Binomial Theorem

- The coefficients of the expansions of a binomial are arranged in an array. This array is called Pascal's triangle. It can be written as

Index	Coefficient(s)
0	${}^{0}C_{0}$ $(=1)$
1	${}^{1}C_{0}$ $(=1)$ ${}^{1}C_{1}$ $(=1)$
2	${}^{2}C_{0}$ $(=1)$ ${}^{2}C_{1}$ $(=2)$ ${}^{2}C_{2}$ $(=1)$
3	${}^{3}C_{0}$ $(=1)$ ${}^{3}C_{1}$ $(=3)$ ${}^{3}C_{2}$ $(=3)$ ${}^{3}C_{3}$ $(=1)$
4	${}^{4}C_{0}$ $(=1)$ ${}^{4}C_{1}$ $(=4)$ ${}^{4}C_{2}$ $(=6)$ ${}^{4}C_{3}$ $(=4)$ ${}^{4}C_{4}$ $(=1)$
5	

- **General Term:** The $(r+1)^{th}$ term (denoted by T_{r+1}) is known as the general term of the expansion $(a+b)^n$ and it is given by $T_{r+1} = {}^{n}C_{r}\, a^{n-r} b^{r}$

  Example 1: In the expansion of $(5x - 7y)^9$, find the general term?

  Solution: $T_{r+1} = {}^{9}C_{r}\,(5x)^{9-r}(-7y)^{r} = (-1)^{r}\,{}^{9}C_{r}\,(5x)^{9-r}(7y)^{r}$

- **Middle term in the expansion of $(a+b)^n$:**
  - If n is even, then the number of terms in the expansion will be $n+1$. Since n is even, $n+1$ is odd. Therefore, the middle term is $\left(\frac{n}{2}+1\right)^{th}$ term.
  - If n is odd, then $n+1$ is even. So, there will be two middle terms in the expansion. They are $\left(\frac{n+1}{2}\right)^{th}$ term and $\left(\frac{n+1}{2}+1\right)^{th}$ term.
- In the expansion of $\left(x+\frac{1}{x}\right)^{2n}$, where $x \neq 0$, the middle term is $\left(\frac{2n}{2}+1\right)^{th}$, i.e., $(n+1)^{th}$ term [since $2n$ is even].

  It is given by ${}^{2n}C_{n}\, x^{n}\left(\frac{1}{x}\right)^{n} = {}^{2n}C_{n}$ which is a constant.
  This term is called the term independent of x or the constant term.
  Note: In the expansion of $(a+b)^n$, r^{th} term from the end $= (n-r+2)^{th}$ term from the beginning

Example 2: In the expansion of $\left(\frac{x^3}{4}-\frac{12}{x}\right)^4$, find the middle term and find the term which is independent of *x*.

Solution: As 4 is even, the middle term in the expansion of $\left(\frac{x^3}{4}-\frac{12}{x}\right)^4$ is the $\left(\frac{4}{2}+1\right)^{\text{th}}$ term, i.e., 3^{rd} term, which is given by

$$T_3 = T_{2+1} = {}^4C_2\left(\frac{x^3}{4}\right)^2\left(\frac{-12}{x}\right)^2$$

$$= 6 \times \frac{x^6}{16} \times \frac{144}{x^2}$$

$$= 54x^4$$

Now, we will find the term in the expansion which is independent of *x*. Suppose $(r + 1)^{\text{th}}$ term is independent of *x*.
The $(r + 1)^{\text{th}}$ term in the expansion of $(a + b)^n$ is given by $T_{r+1} = {}^nC_r\, a^{n-r}\, b^r$

Hence, the $(r + 1)^{\text{th}}$ term in the expansion of $\left(\frac{x^3}{4}-\frac{12}{x}\right)^4$ is given by