Fourier Transform

4

Introduction

Fourier transform provides a frequency domain description of time domain signals and is extension of Fourier series to non-periodic signals.

- Fourier transform

$$\text{F.T}\left[f(t)\right] = F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t} dt$$

- Inverse Fourier transform

$$\text{I.F.T}\left[F(\omega)\right] = f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t} d\omega$$

Dirichlets Conditions of Fourier Transformation

For existence of Fourier transform

- Fourier transform is defined for all stable signals

$$\int_{-\infty}^{\infty} |f(t)|\, dt < \infty$$

- Periodic signals, which are neither absolutely integrable nor square integral over an infinite interval, can be considered to have Fourier transform if impulse functions are permitted in the transform.
- f(t) have a finite number of discontinuities and finite number of maxima and minima within any finite interval.

The Properties of Fourier Transform

Properties	X(f) - form	X(ω) - form
Linearity $ax_1(t)+bx_2(t)$	$aX_1(f)+bX_2(f)$	$aX_1(\omega)+bX_2(\omega)$
Time-scaling $x(at)$	$\frac{1}{\|a\|}X\left(\frac{f}{a}\right)$	$\frac{1}{\|a\|}X\left(\frac{\omega}{a}\right)$
Time-reversal $x(-t)$	$X(-f)$	$X(-\omega)$
Time-shift $x(t\pm t_0)$	$e^{\pm j2\pi f t_0}X(f)$	$e^{\pm j\omega t_0}X(\omega)$
Frequency shift $x(t)e^{\pm j\omega_0 t}$	$X(f\mp f_0)$	$X(\omega\mp\omega_0)$
Differentiation in time	$\frac{d}{dt}x(t)\leftrightarrow j2\pi f X(f)$	$\frac{d}{dt}x(t)\leftrightarrow j\omega X(\omega)$
Frequency Differentiation	$-j2\pi t x(t)\leftrightarrow \frac{d}{df}X(f)$	$-jtx(t)\leftrightarrow \frac{d}{d\omega}X(\omega)$
Convolution in time $x(t)*h(t)$	$X(f)H(f)$	$X(\omega)H(\omega)$
Frequency convolution $x_1(t)*x_2(t)$	$X_1(f)*X_2(f)$	$\frac{1}{2\pi}[X_1(\omega)*X_2(\omega)]$
Integration $\int_{-\infty}^{t}x(\tau)d\tau$	$\frac{X(f)}{j2\pi f}+0.5X(0)\delta(f)$	$\frac{X(\omega)}{j\omega}+\pi X(0)\delta(\omega)$
Parseval's theorem $\int_{-\infty}^{+\infty}x^2(t)dt$	$\int_{-\infty}^{+\infty}\|X(f)\|^2\,df$	$\frac{1}{2\pi}\int_{-\infty}^{+\infty}\|X(\omega)\|^2\,d\omega$

Fourier Transform of Useful Signals

Signal, x(t)	F(f)form	F(ω)form
$e^{-at}u(t)$, a>0	$\frac{1}{a+j2\pi f}$	$\frac{1}{a+j\omega}$
$e^{at}u(-t), a>0$	$\frac{1}{a-j2\pi f}$	$\frac{1}{a-j\omega}$
$\delta(t)$	1	1
A, Constant	$A\delta(f)$	$2\pi A\delta(\omega)$
A rect.(t/T)	$AT\,\text{sinc}(fT)$	$AT\,\text{sinc}\left(\frac{\omega T}{2}\right)$
$e^{-a\|t\|}$	$\frac{2a}{a^2+4\pi^2f^2}$	$\frac{2a}{a^2+\omega^2}$
e^{-at^2}	$\sqrt{\frac{\pi}{a}}\,e^{-\pi^2f^2/a}$	$\sqrt{\frac{\pi}{a}}\,e^{-\omega^2/4a}$
Sgn(t)	$\frac{1}{j\pi f}$	$\frac{2}{j\omega}$
u(t)	$\frac{1}{j2\pi f}+0.5\delta(f)$	$\frac{1}{j\omega}+\pi\delta(\omega)$
$\cos\omega_0 t$	$\frac{\delta(f-f_0)+\delta(f+f_0)}{2}$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$
$\sin\omega_0 t$	$\frac{\delta(f-f_0)-\delta(f+f_0)}{2j}$	$\frac{\pi}{j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$

■■■■

prise Innovations