HOTS (Higher Order Thinking Skills)

Que 1. If $z^2 + \frac{1}{z^2} = 14$, find the value of $z^3 + \frac{1}{z^3}$.

Sol. We have,

$\left(z + \frac{1}{z}\right)^2 = z^2 + \frac{1}{z^2} + 2z\frac{1}{z}$

$\Rightarrow \left(z + \frac{1}{z}\right)^2 = z^2 + \frac{1}{z^2} + 2 \quad \Rightarrow \left(z + \frac{1}{z}\right)^2 = 14 + 2 = 16$

$\Rightarrow \left(z + \frac{1}{z}\right)^2 = 4^2 \quad \Rightarrow z^3 + \frac{1}{Z} = 4$

$\Rightarrow \left(z + \frac{1}{z}\right)^3 = 4^3 \quad \Rightarrow z^3 + \frac{1}{z^3} + 3 \text{ x } z \text{ x } \frac{1}{z}\left(z + \frac{1}{z}\right) = 64$

$\Rightarrow z^3 + \frac{1}{z^3} + 3 \text{ x } 4 = 64 \quad \Rightarrow z^3 + \frac{1}{z^3} = 64 - 12$

$\Rightarrow z^3 + \frac{1}{z^3} = 52$

Que 2. If $x + \frac{1}{x} = 3$, find the value of $x^4 + \frac{1}{x^4}$.

Sol. We have $x + \frac{1}{x} = 3$

Squaring both the sides, we get

$\left(x + \frac{1}{x}\right)^2 = 3^2 \quad \Rightarrow \left(x + \frac{1}{x}\right)^2 = 9$

$\Rightarrow x^2 + \frac{1}{x^2} + 2 \text{ x } x \text{ x } \frac{1}{x} = 9 \quad \Rightarrow x^2 + \frac{1}{x^2} + 2 = 9$

$\Rightarrow x^2 + \frac{1}{x^2} = 9 - 2 \quad \Rightarrow x^2 + \frac{1}{x^2} = 7$

Now, $x^2 + \frac{1}{x^2} = 7 \quad \Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = 7^2 \quad \Rightarrow (x^2)^2 +$

$\Rightarrow x^4 + \frac{1}{x^4} + 2 = 49 \quad \Rightarrow x^4 + \frac{1}{x^4} = 49 - 2 \quad \Rightarrow x^4 + \frac{1}{x^4} = 47$

Que 3. If the polynomials $az^3 + 4z^2 + 3z - 4$ and $z^3 - 4z + a$ leave the same remainder when divided by $z - 3$, find the value of a.

Sol. Let $p(z) = az^3 + 4z^2 + 3z - 4$ and $q(z) = z^3 - 4z + a$

When p(z) is divided by z – 3 the remainder is given by,

P (3) = a x 3^3 + 4 x 3^2 + 3 x 3 – 4 = 27a + 36 + 9 – 4

P (3) = 27a + 41 …(i)

When q(z) is divided by z – 3 the remainder is given by,

q (3) = 3^3 – 4 x 3 + a = 27 – 12 + a

q (3) = 15 + a …(ii)

According to question, p (3) = q (3)

$\Rightarrow$ 27a + 41 = 15 + a $\Rightarrow$ 27a – a = - 41 + 15

26a = - 26

$\Rightarrow$ a = $\frac{-26}{26}$ $\Rightarrow$ a = - 1

Que 4. If $x^2 + \frac{1}{x^2} = 34$, find $x^3 + \frac{1}{x^3} - 9$.

Sol. $\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2 = 34 + 2 = 36$

$\left(x + \frac{1}{x}\right) = 6$

On cubing, we get

$\left(x + \frac{1}{x}\right)^3 = 6^3$ $\Rightarrow$ $x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 216$

$\Rightarrow$ $x^3 + \frac{1}{x^3} + 3 \times 6 = 216$ $\Rightarrow$ $x^3 + \frac{1}{x^3} = 198$

$\Rightarrow$ $x^3 + \frac{1}{x^3} - 9 = 198 - 9 = 189$