate:

$$\lim_{x\to 0}\frac{2^x-1}{x}$$

Formula used:

L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim\limits_{x \to a} f(x) = \lim\limits_{x \to a} g(x) = 0 \text{ or } \pm\infty$ then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

As $x \to 0$, we have

$$\lim_{x \to 0} \frac{2^x - 1}{x} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x \to 0} \frac{2^x - 1}{x} = \lim_{x \to 0} \frac{\frac{d}{dx}(2^x - 1)}{\frac{d}{dx}(x)}$$

$$\lim_{x \to 0} \frac{2^x - 1}{x} = \lim_{x \to 0} \frac{2^x \ln 2}{1}$$

$$\lim_{x \to 0} \frac{2^x - 1}{x} = \ln 2$$

Thus, the value of $\lim\limits_{x \to 0} \frac{2^x - 1}{x}$ is $\ln 2$.

Q. 40. Evaluate

$$\lim_{x \to 0} \left(\frac{3^{2+x} - 9}{x} \right)$$

Answer : To evaluate:

$$\lim_{x \to 0} \frac{3^{2+x} - 9}{x}$$

Formula used:

L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim_{x \to a} f(x) = \lim_{x \to a} g(x) = 0 \text{ or } \pm\infty$ then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

As $x \to 0$, we have

$$\lim_{x \to 0} \frac{3^{2+x} - 9}{x} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x \to 0} \frac{3^{2+x} - 9}{x} = \lim_{x \to 0} \frac{\frac{d}{dx}(3^{2+x} - 9)}{\frac{d}{dx}(x)}$$

$$\lim_{x \to 0} \frac{3^{2+x} - 9}{x} = \lim_{x \to 0} \frac{(2 + x) \ln 3}{1}$$

$$\lim_{x \to 0} \frac{3^{2+x} - 9}{x} = 2 \ln 3$$

Thus, the value of $\lim_{x \to 0} \frac{3^{2+x} - 9}{x}$ is $2 \ln 3$

Exercise 27B

Q. 1. Evaluate the following limits:

$$\lim_{x \to 0} \frac{\sin 4x}{6x}$$

Answer : To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfied any one from 7 indeterminate forms.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim\limits_{x\to0}\frac{\sin x}{x} = 1$

So $\lim\limits_{x\to0}\frac{\sin 4x}{6x} = \lim\limits_{x\to0}(\frac{\sin 4x}{4x})\times\frac{4}{6} = \frac{4}{6} = \frac{2}{3}$

Therefore, $\lim\limits_{x\to0}\frac{\sin 4x}{6x} = \frac{2}{3}$

Q. 2. Evaluate the following limits:

$$\lim_{x\to0}\frac{\sin 5x}{\sin 8x}$$

Answer : To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfied any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim\limits_{x\to0}\frac{\sin x}{x} = 1$

So $\lim\limits_{x\to0}\frac{\sin 5x}{\sin 8x} = \lim\limits_{x\to0}(\frac{\sin 5x}{5x})\times\frac{8x}{\sin 8x}\times\frac{5x}{8x} = \frac{5x}{8x} = \frac{5}{8}$

Therefore, $\lim\limits_{x\to0}\frac{\sin 5x}{\sin 8x} = \frac{5}{8}$

Q. 3. Evaluate the following limits:

$$\lim_{x \to 0} \frac{\tan 3x}{\tan 5x}$$

Answer : To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfied any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim_{x \to 0} \frac{\tan x}{x} = 1$

So $\lim_{x \to 0} \frac{\tan 3x}{\tan 5x} = \lim_{x \to 0} \left(\frac{\tan 3x}{3x}\right) \times \frac{5x}{\sin 5x} \times \frac{3x}{5x} = \frac{3x}{5x} = \frac{3}{5}$

Therefore, $\lim_{x \to 0} \frac{\tan 3x}{\tan 5x} = \frac{3}{5}$

Q. 4. Evaluate the following limits:

$$\lim_{x \to 0} \frac{\tan \alpha x}{\tan \beta x}$$

Answer : To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfied any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim_{x \to 0} \frac{\tan x}{x} = 1$ '

So $\lim\limits_{x\to 0}\frac{\tan\alpha x}{\tan\beta x} = \lim\limits_{x\to 0}(\frac{\tan\alpha x}{\alpha x}) \times \frac{\beta x}{\sin\beta x} \times \frac{\alpha x}{\beta x} = \frac{\alpha x}{\beta x} = \frac{\alpha}{\beta}$

Therefore, $\lim\limits_{x\to 0}\frac{\tan\alpha x}{\tan\beta x} = \frac{\alpha}{\beta}$

Q. 5. Evaluate the following limits:

$$\lim_{x\to 0}\frac{\sin 4x}{\tan 7x}$$

Answer : To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim\limits_{x\to 0}\frac{\sin x}{x} = 1$ and $\lim\limits_{x\to 0}\frac{\tan x}{x} = 1$

So $\lim\limits_{x\to 0}\frac{\sin 4x}{\tan 7x} = \lim\limits_{x\to 0}(\frac{\sin 4x}{4x}) \times \frac{7x}{\sin 7x} \times \frac{4x}{7x} = \frac{4x}{7x} = \frac{4}{7}$

Therefore, $\lim\limits_{x\to 0}\frac{\sin 4x}{\tan 7x} = \frac{4}{7}$

Q. 6. Evaluate the following limits:

$$\lim_{x\to 0}\frac{\tan 3x}{\sin 4x}$$

Answer : To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim\limits_{x\to 0}\frac{\tan x}{x} = 1$ and $\lim\limits_{x\to 0}\frac{\sin x}{x} = 1$

So $\lim\limits_{x\to 0}\frac{\tan 3x}{\sin 4x} = \lim\limits_{x\to 0}(\frac{\tan 3x}{3x}) \times \frac{4x}{\sin 4x} \times \frac{3x}{4x} = \frac{3x}{4x} = \frac{3}{4}$

Therefore, $\lim\limits_{x\to 0}\frac{\tan 3x}{\sin 4x} = \frac{3}{4}$

Q. 7. Evaluate the following limits:

$$\lim_{x\to 0}\frac{\sin mx}{\tan nx}$$

Answer : To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim\limits_{x\to 0}\frac{\sin x}{x} = 1$ and $\lim\limits_{x\to 0}\frac{\tan x}{x} = 1$

So $\lim\limits_{x\to 0}\frac{\sin mx}{\tan nx} = \lim\limits_{x\to 0}(\frac{\sin mx}{mx}) \times \frac{nx}{\tan nx} \times \frac{mx}{nx} = \frac{mx}{nx} = \frac{m}{n}$

Therefore, $\lim\limits_{x\to 0}\frac{\sin mx}{\tan nx} = \frac{m}{n}$

Q. 8. Evaluate the following limits:

$$\lim_{x\to 0}\frac{\sin x - 2\sin 3x + \sin 5x}{x}$$

Answer : To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, inderterminate Form is $\frac{0}{0}$

Formula used: $\lim_{x \to 0} \frac{\sin x}{x} = 1$

So $\lim_{x \to 0} \frac{\sin x - 2\sin 3x + \sin 5x}{x} = \lim_{x \to 0} (\frac{\sin x}{x} - \frac{2\sin 3x}{x} + \frac{\sin 5x}{x}) = \lim_{x \to 0} (\frac{\sin x}{x} - \frac{2\sin 3x}{3x} \times 3 + \frac{5\sin 5x}{5x})$

By using the above formula, we have

$\therefore \lim_{x \to 0} (\frac{\sin x}{x} - \frac{2\sin 3x}{3x} \times 3 + \frac{5\sin 5x}{5x}) = 1 - 2 \times 3 + 5 = 0$

Therefore, $\lim_{x \to 0} \frac{\sin x - 2\sin 3x + \sin 5x}{x} = 0$

Q. 9. Evaluate the following limits:

$$\lim_{x \to \pi/6} \frac{(2\sin^2 x + \sin x - 1)}{(2\sin^2 x - 3\sin x + 1)}$$

Answer : To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim_{x\to 0}\frac{\sin x}{x} = 1$ or we can used L hospital Rule,

So, by using the rule, Differentiate numerator and denominator

$$\lim_{x\to\frac{\pi}{6}}\frac{4\sin x\cos x+\cos x}{4\sin x\cos x-3\cos x} = \lim_{x\to\frac{\pi}{6}}\frac{4\sin x+1}{4\sin x-3} = \frac{2+1}{2-3} = -3$$

Therefore, $\lim_{x\to 0}\frac{2\sin^2 x+\sin x-1}{2\sin^2 x-3\sin x+1} = -3$

Q. 10. Evaluate the following limits:

$$\lim_{x\to 0}\frac{(\sin 2x+3x)}{(2x+\sin 3x)}$$

Answer : To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim_{x\to 0}\frac{\sin x}{x} = 1$ or we can used L hospital Rule,

So, by using the above formula, we have

Divide numerator and denominator by x,

$$\lim_{x\to 0}\frac{\sin 2x+3x}{2x+\sin 3x} = \lim_{x\to 0}\frac{\frac{\sin 2x+3x}{x}}{\frac{2x+\sin 3x}{x}} = \lim_{x\to 0}\frac{\frac{\sin 2x}{x}+3}{2+\frac{\sin 3x}{x}} = \lim_{x\to 0}\frac{\frac{2\sin 2x}{2x}+3}{2+\frac{3\sin 3x}{3x}} = \frac{2+3}{2+3} = 1$$

ALTER: by using the rule, Differentiate numerator and denominator

$$\lim_{x\to 0}\frac{2\cos 2x+3}{2+3\cos 3x}=\frac{5}{5}=1$$

$$\text{Therefore,}\lim_{x\to 0}\frac{\sin 2x+3x}{2x+\sin 3x}=1$$

Q. 11. Evaluate the following limits:

$$\lim_{x\to 0}\frac{(\tan 2x-x)}{(3x-\tan x)}$$

Answer : To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim_{x\to 0}\frac{\tan x}{x}=1$ or we can used L hospital Rule,

So, by using the above formula, we have

Divide numerator and denominator by x,

$$\lim_{x\to 0}\frac{\tan 2x-x}{3x-\tan x}=\lim_{x\to 0}\frac{\frac{\tan 2x-x}{x}}{\frac{3x-\tan x}{x}}=\lim_{x\to 0}\frac{\frac{\tan 2x}{x}-1}{3-\frac{\tan x}{x}}=\lim_{x\to 0}\frac{\frac{2\tan 2x}{2x}-1}{3-\frac{\tan x}{x}}=\frac{2-1}{3-1}=\frac{1}{2}$$

$$\text{Therefore,}\lim_{x\to 0}\frac{\tan 2x-x}{3x-\tan x}=\frac{1}{2}$$

Q. 12. Evaluate the following limits:

$$\lim_{x\to 0}\frac{(x^2-\tan 2x)}{\tan x}$$

Answer : To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim_{x\to 0}\frac{\tan x}{x} = 1$ or we can used L hospital Rule,

So, by using the above formula, we have

Divide numerator and denominator by x,

$$\lim_{x\to 0}\frac{x^2-\tan 2x}{\tan x} = \lim_{x\to 0}\frac{\frac{x^2-\tan 2x}{x}}{\frac{\tan x}{x}} = \lim_{x\to 0}\frac{x-\frac{\tan 2x}{x}}{\frac{\tan x}{x}} = \lim_{x\to 0}\frac{x-\frac{2\tan 2x}{2x}}{\frac{\tan x}{x}} = \frac{0-2}{1} = -2$$

Therefore, $\lim_{x\to 0}\frac{x^2-\tan 2x}{\tan x} = -2$

Q. 13. Evaluate the following limits:

$$\lim_{x\to 0}\frac{x\cos x + \sin x}{x^2 + \tan x}$$

Answer : To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim_{x\to 0}\frac{\sin x}{x} = 1$ and $\lim_{x\to 0}\frac{\tan x}{x} = 1$

So, by using the above formula, we have

Divide numerator and denominator by x,

$$\lim_{x\to0}\frac{x\cos x+\sin x}{x^2+\tan x}=\lim_{x\to0}\frac{\frac{x\cos x+\sin x}{x}}{\frac{x^2+\tan x}{x}}=\lim_{x\to0}\frac{\cos x+\frac{\sin x}{x}}{x+\frac{\tan x}{x}}=\frac{1+1}{0+1}=2$$

Therefore, $\lim_{x\to0}\frac{x\cos x+\sin x}{x^2+\tan x}=2$

Q. 14. Evaluate the following limits:

$$\lim_{x\to0}\frac{\tan x-\sin x}{\sin^3 x}$$

Answer : To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

NOTE : $\tan x - \sin x = \frac{\sin x}{\cos x} - \sin x = \frac{\sin x - \sin x\cos x}{\cos x} = \sin x\,(\frac{1-\cos x}{\cos x})$

$$\lim_{x\to0}\frac{\tan x-\sin x}{\sin^3 x}=\lim_{x\to0}\frac{\frac{1-\cos x}{\cos x}}{\sin^2 x}=\lim_{x\to0}\frac{1-\cos x}{\sin^2 x\cos x}$$

Divide numerator and denominator by x^2,

$$=\lim_{x\to0}\frac{\frac{1-\cos x}{x^2}}{\frac{\sin^2 x\cos x}{x^2}}$$

Formula used: $\lim_{x\to 0}\frac{1-\cos x}{x^2} = 1/2$ and $\lim_{x\to 0}\frac{\sin x}{x} = 1$ or we can used L hospital Rule,

So, by using the above formula, we have

$$\lim_{x\to 0}\frac{\frac{1-\cos x}{x^2}}{\frac{\sin^2 x\cos x}{x^2}} = \frac{\frac{1}{2}}{1} = \frac{1}{2}$$

Therefore, $\lim_{x\to 0}\frac{\tan x-\sin x}{\sin^3 x} = \frac{1}{2}$

Q. 15. Evaluate the following limits:

$$\lim_{x\to 0} x\ \operatorname{cosec} x$$

Answer : To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form are $0 \times \infty$

Formula used: $\lim_{x\to 0}\frac{x}{\sin x} = 1$

So, by using the above formula, we have

$$\lim_{x\to 0} x\ \operatorname{cosec} x = \lim_{x\to 0}\frac{x}{\sin x} = 1$$

Therefore, $\lim_{x\to 0} x\ \operatorname{cosec} x = 1$

Q. 16. Evaluate the following limits:

$$\lim_{x\to 0}(x\cot 2x)$$

Answer : To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $0 \times \infty$

Formula used: $\lim_{x\to 0}\frac{x}{\tan x} = 1$

So, by using the above formula, we have

$$\lim_{x\to 0} x\ \cot 2x = \lim_{x\to 0}\frac{2x}{2\tan 2x} = \frac{1}{2}$$

Therefore, $\lim_{x\to 0} x\ \cot x = \frac{1}{2}$

Q. 17. Evaluate the following limits:

$$\lim_{x\to 0}\frac{\sin x \cos x}{3x}$$

Answer : To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim_{x\to 0}\frac{\sin x}{x} = 1$

So, by using the above formula, we have

$$\lim_{x\to 0}\frac{\sin x\cos x}{3x}=\lim_{x\to 0}\frac{\sin x}{x}\times\frac{\cos x}{3}=\frac{1}{3}$$

Therefore, $\lim_{x\to 0}\frac{\sin x\cos x}{3x}=\frac{1}{3}$

Q. 18. Evaluate the following limits:

$$\lim_{x\to 0}\frac{\sin(x/4)}{x}$$

Answer : To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim_{x\to 0}\frac{\sin x}{x}=1$

So, by using the above formula, we have

$$\lim_{x\to 0}\frac{\sin(x/4)}{x}=\lim_{x\to 0}\frac{\sin(x/4)}{4(x/4)}=\frac{1}{4}$$

Therefore, $\lim_{x\to 0}\frac{\sin(x/4)}{x}=\frac{1}{4}$

Q. 19. Evaluate the following limits:

$$\lim_{x\to 0}\frac{\tan(x/2)}{3x}$$

Answer : To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim\limits_{x\to 0}\frac{\tan x}{x} = 1$

So, by using the above formula, we have

$\lim\limits_{x\to 0}\frac{\tan(x/2)}{3x} = \lim\limits_{x\to 0}\frac{\tan(x/2)}{6(x/2)} = \frac{1}{6}$ [Divide and multiply with 2 on denominator]

Therefore, $\lim\limits_{x\to 0}\frac{\tan(x/2)}{3x} = \frac{1}{6}$

Q. 20. Evaluate the following limits:

$$\lim_{x\to 0}\frac{1-\cos x}{\sin^2 x}$$

Answer : To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

[NOTE: $1 - \cos x = 2\sin^2(x/2)$]

Formula used: $\lim\limits_{x\to 0}\frac{\sin x}{x} = 1$

So, by using the above formula, we have

$$\lim_{x\to 0}\frac{1-\cos x}{\sin^2 x}=\lim_{x\to 0}\frac{2\ \sin 2(x/2)}{\sin^2 x}$$

Divide numerator and denominator by x^2, we have

$$\lim_{x\to 0}\frac{2\ \sin^2(x/2)}{\sin^2 x}=\lim_{x\to 0}\frac{\frac{2\sin^2(\frac{x}{2})}{x^2}}{\frac{\sin^2 x}{x^2}}=\lim_{x\to 0}\frac{\frac{2\sin^2(\frac{x}{2})}{4\frac{x^2}{4}}}{\frac{\sin^2 x}{x^2}}=\frac{\frac{2}{4}}{1}=\frac{2}{4}=\frac{1}{2}$$

[NOTE: $\lim\limits_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2}$]

Therefore, $\lim\limits_{x\to 0}\frac{1-\cos x}{\sin^2 x}=\frac{1}{2}$

Q. 21. Evaluate the following limits:

$$\lim_{x\to 0}\frac{1-\cos 3x}{x^2}$$

Answer : To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim\limits_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2}$

So, by using the above formula, we have

$$\lim_{x\to 0}\frac{1-\cos 3x}{x^2} = \lim_{x\to 0}\frac{9[1-\cos 3x]}{(3x)^2} = \frac{9}{2}$$

Therefore, $\lim_{x\to 0}\frac{1-\cos 3x}{x^2} = \frac{9}{2}$

Q. 22. Evaluate the following limits:

$$\lim_{x\to 0}\frac{1-\cos x}{\sin^2 2x}$$

Answer : To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim_{x\to 0}\frac{1-\cos x}{x^2} = \frac{1}{2}$ and $\lim_{x\to 0}\frac{\sin x}{x} = 1$

Divide numerator and denominator by x^2, we have

So, by using the above formula, we have

$$\lim_{x\to 0}\frac{1-\cos x}{\sin^2 2x} = \lim_{x\to 0}\frac{\frac{[1-\cos x]}{x^2}}{\frac{\sin^2 2x}{x^2}} = \frac{1}{2}$$

Therefore, $\lim_{x\to 0}\frac{1-\cos x}{\sin^2 2x} = \frac{1}{2}$

Q. 23. Evaluate the following limits:

$$\lim_{x\to 0}\frac{1-\cos 2x}{3\tan^2 x}$$

Answer : To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim_{x\to 0}\frac{1-\cos x}{x^2} = \frac{1}{2}$ and $\lim_{x\to 0}\frac{\tan x}{x} = 1$

Divide numerator and denominator by x^2, we have

So, by using the above formula, we have

$$\lim_{x\to 0}\frac{1-\cos 2x}{3\tan^2 x} = \lim_{x\to 0}\frac{\frac{4[1-\cos 2x]}{(4)x^2}}{\frac{3\tan^2 x}{x^2}} = \frac{4}{6} = \frac{2}{3}$$

Therefore, $\lim_{x\to 0}\frac{1-\cos 2x}{3\tan^2 x} = \frac{1}{6}$

Q. 24. Evaluate the following limits:

$$\lim_{x\to 0}\frac{(1-\cos 4x)}{(1-\cos 6x)}$$

Answer :

To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim\limits_{x \to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$

Divide numerator and denominator by x^2, we have

So, by using the above formula, we have

$$\lim_{x \to 0} \frac{1-\cos 4x}{1-\cos 6x} = \lim_{x \to 0} \frac{\frac{16[1-\cos 4x]}{(4x)^2}}{\frac{36[1-\cos 6x]}{(6x)^2}} = \frac{\frac{16}{2}}{\frac{36}{2}} = \frac{8}{18} = \frac{4}{9}$$

Therefore, $\lim\limits_{x \to 0} \frac{1-\cos 4x}{1-\cos 6x} = \frac{4}{9}$

Q. 25. Evaluate the following limits:

$$\lim_{x \to 0} \frac{1-\cos mx}{1-\cos nx}$$

Answer : To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim\limits_{x \to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$

Divide numerator and denominator by m^2 and n^2, we have

So, by using the above formula, we have

$$\lim_{x\to 0}\frac{1-\cos mx}{1-\cos nx}=\lim_{x\to 0}\frac{\frac{m^2[1-\cos mx]}{(mx)^2}}{\frac{n^2[1-\cos nx]}{(nx)^2}}=\frac{m^2}{n^2}$$

Therefore, $\lim_{x\to 0}\frac{1-\cos mx}{1-\cos nx}=\frac{m^2}{n^2}$

Q. 26. Evaluate the following limits:

$$\lim_{x\to 0}\frac{2\sin x-\sin 2x}{x^3}$$

Answer : To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

We know that sin 2x = 2 sin x cosx

Formula used: $\lim_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2}$ and $\lim_{x\to 0}\frac{\sin x}{x}=1$

So, by using the above formula, we have

$$\lim_{x\to 0}\frac{2\sin x-\sin 2x}{x^3}=\lim_{x\to 0}\frac{2\sin x-2\sin x\cos x}{x^3}=\lim_{x\to 0}\frac{2\sin x[1-\cos x]}{x^3}=\lim_{x\to 0}\frac{2\sin x}{x}\times\frac{[1-\cos x]}{x^2}=\frac{2}{2}=1$$

Therefore, $\lim_{x\to 0}\frac{2\sin x-\sin 2x}{x^3}=1$

Q. 27. Evaluate the following limits:

$$\lim_{x\to 0}\frac{(\tan x - \sin x)}{x^3}$$

Answer : To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

NOTE : $\tan x - \sin x = \frac{\sin x}{\cos x} - \sin x = \frac{\sin x - \sin x \cos x}{\cos x} = \sin x \left(\frac{1-\cos x}{\cos x}\right)$

$$\lim_{x\to 0}\frac{\tan x - \sin x}{x^3} = \lim_{x\to 0}\frac{\left(\frac{1-\cos x}{\cos x}\right)\sin x}{x^3} = \lim_{x\to 0}\frac{1-\cos x}{x^2\ \cos x}\times\frac{\sin x}{x}$$

Formula used: $\lim_{x\to 0}\frac{1-\cos x}{x^2} = 1/2$ and $\lim_{x\to 0}\frac{\sin x}{x} = 1$ or we can used L hospital Rule,

So, by using the above formula, we have

$$\lim_{x\to 0}\frac{1-\cos x}{x^2\ \cos x}\times\frac{\sin x}{x} = \frac{1}{2}$$

Therefore, $\lim_{x\to 0}\frac{\tan x - \sin x}{x^3} = \frac{1}{2}$

Q. 28. Evaluate the following limits:

$$\lim_{x\to 0}\frac{\tan 2x - \sin 2x}{x^3}$$

Answer : To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

$$\lim_{x\to 0}\frac{\tan 2x-\sin 2x}{x^3}=\lim_{x\to 0}\frac{\sin 2x+\sin 2x\cos 2x}{x^3}=\lim_{x\to 0}\frac{\sin 2x(1-\cos 2x)}{x^3}=\lim_{x\to 0}\frac{2\sin 2x}{2x}\times\frac{4(1-\cos 2x)}{(2x)^2}=4$$

Therefore, $\lim_{x\to 0}\frac{\tan 2x-\sin\ \ 2x}{x^3}=4$

Q. 29. Evaluate the following limits:

$$\lim_{x\to 0}\frac{\operatorname{cosec} x-\cot x}{x}$$

Answer : To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\infty \times \infty$

cosec x – cot x = (1 – cos x)/sinx

$$\lim_{x\to 0}\frac{\operatorname{cosec} x-\cot x}{x}=\lim_{x\to 0}\frac{1-\cos x}{x\sin x}=\lim_{x\to 0}\frac{\frac{1-\cos x}{x^2}}{\frac{x\sin x}{x^2}}=\lim_{x\to 0}\frac{\frac{1-\cos x}{x^2}}{\frac{\sin x}{x}}$$

Formula used: $\lim_{x\to 0}\frac{1-\cos x}{x^2}=1/2$ and $\lim_{x\to 0}\frac{\sin x}{x}=1$

$$\lim_{x\to 0}\frac{\operatorname{cosec} x-\cot x}{x}=\lim_{x\to 0}\frac{\frac{1-\cos x}{x^2}}{\frac{\sin x}{x}}=\frac{1}{2}$$

Therefore, $\lim_{x\to 0}\frac{\operatorname{cosec} x-\cot x}{x}=\frac{1}{2}$

Q. 30. Evaluate the following limits:

$$\lim_{x\to 0}\frac{\cot 2x - \operatorname{cosec} 2x}{x}$$

Answer : To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form are $\infty \times \infty$

cosec 2x – cot 2x = (1 – cos 2x)/sin 2x

$$\lim_{x\to 0}\frac{\cot 2x - \operatorname{cosec} 2x}{x} = \lim_{x\to 0}\frac{\cos 2x - 1}{x\sin 2x} = \lim_{x\to 0}\frac{\frac{\cos 2x - 1}{x^2}}{\frac{x\sin 2x}{x^2}} = \lim_{x\to 0}\frac{\frac{4[\cos 2x - 1]}{(2x)^2}}{\frac{2\sin 2x}{2x}}$$

Formula used: $\lim_{x\to 0}\frac{1-\cos x}{x^2} = 1/2$ and $\lim_{x\to 0}\frac{\sin x}{x} = 1$

$$\lim_{x\to 0}\frac{\cot 2x - \operatorname{cosec} 2x}{x} = \lim_{x\to 0}\frac{\frac{4[\cos 2x - 1]}{(2x)^2}}{\frac{2\sin 2x}{2x}} = \frac{-4}{2} = -2$$

Therefore, $\lim_{x\to 0}\frac{\cot 2x - \operatorname{cosec} 2x}{x} = -2$

Q. 31. Evaluate the following limits:

$$\lim_{x\to 0}\frac{\sin 2x(1 - \cos 2x)}{x^3}$$

Answer : To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

$$\lim_{x\to0}\frac{\sin 2x(1-\cos 2x)}{x^3}=\lim_{x\to0}\frac{\sin 2x}{x}\times\frac{(1-\cos 2x)}{x^2}=\lim_{x\to0}\frac{2\sin 2x}{2x}\times\frac{4(1-\cos 2x)}{(2x)^2}$$

Formula used: $\lim_{x\to0}\frac{1-\cos x}{x^2}=1/2$ and $\lim_{x\to0}\frac{\sin x}{x}=1$

$$\lim_{x\to0}\frac{\sin 2x(1-\cos 2x)}{x^3}=4$$

Therefore, $\lim_{x\to0}\frac{\sin 2x(1-\cos 2x)}{x^3}=4$

Q. 32 . Evaluate the following limits:

$$\lim_{x\to\frac{\pi}{4}}\frac{\sec^2 x-2}{\tan x-1}$$

Answer : To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

By using L hospital Rule,

Differtiate both sides w.r.t x

So $\lim_{x\to\frac{\pi}{4}}\frac{\sec^2 x-2}{\tan x-1}$ = So $\lim_{x\to\frac{\pi}{4}}\frac{2\sec x(\sec x\tan x)-0}{\sec^2 x-0}=\lim_{x\to\frac{\pi}{4}}\frac{2\sec x(\sec x\tan x)}{\sec^2 x}=\lim_{x\to\frac{\pi}{4}}2\tan x=2$

Therefore, $\lim_{x \to \frac{\pi}{4}} \frac{\sec^2 x - 2}{\tan x - 1} = 2$

Q. 33. Evaluate the following limits:

$$\lim_{x \to \frac{\pi}{4}} \frac{\text{cosec}^2 x - 2}{\cot x - 1}$$

Answer : To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

By using L hospital Rule,

Differtiate both sides w.r.t x

So $\lim_{x \to \frac{\pi}{4}} \frac{\text{cosec}^2 x - 2}{\cot x - 1} = \lim_{x \to \frac{\pi}{4}} \frac{2\text{cosec}\, x(-\text{cosec}\, x \cot x) - 0}{-\text{cosec}^2 x - 0} = \lim_{x \to \frac{\pi}{4}} \frac{2\text{cosec}\, x(-\text{cosec}\, x \cot x)}{-\text{cosec}^2 x} = \lim_{x \to \frac{\pi}{4}} 2 \cot x = 2$

Therefore, $\lim_{x \to \frac{\pi}{4}} \frac{\text{cosec}^2 x - 2}{\cot x - 1} = 2$

Q. 34. Evaluate the following limits:

$$\lim_{x \to \frac{\pi}{4}} \frac{1 - \tan x}{x - \frac{\pi}{4}}$$

Answer : To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Formis $\frac{0}{0}$

By using L hospital Rule,

Differtiate both sides w.r.t x

So $\lim_{x \to \frac{\pi}{4}} \frac{1-\tan x}{x - \frac{\pi}{4}} = \lim_{x \to \frac{\pi}{4}} \frac{0-\sec^2 x}{1-0} = \lim_{x \to \frac{\pi}{4}} \frac{-\sec^2 x}{1} =$ -2

Therefore, $\lim_{x \to \frac{\pi}{4}} \frac{1-\tan x}{x - \frac{\pi}{4}} = -2$

Q. 35. Evaluate the following limits:

$$\lim_{x \to \pi} \frac{\sin 3x - 3\sin x}{(\pi - x)^3}$$

Answer : To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

By using L hospital Rule,

Differtiate both sides w.r.t x

So $\lim_{x \to \pi} \frac{\sin 3x - 3\sin x}{(\pi - x)^3} = \lim_{x \to \pi} \frac{3\cos 3x - 3\cos x}{-3(\pi - x)^2}$

Again, indeterminate Form is $\frac{0}{0}$

So, Differtiate both sides w.r.t x again, we have

$$\lim_{x\to\pi}\frac{\sin 3x-3\sin x}{(\pi-x)^3}=\lim_{x\to\pi}\frac{-9\sin 3x+3\sin x}{6(\pi-x)}$$

Again, indeterminate Form is $\frac{0}{0}$

So, Differtiate both sides w.r.t x again, we have

$$\lim_{x\to\pi}\frac{\sin 3x-3\sin x}{(\pi-x)^3}=\lim_{x\to\pi}\frac{-27\cos 3x+3\cos x}{-6}=\frac{-27\cos 3\pi+3\cos \pi}{-6}=\frac{27-3}{-6}=-4$$

Therefore, $\lim_{x\to\pi}\frac{\sin 3x-3\sin x}{(\pi-x)^3}=-4$

Q. 36. Evaluate the following limits:

$$\lim_{x\to\frac{\pi}{2}}\frac{1+\cos 2x}{(\pi-2x)^2}$$

Answer : To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

By using L hospital Rule,

Differtiate both sides w.r.t x

So $\lim\limits_{x\to\frac{\pi}{2}}\frac{1+\cos 2x}{(\pi-2x)^2}=\lim\limits_{x\to\frac{\pi}{2}}\frac{0+(-2\sin 2x)}{2(\pi-2x)(-2)}=\lim\limits_{x\to\frac{\pi}{2}}\frac{-2\sin 2x}{-4(\pi-2x)}=\lim\limits_{x\to\frac{\pi}{2}}\frac{2\sin 2x}{4(\pi-2x)}$

Again, indeterminate Form is $\frac{0}{0}$

So, Differtiate both sides w.r.t x again, we have

$$\lim_{x\to\frac{\pi}{2}}\frac{1+\cos 2x}{(\pi-2x)^2}=\lim_{x\to\frac{\pi}{2}}\frac{4\cos 2x}{4(-2)}=\lim_{x\to\frac{\pi}{2}}\frac{\cos 2x}{(-2)}=\frac{\cos\pi}{(-2)}=\frac{-1}{(-2)}=\frac{1}{2}$$

Therefore, $\lim\limits_{x\to\frac{\pi}{2}}\frac{1+\cos 2x}{(\pi-2x)^2}=\frac{1}{2}$

Q. 37. Evaluate the following limits:

$$\lim_{x\to a}\frac{(\cos x-\cos a)}{(x-a)}$$

Answer :

$$=\lim_{x\to a}\frac{(\cos x-\cos a)}{(x-a)}$$

$$=\lim_{x\to a}\frac{-2\times\sin\left(\frac{x+a}{2}\right)\sin\left(\frac{x-a}{2}\right)}{x-a}\ [\because \cos x-\cos a=-2\times\sin\frac{x+a}{2}\sin\frac{x-a}{2}]$$

$$=\lim_{x\to a}\sin\frac{x+a}{2}\times-\frac{\sin\left(\frac{x-a}{2}\right)}{\frac{(x-a)}{2}}$$

$$=-1\times\lim_{x\to a}\sin\frac{x+a}{2}\left[\because \lim_{x\to a}\frac{\sin\theta}{\theta}=1\right]$$

$$=-1\times\sin\frac{(a+a)}{2}$$

$$= -1 \times \sin\frac{2a}{2}$$

= -sin (a)

$$\therefore \lim_{x \to a}\frac{(\cos x - \cos a)}{(x - a)} = -\sin a$$

Q. 38. Evaluate the following limits:

$$\lim_{x \to a}\frac{(\sin x - \sin a)}{(x - a)}$$

Answer :

$$= \lim_{x \to a}\frac{(\sin x - \sin a)}{(x - a)}$$

$$= \lim_{x \to a}\frac{\left(2 \times \cos\frac{x+a}{2}\sin\frac{x-a}{2}\right)}{(x - a)} \left[\because \sin x - \sin a = 2 \times \cos\frac{x+a}{2}\sin\frac{x-a}{2}\right]$$

$$= 1 \times \lim_{x \to a}\cos\frac{x+a}{2} \left[\because \lim_{x \to a}\frac{\sin\theta}{\theta} = 1\right]$$

$$= \cos\frac{a+a}{2}$$

=cosa

$$\therefore \lim_{x \to a}\frac{\sin x - \sin a}{x - a} = \cos a$$

Q. 39. Evaluate the following limits:

$$\lim_{x \to a}\frac{(\sin x - \sin a)}{(\sqrt{x} - \sqrt{a})}$$

Answer :

$$= \lim_{x \to a}\frac{(\sin x - \sin a)}{(\sqrt{x} - \sqrt{a})}$$

$$= \lim_{x \to a} \frac{(\sin x - \sin a)}{(\sqrt{x} - \sqrt{a})} \times \frac{(\sqrt{x} + \sqrt{a})}{(\sqrt{x} + \sqrt{a})}$$ [Multiply and divide by √x-√a]

$$= \lim_{x \to a} \frac{(\sin x - \sin a) \times (\sqrt{x} + \sqrt{a})}{(x - a)}$$

$$= \cos a \times \lim_{x \to a} (\sqrt{x} + \sqrt{a}) \left[\because \lim_{x \to a} \frac{\sin x - \sin a}{x - a} = \cos a\right]$$

=2√a×cosa

=2√acosa

$$\therefore \lim_{x \to a} \frac{(\sin x - \sin a)}{(\sqrt{x} - \sqrt{a})} = 2\sqrt{a} \cos a$$

Q. 40. Evaluate the following limits:

$$\lim_{x \to 0} \frac{\sin 5x - \sin 3x}{\sin x}$$

Answer :

$$= \lim_{x \to 0} \frac{\left(2 \sin \frac{5x - 3x}{2} \cos \frac{5x + 3x}{2}\right)}{\sin x} \left[Applying \sin C - \sin D = 2 \sin \frac{C - D}{2} \cos \frac{C + D}{2}\right]$$

$$= \lim_{x \to 0} 2 \cos 4x$$

= 2 × 1

$$\therefore \lim_{x \to 0} \frac{\sin 5x - \sin 3x}{\sin x} = 2$$

Q. 41. Evaluate the following limits:

$$\lim_{x \to 0} \frac{(\cos 3x - \cos 5x)}{x^2}$$

Answer :

$$= \lim_{x \to 0} \frac{\left(1 - \cos 5x - (1 - \cos 3x)\right)}{x^2}$$

$$= \lim_{x \to 0} \left(\frac{1 - \cos 5x}{x^2} - \frac{1 - \cos 3x}{x^2}\right)$$

$$= \left(\lim_{x \to 0} \frac{1 - \cos 5x}{x^2} \times \frac{25}{25}\right) - \left(\lim_{x \to 0} \frac{1 - \cos 3x}{x^2} \times \frac{9}{9}\right)$$

$$= \frac{25}{2} - \frac{9}{2} \left[\because \lim_{x \to 0} \frac{1 - \cos ax}{(ax)^2} = \frac{1}{2}\right]$$

$$= \frac{16}{2}$$

=8

$$\therefore \lim_{x \to 0} \frac{(\cos 3x - \cos 5x)}{x^2} = 8$$

Q. 42. Evaluate the following limits:

$$\lim_{x \to 0} \frac{(\sin 3x + \sin 5x)}{(\sin 6x - \sin 4x)}$$

Answer :

$$= \lim_{x \to 0} \frac{\left(2 \times \sin \frac{3x + 5x}{2} \times \cos \frac{3x - 5x}{2}\right)}{\left(2 \times \cos \frac{6x + 4x}{2} \sin \frac{6x - 4x}{2}\right)}$$

$$= \lim_{x \to 0} \frac{\sin 4x \cos x}{\cos 5x \sin x}$$

$$= \lim_{x \to 0} \frac{\sin 4x}{\cos 5x \times \frac{\sin x}{\cos x}} \times \frac{4x}{4x}$$

$$= 4 \times \lim_{x \to 0} \frac{\sin 4x}{4x} \times \frac{1}{\cos 5x} \times \frac{x}{\tan x} \left[\begin{array}{l} \because \lim_{x \to 0} \frac{\sin \theta}{\theta} = 1 \\ \lim_{x \to 0} \frac{\theta}{\tan \theta} = 1 \end{array}\right]$$

= 4

$$\therefore \lim_{x\to 0}\frac{(\sin 3x+\sin 5x)}{(\sin 6x-\sin 4x)}=4$$

Q. 43. Evaluate the following limits:

$$\lim_{x\to 0}\frac{[\sin(2+x)-\sin(2-x)]}{x}$$

Answer :

$$=\lim_{x\to 0}\frac{[\sin(2+x)-\sin(2-x)]}{x}$$

$$=\lim_{x\to 0}\frac{\left[2\times\cos\frac{(2+x+2-x)}{2}\times\sin\frac{(2+x-2+x)}{2}\right]}{x}$$

$$=\lim_{x\to 0}\frac{(2\times\cos 2\times\sin x)}{x}$$

$$=2\cos 2\lim_{x\to 0}\frac{\sin x}{x}$$

=2cos2

$$\lim_{x\to 0}\frac{[\sin(2+x)-\sin(2-x)]}{x}=2\cos 2$$

Q. 44. Evaluate the following limits:

$$\lim_{x\to 0}\frac{(1-\cos 2x)}{(\cos 2x-\cos 8x)}$$

Answer :

$$=\lim_{x\to 0}\frac{1-\cos 2x}{(\cos 2x-\cos 8x)}$$

$$=\lim_{x\to 0}\frac{2\times\sin x\times\sin x}{2\times\sin 3x\times\sin 5x}\times\frac{5x\times 3x}{x\times x}\times\frac{1}{15}$$

$$= \frac{1}{15} \times 1 \times 1 \times 1 \times 1 \left[\because \lim_{x \to 0} \frac{\sin\theta}{\theta} = 1\right]$$

$$= \frac{1}{15}$$

$$\lim_{x \to 0} \frac{1 - \cos 2x}{(\cos 2x - \cos 8x)} = \frac{1}{15}$$

Q. 45. Evaluate the following limits:

$$\lim_{x \to \frac{\pi}{2}} \left(\frac{\pi}{2} - x\right) \tan x$$

Answer :

$$= \lim_{x \to \frac{\pi}{2}} \left(\frac{\pi}{2} - x\right) \tan x$$

$$= -1 \times \lim_{y \to 0} y \tan\left(y + \frac{\pi}{2}\right) \left[x - \frac{\pi}{2} = y\right]$$

$$= -1 \times \lim_{y \to 0} y \cot y \times -1$$

$$= \lim_{y \to 0} \frac{y}{\tan y}$$

=1

$$\therefore \lim_{x \to \frac{\pi}{2}} \left(\frac{\pi}{2} - x\right) \tan x = 1$$

Q. 46. Evaluate the following limits:

$$\lim_{x \to 0} \frac{(\sqrt{1 + 2x} - \sqrt{1 - 2x}}{\sin x}$$

Answer :

$$= \lim_{x\to 0} \frac{\left(\sqrt{1+2x} - \sqrt{1-2x}\right)}{\sin x}$$

$$= \lim_{x\to 0} \frac{\left(\sqrt{1+2x} - \sqrt{1-2x}\right)}{\sin x} \times \frac{\sqrt{1+2x} + \sqrt{1-2x}}{\left(\sqrt{1+2x} + \sqrt{1-2x}\right)}$$

$$= \lim_{x\to 0} \frac{1+2x-1+2x}{\sin x} \times \frac{1}{\sqrt{1+2x} + \sqrt{1-2x}}$$

$$= 4 \times \lim_{x\to 0} \frac{x}{\sin x} \times \frac{1}{\sqrt{1+2x} + \sqrt{1-2x}}$$

$$= 4 \times \frac{1}{2} \times 1$$

=2

$$\therefore \lim_{x\to 0} \frac{\left(\sqrt{1+2x} - \sqrt{1-2x}\right)}{\sin x} = 2$$

Q. 47. Evaluate the following limits:

$$\lim_{h\to 0} \frac{(a+h)^2 \sin(a+h) - a^2 \sin a}{h}$$

Answer :

$$= \lim_{h\to 0} \frac{(a+h)^2 \sin(a+h) - a^2 \sin a}{h}$$

$$= \lim_{h\to 0} \frac{a^2(\sin(a+h) - \sin a) + 2ah\sin(a+h) + h^2 \sin(a+h)}{h}$$

$$= 2a\sin a + 0 + \lim_{h\to 0} \frac{a^2 \times 2 \times \cos\left(a + \frac{h}{2}\right) \times \sin h}{h}$$

$$= 2a\sin a + 2a^2 \cos a$$

$=2a^2\cos a+2a\sin a$

$$\therefore \lim_{h\to 0}\frac{(a+h)^2\sin(a+h)-a^2\sin a}{h}=2a^2\cos a+2a\sin a$$

Q. 48. Evaluate the following limits:

$$\lim_{h\to 0}\frac{(e^{3+x}-\sin x-e^3)}{x}$$

Answer :

$$=\lim_{x\to 0}\frac{(e^{3+x}-\sin x-e^3)}{x}$$

$$=-\lim_{x\to 0}\frac{\sin x}{x}+\lim_{x\to 0}\frac{e^{3+x}-e^3}{x}$$

$$=-1+\lim_{x\to 0}\frac{e^3(e^x-1)}{x}$$

$=-1+e^3$

$$\therefore \lim_{x\to 0}\frac{(e^{3+x}-\sin x-e^3)}{x}=e^3-1$$

Q. 49. Evaluate the following limits:

$$\lim_{x\to 0}\frac{(e^{\tan x}-1)}{\tan x}$$

Answer :

$$=\lim_{x\to 0}\frac{(e^{\tan x}-1)}{\tan x}$$

As x tends to 0, tan(x) also tends to zero,

So,

$$\lim_{x\to 0}\frac{(e^{\tan x}-1)}{\tan x}=\lim_{\tan x\to 0}\frac{(e^{\tan x}-1)}{\tan x}$$

=1

$$\therefore \lim_{x\to 0}\frac{(e^{\tan x}-1)}{\tan x}=1$$

Q. 50. Evaluate the following limits:

$$\lim_{x\to 0}\frac{(e^{\tan x}-1)}{x}$$

Answer :

$$=\lim_{x\to 0}\frac{e^{\tan x}-1}{x}$$

$$=\lim_{x\to 0}\frac{e^{\tan x}-1}{x}\times\frac{\tan x}{\tan x}$$

$$=\lim_{x\to 0}\frac{e^{\tan x}-1}{\tan x}\times\frac{\tan x}{x}$$

=1×1

=1

$$\therefore \lim_{x\to 0}\frac{e^{\tan x}-1}{x}=1$$

Q. 51. Evaluate the following limits:

$$\lim_{x\to 0}\frac{ax+x\cos x}{b\sin x}$$

Answer :

$$=\lim_{x\to 0}\frac{ax+x\cos x}{b\sin x}$$

$$= \lim_{x \to 0} \frac{ax}{b \sin x} + \frac{x\cos x}{b \sin x}$$

$$= \lim_{x \to 0} \frac{ax}{b \sin x} + \lim_{x \to 0} \frac{x\cos x}{b \sin x}$$

$$= \frac{a}{b} + \frac{1}{b} \left[\begin{array}{l} \because \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1 \\ \lim_{x \to 0} \cos \theta = 1 \end{array} \right]$$

$$= \frac{a+1}{b}$$

$$\therefore \lim_{x \to 0} \frac{ax + x \cos x}{b \sin x} = \frac{a+1}{b}$$

Q. 52. Evaluate the following limits:

$$\lim_{x \to 0} \frac{\sin ax + bx}{ax + \sin bx}, \text{where } a, b, a + b \neq 0$$

Answer :

$$= \lim_{x \to 0} \frac{\sin(ax) + bx}{ax + \sin(bx)}$$

$$= \lim_{x \to 0} \frac{\sin(ax) + bx}{ax + \sin(bx)} \times \frac{bx}{ax} \times \frac{a}{b}$$

$$= \lim_{x \to 0} \frac{\frac{\sin ax + bx}{ax}}{\frac{ax + \sin bx}{bx}} \times \frac{a}{b}$$

$$= \frac{a}{b} \times \frac{\lim_{x \to 0} \frac{\sin ax + bx}{ax}}{\lim_{x \to 0} \frac{ax + \sin bx}{bx}}$$

$$= \frac{a}{b} \times \frac{1 + \frac{b}{a}}{1 + \frac{a}{b}}$$

$$= \frac{a}{b} \times \frac{b}{a}$$

=1

$$\therefore \lim_{x \to 0} \frac{\sin(ax) + bx}{ax + \sin(bx)} = 1$$

Q. 53. Evaluate the following limits:

$$\lim_{x \to 0} \frac{\sin(\pi - x)}{\pi(\pi - x)}$$

Answer :

$$= \lim_{x \to 0} \frac{\sin(\pi - x)}{\pi(\pi - x)}$$

$$= \lim_{x \to 0} \frac{\sin x}{\pi(\pi - x)} \times \frac{x}{x}$$

$$= \lim_{x \to 0} \frac{\sin x}{x} \times \lim_{x \to 0} \frac{\pi - (\pi - x)}{\pi(\pi - x)}$$

$$= 1 \times \lim_{x \to 0} \left(\frac{1}{\pi - x} - \frac{1}{\pi} \right)$$

$$= \frac{1}{\pi} - \frac{1}{\pi}$$

$$= 0$$

$$\therefore \lim_{x \to 0} \frac{\sin(\pi - x)}{\pi(\pi - x)} = 0$$

Q. 54. Evaluate the following limits:

$$\lim_{x \to \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}}$$

Answer :

$$= \lim_{x \to \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}}$$

As x tends to $\pi/2, x - \pi/2$ tends to zero.

Let,

$$y = x - \frac{\pi}{2}$$

$$= \lim_{y \to 0} \frac{\tan\left(2y + \frac{\pi}{2} \times 2\right)}{y}$$

$$= \lim_{y \to 0} \frac{\tan(\pi + 2y)}{y}$$

$$= \lim_{y \to 0} \frac{\tan 2y}{2y} \times 2$$

=2

$$\therefore \lim_{x \to \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}} = 2$$

Q. 55. Evaluate the following limits:

$$\lim_{x \to 0} \frac{\cos 2x - 1}{\cos x - 1}$$

Answer :

$$= \lim_{x \to 0} \frac{\cos 2x - 1}{\cos x - 1}$$

$$= \lim_{x \to 0} \frac{\cos 2x - 1}{\cos x - 1} \times \frac{4 \times x \times x}{2x \times 2x}$$

$$= 4 \times \lim_{x \to 0} \frac{\frac{\cos 2x - 1}{2x \times 2x}}{\frac{\cos x - 1}{x \times x}}$$

$$= 4 \times \frac{\lim_{x \to 0} \frac{\cos 2x - 1}{(2x)^2}}{\lim_{x \to 0} \frac{\cos x - 1}{x^2}}$$

$$= 4 \times \frac{\frac{1}{2}}{\frac{1}{2}} = 4$$

$$\therefore \lim_{x \to 0} \frac{\cos 2x - 1}{\cos x - 1} = 4$$

Q. 56. Evaluate the following limits:

$$\lim_{x \to 0} (\operatorname{cosec} x - \cot x)$$

Answer :

$$= \lim_{x \to 0} (\csc x - \cot x)$$

$$= \lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{\cos x}{\sin x} \right)$$

$$= \lim_{x \to 0} \left(\frac{1 - \cos x}{\sin x} \right)$$

$$= \lim_{x \to 0} \left(\frac{2 \times \sin\frac{x}{2} \times \sin\frac{x}{2}}{2 \times \sin\frac{x}{2} \times \cos\frac{x}{2}} \right) \; [\because 1 - \cos\theta = 2\sin\theta \times \sin\theta]$$

$$= \lim_{x\to 0}\left(\tan\frac{x}{2}\right)$$

=0

$$\therefore \lim_{x\to 0}(\csc x - \cot x) = 0$$

Q. 57. Evaluate the following limits:

$$\lim_{x\to 0}\frac{1-\cos 2mx}{1-\cos 2nx}$$

Answer :

$$= \lim_{x\to 0}\frac{1-\cos 2mx}{1-\cos 2nx}$$

$$= \lim_{x\to 0}\frac{\frac{1-\cos 2mx}{(2mx)^2}\times(2mx)^2}{\frac{1-\cos 2nx}{(2nx)^2}\times(2nx)^2}$$

$$= \frac{\frac{1}{2}}{\frac{1}{2}}\times\frac{m\times m}{n\times n}\left[\because \frac{1-\cos\theta}{\theta\times\theta} = \frac{1}{2}\right]$$

$$= \frac{m^2}{n^2}$$

$$\therefore \lim_{x\to 0}\frac{1-\cos 2mx}{1-\cos 2nx} = \frac{m^2}{n^2}$$

Q. 58. Evaluate the following limits:

$$\lim_{x\to 0}\frac{1-\cos mx}{1-\cos nx}$$

Answer :

$$= \lim_{x\to 0}\frac{1-\cos mx}{1-\cos nx}$$

$$= \lim_{x \to 0} \frac{\frac{1 - \cos mx}{mx \times mx}}{\frac{1 - \cos nx}{nx \times nx}} \times \frac{m \times m}{n \times n}$$

$$= \frac{m^2}{n^2}$$

$$\therefore \lim_{x \to 0} \frac{1 - \cos mx}{1 - \cos nx} = \frac{m^2}{n^2}$$

Q. 59. Evaluate the following limits:

$$\lim_{x \to 0} \frac{\sin^2 mx}{\sin^2 nx}$$

Answer :

$$= \lim_{x \to 0} \frac{\sin mx \times \sin mx}{\sin nx \times \sin nx}$$

$$= \lim_{x \to 0} \frac{\frac{\sin mx \times \sin mx}{mx \times mx}}{\frac{\sin nx \times \sin nx}{nx \times nx}} \times \frac{m^2}{n^2}$$

$$= \frac{m^2}{n^2}$$

$$\therefore \lim_{x \to 0} \frac{\sin mx \times \sin mx}{\sin nx \times \sin nx} = \frac{m^2}{n^2}$$

Q. 60. Evaluate the following limits:

$$\lim_{x \to 0} \frac{\sin 2x + \sin 3x}{2x + \sin 3x}$$

Answer :

$$= \lim_{x \to 0} \frac{\sin 2x + \sin 3x}{2x + \sin 3x} \times \frac{3x}{3x}$$

$$= \lim_{x \to 0} \frac{\frac{\sin 2x + \sin 3x}{3x}}{\frac{2x + \sin 3x}{3x}}$$

$$= \frac{\frac{2}{3} + 1}{\frac{2}{3} + 1}$$

=1

$$\therefore \lim_{x \to 0} \frac{\sin 2x + \sin 3x}{2x + \sin 3x} = 1$$

Q. 61. Evaluate the following limits:

$$\lim_{x \to 0} \frac{\sec 4x - \sec 2x}{\sec 3x - \sec x}$$

Answer :

$$= \lim_{x \to 0} \frac{\sec 4x - \sec 2x}{\sec 3x - \sec x}$$

$$= \lim_{x \to 0} \frac{\frac{1}{\cos 4x} - \frac{1}{\cos 2x}}{\frac{1}{\cos 3x} - \frac{1}{\cos x}}$$

$$= \lim_{x \to 0} \frac{(\cos 2x - \cos 4x) \times \cos x \times \cos 3x}{(\cos x - \cos 3x) \times \cos 2x \times \cos 4x}$$

$$= \lim_{x \to 0} \frac{2 \times \sin 3x \times \sin x \times \cos x \times \cos 3x}{2 \times \sin 2x \times \sin x \times \cos 2x \times \cos 4x}$$

$$= \lim_{x \to 0} \frac{\sin 6x \times \cos x}{\sin 4x \times \cos 4x} \times \frac{2}{2}$$

$$= 2 \times \lim_{x \to 0} \frac{\sin 6x \times \cos x}{\sin 8x} \times \frac{8x}{6x} \times \frac{6}{8}$$

$$= \frac{3}{2} \times \lim_{x \to 0} \frac{\frac{\sin 6x}{6x} \times \cos x}{\frac{\sin 8x}{8x}}$$

$$= \frac{3}{2} \times \frac{1 \times 1}{1}$$

$$= \frac{3}{2}$$

$$\therefore \lim_{x \to 0} \frac{\sec 4x - \sec 2x}{\sec 3x - \sec x} = \frac{3}{2}$$

Q. 62. Evaluate the following limits:

$$\lim_{x \to 0} \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin^2 x}$$

Answer :

$$= \lim_{x \to 0} \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin x \times \sin x}$$

$$= \lim_{x \to 0} \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin x \times \sin x} \times \frac{\sqrt{2} + \sqrt{1 + \cos x}}{\sqrt{2} + \sqrt{1 + \cos x}}$$

$$= \lim_{x \to 0} \frac{2 - (1 - \cos x)}{\sin x \times \sin x \times \sqrt{2} + \sqrt{1 + \cos x}}$$

$$= \lim_{x \to 0} \frac{1 - \cos x}{2 \times \sin\frac{x}{2} \cos\frac{x}{2} \sin x \left(\sqrt{2} + \sqrt{1 + \cos x}\right)}$$

$$= \lim_{x \to 0} \frac{2 \times \sin\frac{x}{2} \times \sin\frac{x}{2}}{2 \times \sin\frac{x}{2} \cos\frac{x}{2} \sin x \left(\sqrt{2} + \sqrt{1 + \cos x}\right)}$$

$$= \lim_{x \to 0} \frac{\frac{\sin\frac{x}{2}}{\frac{x}{2}} \times \frac{1}{2}}{\frac{\sin x}{x}} \times \lim_{x \to 0} \frac{1}{\cos\frac{x}{2} \times \left(\sqrt{2} + \sqrt{1 + \cos x}\right)}$$

$$= \frac{1}{2} \times \frac{1}{(\sqrt{2} + \sqrt{2})}$$

$$= \frac{1}{4\sqrt{2}}$$

$$\therefore \lim_{x \to 0} \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin x \times \sin x} = \frac{1}{4\sqrt{2}}$$

Q. 63. Evaluate the following limits:

$$\lim_{x \to 0} \frac{\sqrt{1 + \sin x} - \sqrt{1 - \cos x}}{x}$$

Answer :

$$= \lim_{x \to 0} \frac{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}{x}$$

$$= \lim_{x \to 0} \frac{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}{x} \times \frac{\sqrt{1 + \sin x} + \sqrt{1 - \sin x}}{\sqrt{1 + \sin x} + \sqrt{1 - \sin x}}$$

$$= \lim_{x \to 0} \frac{1 + \sin x - (1 - \sin x)}{x(\sqrt{1 + \sin x} + \sqrt{1 - \sin x})}$$

$$= \lim_{x \to 0} \frac{2 \times \sin x}{x(\sqrt{1 + \sin x} + \sqrt{1 - \sin x})}$$

$$= 2 \times \lim_{x \to 0} \frac{\sin x}{x} \times \lim_{x \to 0} \frac{1}{(\sqrt{1 + \sin x} + \sqrt{1 - \sin x})}$$

$$= 2 \times 1 \times \frac{1}{2}$$

=1

$$\therefore \lim_{x \to 0} \frac{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}{x} = 1$$

Q. 64. Evaluate the following limits:

$$\lim_{x \to \frac{\pi}{6}} \frac{2 - \sqrt{3}\cos x - \sin x}{(6x - \pi)^2}$$

Answer :

$$= \lim_{x \to \frac{\pi}{6}} \frac{2 - \sqrt{3}\cos x - \sin x}{(6x - \pi)^2}$$

$$= \lim_{y \to 0} \frac{2 - \sqrt{3}\cos\left(y + \frac{\pi}{6}\right) - \sin\left(y + \frac{\pi}{6}\right)}{y^2 \times 36}$$

$$= \frac{1}{36} \times \lim_{y \to 0} \frac{2 - \frac{3}{2}\cos y + \frac{\sqrt{3}}{2}\sin y - \frac{1}{2}\cos y - \frac{\sqrt{3}}{2}\sin y}{y^2}$$

$$= \frac{1}{36} \times \lim_{y \to 0} \frac{2(1 - \cos y)}{y^2}$$

$$= 2 \times \frac{1}{2} \times \frac{1}{36}$$

$$= \frac{1}{36}$$

$$\therefore \lim_{x \to \frac{\pi}{6}} \frac{2 - \sqrt{3}\cos x - \sin x}{(6x - \pi)^2} = \frac{1}{36}$$

Q. 65. Evaluate the following limits:

$$\lim_{x \to 0} \frac{\cos ax - \cos bx}{\cos cx - 1}$$

Answer :

$$= \lim_{x\to 0}\frac{\cos ax - \cos bx}{\cos cx - 1}$$

$$= \lim_{x\to 0}\frac{1 - \cos bx - (1 - \cos ax)}{\cos cx - 1}$$

$$= \lim_{x\to 0}\frac{\frac{(1 - \cos bx)}{(bx)^2} \times b^2 - \frac{(1 - \cos ax)}{(ax)^2} \times a^2}{\frac{-(1 - \cos cx)}{(cx)^2} \times c^2}$$

$$= \frac{a^2 - b^2}{c^2}$$

$$\therefore \lim_{x\to 0}\frac{\cos ax - \cos bx}{\cos cx - 1} = \frac{a^2 - b^2}{c^2}$$

Q. 66. Evaluate the following limits:

$$\lim_{x\to a}\frac{\cos x - \cos a}{\cot x - \cot a}$$

Answer :

$$= \lim_{x\to a}\frac{\cos x - \cos a}{\cot x - \cot a}$$

$$= \lim_{x\to a}\frac{(\cos x - \cos a)}{\frac{\sin(a - x)}{\sin x \sin a}}$$

$$= \sin a \times \lim_{x\to a}\frac{\sin\left(\frac{x + a}{2}\right) \times \sin x}{\cos\left(\frac{x - a}{2}\right)}$$

$=\sin^3 a$

$$\therefore \lim_{x\to a}\frac{\cos x - \cos a}{\cot x - \cot a} = \sin a \times \sin a \times \sin a$$

Q. 67. Evaluate the following limits:

$$\lim_{x \to \frac{\pi}{4}} \frac{\tan^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$$

Answer :

$$= \lim_{x \to \frac{\pi}{4}} \frac{\left(\frac{-\sin x\,(\cos 2x)}{\cos x \cos x \cos x}\right)}{\left(\frac{\cos x - \sin x}{\sqrt{2}}\right)}$$

$$= -\sqrt{2} \times \lim_{x \to \frac{\pi}{4}} \frac{\sin x\,(\cos x + \sin x)}{\cos x \times \cos x \times \cos x}$$

$$= -\sqrt{2} \times \frac{\frac{1}{\sqrt{2}} \times \sqrt{2}}{\frac{1}{\sqrt{2}}^{3}}$$

$$= -4$$

Q. 68. Evaluate the following limits:

$$\lim_{x \to \frac{\pi}{2}} \frac{\sqrt{2} - \sqrt{1 + \sin x}}{\sqrt{2} \cos^2 x}$$

Answer :

$$= \lim_{x \to \frac{\pi}{2}} \frac{\sqrt{2} - \sqrt{1 + \sin x}}{\sqrt{2} \cos x \cos x}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\sqrt{2} - \sqrt{1 + \sin x}}{\sqrt{2} \cos x \cos x} \times \frac{\sqrt{2} + \sqrt{1 + \sin x}}{\sqrt{2} + \sqrt{1 + \sin x}}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{1 - \sin x}{\sqrt{2} + \sqrt{1 + \sin x}\left(\sqrt{2} \cos x \cos x\right)}$$

Let,

$y = x - \frac{\pi}{2}$

$$= \lim_{y \to 0} \frac{1 - \cos y}{\sqrt{2} + \sqrt{1 + \cos y}\left(\sqrt{2} \sin y \sin y\right)}$$

$$= \frac{1}{2\sqrt{2}} \times \frac{1}{2\sqrt{2}}$$

$$= \frac{1}{8}$$

$$\therefore \lim_{x \to \frac{\pi}{2}} \frac{\sqrt{2} - \sqrt{1 + \sin x}}{\sqrt{2} \cos x \cos x} = \frac{1}{8}$$

Q. 69. Evaluate the following limits:

$$\lim_{x \to \frac{\pi}{6}} \frac{\cot^2 x - 3}{\operatorname{cosec} x - 2}$$

Answer :

$$= \lim_{x \to \frac{\pi}{6}} \frac{\cot x \times \cot x - 3}{\csc x - 2}$$

$$= \lim_{x \to \frac{\pi}{6}} \frac{(\cos x \times \cos x) - 3 \times \sin x \times \sin x}{\sin x (1 - 2 \sin x)}$$

$$= \lim_{x \to \frac{\pi}{6}} \frac{1 - 4 \times \sin x \times \sin x}{\sin x (1 - 2 \sin x)}$$

$$= \lim_{x \to \frac{\pi}{6}} \frac{(1 - 2\sin x) \times (1 + 2\sin x)}{\sin x\,(1 - 2\sin x)}$$

=4

$$\therefore \lim_{x \to \frac{\pi}{6}} \frac{\cot x \times \cot x - 3}{\csc x - 2} = 4$$

Q. 70. Evaluate the following limits:

$$\lim_{x \to \pi} \frac{\sqrt{2 + \cos x} - 1}{(\pi - x)^2}$$

Answer :

$$= \lim_{x \to \pi} \frac{\sqrt{2 + \cos x} - 1}{(\pi - x)^2}$$

$$= \lim_{x \to \pi} \frac{\sqrt{2 + \cos x} - 1}{(\pi - x)^2} \times \frac{\sqrt{2 + \cos x} + 1}{\sqrt{2 + \cos x} + 1}$$

$$= \lim_{x \to \pi} \frac{1 + \cos x}{(\pi - x)^2} \times \frac{1}{\sqrt{2 + \cos x} + 1}$$

Let,

$$y = x - \pi$$

$$= \lim_{y \to 0} \frac{1 - \cos y}{x^2 \times \sqrt{2 - \cos y} + 1}$$

$$= \frac{1}{4}$$

$$\therefore \lim_{x \to \pi} \frac{\sqrt{2 + \cos x} - 1}{(\pi - x)^2} = \frac{1}{4}$$

Q. 71. Evaluate the following limits:

$$\lim_{x \to \frac{\pi}{4}} \frac{1 - \tan x}{1 - \sqrt{2} \sin x}$$

Answer :

$$= \lim_{x \to \frac{\pi}{4}} \frac{1 - \tan x}{1 - \sqrt{2} \sin x}$$

Let,

$$y = x - \frac{\pi}{4}$$

$$= \lim_{y \to 0} \frac{2 \tan x}{1 - \cos x + \sin x}$$

$$= \lim_{y \to 0} \frac{\frac{2 \cos \frac{x}{2}}{\cos x}}{\sin \frac{x}{2} + \cos \frac{x}{2}}$$

= 2

$$\therefore \lim_{x \to \frac{\pi}{4}} \frac{1 - \tan x}{1 - \sqrt{2} \sin x} = 2$$

Q. 72. Evaluate the following limits:

$$\lim_{x \to \frac{\pi}{6}} \frac{2 \sin^2 x + \sin x - 1}{2 \sin^2 x - 3 \sin x + 1}$$

Answer :

$$= \lim_{x \to \frac{\pi}{6}} \frac{2 \times \sin x \times \sin x + \sin x - 1}{2 \times \sin x \times \sin x - 3 \sin x + 1}$$

$$= \lim_{x \to \frac{\pi}{6}} \frac{(2\sin x - 1) \times (\sin x + 1)}{(2\sin x - 1)(\sin x - 1)}$$

= -3

$$\therefore \lim_{x \to \frac{\pi}{6}} \frac{2 \times \sin x \times \sin x + \sin x - 1}{2 \times \sin x \times \sin x - 3\sin x + 1} = -3$$

Exercise 27C

Q. 1.

If $f(x) = |x| - 3$**, find** $\lim_{x \to 3} f(x)$

Answer :

Left Hand Limit(L.H.L.):

$$\lim_{x \to 3^-} f(x)$$

$$= \lim_{x \to 3^-} |x| - 3$$

$$= \lim_{x \to 3^-} -(x - 3)$$

= - (3 - 3)

= 0

Right Hand Limit(R.H.L.):

$$\lim_{x \to 3^+} f(x)$$

$$= \lim_{x \to 3^+} |x| - 3$$

$$= \lim_{x \to 3^+} (x - 3)$$

= 3-3

= 0

Since,

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x)$$

We can say that the limit exists and

$\lim_{x\to 3} f(x) = 0$

Q. 2.

$$\text{Let } f(x) = \begin{cases} \frac{x}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Show that $\lim_{x \to 0} f(x)$ does not exist.

Answer :

Left Hand Limit(L.H.L.):

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{x}{|x|}$$

$$= \lim_{x\to 0^-} \frac{x}{(-x)}$$

$$= \lim_{x\to 0^-} -1$$

= -1

Right Hand Limit(R.H.L.):

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{x}{|x|}$$

$$= \lim_{x\to 0^+} \frac{x}{(+x)}$$

$$= \lim_{x\to 0^+} 1$$

=1

Since $\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$, $\lim_{x\to 0} f(x)$ does not exist

Q. 3.

Let $f(x)=\begin{cases}\dfrac{|x-3|}{(x-3)}, & x\neq 3\\ 0, & x=3\end{cases}$

Show that $\lim\limits_{x\to 3} f(x)$ does not exist.

Answer :

Left Hand Limit(L.H.L.):

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-}\frac{|x-3|}{x-3}$$

$$= \lim_{x\to 3^-}\frac{-(x-3)}{x-3}$$

$$= \lim_{x\to 3^-} -1$$

= -1

Right Hand Limit(R.H.L.):

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+}\frac{|x-3|}{x-3}$$

$$= \lim_{x\to 3^+}\frac{(x-3)}{x-3}$$

$$= \lim_{x\to 3^+} 1$$

= 1

$$\lim_{x\to 3^-} f(x) \neq \lim_{x\to 3^+} f(x)$$

Thus, $\lim\limits_{x\to 3} f(x)$ does not exist.

Q. 4.

Let $f(x)=\begin{cases}1+x^2, 0\le x\le 1\\ 2-x,\ x>1\end{cases}$

Show that $\lim_{x\to 1} f(x)$ does not exist.

Answer : Left Hand Limit(L.H.L.):

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} 1+x^2$$

$= 1 + (1)^2$

$= 1 + 1$

$= 2$

Right Hand Limit(R.H.L.):

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 2-x$$

$= 2 – (1)$

$= 2 – 1$

$=1$

$$\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$$

Thus, $\lim_{x\to 1} f(x)$ does not exist.

Q. 5.

Let $f(x)=\begin{cases}\dfrac{x-|x|}{x}, x\neq 0\\ 2,\qquad x=0\end{cases}$

Show that $\lim_{x\to 0} f(x)$ does not exist

Answer :

Left Hand Limit(L.H.L.):

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{x - |x|}{x}$$

$$= \lim_{x \to 0^-} \frac{x - (-x)}{x}$$

$$= \lim_{x \to 0^-} \frac{x + x}{x}$$

$$= \lim_{x \to 0^-} \frac{2x}{x}$$

$$= \lim_{x \to 0^-} 2$$

= 2

Right Hand Limit(R.H.L.):

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{x - |x|}{x}$$

$$= \lim_{x \to 0^+} \frac{x - (x)}{x}$$

$$= \lim_{x \to 0^+} \frac{0}{x}$$

$$= \lim_{x \to 0^+} 0$$

= 0

$$\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$$

Thus, $\lim_{x \to 0} f(x)$ does not exist.

Q. 6

Let $f(x)=\begin{cases}5x-4, & 0<x\le 1\\ 4x^3-3x, & 1<x<2\end{cases}$

Find $\lim\limits_{x\to 1} f(x)$

Answer :

Left Hand Limit(L.H.L.):

$\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-} 5x-4$

= 5(1) – 4

= 5 – 4

= 1

Right Hand Limit(R.H.L.):

$\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} 4x^3 - 3x$

= 4 $(1)^3$ – 3 (1)

= 4 – 3

= 1

$\therefore \lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^+} f(x)$

Thus, $\lim\limits_{x\to 1} f(x) = 1$

Q. 7

Let $f(x) = \begin{cases} 4x - 5, x \leq 2 \\ x - a, \ x > 2 \end{cases}$

If $\lim\limits_{x \to 2} f(x)$ **exists then find the value of a.**

Answer :

Left Hand Limit(L.H.L.):

$\lim\limits_{x \to 2^-} f(x) \ = \ \lim\limits_{x \to 2^-} 4x - 5$

= 4 (2) – 5

= 8 – 5

= 3

Right Hand Limit(R.H.L.):

$\lim\limits_{x \to 2^+} f(x) \ = \ \lim\limits_{x \to 2^+} x - a$

= 2 – a

Since $\lim\limits_{x \to 2} f(x)$ it exists,

$\lim\limits_{x \to 2^-} f(x) = \ \lim\limits_{x \to 2^+} f(x)$

→ 3 = 2 – a

→ a = 2 – 3

→ a = -1

Q. 8

Let $f(x)=\begin{cases}\dfrac{3x}{|x|+2x}, & x\neq 0\\ 0, & x=0\end{cases}$

Show that $\lim\limits_{x\to 0} f(x)$ does not exist.

Answer :

Left Hand Limit(L.H.L.):

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{3x}{|x| + 2x}$$

$$= \lim_{x\to 0^-} \frac{3x}{(-x) + 2x}$$

$$= \lim_{x\to 0^-} \frac{3x}{x}$$

$$= \lim_{x\to 0^-} 3$$

= 3

Right Hand Limit(R.H.L.):

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{3x}{|x| + 2x}$$

$$= \lim_{x\to 0^+} \frac{3x}{(x) + 2x}$$

$$= \lim_{x\to 0^+} \frac{3x}{3x}$$

$$= \lim_{x\to 0^+} 1$$

= 1

Since

$$\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$$

Thus, $\lim_{x\to 0} f(x)$ does not exist.

Q. 9

Let $f(x) = \begin{cases} \cos x, x \geq 0 \\ x + k, x < 0 \end{cases}$

Find the value of k for which $\lim_{x\to 0} f(x)$ **exist.**

Answer :

Left Hand Limit(L.H.L.):

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x + k$$

= 0 + k

= k

Right Hand Limit(R.H.L.):

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \cos x$$

= cos (0)

= 1

It is given that $\lim_{x\to 0} f(x)$ exists. Therefore,

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$$

→ k = 1

Q. 10

Show that $\lim\limits_{x \to 0} \frac{1}{x}$ does not exist.

Answer : Let x = 0+h for x tending to 0^+

Since x→ 0, h also tends to 0

Right Hand Limit(R.H.L.):

$$\lim_{x \to 0^+} f(x)$$

$$= \lim_{x \to 0^+} \frac{1}{x}$$

$$= \lim_{h \to 0^+} \frac{1}{0+h}$$

$$= \lim_{h \to 0^+} \frac{1}{+h}$$

$$= +\frac{1}{0}$$

$$= +\infty$$

Let x=0 -h for x tending to 0^-

Since x→0, h also tends to 0.

Left Hand Limit(L.H.L.):

$$= \lim_{x \to 0^-} f(x)$$

$$= \lim_{x \to 0^-} \frac{1}{x}$$

$$= \lim_{h \to 0^-} \frac{1}{0-h}$$

$$= \lim_{h\to 0^-} \frac{1}{-h}$$

$$= -\frac{1}{0}$$

$$= -\infty$$

Since,

$$\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$$

Thus, $\lim_{x\to 0} \frac{1}{x}$ does not exist.

Q. 11

Show that $\lim_{x\to 0} \frac{1}{|x|} = \infty$.

Answer : Let x = 0 + h, when x is tends to 0^+

Since x tends to 0, h will also tend to 0.

Right Hand Limit(R.H.L):

$$\lim_{x\to 0^+} f(x)$$

$$= \lim_{x\to 0^+} \frac{1}{|x|}$$

$$= \lim_{x\to 0^+} \frac{1}{(x)}$$

$$= \lim_{h \to 0^+} \frac{1}{(0+h)}$$

$$= \frac{1}{0}$$

$=\infty$

Let x = 0 - h, when x is tends to 0^-

ince x tends to 0, h will also tend to 0.

Left Hand Limit(L.H.L.):

$$\lim_{x \to 0^-} f(x)$$

$$= \lim_{x \to 0^-} \frac{1}{|x|}$$

$$= \lim_{x \to 0^-} \frac{1}{(-x)}$$

$$= \lim_{h \to 0^-} \frac{1}{-(0-h)}$$

$$= \lim_{h \to 0^-} \frac{1}{h}$$

$$= \frac{1}{0}$$

$= \infty$

Thus,

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$$

$$\therefore \lim_{x \to 0} \frac{1}{|x|} = \infty.$$

Q. 12.

Show that $\lim_{x \to 0} e^{-1/x}$ does not exist.

Answer : Left Hand Limit(L.H.L.):

$$\lim_{x \to 0^-} f(x)$$

$$= \lim_{x \to 0^-} e^{\frac{-1}{(-x)}}$$

$$= \lim_{x \to 0^-} e^{\frac{1}{x}}$$

$$= e^{\frac{1}{0}}$$

$$= e^{\infty}$$

$$\lim_{x \to 0^+} f(x)$$

Right Hand Limit(R.H.L.):

$$= \lim_{x \to 0^+} e^{\frac{-1}{x}}$$

$$= e^{\frac{-1}{0}}$$

$$= e^{-\infty}$$

$= \frac{1}{e^{\infty}}$

[Formula $\frac{1}{\infty} = 0$, anything to the power infinity is also infinity. Thus $\frac{1}{e\infty} = \frac{1}{\infty} = 0$]

$=0$

Since

$\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$

$\therefore \lim_{x \to 0} e^{-1/x}$ does not exist.

Q. 13.

Show that $\lim_{x \to 0} \sin \frac{1}{x}$ does not exist.

Answer : Let x = 0 + h, when x is tends to 0^+

Since x tends to 0, h will also tend to 0.

Right Hand Limit(R.H.L.):

$\lim_{x \to 0^+} f(x)$

$= \lim_{x \to 0^+} \sin \frac{1}{x}$

$= \lim_{h \to 0^+} \sin \frac{1}{0 + h}$

$= \sin \frac{1}{0}$

$= \sin \infty$

$= \infty$

Let x = 0 - h, when x is tends to 0^-

Since x tends to 0, h will also tend to 0.

Left Hand Limit(L.H.L.):

$$\lim_{x \to 0^-} f(x)$$

$$= \lim_{x \to 0^-} \sin \frac{1}{x}$$

$$= \lim_{h \to 0^-} \sin \frac{1}{0-h}$$

$$= \sin \frac{1}{-0}$$

$$= -\sin \frac{1}{0}$$

$$= - \sin \infty$$

$$= -\infty$$

Since,

$$\lim_{x - 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$$

$\therefore \lim_{x \to 0} \sin \frac{1}{x}$ does not exist.

Q. 14

Show that $\lim_{x \to 0} \frac{x}{|x|}$ does not exist.

Answer :

Left Hand Limit(L.H.L.):

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{x}{|x|}$$

$$= \lim_{x \to 0^-} \frac{x}{(-x)}$$

$$= \lim_{x \to 0^-} -1$$

= -1

Right Hand Limit(R.H.L.):

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{x}{|x|}$$

$$= \lim_{x \to 0^+} \frac{x}{(x)}$$

$$= \lim_{x \to 0^+} 1$$

= 1

Since

$$\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$$

Thus, $\lim_{x \to 0} \frac{x}{|x|}$ does not exist.

Q. 15

Let $f(x)=\begin{cases}\dfrac{k\cos x}{\pi-2x}, & x\neq\dfrac{\pi}{2}\\ 3, & x=\dfrac{\pi}{2}\end{cases}$

If $\lim\limits_{x\to\frac{\pi}{2}} f(x)=f\left(\dfrac{\pi}{2}\right)$**, find the value of k.**

Answer :

$$\lim_{x\to\frac{\pi}{2}} f(x) = \lim_{x\to\frac{\pi}{2}}\frac{k\cos x}{\pi-2x}$$

Let $h = x - \frac{\pi}{2}$

$\rightarrow x = h + \frac{\pi}{2}$

$x \to \dfrac{\pi}{2}$

or, $h + \frac{\pi}{2} \to \frac{\pi}{2}$

or , $h \rightarrow 0$

Putting this in the original sum,

$$= \lim_{h \to 0} \frac{k \cos\left(\frac{\pi}{2} + h\right)}{\pi - 2\left(\frac{\pi}{2} + h\right)}$$

$$= \lim_{h \to 0} \frac{-k \sin h}{\pi - \pi + h}$$

$$= \lim_{h \to 0} \frac{-k \sin h}{h}$$

$$= -k \lim_{h \to 0} \frac{\sin h}{h}$$

[Applying formula $\lim_{x \to 0} \frac{\sin x}{x} = 1$]

= -k × 1

= -k

$$f\left(\frac{\pi}{2}\right) = 3$$

It is given that $\lim_{x \to \frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$

∴ -k = 3

→ k = -3