6

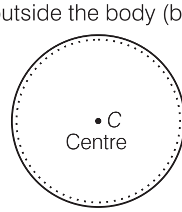
System of Particles and Rotational Motion

Multiple Choice Questions (MCQs)

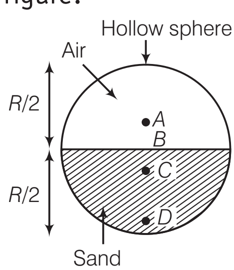
Q. 1 For which of the following does the centre of mass lie outside the body?

(a) A pencil (b) A shotput (c) A dice (d) A bangle

Ans. *(d)* A bangle is in the form of a ring as shown in the adjacent diagram. The centre of mass lies at the centre, which is outside the body (boundary).



Q. 2 Which of the following points is the likely position of the centre of mass of the system shown in figure?



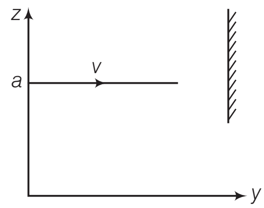
(a) A (b) B (c) C (d) D

Thinking Process

In a system of particles, the centre of mass of a body lies closer to heavier mass or masses.

Ans. *(c)* Centre of mass of a system lies towards the part of the system, having bigger mass. In the above diagram, lower part is heavier, hence CM of the system lies below the horizontal diameter.

Q. 3 A particle of mass m is moving in yz-plane with a uniform velocity v with its trajectory running parallel to +ve y-axis and intersecting z-axis at z = a in figure. The change in its angular momentum about the origin as it bounces elastically from a wall at y = constant is



(a) $mva\,\hat{\mathbf{e}}_x$ (b) $2\,mva\,\hat{\mathbf{e}}_x$ (c) $ymv\,\hat{\mathbf{e}}_x$ (d) $2\,ymv\,\hat{\mathbf{e}}_x$

Thinking Process

In elastic collision, KE of the system remains conserved. Therefore, the ball will bounce back with the same speed v but in opposite direction i.e., along -ve y -axis.

Ans. *(b)* The initial velocity is $\mathbf{v}_i = \mathbf{v}\,\hat{\mathbf{e}}_y$ and after reflection from the wall, the final velocity is $\mathbf{v}_f = -\,v\hat{\mathbf{e}}_y$. The trajectory is described as position vector $\mathbf{r} = y\hat{\mathbf{e}}_y + a\hat{\mathbf{e}}_z$.

Hence, the change in angular momentum is $\mathbf{r} \times m(\mathbf{v}_f - \mathbf{v}_i) = 2mva\hat{\mathbf{e}}_x$.

Q. 4 When a disc rotates with uniform angular velocity, which of the following is not true?

(a) The sense of rotation remains same
(b) The orientation of the axis of rotation remains same
(c) The speed of rotation is non-zero and remains same
(d) The angular acceleration is non-zero and remains same

Ans. *(d)* We know that angular acceleration

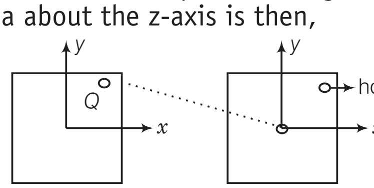
$$\alpha = \frac{d\omega}{dt}, \text{ given } \omega = \text{constant}$$

where ω is angular velocity of the disc

$$\Rightarrow \qquad \alpha = \frac{d\omega}{dt} = \frac{0}{dt} = 0$$

Hence, angular acceleration is zero.

Q. 5 A uniform square plate has a small piece Q of an irregular shape removed and glued to the centre of the plate leaving a hole behind in figure. The moment of inertia about the z-axis is then,



(a) increased (b) decreased
(c) the same (d) changed in unpredicted manner

Thinking Process

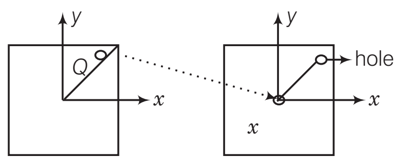
For two bodies having same mass, the body having mass distributed at greater distance from an axis, will have more moment of inertia.

Ans. *(b)* In the given diagrams, when the small piece Q removed and glued to the centre of the plate, the mass comes closer to the z-axis, hence, moment of inertia decreases.

Q. 6 In problem 5, the CM of the plate is now in the following quadrant of x-y plane.

(a) I (b) II (c) III (d) IV

Ans. *(c)* Consider the adjacent diagram, there is a line shown in the figure drawn along the diagonal. First, centre of mass of the system was on the dotted line and was shifted towards Q from the centre (Ist quadrant).



When mass is removed, it will be on the same line but shifted away from the centre and below (IIIrd quadrant). Position of CM is shown by X in the diagram.

Q. 7 The density of a non-uniform rod of length 1m is given by $\rho(x) = a(1 + bx^2)$ where, a and b are constants and $0 \le x \le 1$. The centre of mass of the rod will be at

(a) $\frac{3(2+b)}{4(3+b)}$ (b) $\frac{4(2+b)}{3(3+b)}$ (c) $\frac{3(3+b)}{4(2+b)}$ (d) $\frac{4(3+b)}{3(2+b)}$

Ans. *(a)* Density is given as $\rho(x) = a(1 + bx^2)$

where a and b are constants and $0 \le x \le 1$.

Let $b \to 0$,in this case

$$\rho(x) = a = \text{constant}$$

Hence, centre of mass will be at $x = 0.5$ m. (middle of the rod)

Putting, $b = 0$ in all the options, only (a) gives 0.5.

***Note** We should not check options by putting $a = 0$, because $\rho = 0$ for $a = 0$.*

Q. 8 A merry-go-round, made of a ring-like platform of radius R and mass M, is revolving with angular speed ω. A person of mass M is standing on it. At one instant, the person jumps off the round, radially away from the centre of the round (as seen from the round). The speed of the round of afterwards is

(a) 2ω (b) ω (c) $\frac{\omega}{2}$ (d) 0

Ans. *(a)* As no external torque acts on the system, angular momentum should be conserved.

Hence $I\omega =$ constant ...(i)

where, I is moment of inertia of the system and ω is angular velocity of the system.

From Eq. (i) $I_1\omega_1 = I_2\omega_2$

(where ω_1 and ω_2 are angular velocities before and after jumping)

$$\Rightarrow \quad I\omega = \frac{I}{2} \times \omega_2$$

(as mass reduced to half, hence, moment of inertia also reduced to half)

$$\Rightarrow \quad \omega_2 = 2\omega$$

Multiple Choice Questions (More Than One Options)

Q. 9 Choose the correct alternatives

(a) For a general rotational motion, angular momentum **L** and angular velocity ω need not be parallel.

(b) For a rotational motion about a fixed axis, angular momentum **L** and angular velocity ω are always parallel.

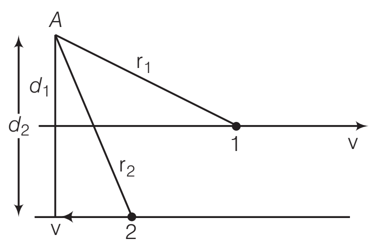
(c) For a general translational motion, momentum **p** and velocity **v** are always parallel.

(d) For a general translational motion, acceleration **a** and velocity **v** are always parallel.

Ans. *(a, c)*

For a general rotational motion ,where axis of rotation is not symmetric. Angular momentum **L** and angular velocity ω need not be parallel. For a general translational motion momentum $\mathbf{p} = m\mathbf{v}$, hence, **p** and **v** are always parallel.

Q. 10 Figure shows two identical particles 1 and 2, each of mass m, moving in opposite directions with same speed **v** along parallel lines. At a particular instant $\mathbf{r}_1$ and $\mathbf{r}_2$ are their respective position vectors drawn from point A which is in the plane of the parallel lines. Choose the correct options.



(a) Angular momentum I_1 of particle 1 about A is $I = mv(d_1)$

(b) Angular momentum I_2 of particle 2 about A is $I_2 = mv\mathbf{r}_2$

(c) Total angular momentum of the system about A is $I = mv(\mathbf{r}_1 + \mathbf{r}_2)$

(d) Total angular momentum of the system about A is $I = mv(d_2 - d_1) \otimes$

represents a unit vector coming out of the page.

$\otimes$ *represents a unit vector going into the page.*

Ans. *(a, b)*

The angular momentum **L** of a particle with respect to origin is defined to be $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ where, **r** is the position vector of the particle and **p** is the linear momentum. The direction of **L** is perpendicular to both d **r** and **p** by right hand rule.

For particle 1, $I_1 = \mathbf{r}_1 \times m\mathbf{v}$, is out of plane of the paper and perpendicular to $\mathbf{r}_1$ and $\mathbf{p}(m\,\mathbf{v})$ Similarly $I_2 = \mathbf{r}_2 \times m(-\mathbf{v})$ is into the plane of the paper and perpendicular to $\mathbf{r}_2$ and $-\mathbf{p}$.

Hence, total angular momentum

$$l = l_1 + l_2 = \mathbf{r}_1 \times m\,\mathbf{v} + (-\mathbf{r}_2 \times m\,\mathbf{v})$$

$|l| = mv\,d_1 - mvd_2$ as $d_2 > d_1$, total angular momentum will be inward

Hence, $$I = m\,\mathbf{v}\,(d_2 - d_1) \otimes$$

Note *In the expression of angular momentum $I = \mathbf{r} \times \mathbf{p}$ the direction of l is taken by right hand rule.*

Q. 11 **The net external torque on a system of particles about an axis is zero. Which of the following are compatible with it?**

(a) The forces may be acting radially from a point on the axis

(b) The forces may be acting on the axis of rotation

(c) The forces may be acting parallel to the axis of rotation

(d) The torque caused by some forces may be equal and opposite to that caused by other forces

Ans. *(a, b, c, d)*

We know that torque on a system of particles $\tau = \mathbf{r} \times \mathbf{F} = \mathbf{F} \sin \theta \hat{\mathbf{n}}$ ...(i)

where, θ is angle between $\mathbf{r}$ and $\mathbf{F}$, and $\hat{\mathbf{n}}$ is a unit vector perpendicular to both $\mathbf{r}$ and $\mathbf{F}$.

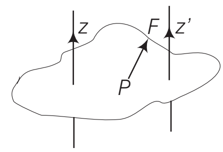
(a) When forces act radially, $\theta = 0$ hence $|\tau| = 0$ [from Eq. (i)]

(b) When forces are acting on the axis of rotation, $\mathbf{r} = 0, |\tau| = 0$ [from Eq. (i)]

(c) When forces acting parallel to the axis of rotation $\theta = 0°, |\tau| = 0$ [from Eq. (i)]

(d) When torque by forces are equal and opposite, $\tau_{net} = \tau_1 - \tau_2 = 0$

Q. 12 **Figure shows a lamina in *xy*-plane. Two axes *z* and *z'* pass perpendicular to its plane. A force F acts in the plane of lamina at point *P* as shown. Which of the following are true? (The point *P* is closer to *z'*-axis than the *z*-axis.)**



(a) Torque τ caused by $\mathbf{F}$ about z -axis is along $-\hat{\mathbf{k}}$

(b) Torque τ' caused by $\mathbf{F}$ about z'-axis is along $-\hat{\mathbf{k}}$

(c) Torque τ caused by $\mathbf{F}$ about z-axis is greater in magnitude than that about z-axis

(d) Total torque is given be $\tau = \tau + \tau'$

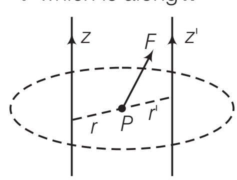
Thinking Process

Torque of a force $\mathbf{F}$ *about an axis is* $\mathbf{r} \times \mathbf{F}$ *which is perpendicular to the plane containing* $\mathbf{r}$ *and* $\mathbf{F}$.

Ans. *(b, c)*

(a) Consider the adjacent diagram, where $r > r'$

Torque τ about z-axis $\tau = \mathbf{r} \times \mathbf{F}$ which is along $\hat{\mathbf{k}}$



(b) $\tau' = \mathbf{r}' \times \mathbf{F}$ which is along $-\hat{\mathbf{k}}$

(c) $|\tau|_z = Fr_\perp$ = magnitude of torque about z-axis where $r_\perp$ is perpendicular distance between F and z-axis.

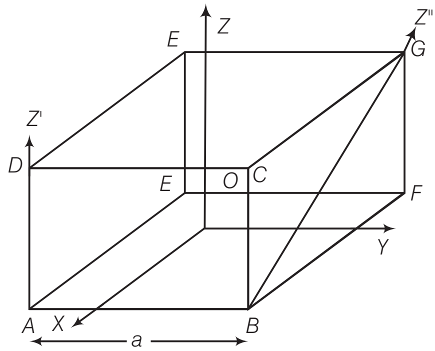
Similarly, $|\tau|_{z'} = F r_\perp'$

Clearly $r_\perp > r_\perp' \Rightarrow |\tau|_z > |\tau|_{z'}$

(d) We are always calculating resultant torque about a common axis.

Hence, total torque $\tau \neq \tau + \tau'$, because τ and τ' are not about common axis.

Q. 13 With reference to figure of a cube of edge a and mass m, state whether the following are true or false. (O is the centre of the cube.)



(a) The moment of inertia of cube about z-axis is $I_z = I_x + I_y$

(b) The moment of inertia of cube about z'-axis is $I_z = I_z + \frac{ma^2}{2}$

(c) The moment of inertia of cube about z''-axis is $= I_z + \frac{ma^2}{2}$

(d) $I_x = I_y$

Thinking Process

Moment of inertia about two symmetrical axes are same. To calculate net moment of inertia we can apply the concept of symmetry.

Ans. ***(a, b, d)***

(a) Theorem of perpendicular axes is applicable only for laminar (plane) objects. Thus, option (a) is false.

(b) As $z' \parallel z$ and distance between them $= a\frac{\sqrt{2}}{2} = \frac{a}{\sqrt{2}}$

Now, by theorem of parallel axes

$$I_{z'} = I_z + m\left(\frac{a}{\sqrt{2}}\right)^2 = I_z + \frac{ma^2}{2}$$

Hence, choice (b) is true.

(c) z'' is not parallel to z hence, theorem of parallel axis cannot be applied.

Thus, option (c) is false.

(d) As x and y-axes are symmetrical.

Hence, $I_x = I_y$

Thus, option (d) is true.

Very Short Answer Type Questions

Q. 14 The centre of gravity of a body on the earth coincides with its centre of mass for a small object whereas for an extended object it may not. What is the qualitative meaning of small and extended in this regard?
For which of the following two coincides? A building, a pond, a lake, a mountain?

Thinking Process

Centre of gravity is centre of a given structure but centre of mass is a point where whole mass of the body can be assumed to be concentrated.

Ans. When the vertical height of the object is very small as compared to the earth's radius, we call the object small, otherwise it is extended.

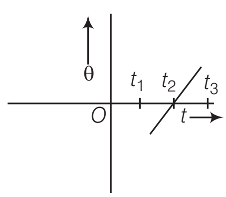
(i) Building and pond are small objects.
(ii) A deep lake and a mountain are examples of extended objects.

Q. 15 Why does a solid sphere have smaller moment of inertia than a hollow cylinder of same mass and radius, about an axis passing through their axes of symmetry?

Ans. The moment of inertia of a body is given by $I = \Sigma m_i r_i^2$ [sum of moment of inertia of each constituent particles]

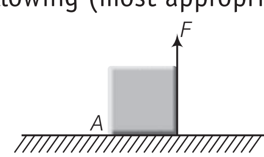
All the mass in a cylinder lies at distance R from the axis of symmetry but most of the mass of a solid sphere lies at a smaller distance than R.

Q. 16 The variation of angular position θ, of a point on a rotating rigid body, with time t is shown in figure. Is the body rotating clockwise or anti-clockwise?



Ans. As the slope of θ-t graph is positive and positive slope indicates anti-clockwise rotation which is traditionally taken as positive.

Q. 17 A uniform cube of mass m and side a is placed on a frictionless horizontal surface. A vertical force F is applied to the edge as shown in figure. Match the following (most appropriate choice)



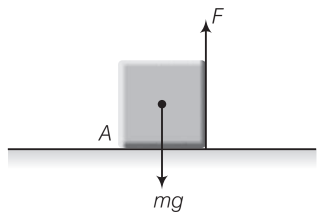
(a)	$mg/4 < F < mg/2$	(i)	Cube will move up.
(b)	$F > mg/2$	(ii)	Cube will not exhibit motion.
(c)	$F > mg$	(iii)	Cube will begin to rotate and slip at A.
(d)	$F = mg/4$	(iv)	Normal reaction effectively at $a/3$ from A, no motion.

Ans. Consider the below diagram

Moment of the force F about point A, $\tau_1 = F \times a$ (anti-clockwise)

Moment of weight mg of the cube about point A,

$$\tau_2 = mg \times \frac{a}{2} \text{ (clockwise)}$$



Cube will not exhibit motion, if $\tau_1 = \tau_2$

($\because$ In this case, both the torque will cancel the effect of each other)

$$\therefore \quad F \times a = mg \times \frac{a}{2} \Rightarrow F = \frac{mg}{2}$$

Cube will rotate only when, $\tau_1 > \tau_2$

$$\Rightarrow \quad F \times a > mg \times \frac{a}{2} \Rightarrow F > \frac{mg}{2}$$

Let normal reaction is acting at $\frac{a}{3}$ from point A, then

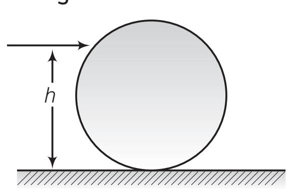
$$mg \times \frac{a}{3} = F \times a \text{ or } F = \frac{mg}{3} \qquad \text{(For no motion)}$$

When $F = \frac{mg}{4}$ which is less than $\frac{mg}{3}$,. $\left(F < \frac{mg}{3}\right)$

there will be no motion.

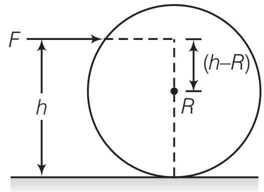
$\therefore$ (a) $\rightarrow$ (ii) (b) $\rightarrow$ (iii) (c) $\rightarrow$ (i) (d) $\rightarrow$ (iv)

Q. 18 A uniform sphere of mass m and radius R is placed on a rough horizontal surface (figure). The sphere is struck horizontally at a height h from the floor. Match the following



(a)	$h = R/2$	(i)	Sphere rolls without slipping with a constant velocity and no loss of energy.
(b)	$h = R$	(ii)	Sphere spins clockwise, loses energy by friction.
(c)	$h = 3R/2$	(iii)	Sphere spins anti-clockwise, loses energy by friction.
(d)	$h = 7R/5$	(iv)	Sphere has only a translational motion, looses energy by friction.

Ans. Consider the diagram where a sphere of m and radius R, struck horizontally at height h above the floor



The sphere will roll without slipping when $\omega = \frac{v}{r}$, where, v is linear velocity and ω is angular velocity of the sphere.

Now, angular momentum of sphere, about centre of mass

[We are applying conservation of angular momentum just before and after struck]

$$mv(h - R) = I\omega = \left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right)$$

$$\Rightarrow \qquad mv(h - R) = \frac{2}{5}mvR$$

$$h - R = \frac{2}{5}R \quad \Rightarrow \quad h = \frac{7}{5}R$$

Therefore, the sphere will roll without slipping with a constant velocity and hence, no loss of energy, so (d) $\rightarrow$(i)

Torque due to applied force, F about centre of mass

$$\tau = F(h - R) \qquad \text{(clockwise)}$$

For $\tau = 0$, $h = R$, sphere will have only translational motion. It would lose energy by friction.

Hence, (b) $\rightarrow$ (iv)

The sphere will spin clockwise when $\tau > 0 \Rightarrow h > R$

Therefore, (c) $\rightarrow$ (ii)

The sphere will spin anti-clockwise when $\tau < 0 \Rightarrow h < R$, (a) $\rightarrow$ (iii)

Short Answer Type Questions

Q. 19 The vector sum of a system of non-collinear forces acting on a rigid body is given to be non-zero. If the vector sum of all the torques due to the system of forces about a certain point is found to be zero, does this mean that it is necessarily zero about any arbitrary point?

Ans. *No*, not necessarily.

Given, $$\sum_i F_i \neq 0$$

The sum of torques about a certain point O, $\sum_i \mathbf{r}_i \times \mathbf{F}_i = 0$

The sum of torques about any other point O′

$$\sum_i (\mathbf{r}_i - \mathbf{a}) \times \mathbf{F}_i = \sum_i \mathrm{r}_i \times \mathbf{F}_i - \mathbf{a} \times \sum_i \mathrm{F}_i$$

Here, the second term need not vanish.

Therefore, sum of all the torques about any arbitrary point need not be zero necessarily.

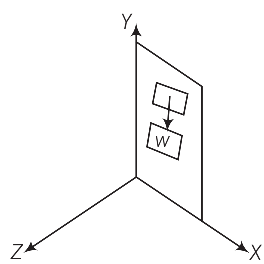
Q. 20 A wheel in uniform motion about an axis passing through its centre and perpendicular to its plane is considered to be in mechanical (translational plus rotational) equilibrium because no net external force or torque is required to sustain its motion. However, the particles that constitute the wheel do experience a centripetal the acceleration directed towards the centre. How do you reconcile this fact with the wheel being in equilibrium?

How would you set a half wheel into uniform motion about an axis passing through the centre of mass of the wheel and perpendicular to its plane? Will you require external forces to sustain the motion?

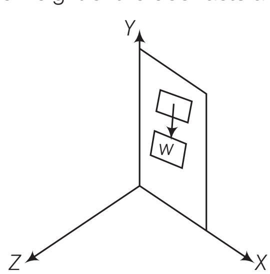
Ans. Wheel is a rigid body. The particles that constitute the wheel do experience a centripetal acceleration directed towards the centre. This acceleration arises due to internal elastic forces, which cancel out in pairs.

In a half wheel, the distribution of mass about its centre of mass (through which axis of rotation passes) is not symmetrical. Therefore, the direction of angular momentum of the wheel does not coincide with the direction of its angular velocity. Hence, an external torque is required to maintain the motion of the half wheel.

Q. 21 A door is hinged at one end and is free to rotate about a vertical axis (figure). Does its weight cause any torque about this axis? Give reason for your answer.



Ans. Consider the diagram, where weight of the door acts along negative y-axis.



A force can produce torque only along a direction normal to itself as $\tau = \mathbf{r} \times \mathbf{F}$. So, when the door is in the xy-plane, the torque produced by gravity can only be along $\pm z$ direction, never about an axis passing through y-direction.

Hence, the weight will not produce any torque about y-axis.

Q. 22 $(n-1)$ equal point masses each of mass m are placed at the vertices of a regular n-polygon. The vacant vertex has a position vector **a** with respect to the centre of the polygon. Find the position vector of centre of mass.

Thinking Process

The centre of mass of a regular n-polygon lies at its geometrical centre.

Ans. Let **b** be the position vector of the centre of mass of a regular n-polygon.

$(n-1)$ equal point masses are placed at $(n-1)$ vertices of the regular n-polygon, therefore, for its centre of mass

$$r_{CM} = \frac{(n-1)\,mb + ma}{(n-1)\,m + m} = 0 \qquad (\because \text{ Centre of mass lies at centre})$$

$$\Rightarrow \qquad (n-1)\,mb + ma = 0$$

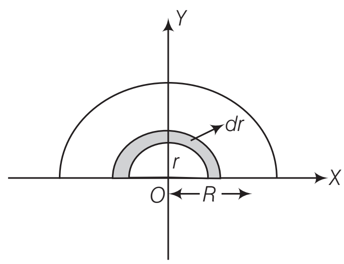
$$\Rightarrow \qquad b = -\frac{a}{(n-1)}$$

Long Answer Type Questions

Q. 23 Find the centre of mass of a uniform (a) half-disc, (b) quarter-disc.

Ans. Let M and R be the mass and radius of the half-disc, mass per unit area of the half-disc

$$m = \frac{M}{\frac{1}{2}\pi R^2} = \frac{2M}{\pi R^2}$$



(a) The half-disc can be supposed to be consists of a large number of semicircular rings of mass dm and thickness dr and radii ranging from $r = 0$ to $r = R$.

Surface area of semicircular ring of radius r and of thickness $dr = \frac{1}{2} 2\pi r \times dr = \pi r dr$

$\therefore$ Mass of this elementary ring, $dm = \pi r dr \times \dfrac{2M}{\pi R^2}$

$$dm = \frac{2M}{R^2} r dr$$

If (x, y) are coordinates of centre of mass of this element,

then, $\qquad (x, y) = \left(0, \dfrac{2r}{\pi}\right)$

Therefore, $\qquad x = 0$ and $y = \dfrac{2r}{\pi}$

Let x_{CM} and y_{CM} be the coordinates of the centre of mass of the semicircular disc.

Then $$x_{CM} = \frac{1}{M}\int_0^R x\,dm = \frac{1}{M}\int_0^R 0\,dm = 0$$

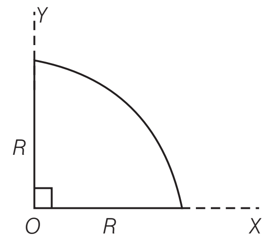
$$y_{CM} = \frac{1}{M}\int_0^R y\,dm = \frac{1}{M}\int_0^R \frac{2r}{\pi} \times \left(\frac{2M}{R^2}\,rdr\right)$$

$$= \frac{4}{\pi R^2}\int_0^R r^2 dr = \frac{4}{\pi R^2}\left[\frac{r^3}{3}\right]_0^R$$

$$= \frac{4}{\pi R^2} \times \left(\frac{R^3}{3} - 0\right) = \frac{4R}{3\pi}$$

$\therefore$ Centre of mass of the semicircular disc $= \left(0, \frac{4R}{3\pi}\right)$

(b) Centre of mass of a uniform quarter disc.



Mass per unit area of the quarter disc $= \dfrac{M}{\frac{\pi R^2}{4}} = \dfrac{4M}{\pi R^2}$

Using symmetry

For a half-disc along y-axis centre of mass will be at $x = \dfrac{4R}{3\pi}$

For a half-disc along x-axis centre of mass will be at $x = \dfrac{4R}{3\pi}$

Hence, for the quarter disc centre of mass $= \left(\dfrac{4R}{3\pi}, \dfrac{4R}{3\pi}\right)$

Q. 24 Two discs of moments of inertia I_1 and I_2 about their respective axes (normal to the disc and passing through the centre), and rotating with angular speed ω_1 and ω_2 are brought into contact face to face with their axes of rotation coincident.

(a) Does the law of conservation of angular momentum apply to the situation? Why?

(b) Find the angular speed of the two discs system.

(c) Calculate the loss in kinetic energy of the system in the process.

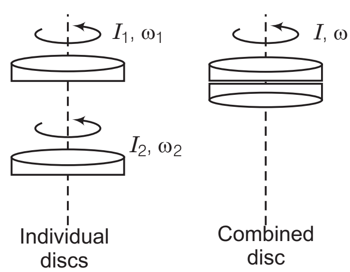
(d) Account for this loss.

Thinking Process

Due to friction between the two discs, the system will acquire common angular speed after sometime.

Ans. Consider the diagram below

Let the common angular velocity of the system is ω.



(a) *Yes*, the law of conservation of angular momentum can be applied. Because, there is no net external torque on the system of the two discs.

External forces, gravitation and normal reaction, act through the axis of rotation, hence, produce no torque.

(b) By conservation of angular momentum

$$L_f = L_i$$

$$\Rightarrow \quad I\omega = I_1\omega_1 + I_2\omega_2$$

$$\Rightarrow \quad \omega = \frac{I_1\omega_1 + I_2\omega_2}{I} = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2} \qquad (\because I = I_1 + I_2)$$

(c) $K_f = \frac{1}{2}(I_1 + I_2)\frac{(I_1\omega_1 + I_2\omega_2)^2}{(I_1 + I_2)^2} = \frac{1}{2}\frac{(I_1\omega_1 + I_2\omega_2)^2}{(I_1 + I_2)}$

$K_i = \frac{1}{2}(I_1\omega_1^2 + I_2\omega_2^2)$

$\Delta K = K_f - K_i = -\frac{I_1 I_2}{2(I_1 + I_2)}(\omega_1 - \omega_2)^2 < 0$

(d) Hence, there is loss in KE of the system. The loss in kinetic energy is mainly due to the work against the friction between the two discs.

Q. 25 A disc of radius R is rotating with an angular ω_0 about a horizontal axis. It is placed on a horizontal table. The coefficient of kinetic friction is μ_κ.

(a) What was the velocity of its centre of mass before being brought in contact with the table?

(b) What happens to the linear velocity of a point on its rim when placed in contact with the table?

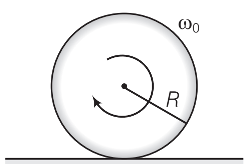
(c) What happens to the linear speed of the centre of mass when disc is placed in contact with the table?

(d) Which force is responsible for the effects in (b) and (c)?

(e) What condition should be satisfied for rolling to begin?

(f) Calculate the time taken for the rolling to begin.

Ans. (a) Before being brought in contact with the table the disc was in pure rotational motion hence, $v_{CM} = 0$.

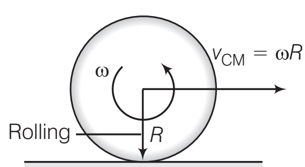


(b) When the disc is placed in contact with the table due to friction velocity of a point on the rim decreases.

(c) When the rotating disc is placed in contact with the table due to friction centre of mass acquires some linear velocity.

(d) Friction is responsible for the effects in (b) and (c) .

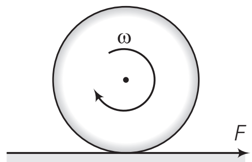
(e) When rolling starts $v_{CM} = \omega R$.



where ω is angular speed of the disc when rolling just starts.

(f) Acceleration produced in centre of mass due to friction

$$a_{CM} = \frac{F}{m} = \frac{\mu_k mg}{m} = \mu_k g.$$



Angular retardation produced by the torque due to friction.

$$\alpha = \frac{\tau}{I} = \frac{\mu_k mgR}{I} \qquad [\because \tau = (\mu_k N)R = \mu_k mgR]$$

$\therefore$ $$v_{CM} = u_{CM} + a_{CM}t$$

$\Rightarrow$ $$v_{CM} = \mu_k gt \qquad (\because u_{CM} = 0)$$

and $$\omega = \omega_0 + \alpha t$$

$\Rightarrow$ $$\omega = \omega_0 - \frac{\mu_k mgR}{I}t$$

For rolling without slipping, $\frac{v_{CM}}{R} = \omega$

$\Rightarrow$ $$\frac{v_{CM}}{R} = \omega_0 - \frac{\mu_k mgR}{I}t$$

$$\frac{\mu_k gt}{R} = \omega_0 - \frac{\mu_k mgR}{I}t$$

$$t = \frac{R\omega_0}{\mu_k g\left(1 + \frac{mR^2}{I}\right)}$$

Note *In this problem, frictional force helps in setting pure rolling motion.*

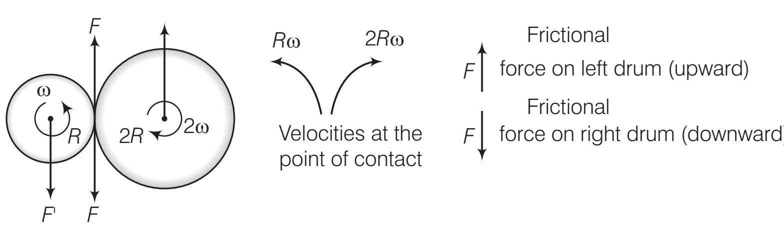
Q. 26 Two cylindrical hollow drums of radii R and $2R$, and of a common height h, are rotating with angular velocities ω (anti-clockwise) and ω (clockwise), respectively. Their axes, fixed are parallel and in a horizontal plane separated by $3R + \delta$. They are now brought in contact $(\delta \to 0)$

(a) Show the frictional forces just after contact.

(b) Identify forces and torques external to the system just after contact.

(c) What would be the ratio of final angular velocities when friction ceases?

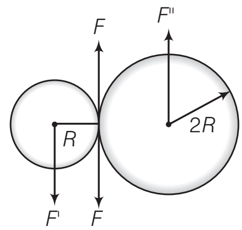
Ans. (a) Consider the situation shown below, we have shown the frictional forces.



(b) $F' = F = F''$ where F' and F'' are external forces through support.

$\Rightarrow$ $F_{net} = 0$ (one each cylinder)

External torque $= F \times 3R$, (anti-clockwise)



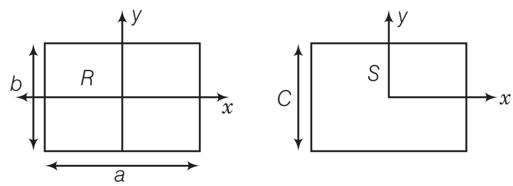
(c) Let ω_1 and ω_2 be final angular velocities of smaller and bigger drum respectively (anti-clockwise and clockwise respectively)

Finally, there will be no friction.

Hence, $R\omega_1 = 2R\omega_2 \Rightarrow \dfrac{\omega_1}{\omega_2} = 2$

***Note** We should be very careful while indicating direction of frictional forces.*

Q. 27 A uniform square plate S (side c) and a uniform rectangular plate R (sides b, a) have identical areas and masses.



Show that

(a) $I_{xR} / I_{xS} < 1$ (b) $I_{yR} / I_{yS} > 1$ (c) $I_{zR} / I_{zS} > 1$

Ans. By given question

Area of square = Area of rectangular plate

$\Rightarrow$ $c^2 = a \times b \Rightarrow c^2 = ab$

Now by definition

(a) $$\frac{I_{xR}}{I_{xS}} = \frac{b^2}{c^2} \qquad [\because I \propto (\text{area})^2]$$

From the diagram $b < c$

$\Rightarrow$ $$\frac{I_{xR}}{I_{xS}} = \left(\frac{b}{c}\right)^2 < 1 \Rightarrow I_{xR} < I_{xS}$$

(b) $$\frac{I_{yR}}{I_{yS}} = \frac{a^2}{c^2}$$

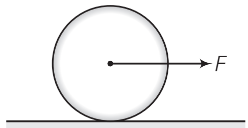
as $a > c$

$$\frac{I_{yR}}{I_{yS}} = \left(\frac{a}{c}\right)^2 > 1$$

(c) $I_{zR} - I_{zS} \propto (a^2 + b^2 - 2c^2) = a^2 + b^2 - 2ab = (a-b)^2 \quad [\because c^2 = ab]$

$\Rightarrow$ $$(I_{zR} - I_{zS}) > 0 \Rightarrow \frac{I_{zR}}{I_{zS}} > 1$$

Q. 28 A uniform disc of radius R, is resting on a table on its rim. The coefficient of friction between disc and table is μ (figure). Now, the disc is pulled with a force F as shown in the figure. What is the maximum value of F for which the disc rolls without slipping?



Thinking Process

Frictional force on the disc will be in opposite direction of F and supports the rotation of the disc in clockwise direction.

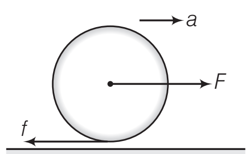
Ans. Consider the diagram below

Frictional force (f) is acting in the opposite direction of F.

Let the acceleration of centre of mass of disc be a then

$$F - f = Ma \quad \text{...(i)}$$

where M is mass of the disc



The angular acceleration of the disc is

$$\alpha = a/R \quad \text{(for pure rolling)}$$

from $\tau = I\alpha$

$\Rightarrow$ $$fR = \left(\frac{1}{2}MR^2\right)\alpha \Rightarrow fR = \left(\frac{1}{2}MR^2\right)\left(\frac{a}{R}\right)$$

$\Rightarrow$ $$Ma = 2f \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$f = F/3 \quad [\because N = Mg]$$

$\because$ $$f \le \mu N = \mu Mg$$

$\Rightarrow$ $$\frac{F}{3} \le \mu Mg \Rightarrow F \le 3\mu Mg$$

$\Rightarrow$ $$F_{max} = 3\mu Mg$$