1. **$\cos^{-1}x$ ની મૂળભૂત કિંમતોને સમાવતો ગણ ..... છે.**

(A) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (B) $(0, \pi)$ (C) $[0, \pi]$ (D) $(0, \pi) - \left\{\frac{\pi}{2}\right\}$

જવાબ (C) $[0, \pi]$

➥ $\cos^{-1}$ ની વ્યાખ્યા મુજબ તેનો વિસ્તાર ગણ $[0, \pi]$ છે.
$\therefore$ $\cos^{-1}x$ ની મૂળભૂત કિંમતો $[0, \pi]$ માં સમાયેલ હોય.

2. **$\text{cosec}^{-1}x$ ની મૂળભૂત કિંમતોને સમાવતો ગણ ..... હોય.**

(A) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (B) $[0, \pi] - \left\{\frac{\pi}{2}\right\}$ (C) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (D) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

જવાબ (D) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

➥ $\text{cosec}^{-1}x$ ની વ્યાખ્યા મુજબ મૂળભૂત કિંમતોને સમાવતો ગણ $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$ થાય. જે $\text{cosec}^{-1}x$ નો સહપ્રદેશ ગણ છે.

3. **જો $3\tan^{-1}x + \cot^{-1}x = \pi$ હોય તો x = ...**

(A) 0 (B) 1 (C) -1 (D) $\frac{1}{2}$

જવાબ (B) 1

➥ $3\tan^{-1}x + \cot^{-1}x = \pi$

$\therefore$ $2\tan^{-1}x + \tan^{-1}x + \cot^{-1}x = \pi$

$\therefore$ $2\tan^{-1}x + \frac{\pi}{2} = \pi$ ($\because$ કોટી સંખ્યાના સૂત્ર મુજબ)

$\therefore$ $2\tan^{-1}x = \frac{\pi}{2}$

$\therefore$ $\tan^{-1}x = \frac{\pi}{4}$

$\therefore$ $x = \tan\frac{\pi}{4}$

$\therefore$ $x = 1$

4. **$\sin^{-1}\left(\cos\left(\frac{33\pi}{5}\right)\right)$ = ...**

(A) $\frac{3\pi}{5}$ (B) $\frac{-7\pi}{5}$ (C) $\frac{\pi}{10}$ (D) $\frac{-\pi}{10}$

જવાબ (C) $\frac{\pi}{10}$

➥ અહીં $\cos\left(\frac{33\pi}{5}\right) = \cos\left(\frac{30\pi + 3\pi}{5}\right)$

$= \cos\left(6\pi + \frac{3\pi}{5}\right)$

$= \cos\left(\frac{3\pi}{5}\right)$

$$= \cos\left(\frac{\pi}{2} - \frac{\pi}{10}\right) = \sin\left(\frac{\pi}{10}\right)$$

$\therefore \cos\left(\frac{33\pi}{5}\right) = \sin\left(\frac{\pi}{10}\right)$

$\therefore \sin^{-1}\left(\cos\left(\frac{33\pi}{5}\right)\right) = \sin^{-1}\left(\sin\frac{\pi}{10}\right)$

$$= \sin^{-1}\left(\sin\frac{\pi}{10}\right)$$

$$= \frac{\pi}{10}$$

5. **વિધેય $\cos^{-1}(2x - 1)$ નો પ્રદેશ ગણ ..... છે.**

(A) [0, 1] (B) [–1, 1] (C) (–1, 1] (D) [0, π]

જવાબ (A) [0, 1]

➥ ધારો કે $\cos^{-1}(2x - 1) = \theta$

$\therefore 2x - 1 = \cos\theta$

હવે $-1 \le \cos\theta \le 1$ હોય.

$\therefore -1 \le 2x - 1 \le 1$

$\therefore -1 + 1 \le 2x - 1 + 1 \le 1 + 1$

$\therefore 0 \le 2x \le 2$

$\therefore 0 \le x \le 1$

$\therefore x \in [0, 1]$

$\therefore \cos^{-1}(2x - 1)$ નો પ્રદેશ ગણ = [0, 1] થાય.

6. **વિધેય $f(x) = \sin^{-1}\left(\sqrt{x-1}\right)$ નો પ્રદેશ ગણ ..... છે.**

(A) [1, 2] (B) [–1, 1] (C) [0, 1] (D) આ પૈકી કોઈ નહીં.

જવાબ (A) [1, 2]

➥ ધારો કે, $\sin^{-1}\left(\sqrt{x-1}\right) = \theta$

$\therefore \sqrt{x-1} = \sin\theta$

$\therefore \sin^2\theta = x - 1$

હવે $0 \le \sin^2\theta \le 1$

$\therefore 0 \le x - 1 \le 1$

$\therefore 0 + 1 \le (x - 1) + 1 \le 1 + 1$

$\therefore 1 \le x \le 2$

$\therefore x \in [1, 2]$

$\therefore \sin^{-1}\left(\sqrt{x-1}\right)$ નો પ્રદેશગણ = [1, 2] થાય.

7. **$\cos\left(\sin^{-1}\frac{2}{5} + \cos^{-1}x\right) = 0$ તો x = ...**

(A) $\frac{1}{5}$ (B) $\frac{2}{5}$ (C) 0 (D) 1

જવાબ (B) $\frac{2}{5}$

➥ અહીં $\cos\left(\sin^{-1}\frac{2}{5} + \cos^{-1}x\right) = 0$

$\therefore \cos\left(\sin^{-1}\frac{2}{5} + \cos^{-1}x\right) = \cos\frac{\pi}{2}$

$\therefore \sin^{-1}\left(\frac{2}{5}\right) + \cos^{-1}x = \frac{\pi}{2}$

$\therefore \sin^{-1}\left(\frac{2}{5}\right) = \frac{\pi}{2} - \cos^{-1}x$

$\therefore \sin^{-1}\left(\frac{2}{5}\right) = \sin^{-1} x$ ($\because$ કોટી સંખ્યાનું પરિણામ)

$\therefore x = \frac{2}{5}$

8. $\sin\left[2\tan^{-1}(0.75)\right]$ **= .....**

(A) 0.75 (B) 1.5 (C) 0.96 (D) sin(15°)

જવાબ **(C) 0.96**

➥ $\sin\left[2\tan^{-1}(0.75)\right]$

$= \sin\left(2\tan^{-1}\left(\frac{3}{4}\right)\right)$

ધારો કે $\tan^{-1}\frac{3}{4} = \theta$

$\therefore \frac{3}{4} = \tan\theta$

$\therefore \sin\left(2\tan^{-1}\frac{3}{4}\right) = \sin(2\theta)$

$= \frac{2\tan\theta}{1+\tan^2\theta}$

$= \frac{2\left(\frac{3}{4}\right)}{1+\frac{9}{16}}$

$= \frac{2\left(\frac{3}{4}\right)(16)}{16+9}$

$= \frac{24}{25}$

$= 0.96$

9. $\cos^{-1}\left(\cos\frac{3\pi}{2}\right)$ **= .....**

(A) $\frac{\pi}{2}$ (B) $\frac{3\pi}{2}$ (C) $\frac{5\pi}{2}$ (D) $\frac{7\pi}{2}$

જવાબ **(A)** $\frac{\pi}{2}$

➥ $\cos^{-1}\left(\cos\frac{3\pi}{2}\right) = \cos^{-1}(0)$ $\left(\because \cos\frac{3\pi}{2} = 0 \text{ હોય.}\right)$

$= \cos^{-1}\left(\cos\frac{\pi}{2}\right)$

$\therefore \cos^{-1}\left(\cos\frac{3\pi}{2}\right) = \frac{\pi}{2}$

10. $2\sec^{-1}(2) + \sin^{-1}\left(\frac{1}{2}\right)$ **= .....**

(A) $\frac{\pi}{6}$ (B) $\frac{5\pi}{6}$ (C) $\frac{7\pi}{6}$ (D) 1

જવાબ **(B)** $\frac{5\pi}{6}$

➥ $2\sec^{-1}(2) + \sin^{-1}\left(\frac{1}{2}\right)$

$= 2\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right)$

$= 2\left(\frac{\pi}{3}\right) + \frac{\pi}{6}$

$= \frac{4\pi + \pi}{6} = \frac{5\pi}{6}$

11. **જો $\tan^{-1}x + \tan^{-1}y = \frac{4\pi}{5}$ હોય તો $\cot^{-1}x + \cot^{-1}y$ = ...**

(A) $\frac{\pi}{5}$ (B) $\frac{2\pi}{5}$ (C) $\frac{3\pi}{5}$ (D) π

જવાબ (A) $\frac{\pi}{5}$

➥ અહીં $\tan^{-1}x + \tan^{-1}y = \frac{4\pi}{5}$

$\therefore \frac{\pi}{2} - \cot^{-1}x + \frac{\pi}{2} - \cot^{-1}y = \frac{4\pi}{5}$

($\because$ કોટી સંખ્યાના સૂત્ર મુજબ)

$\therefore \pi - \frac{4\pi}{5} = \cot^{-1}x + \cot^{-1}y$

$\therefore \frac{\pi}{5} = \cot^{-1}x + \cot^{-1}y$

12. **જો $\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ હોય તો x = ..., જ્યાં $a, x \in [0, 1]$.**

(A) 0 (B) $\frac{a}{2}$ (C) a (D) $\frac{2a}{1-a^2}$

જવાબ (D) $\frac{2a}{1-a^2}$

➥ $\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

અહીં, $a = \tan\theta$ મૂકો.

$\therefore \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) + \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$\therefore \sin^{-1}(\sin 2\theta) + \cos^{-1}(\cos 2\theta) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$\therefore 2\theta + 2\theta = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$\therefore 2(2\theta) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$\therefore 2(2\tan^{-1}a) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$\therefore 2\left(\tan^{-1}\left(\frac{2a}{1-a^2}\right)\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$\therefore \tan^{-1}\left(\frac{2\left(\frac{2(2a)}{1-a^2}\right)}{1-\left(\frac{2a}{1-a^2}\right)^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$\therefore x = \dfrac{2a}{1-a^2}$ થાય.

13. $\cot\left(\cos^{-1}\left(\dfrac{7}{25}\right)\right) =$ .....

(A) $\dfrac{25}{24}$ (B) $\dfrac{25}{7}$ (C) $\dfrac{24}{25}$ (D) $\dfrac{7}{24}$

જવાબ (D) $\dfrac{7}{24}$

➥ ધારો કે $\cos^{-1}\left(\dfrac{7}{25}\right) = \theta$

$\therefore \cos\theta = \dfrac{7}{25}$

હવે $\sin\theta = \sqrt{1-\cos^2\theta}$

$= \sqrt{1-\dfrac{49}{625}}$

$= \sqrt{\dfrac{625-49}{625}}$

$= \sqrt{\dfrac{576}{625}}$

$\therefore \sin\theta = \dfrac{24}{25}$

$\therefore \cot\theta = \dfrac{\cos\theta}{\sin\theta} = \dfrac{\frac{7}{25}}{\frac{24}{25}} = \dfrac{7}{24}$

$\Rightarrow \theta = \cot^{-1}\left(\dfrac{7}{24}\right)$

$\therefore \cot\left(\cos^{-1}\left(\dfrac{7}{25}\right)\right) = \cot\left(\cot^{-1}\dfrac{7}{24}\right)$

$= \dfrac{7}{24}$

14. $\tan\left(\dfrac{1}{2}\cos^{-1}\dfrac{2}{\sqrt{5}}\right) =$ .....

(A) $2+\sqrt{5}$ (B) $\sqrt{5}-2$ (C) $\dfrac{\sqrt{5}+2}{2}$ (D) $5+\sqrt{2}$

જવાબ (B) $\sqrt{5}-2$

➥ $\tan\left(\dfrac{1}{2}\cos^{-1}\dfrac{2}{\sqrt{5}}\right)$

ધારો કે, $\cos^{-1}\left(\dfrac{2}{\sqrt{5}}\right) = \theta$

$\therefore \cos\theta = \dfrac{2}{\sqrt{5}}$

$\therefore \tan\left(\dfrac{1}{2}\cos^{-1}\dfrac{2}{\sqrt{5}}\right) = \tan\left(\dfrac{\theta}{2}\right)$

$= \sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}}$

$$= \sqrt{\frac{1 - \frac{2}{\sqrt{5}}}{1 + \frac{2}{\sqrt{5}}}}$$

$$= \sqrt{\frac{\sqrt{5} - 2}{\sqrt{5} + 2}}$$

$$= \sqrt{\frac{(\sqrt{5} - 2)^2}{(\sqrt{5} + 2) \cdot (\sqrt{5} - 2)}}$$

$$= \sqrt{\frac{(\sqrt{5} - 2)^2}{5 - 4}}$$

$$= \sqrt{(\sqrt{5} - 2)^2}$$

$$= \sqrt{5} - 2$$

15. જો $|x| \leq 1$ હોય તો $2\tan^{-1}x + \sin^{-1}\left(\frac{2x}{1+x^2}\right) =$ .....

(A) $4\tan^{-1}x$ (B) 0 (C) $\frac{\pi}{2}$ (D) π

જવાબ (A) $4\tan^{-1}x$

➥ $2\tan^{-1}x + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

અહીં, $x = \tan\theta$ મૂકો.

$= 2\tan^{-1}(\tan\theta) + \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$= 2\theta + \sin^{-1}(\sin 2\theta)$

$= 2\theta + 2\theta = 4\theta = 4\tan^{-1}x$

16. જો $\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$ હોય તો $\alpha(\beta+\gamma) + \beta(\gamma+\alpha) + \gamma(\alpha+\beta) =$ .....

(A) 0 (B) 1 (C) 6 (D) 12

જવાબ (C) 6

➥ $\cos^{-1}$ નો વિસ્તાર ગણ $= [0, \pi]$ છે.

$\therefore$ આમ $0 \leq \cos^{-1}\alpha \leq \pi$ થાય.

$\therefore \cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$

$= \pi + \pi + \pi$

$\therefore \cos^{-1}\alpha = \pi \therefore \alpha = \cos\pi$

$\therefore \alpha = -1$

આજ રીતે $\beta = -1$ અને $\gamma = -1$ મળે.

$\therefore \alpha(\beta+\gamma) + \beta(\gamma+\alpha) + \gamma(\alpha+\beta)$

$= -1(-1-1) + (-1)(-1-1) + (-1)(-1-1)$

$= -1(-2) + (-1)(-2) + (-1)(-2)$

$= 2 + 2 + 2$

$= 6$

17. અંતરાલ $\left[\frac{\pi}{2}, \pi\right]$ માં સમીકરણ $\sqrt{1+\cos 2x} = \sqrt{2}\ \cos^{-1}(\cos x)$ ના ઉકેલની સંખ્યા ..... છે.

(A) શૂન્ય (B) એક (C) બે (D) અનંત

જવાબ (A) શૂન્ય

➥ $\sqrt{1+\cos 2x} = \sqrt{2}\ \cos^{-1}(\cos x)$

$\therefore \sqrt{2\cos^2 x} = \sqrt{2} \cdot x \quad (\because\ 1 + \cos(2\theta) = 2\cos^2\theta)$

$\therefore \sqrt{2}\cos x = \sqrt{2} \cdot x$

$\therefore \cos x = x$

જે x ની કોઈપણ વાસ્તવિક કિંમત માટે સત્ય નથી.

$\therefore$ આપેલ સમીકરણને એકપણ ઉકેલ ન હોય.

18. જો $\cos^{-1}x > \sin^{-1}x$ હોય તો નીચેના પૈકી ..... સત્ય હોય.

(A) $\frac{1}{\sqrt{2}} < x \leq 1$ (B) $0 \leq x < \frac{1}{\sqrt{2}}$ (C) $-1 \leq x < \frac{1}{\sqrt{2}}$ (D) $x > 0$

જવાબ (C) $\mathbf{-1 \leq x < \frac{1}{\sqrt{2}}}$

➥ $\cos^{-1}x > \sin^{-1}x$

$\therefore \cos^{-1}x > \frac{\pi}{2} - \cos^{-1}x \quad (\because$ કોટી સંખ્યાનું પરિણામ)

$\therefore 2\cos^{-1}x > \frac{\pi}{2}$

$\therefore \cos^{-1}x > \frac{\pi}{4}$

$\therefore \cos^{-1}x > \frac{1}{\sqrt{2}}$

$\therefore \frac{1}{\sqrt{2}} < \cos^{-1}x$ અને $x \in [-1, 1]$ અર્થાત્ $-1 \leq x \leq 1$

$\therefore -1 \leq x < \frac{1}{\sqrt{2}}$

$(\because$ પ્રથમ ચરણમાં cos ઘટતું વિધેય છે.)