CHAPTER

8

Term-II

APPLICATIONS OF THE INTEGRALS

Syllabus

- ***Applications in finding the area under simple curves, especially lines, parabolas; area of circles/ parabolas/ellipses (in standard form only). (the region should be clearly identifiable).***

STAND ALONE MCQs (1 Mark each)

Q. 1. The area of the region bounded by the y-axis, $y = \cos x$ and $y = \sin x$, $0 \le x \le \pi/2$ is

(A) $\sqrt{2}$ sq. units (B) $(\sqrt{2}+1)$ sq. units

(C) $(\sqrt{2}-1)$ sq. units (D) $(2\sqrt{2}-1)$ sq. units

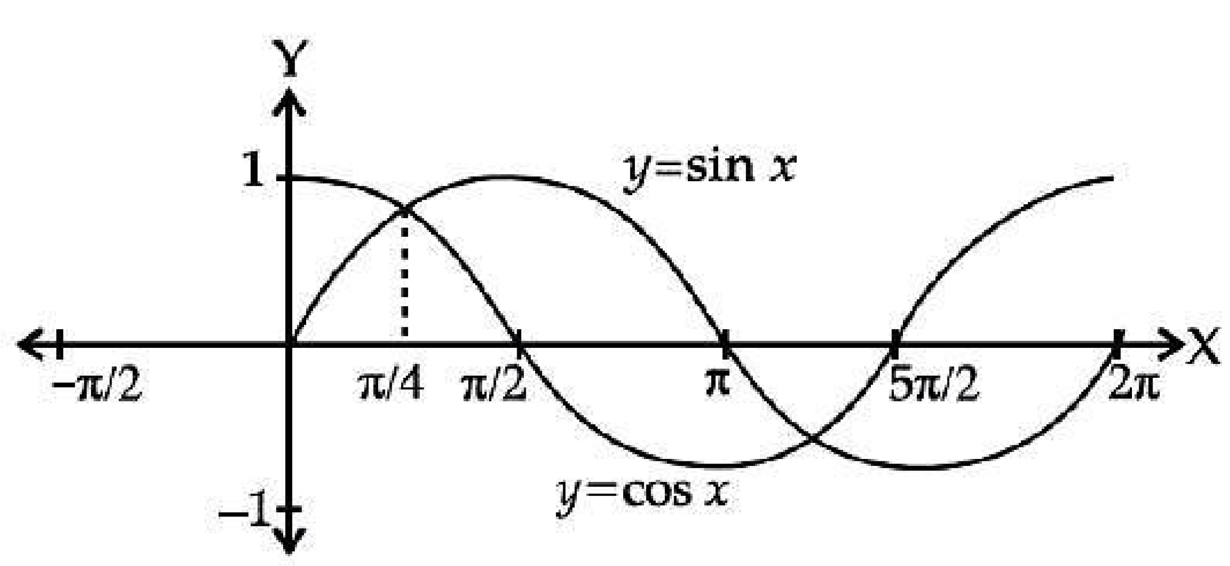
Ans. Option (C) is correct.

Explanation : We have $y = \cos x$ and $y = \sin x$, where $0 \le x \le \frac{\pi}{2}$.

We get $\cos x = \sin x$

$$\Rightarrow \quad x = \frac{\pi}{4}$$

From the figure, area of the shaded region,



$$A = \int_0^{\pi/4} (\cos x + \sin x)dx$$

$$= [\sin x + \cos x]_0^{\pi/4}$$

$$= \left[\sin\frac{\pi}{4} + \cos\frac{\pi}{4} - \sin 0 - \cos 0\right]$$

$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1$$

$$= (\sqrt{2}-1) \text{ sq. units}$$

Q. 2. The area of the region bounded by the curve $x^2 = 4y$ and the straight-line $x = 4y - 2$ is

(A) $\frac{3}{8}$ sq. units (B) $\frac{5}{8}$ sq. units

(C) $\frac{7}{8}$ sq. units (D) $\frac{9}{8}$ sq. units

Ans. Option (D) is correct.

Explanation:

$$x^2 = x + 2$$

$$x^2 - x - 2 = 0$$

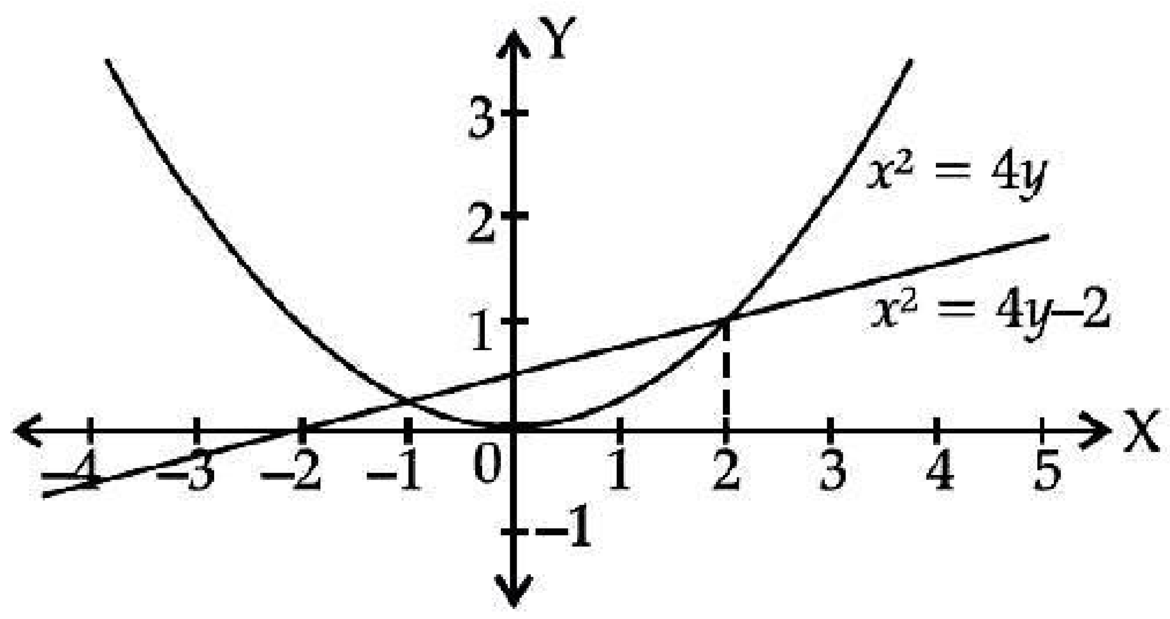
$$(x-2)(x+1) = 0$$

$$x = -1, 2$$

For $x = -1$, $y = \frac{1}{4}$ and for $x = 2$, $y = 1$

Points of intersection are $(-1, \frac{1}{4})$ and $(2, 1)$.

Graphs of parabola $x^2 = 4y$ and $x = 4y - 2$ are shown in the following figure :



$$A = \int_{-1}^{2}\left[\frac{x+2}{4}-\frac{x^2}{4}\right]dx$$

$$= \frac{1}{4}\left[\frac{x^2}{2}+2x-\frac{x^3}{3}\right]_{-1}^{2}$$

$$= \frac{1}{4}\left[8-\frac{1}{2}-3\right]$$

$$= \frac{1}{4}\left[8-\frac{1}{2}-3\right]$$

$$= \frac{9}{8} \text{ sq. units}$$

Q. 3. Area of the region in the first quadrant enclosed by the x-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$ is

(A) 16π sq. units (B) 4π sq. units
(C) 32π sq. units (D) 24π sq. units

Ans. Option (B) is correct.

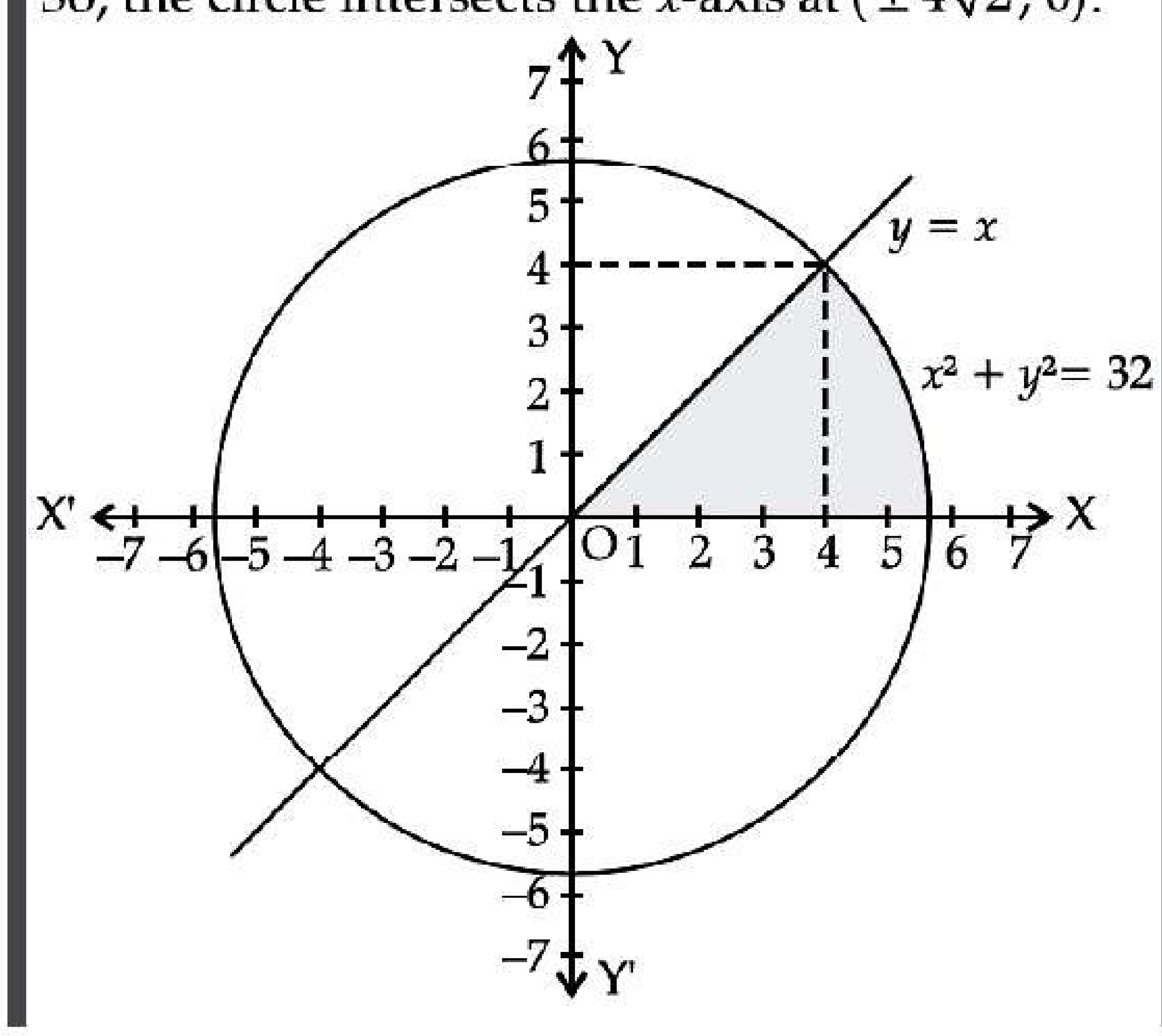
Explanation: We have $y = 0$, $y = x$ and the circle $x^2 + y^2 = 32$ in the first quadrant.
Solving $y = x$ with the circle

$$x^2 + x^2 = 32$$
$$x^2 = 16$$
$$x = 4 \quad \text{(In the first quadrant)}$$

When $x = 4$, $y = 4$ for the point of intersection of the circle with the x-axis.
Put $y = 0$

$$x^2 + 0 = 32$$
$$x = \pm 4\sqrt{2}$$

So, the circle intersects the x-axis at $(\pm 4\sqrt{2}, 0)$.



From the above figure, area of the shaded region,

$$A = \int_0^4 x\,dx + \int_4^{4\sqrt{2}} \sqrt{(4\sqrt{2})^2 - x^2}\,dx$$

$$= \left[\frac{x^2}{2}\right]_0^4 + \left[\frac{x}{2}\sqrt{(4\sqrt{2})^2 - x^2} + \frac{(4\sqrt{2})^2}{2}\sin^{-1}\frac{x}{4\sqrt{2}}\right]_4^{4\sqrt{2}}$$

$$= \left[\frac{16}{2}\right] + \left[\begin{array}{l} 0 + 16\sin^{-1}1 - \frac{4}{2}\sqrt{(4\sqrt{2})^2 - 16^2} \\ -16\sin^{-1}\frac{4}{4\sqrt{2}} \end{array}\right]$$

$$= 8 + \left[\frac{16\pi}{2} - 2\sqrt{16} - 16\frac{\pi}{4}\right]$$

$$= 8 + [8\pi - 8 - 4\pi]$$

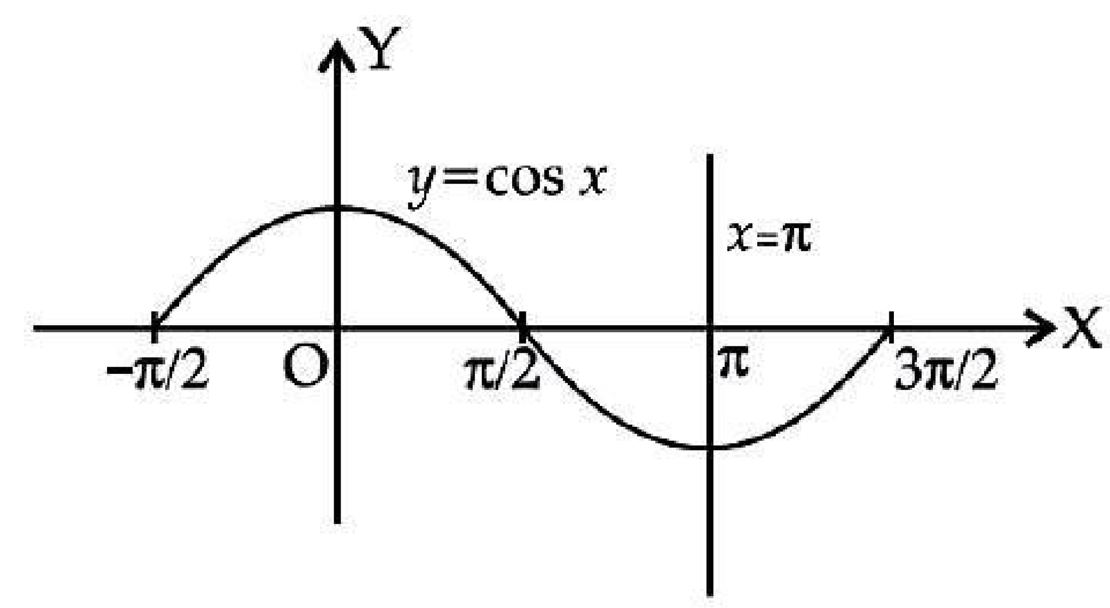
$$= 4\pi \text{ sq. units}$$

Q. 4. Area of the region bounded by the curve $y = \cos x$ between $x = 0$ and $x = \pi$ is

(A) 2 sq. units (B) 4 sq. units
(C) 3 sq. units (D) 1 sq. unit

Ans. Option (A) is correct.

Explanation : We have $y = \cos x$, $x = 0$, $x = \pi$



From the figure, area of the shaded region,

$$A = \int_0^{\pi}|\cos x|dx + \int_0^{\pi/2}\cos x dx$$

$$= 2[\sin x]_0^{\pi/2}$$

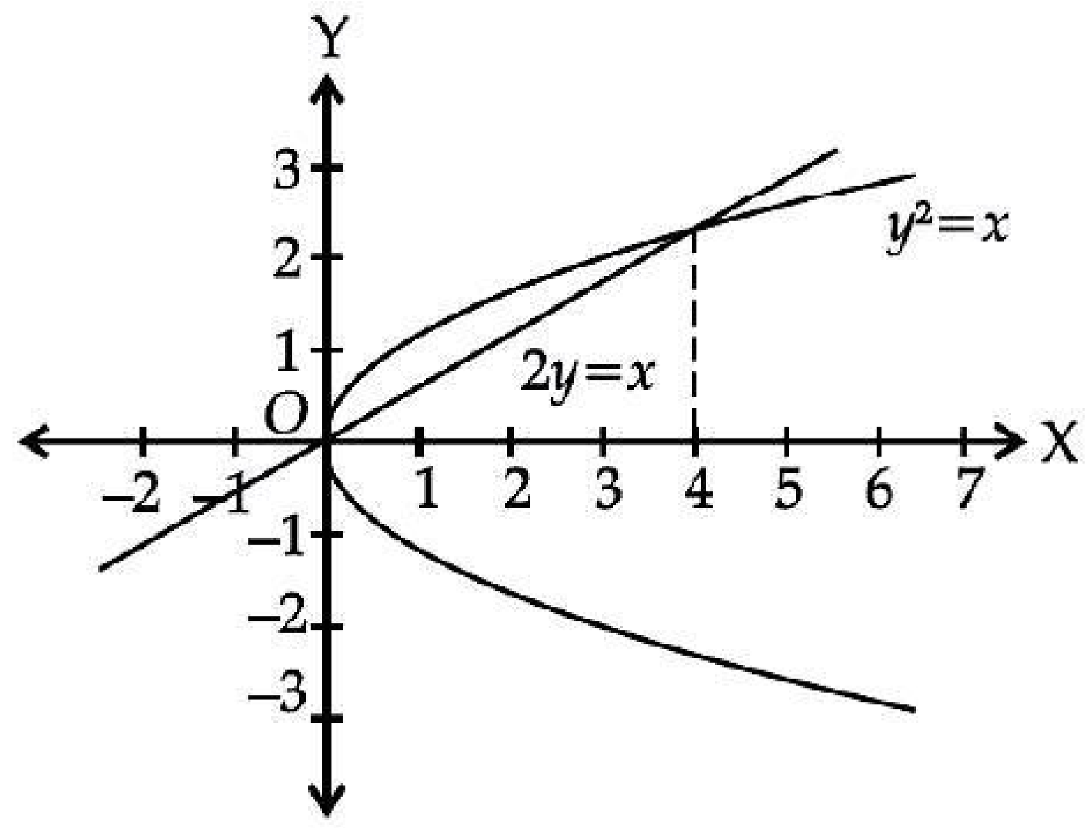
$$= 2 \text{ sq. units}$$

Q. 5. The area of the region bounded by parabola $y^2 = x$ and the straight line $2y = x$ is

(A) $\frac{4}{3}$ sq. units (B) 1 sq. unit
(C) $\frac{2}{3}$ sq. unit (D) $\frac{1}{3}$ sq. unit

Ans. Option (A) is correct.

Explanation : When $y^2 = x$ and $2y = x$
Solving we get $y^2 = 2y$
$\Rightarrow y = 0,\ 2$ and when $y = 2$, $x = 4$
So, points of intersection are (0, 0) and (4, 2).
Graphs of parabola $y^2 = x$ and $2y = x$ are as shown in the following figure :



From the figure, area of the shaded region,

$$A = \int_0^4 \left[\sqrt{x} - \frac{x}{2}\right] dx$$

$$= \left[\frac{2}{3}x^{3/2} - \frac{1}{2}\cdot\frac{x^2}{2}\right]_0^4$$

$$= \frac{2}{3}.(4)^{3/2} - \frac{16}{4} - 0$$

$$= \frac{16}{3} - 4$$

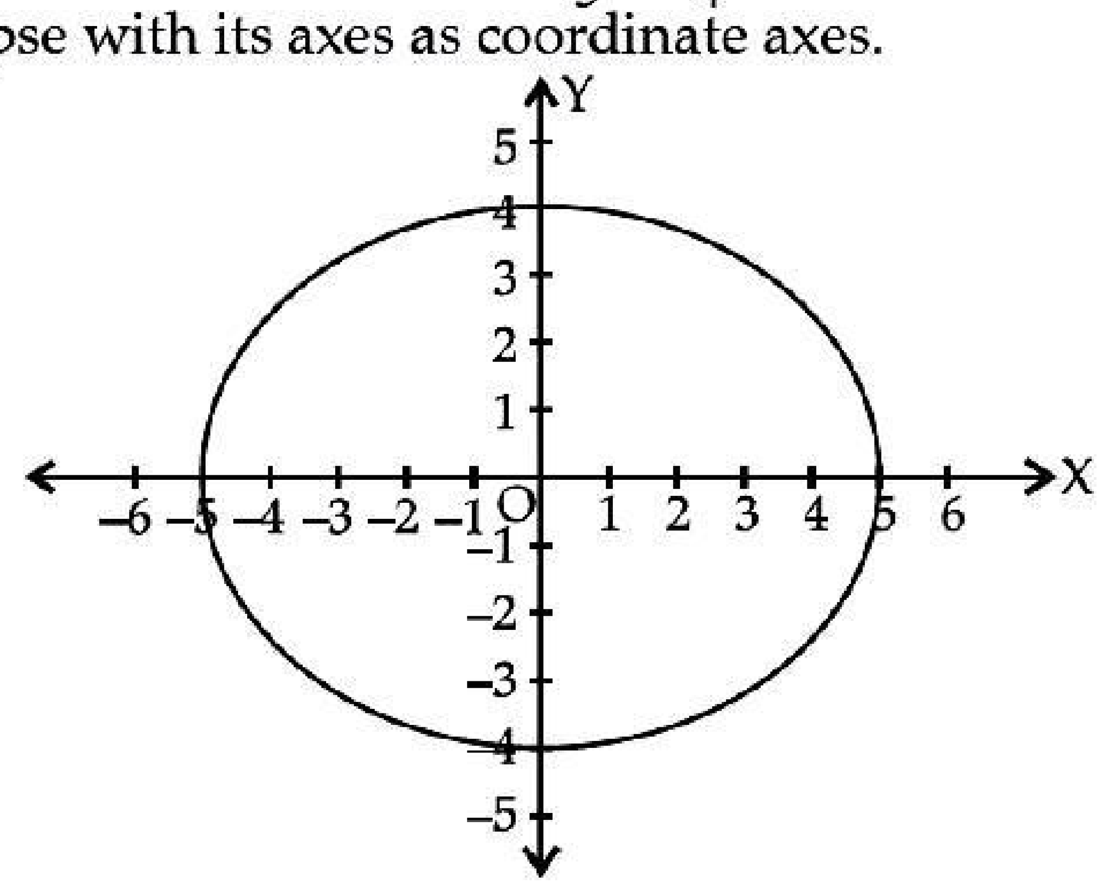
$$= \frac{4}{3} \text{ sq. unit}$$

Q. 6. The area of the region bounded by the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ is

(A) 20π sq. units **(B)** $20\pi^2$ sq. units
(C) $16\pi^2$ sq. units **(D)** 25π sq. units

Ans. Option (A) is correct.

Explanation: We have $\frac{x^2}{5^2} + \frac{y^2}{4^2} = 1$, which is ellipse with its axes as coordinate axes.



$$\frac{y^2}{4^2} = 1 - \frac{x^2}{5^2}$$

$$y^2 = 16\left(1 - \frac{x^2}{25}\right)$$

$$y = \frac{4}{5}\sqrt{5^2 - x^2}$$

From the figure, area of the shaded region,

$$A = 4\int_0^5 \frac{4}{5}\sqrt{5^2 - x^2}\, dx$$

$$= \frac{16}{5}\left[\frac{x}{2}\sqrt{5^2 - x^2} - \frac{5^2}{2}\sin^{-1}\frac{x}{5}\right]_0^5$$

$$= \frac{16}{5}\left[0 + \frac{5^2}{2}\sin^{-1}1 - 0 - 0\right]$$

$$= \frac{16}{5}\cdot\frac{25}{2}\cdot\frac{\pi}{2}$$

$$= 20\pi \text{ sq. units}$$

Q. 7. The area of the region bounded by the circle $x^2 + y^2 = 1$ is

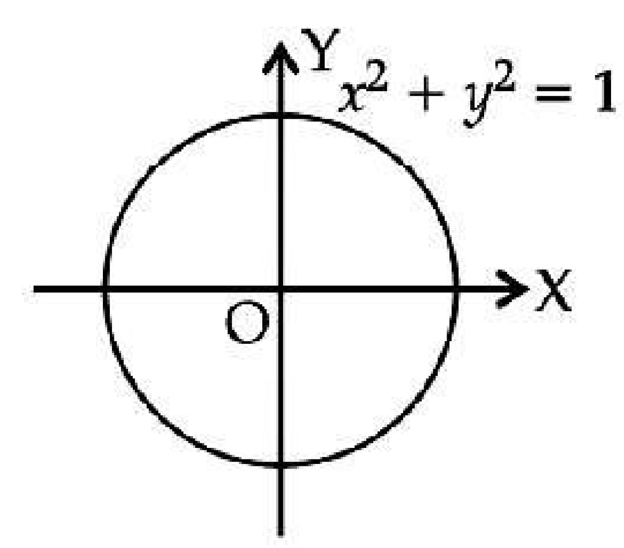
(A) 2π sq. units **(B)** π sq. units
(C) 3π sq. units **(D)** 4π sq. units

Ans. Option (B) is correct.

Explanation : We have, $x^2 + y^2 = 1$, which is a circle having centre at (0, 0) and radius '1' unit.

$$\Rightarrow \quad y^2 = 1 - x^2$$

$$y = \sqrt{1 - x^2}$$



From the figure, area of the shaded region,

$$A = 4\int_0^1 \sqrt{1^2 - x^2}\, dx$$

$$= 4\left[\frac{x}{2}\sqrt{1^2 - x^2} - \frac{1^2}{2}\sin^{-1}\frac{x}{1}\right]_0^1$$

$$= 4\left[0 + \frac{1^2}{2} \times \frac{\pi}{2} - 0 - 0\right]$$

$$= \pi \text{ sq. units}$$

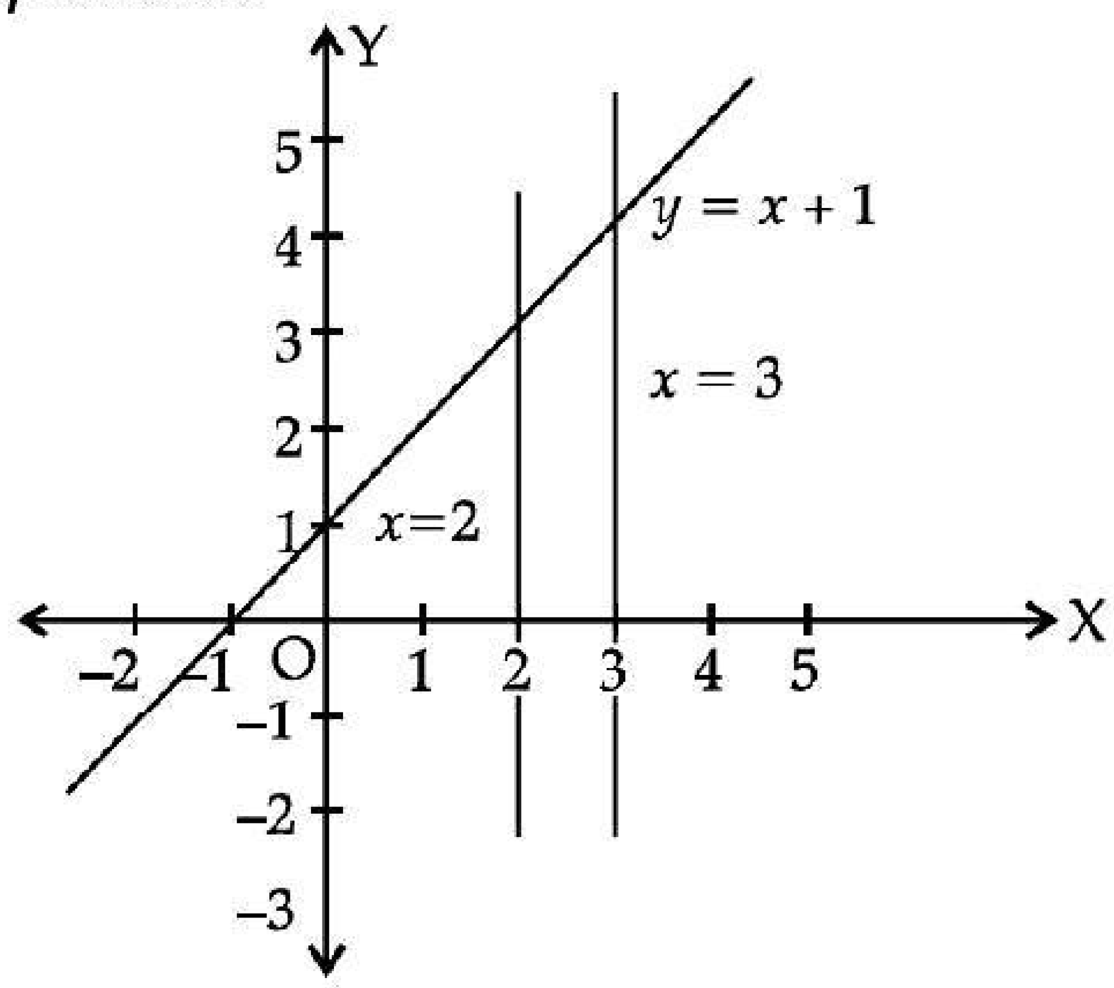
Q. 8. The area of the region bounded by the curve $y = x + 1$ and the lines $x = 2$ and $x = 3$ is

(A) $\frac{7}{2}$ sq. units **(B)** $\frac{9}{2}$ sq. units

(C) $\frac{11}{2}$ sq. units **(D)** $\frac{13}{2}$ sq. units

Ans. Option (A) is correct.

Explanation:



From the figure, area of the shaded region,

$$A = \int_2^3 (x+1)dx$$

$$= \left[\frac{x^2}{2} + x\right]_2^3$$

$$= \left[\frac{9}{2} + 3 - \frac{4}{2} - 2\right]$$

$$= \frac{7}{2} \text{ sq. units}$$

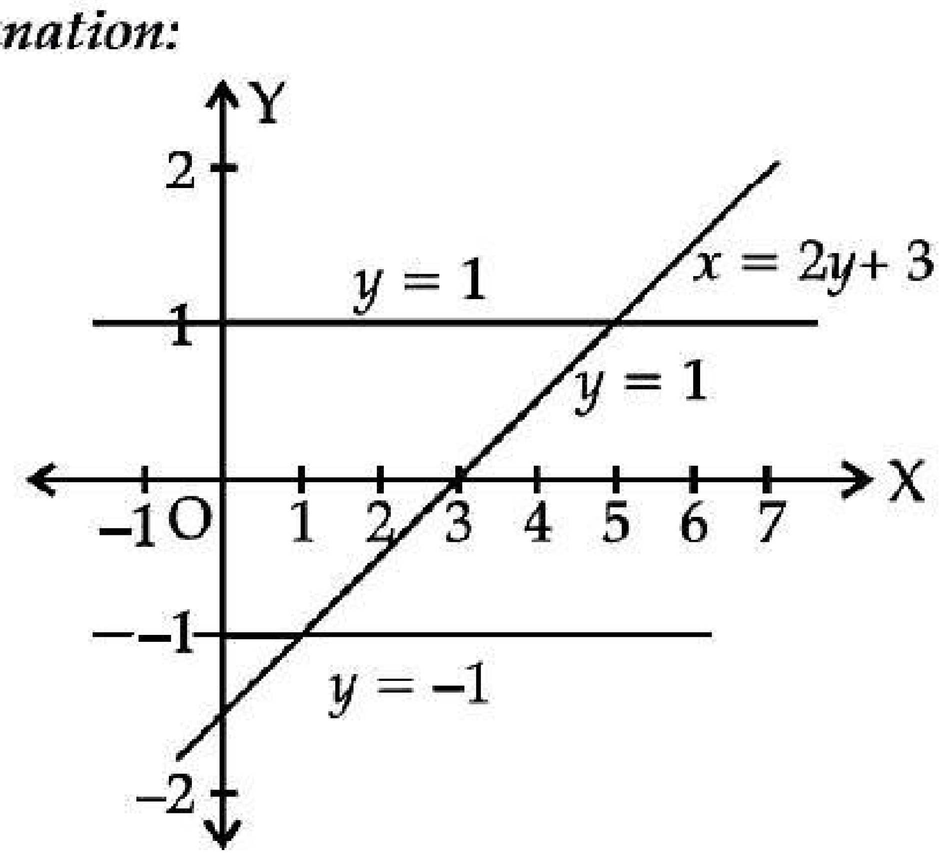
Q. 9. The area of the region bounded by the curve $x = 2y + 3$ and the y lines $y = 1$ and $y = -1$ is,

(A) 4 sq. units **(B)** $\frac{3}{2}$ sq. units

(C) 6 sq. units **(D)** 8 sq. units

Ans. Option (C) is correct.

Explanation:



From the figure, area of the shaded region,

$$A = \int_{-1}^{1} (2y+3)dy$$

$$= \left[y^2 + 3y\right]_{-1}^{1}$$

$$= [1+3-1+3]$$

$$= 6 \text{ sq. units}$$

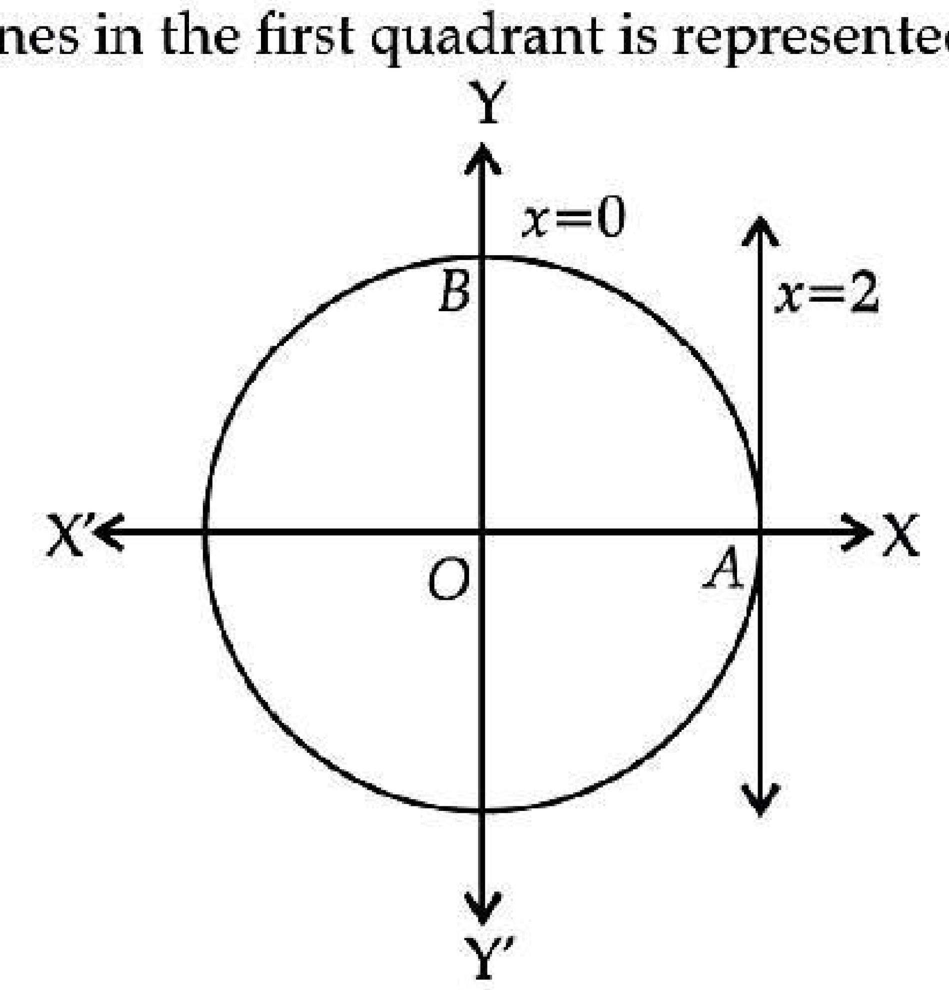
Q. 10. Area lying in the first quadrant and bounded by circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $x = 2$ is

(A) π **(B)** $\frac{\pi}{2}$

(C) $\frac{\pi}{3}$ **(D)** $\frac{\pi}{4}$

Ans. Option (A) is correct.

Explanation : The area bounded by the circle and the lines in the first quadrant is represented as :



$$A = \int_0^2 y dx$$

$$= \int_0^2 \sqrt{4-x^2}\,dx$$

$$= \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2$$

$$= \pi \text{ sq. units}$$

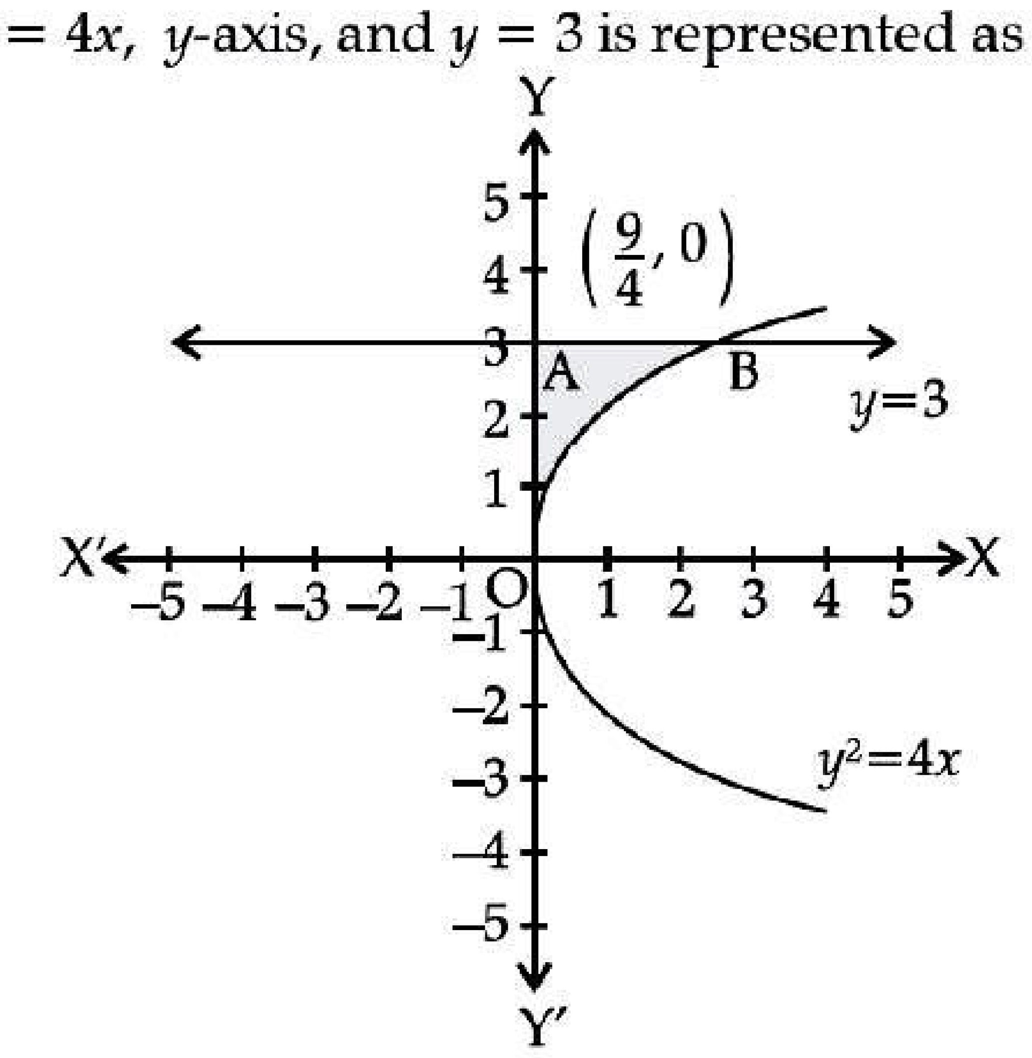
Q. 11. Area of the region bounded by the curve $y^2 = 4x$, y-axis and the line $y = 3$ is

(A) 2 **(B)** $\frac{9}{4}$

(C) $\frac{9}{3}$ **(D)** $\frac{9}{2}$

Ans. Option (B) is correct.

Explanation: The area bounded by the curve, $y^2 = 4x$, y-axis, and $y = 3$ is represented as :



$$\text{Area of } OAB = \int_0^3 x dy$$

$$= \int_0^3 \frac{y^2}{4} dy$$

$$= \frac{1}{4}\left[\frac{y^3}{4}\right]_0^3$$

$$= \frac{1}{12} \times 27$$

$$= \frac{9}{4} \text{ sq. units}$$

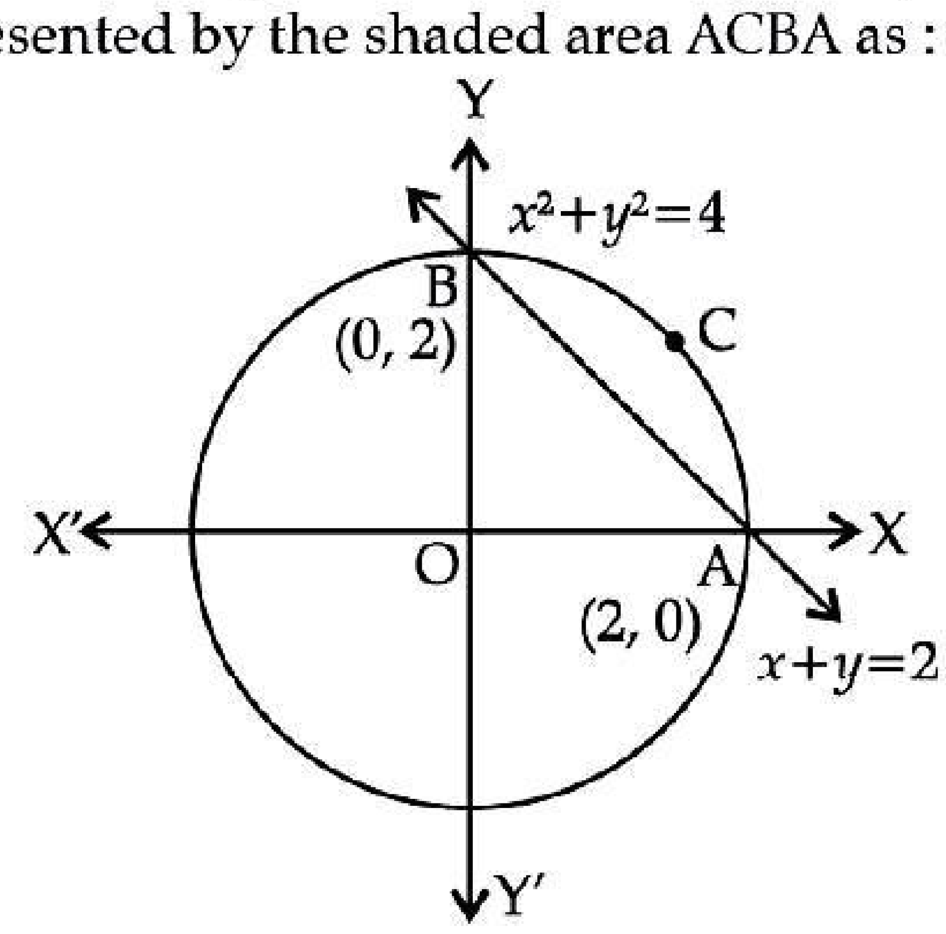
Q. 12. Smaller area enclosed by the circle $x^2 + y^2 - 4$ and the line $x + y = 2$

(A) $2(\pi - 2)$ **(B)** $\pi - 2$

(C) $2\pi - 1$ **(D)** $2(\pi + 2)$

Ans. Option (B) is correct.

Explanation: The smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line, $x + y = 2$ is represented by the shaded area ACBA as :



It can be observed that

Area of $ACBA$ = Area of $OACBO$ −Area of ΔAOB

$$A = \int_0^2 \sqrt{4-x^2}\,dx - \int_0^2 (2-x)dx$$

$$= \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2 - \left[2x - \frac{x}{2}\right]_0^2$$

$$= \left[2 \times \frac{\pi}{2}\right] - [4-2]$$

$$= \pi \quad 2 \text{ sq. units}$$

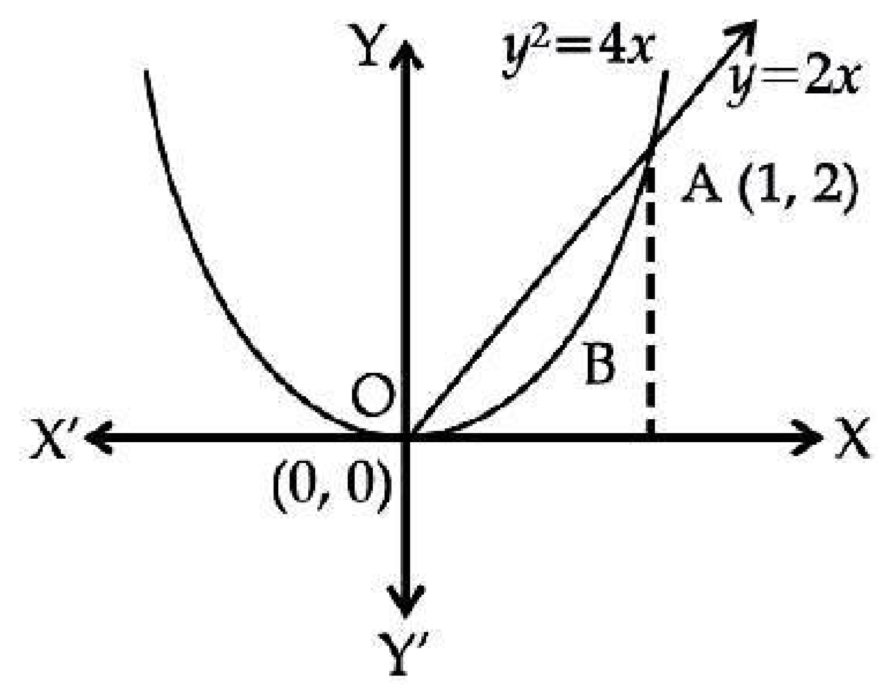
Q. 13. Area lying between the curve $y^2 = 4x$ and $y = 2x$

(a) $\frac{2}{3}$ **(B)** $\frac{1}{3}$

(C) $\frac{1}{4}$ **(D)** $\frac{3}{4}$

Ans. Option (B) is correct.

Explanation: The area lying between the curve $y^2 = 4x$ and $y = 2x$ is represented by the shaded area $OBAO$ as



The points of intersection of the curves are $O(0, 0)$ and $A(1, 2)$.

We draw AC perpendicular to x-axis such that coordinate of C is $(1, 0)$.

Area of $OBAO$=Area of ΔOCA −Area of $OCABO$

$$A = \int_0^1 2x\,dx - \int_0^1 2\sqrt{x}\,dx$$

$$= 2\left[\frac{x^2}{2}\right]_0^1 - 2\left[\frac{x^{3/2}}{\frac{3}{2}}\right]_0^1$$

$$= \left[\left[1 - \frac{4}{3}\right]\right]$$

$$= \left|-\frac{1}{3}\right|$$

$$= \frac{1}{3} \text{ sq. unit}$$

Q. 14. Area bounded by the curve $y = x^3$, the x-axis and the ordinates $x = -2$ and $x = 1$ is

(A) -9 **(B)** $-\frac{15}{4}$

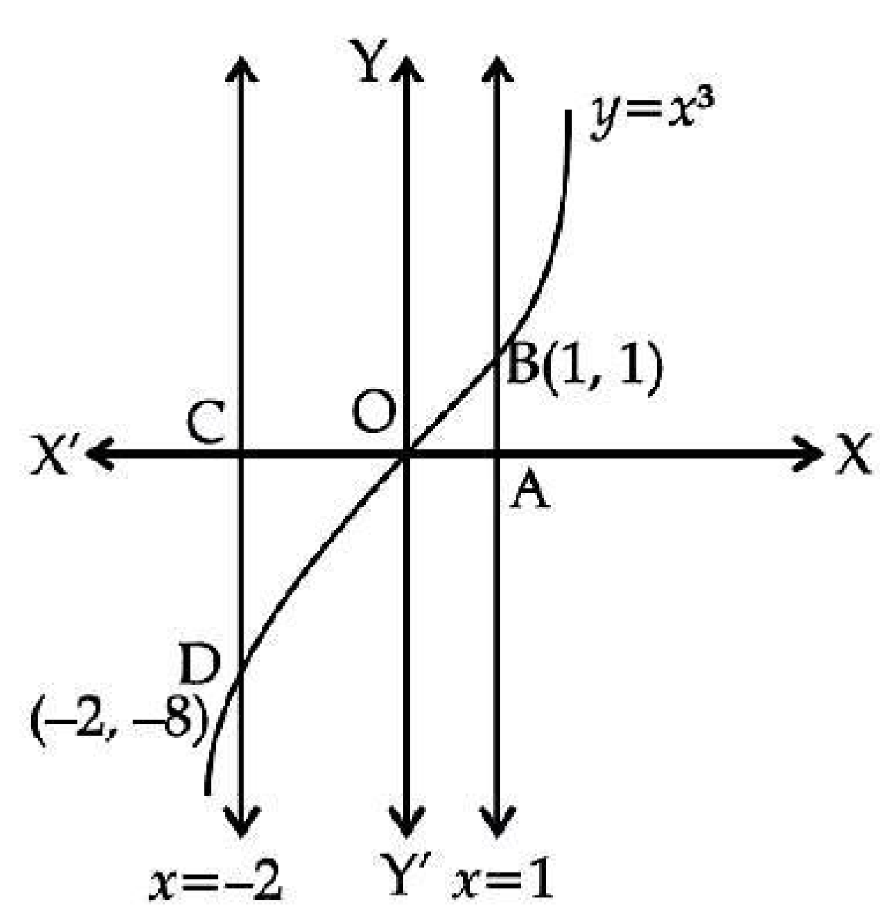
(C) $\frac{15}{4}$ **(D)** $\frac{17}{4}$

Ans. Option (C) is correct.

Explanation: Required area,

$$A = \int_{-2}^{1} y\,dx$$

$$= \int_{-2}^{1} x^3 dx$$



$$= \left[\frac{x^4}{4}\right]_{-2}^{1}$$
$$= \left(\frac{1}{4} - 4\right)$$
$$= -\frac{15}{4}$$
$$\therefore \quad \text{Area} = \left|-\frac{15}{4}\right|$$
$$= \frac{15}{4} \text{ sq. units}$$

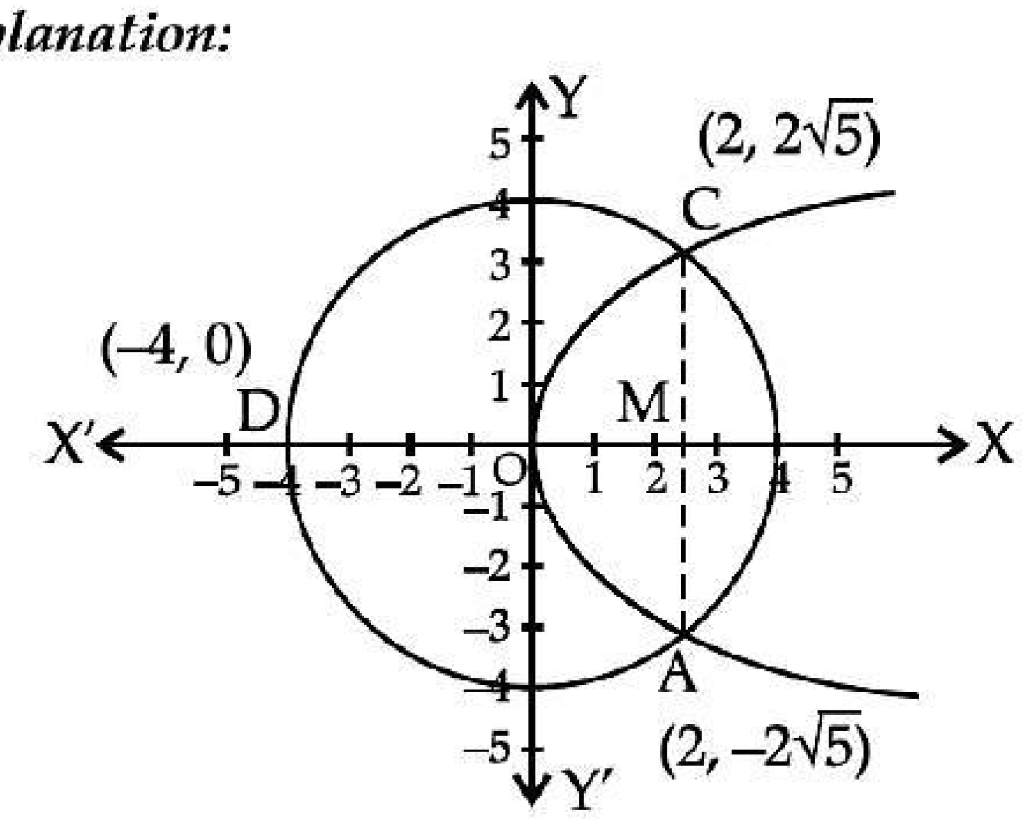
Q. 15. The area of the circle $x^2 + y^2 = 16$ exterior to the parabola $y^2 = 6x$ is

(A) $\frac{4}{3}(4\pi - \sqrt{3})$ **(B)** $\frac{4}{3}(4\pi + \sqrt{3})$

(C) $\frac{4}{3}(8\pi - \sqrt{3})$ **(D)** $\frac{4}{3}(8\pi + \sqrt{3})$

Ans. Option (C) is correct.

Explanation:



Area bounded by the circle and parabola

$= 2[\text{area } (OAMO) + \text{area } (AMBA)]$

$$= 2\left[\int_0^2 \sqrt{6x}dx + \int_2^4 \sqrt{16 - x^2}dx\right]$$
$$= 2\int_0^2 \sqrt{6x}dx + 2\int_2^4 \sqrt{16 - x^2}dx$$
$$= 2\sqrt{6}\int_0^2 \sqrt{x}dx + 2\int_2^4 \sqrt{16 - x^2}dx$$
$$= 2\sqrt{6} \times \frac{2}{3}\left[x^{3/2}\right]_0^2 + 2\left[\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_2^4$$
$$= \frac{4\sqrt{6}}{2}\left(2\sqrt{2} - 0\right) + 2\left[\left\{0 + 8\sin^{-1}(1)\right\} - \left\{2\sqrt{3} + 8\sin^{-1}\left(\frac{1}{2}\right)\right\}\right]$$
$$= \frac{16\sqrt{3}}{3} + 2\left[8 \times \frac{\pi}{2} - 2\sqrt{3} - 8 \times \frac{\pi}{6}\right]$$
$$= \frac{16\sqrt{3}}{3} + 2\left(4\pi - 2\sqrt{3} - \frac{4\pi}{3}\right)$$
$$= \frac{16\sqrt{3}}{3} + 8\pi - 4\sqrt{3} - \frac{8\pi}{3}$$
$$= \frac{16\sqrt{3} + 24\pi - 4\sqrt{3} - 8\pi}{3}$$
$$= \frac{16\pi + 12\sqrt{3}}{3}$$
$$= \frac{4}{3}\left[4\pi + \sqrt{3}\right] \text{ sq. units}$$

$$\text{Area of circle} = \pi(r)^2$$
$$= \pi(4)^2$$
$$= 16\pi \text{ sq. units}$$
$$\therefore \quad \text{Required area} = 16\pi - \frac{4}{3}\left(4\pi + \sqrt{3}\right)$$
$$= 16\pi - \frac{16\pi}{3} - \frac{4\sqrt{3}}{3}$$
$$= \frac{32\pi}{3} - \frac{4\sqrt{3}}{3}$$
$$= \frac{4}{3}\left[8\pi - \sqrt{3}\right] \text{ sq. units}$$

ASSERTION AND REASON BASED MCQs (1 Mark each)

Directions : In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as

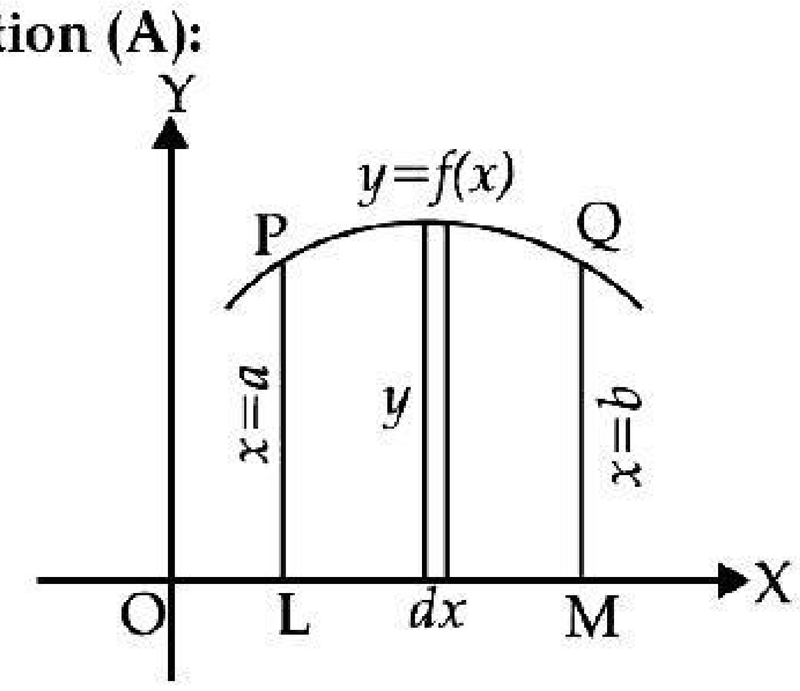
(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

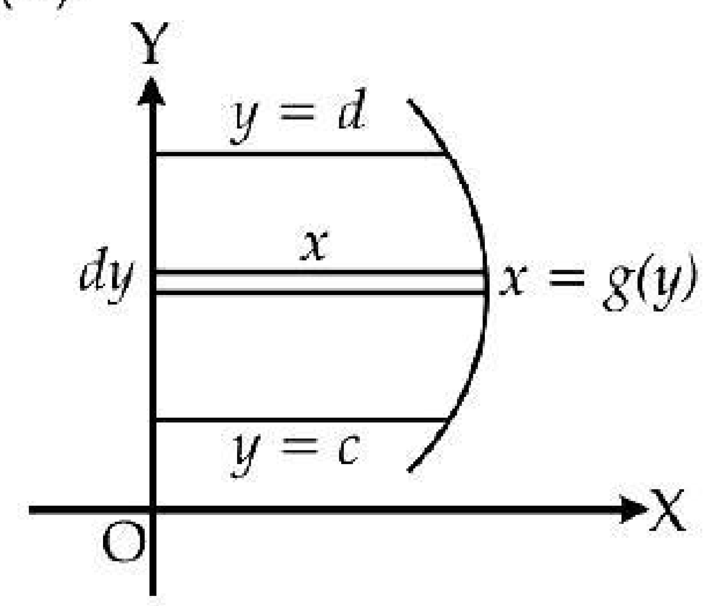
(D) A is false and R is True

Q. 1. Assertion (A):



The area of region $PQML = \int_a^b y\,dx = \int_a^b f(x)dx$

Reason (R):



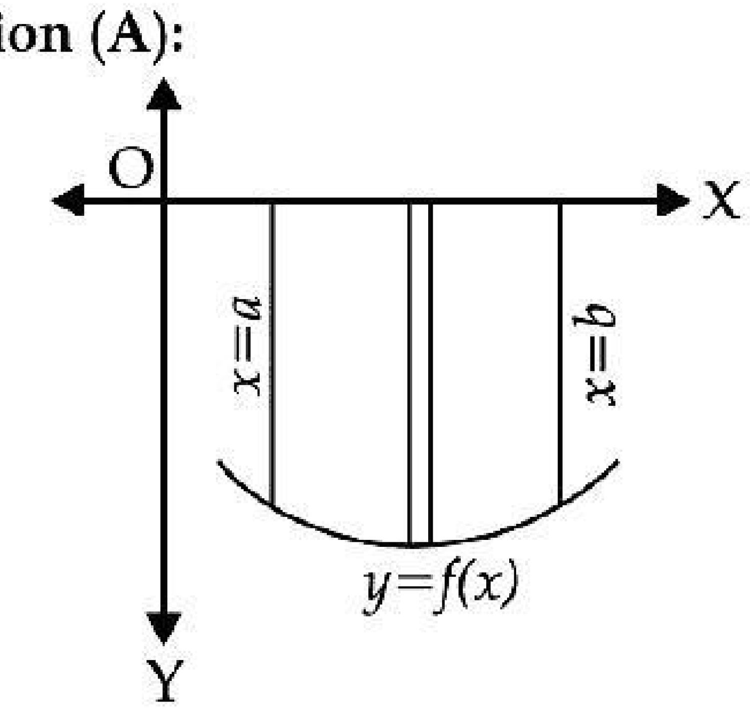
The area A of the region bounded by curve $x = g(y)$, y-axis and the lines $y = c$ and $y = d$ is given by

$$A = \int_c^d x dy$$

Ans. Option (B) is correct.

> *Explanation:* Assertion (A) and Reason (R) both are individually correct.

Q. 2. Assertion (A):



$$\text{Area} = \left|\int_a^b f(x)dx\right|$$

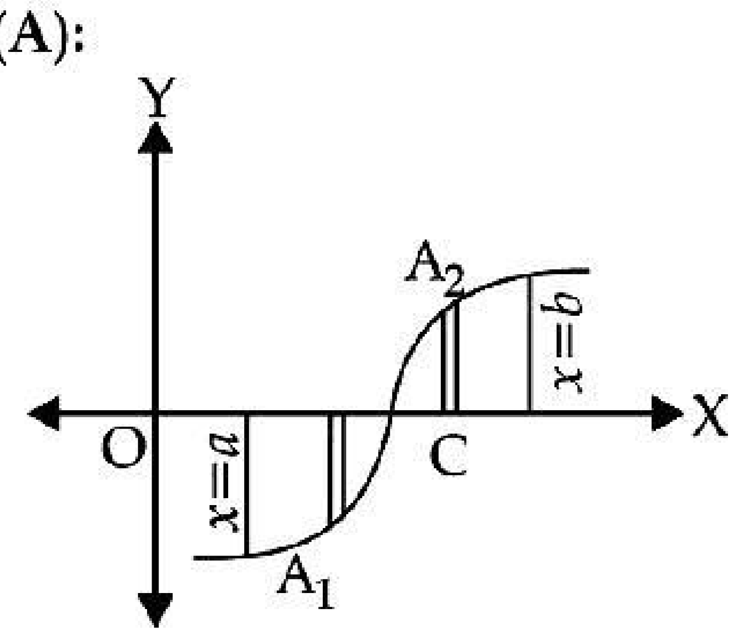
Reason (R): If the curve under consideration lies below x-axis, then $f(x) < 0$ from $x = a$ to $x = b$, the area bounded by the curve $y = f(x)$ and the ordinates $x = a$, $x = b$ and x-axis is negative. But, if the numerical value of the area is to be taken into consideration, then

$$\text{Area} - \left|\int_a^b f(x)dx\right|$$

Ans. Option (A) is correct.

> *Explanation:* Assertion (A) and Reason (R) both are correct, Reason (R) is the correct explanation of Assertion (A).

Q. 3. Assertion (A):



$$\text{Area} = |A_1| + |A_2|$$

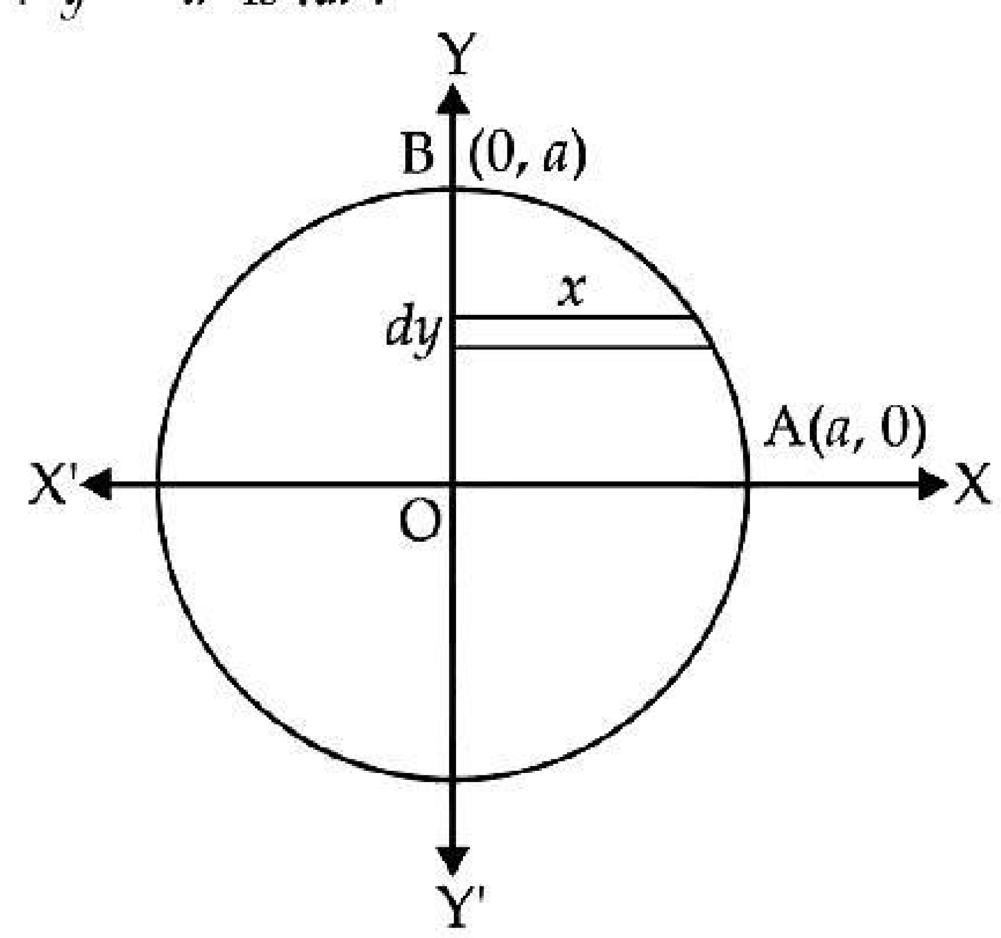
Reason (R): It may happen that some portion of the curve is above x-axis and some portion is below x-axis as shown in the figure. Let A_1 be the area below x-axis and A_2 be the area above the x-axis. Therefore, area bounded by the curve $y = f(x)$, x-axis and the ordinates $x = a$ and $x = b$ is given by

$$\text{Area} = |A_1| + |A_2|$$

Ans. Option (A) is correct.

> *Explanation:* Assertion (A) and Reason (R) both are correct, Reason (R) is the correct explanation of Assertion (A).

Q. 4. Assertion (A): The area enclosed by the circle $x^2 + y^2 = a^2$ is πa^2.



Reason (R): The area enclosed by the circle

$$= 4\int_0^a x dy$$

$$= 4\int_0^a \sqrt{a^2 - y^2}\, dy$$

$$= 4\left[\frac{y}{2}\sqrt{a^2 - y^2} + \frac{a^2}{2}\sin^{1}\frac{y}{a}\right]_0^a$$

$$= 4\left[\left(\frac{a}{2}\times 0 + \frac{a^2}{2}\sin^{-1}1\right) - 0\right]$$

$$= 4\frac{a^2}{2}\frac{\pi}{2}$$

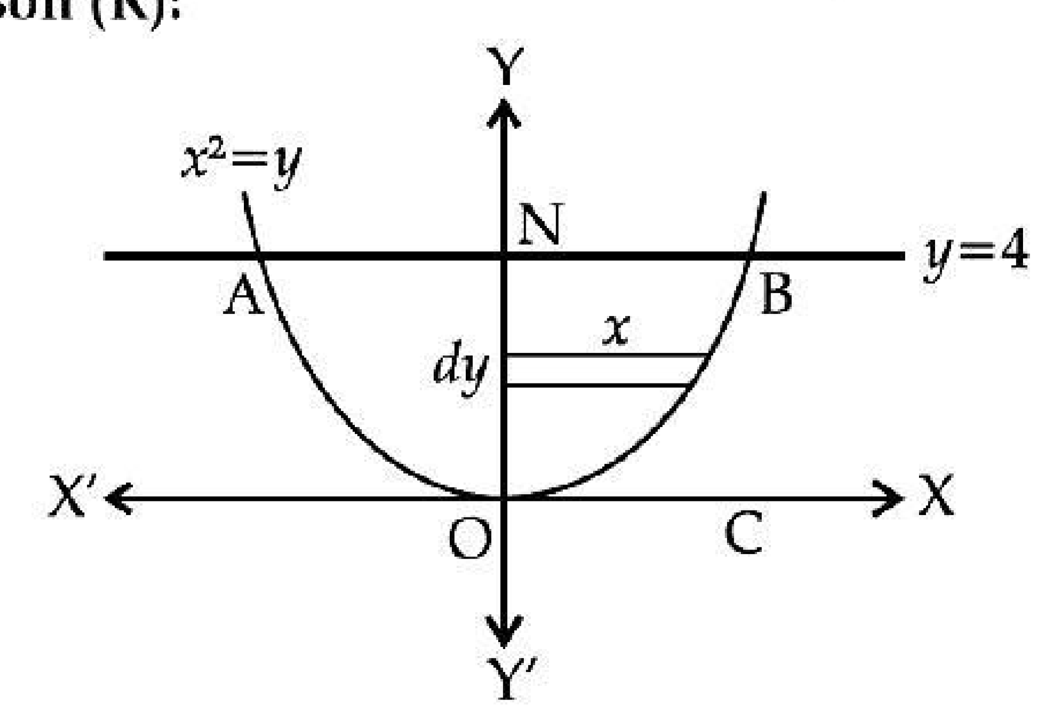
$$= \pi a^2$$

Ans. Option (A) is correct.

> *Explanation:* Assertion (A) and Reason (R) both are correct, Reason (R) is the correct explanation of Assertion (A).

Q. 5. Assertion (A): The area of the region bounded by the curve $y = x^2$ and the line $y = 4$ is $\frac{3}{32}$.

Reason (R):



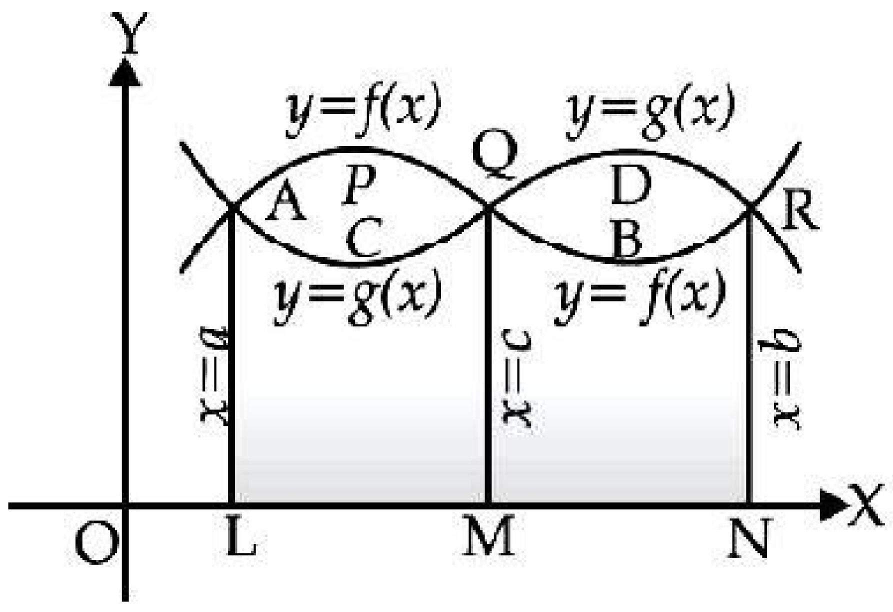
Since the given curve represented by the equation $y = x^2$ is a parabola symmetrical about y-axis only, therefore, from figure, the required area of the region $AOBA$ is given by

$$A = 2\int_0^4 x\,dy$$
$$= 2\int_0^4 \sqrt{y}\,dy$$
$$= 2\times\frac{2}{3}\left[y^{3/2}\right]_0^4$$
$$= \frac{4}{3}\times 8$$
$$= \frac{32}{3}$$

Ans. Option (D) is correct.

> ***Explanation:*** Assertion (A) is wrong. Reason (R) is the correct solution of Assertion (A).

Q. 6. Assertion (A): If the two curves $y = f(x)$ and $y = g(x)$ intersect at $x = a$, $x = c$ and $x = b$, such that $a < c < b$.

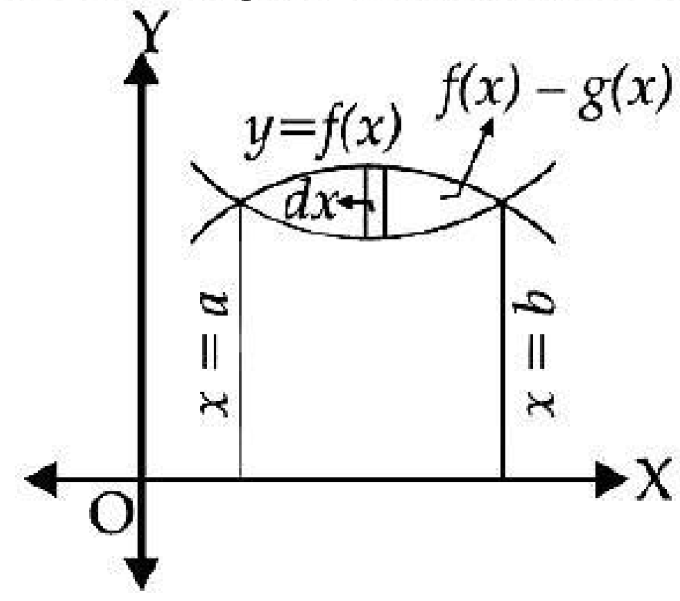


If $f(x) > g(x)$ in $[a, c]$ and $g(x) \leq f(x)$ in $[c, b]$, then Area of the regions bounded by the curve

= Area of region $PACQP$ + Area of region $QDRBQ$.

$$= \int_a^c |f(x)-g(x)|\,dx + \int_c^b |g(x)-f(x)|\,dx.$$

Reason (R): Let the two curves by $y = f(x)$ and $y = g(x)$, as shown in the figure. Suppose these curves intersect at $f(x)$ with width dx.



$$\text{Area} = \int_a^b [f(x)-g(x)]\,dx$$
$$= \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

= Area bounded by the curve $\{y = f(x)\}$

–Area bounded by the curve $\{y = g(x)\}$,

where $f(x) > g(x)$.

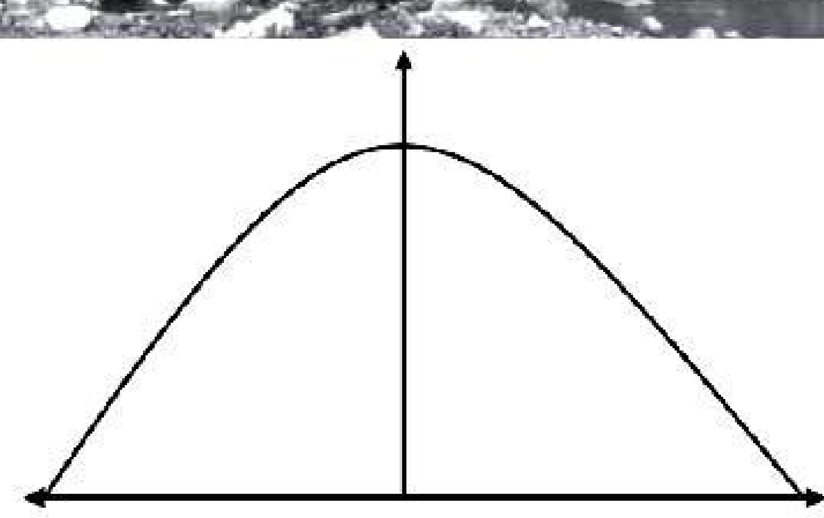
Ans. Option (B) is correct.

> ***Explanation:*** Assertion (A) and Reason (R) both are individually correct.

CASE-BASED MCQs

Attempt any four sub-parts from each question. Each sub-part carries 1 mark.

I. Read the following text and answer the following questions on the basis of the same:



The bridge connects two hills 100 feet apart. The arch on the bridge is in a parabolic form. The highest point on the bridge is 10 feet above the road at the middle of the bridge as seen in the figure.

[CBSE QB-2021]

Q. 1. The equation of the parabola designed on the bridge is

(A) $x^2 = 250y$ (B) $x^2 = -250y$
(C) $y^2 = 250x$ (D) $y^2 = 250y$

Ans. Option (C) is correct.

Q. 2. The value of the integral $\int_{-50}^{50} \frac{x^2}{250}$ is

(A) $\frac{1000}{3}$ (B) $\frac{250}{3}$
(C) 1200 (D) 0

Ans. Option (A) is correct.

> ***Explanation:***
>
> $$\int_{-50}^{50}\frac{x^2}{250} = \frac{1}{250}\left[\frac{x^3}{3}\right]_{-50}^{50}$$
> $$= \frac{1}{250}\times\frac{1}{3}\left[(50)^3-(-50)^3\right]$$
> $$= \frac{1}{750}[125000+125000]$$
> $$= \frac{1000}{3}$$

Q. 3. The integrand of the integral $\int_{-50}^{50} x^2\,dx$ is ________ function.

(A) Even (B) Odd
(C) Neither odd nor even (D) None of these

Ans. Option (A) is correct.

> *Explanation:*
>
> $$f(x) = x^2$$
> $$f(-x) = x^2$$
>
> $\therefore f(x)$ is even function.

Q. 4. The area formed by the curve $x^2 = 250y$, x-axis, $y = 0$ and $y = 10$ is

(A) $\frac{1000\sqrt{2}}{3}$ **(B)** $\frac{4}{3}$

(C) $\frac{1000}{3}$ **(D)** 0

Ans. Option (C) is correct.

> *Explanation:*
>
> $$x^2 = 250y$$
> $$y = \frac{1}{250}x^2$$
>
> at $y = 0$ $\quad x = 0$
>
> at $y = 10$ $\quad x = 50, -50$
>
> $\therefore$ Area formed by curve
>
> $$= \int_{-50}^{50} \frac{1}{250}x^2 dx$$
> $$= \frac{1}{250} \times \frac{1}{3}\left[x^3\right]_0^{50}$$
> $$= \frac{1}{750}[250,000]$$
> $$= \frac{1000}{3} \text{ sq. units}$$

Q. 5. The area formed between $x^2 = 250y$, y-axis, $y = 2$ and $y = 4$ is

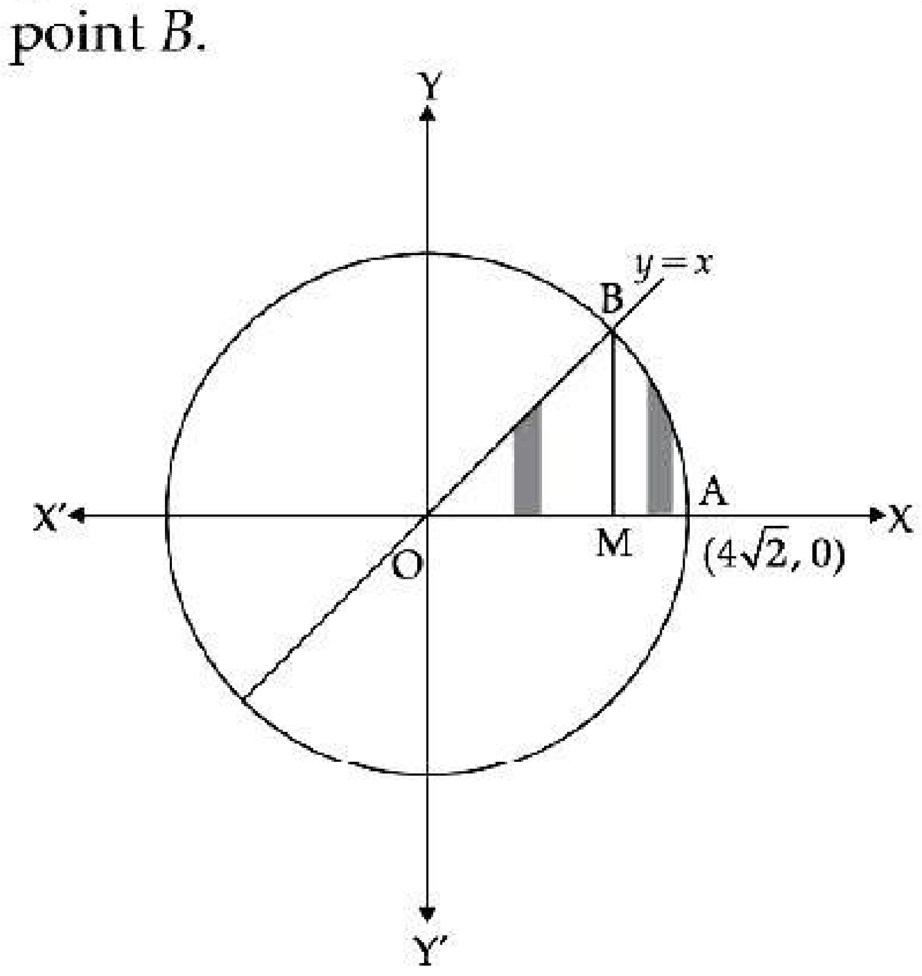
(A) $\frac{1000}{3}$ **(B)** 0

(C) $\frac{1000\sqrt{2}}{3}$ **(D)** None of these

Ans. Option (D) is correct.

II. Read the following text and answer the following questions on the basis of the same:

In the figure $O(0, 0)$ is the centre of the circle. The line $y = x$ meets the circle in the first quadrant at the point B.



Q. 1. The equation of the circle is _______.

(A) $x^2 + y^2 = 4\sqrt{2}$ **(B)** $x^2 + y^2 = 16$

(C) $x^2 + y^2 = 32$ **(D)** $(x - 4\sqrt{2})^2 + 0$

Ans. Option (C) is correct.

> *Explanation:*
>
> Centre = (0, 0),
>
> $r = 4\sqrt{2}$
>
> Equation of circle is
>
> $$x^2 + y^2 = (4\sqrt{2})^2$$
> $$\Rightarrow \quad x^2 + y^2 = 32$$

Q. 2. The co-ordinates of B are _______.

(A) (1, 1) **(B)** (2, 2)

(C) $(4\sqrt{2}, 4\sqrt{2})$ **(D)** (4, 4)

Ans. Option (D) is correct.

> *Explanation:*
>
> $$x^2 + y^2 = 32 \quad \text{...(i)}$$
> $$y = x \quad \text{...(ii)}$$
>
> Solving (i) and (ii),
>
> $\Rightarrow \quad x^2 + y^2 = 32$
>
> $\Rightarrow \quad x^2 = 16$
>
> $\Rightarrow \quad x = 4,$
>
> $\Rightarrow \quad y = x = 4$
>
> $\therefore \quad B = (4, 4)$

Q. 3. Area of ΔOBM is _______ sq. units

(A) 8 **(B)** 16

(C) 32 **(D)** 32π

Ans. Option (A) is correct.

> *Explanation:*
>
> $$\text{Ar}(\Delta OBM) = \int_0^4 x dx$$
> $$= \left[\frac{x^2}{2}\right]_0^4$$
> $$= 8 \text{ sq. units}$$

Q. 4. $Ar\,(BAMB)$ = _______ sq. units

(A) 32π **(B)** 4π

(C) 8 **(D)** $4\pi - 8$

Ans. Option (D) is correct.

> *Explanation:*
>
> $$Ar\,(BAMB) = \int_4^{4\sqrt{2}} \sqrt{32 - x^2} dx$$
> $$= \left[\frac{x}{2}\sqrt{32 - x^2} + 16\sin^{-1}\frac{x}{4\sqrt{2}}\right]_4^{4\sqrt{2}}$$
> $$= (4\pi - 8) \text{ sq. units.}$$

Q. 5. Area of the shaded region is _______ sq. units.

(A) 32π **(B)** 4π

(C) 8 **(D)** $4\pi - 8$

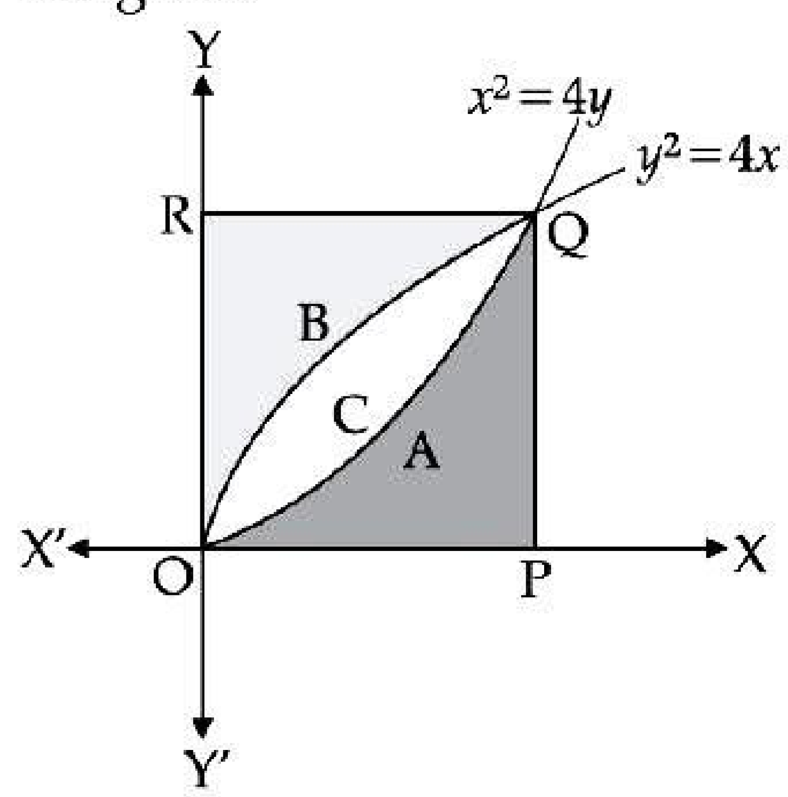
Ans. Option (B) is correct.

Explanation:
Area of shaded region

$$= Ar(\Delta OBM) + Ar(BAMB)$$
$$= 8 + 4\pi - 8$$
$$= 4\pi \text{ sq. units}$$

III. Read the following text and answer the following questions on the basis of the same:

A farmer has a square plot of land. Three of its boundaries are $x = 0$, $y = 0$ and $y = 4$. He wants to divide this land among his three sons A, B and C as shown in figure.



Q. 1. Equation of PQ is ________.

(A) $x = 0$ **(B)** $x = 2$
(C) $x = 4$ **(d)** $y = 4$

Ans. Option (C) is correct.

Explanation: Equation of PQ is $x = 4$.

Q. 2. The co-ordinates of Q are ________.

(A) (2, 2) **(B)** (4, 4)
(C) (1, 1) **(D)** (5, 5)

Ans. Option (B) is correct.

Explanation: Q = (4, 4)

Q. 3. Area received by son B is ________ sq. units.

(A) 4 **(B)** 16
(C) $\frac{16}{3}$ **(D)** $\frac{8}{3}$

Ans. Option (C) is correct.

Explanation:

$$\text{Ar (son B)} = \int_0^4 x\,dy$$
$$= \int_0^4 \frac{y^2}{4}\,dy$$
$$= \left[\frac{y^3}{12}\right]_0^4$$
$$= \frac{1}{12}[4^3 - 0]$$
$$= \frac{64}{12}$$
$$= \frac{16}{3} \text{ sq. units}$$

Q. 4. Area received by son A is ________ sq. units.

(A) 4 **(B)** 16
(C) $\frac{16}{3}$ **(D)** $\frac{8}{3}$

Ans. Option (C) is correct.

Explanation:

$$\text{Ar(son A)} = \int_0^4 y\,dx$$
$$= \int_0^4 \frac{x^2}{4} dx$$
$$= \frac{1}{12}[x^3]_0^4$$
$$= \frac{16}{3} \text{ sq. units}$$

Q. 5. Total area of the square field is ________ sq. units.

(A) 4 **(B)** 16
(C) $\frac{16}{3}$ **(D)** $\frac{8}{3}$

Ans. Option (B) is correct.

Explanation:

$$\text{Total area} = 4 \times 4$$
$$= 16 \text{ sq. units}$$