

CBSE Board
Class IX Mathematics

Time: 3 hrs

Total Marks: 80

General Instructions:

1. All questions are **compulsory**.
 2. The question paper consists of **30** questions divided into **four sections** A, B, C, and D. **Section A** comprises of **6** questions of 1 mark each, **Section B** comprises of **6** questions of 2 marks each, **Section C** comprises of **10** questions of 3 marks each and **Section D** comprises of **8** questions of 4 marks each.
 3. Question numbers **1 to 6** in **Section A** are multiple choice questions where you are to select **one** correct option out of the given four.
 4. Use of calculator is **not** permitted.
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Section A
(Questions 1 to 6 carry 1 mark each)

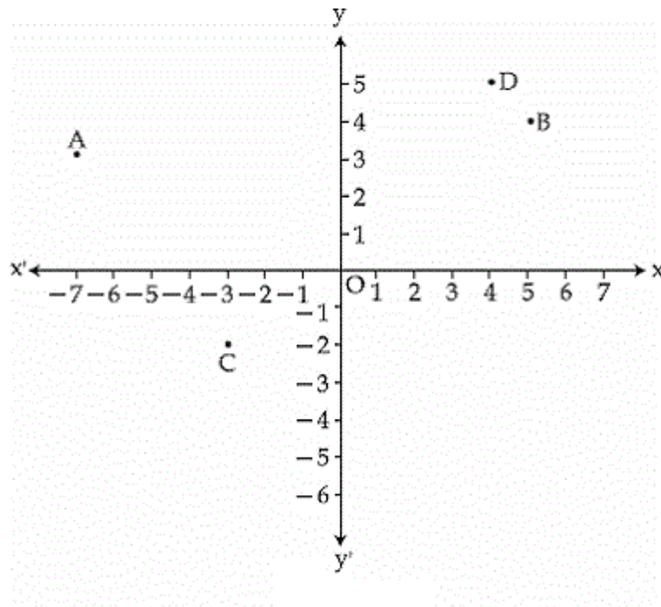
1. Write the decimal form of $\frac{56}{1000}$.
2. What is Linear Equation in two variables?
3. If a triangle and a parallelogram are on the same base and between same parallels, then find the ratio of the area of the triangle to the area of parallelogram?
4. Find the value of p such that $(x - 1)$ is a factor of the polynomial $x^3 + 10x^2 + px$?
5. 70, 65, 75, 71, 36, 55, 61, 62, 41, 40, 39, 35. Find the range of the data?
6. Three angles of a quadrilateral are 60° , 110° and 86° . What is the measure of the fourth angle of the quadrilateral?

Section B
(Questions 7 to 12 carry 2 marks each)

7. Simplify: $\left(\frac{12^{\frac{1}{5}}}{27^{\frac{1}{5}}}\right)^{\frac{5}{2}}$

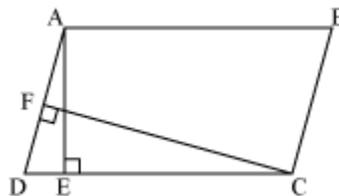
8. How many integral zeroes does the polynomial $3z^3 + 8z^2 - 1$ have?

9. See the given figure and answer the following:



- (i) Coordinates of point A
- (ii) Abscissa of point D
- (iii) The point identified by the coordinates (5, 4)
- (iv) Coordinates of point C

10. In the given figure, ABCD is parallelogram, $AE \perp DC$ and $CF \perp AD$. If $AB = 16$ cm, $AE = 8$ cm and $CF = 10$ cm, find AD.



11. How many litres of water flow out through a pipe having 5 cm^2 area of cross section in one minute, if the speed of water in the pipe is 30 cm/sec ?

12. Two angles are complementary. The larger angle is 3° less than twice the measure of the smaller angle. Find the measure of each angle.

Section C
(Questions 13 to 22 carry 3 marks each)

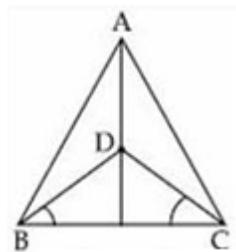
13. Express $\overline{0.001}$ as a fraction in the simplest form.
14. Which of the following expression are polynomials in one variable? State reasons for your answers:

(i) $\frac{(x+1)(x+2)}{x}$ (ii) $t^2(t^2-3)$

(iii) $\frac{1}{2}(x^2+4x+5)$ (iv) $\sqrt{3}x^2+6\sqrt{x}$

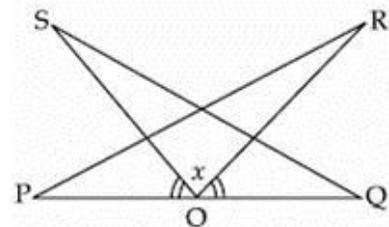
(v) $z+\frac{1}{z}$

15. The polynomials $p(x) = ax^3 + 3x^2 - 3$ and $q(x) = 2x^3 - 5x + a$ when divided by $(x - 4)$ leave the remainders R_1 and R_2 . Find 'a' if $R_1 + R_2 = 0$.
16. In figure, $AB = AC$, D is the point in the interior of $\triangle ABC$ such that $\angle DBC = \angle DCB$. Prove that AD bisects $\angle BAC$ of $\triangle ABC$.



17. In the figure, PQ is a line segment and O is the mid-point of PQ. R and S are on the same side of PQ such that $\angle PQS = \angle QPR$ and $\angle POS = \angle QOR$. Prove that

- (i) $\triangle PQR \cong \triangle QOS$
(ii) $PR = QS$



18. The distribution of weight (in kg) of 100 people is given below.

Weight in kg	Frequency
40-45	13
45-50	25
50-55	28
55-60	15
60-65	12
65-70	5
70-75	2

Construct a histogram for the above distribution.

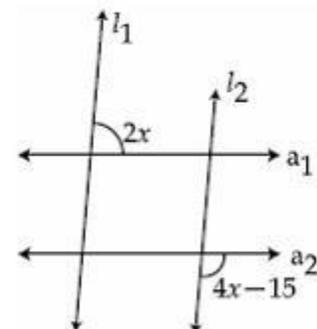
19. Prove that the quadrilateral formed by the bisectors of internal angles of a cyclic quadrilateral is also cyclic.
20. In a parallelogram, show that the angle bisectors of two adjacent angles intersect at right angles.
21. If a dice is rolled once, what is the probability that it will show
i. A multiple of 1? ii. A multiple of 7?
22. The diameter of a roller is 84 cm and its length is 120 cm. It takes 500 complete revolutions to move over once to level a playground. Find the area of the playground in m^2 ? $\left[\pi = \frac{22}{7} \right]$

Section D

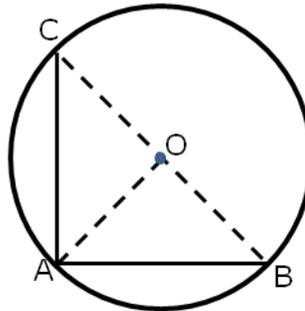
(Questions 23 to 30 carry 4 marks each)

23. If $m = \frac{1}{2 - \sqrt{3}}$ and $n = \frac{1}{2 + \sqrt{3}}$, then what is the value of $7m^2 + 11mn - 7n^2$?

24. In the figure, $l_1 \parallel l_2$ and $a_1 \parallel a_2$. Find the value of x .



25. Without actual division, prove that $2x^4 + x^3 - 14x^2 - 19x - 6$ is exactly divisible by $x^2 + 3x + 2$.
26. There is a triangular field PQR whose corner angles P, Q and R have been measured as 50° , 60° and 70° , respectively. Three friends Anuja, Nikita and Raghav daily go on morning walk and walk along AB, BC and AC, respectively. Who walk the maximum distance among these three? Who walks the least? What value is indicated from this action?
27. Find:
- The lateral or curved surface area of a closed cylindrical petrol storage tank that is 4.2 m in diameter and 4.5 m high.
 - How much steel was actually used for the constructing the above tank if $\frac{1}{12}$ of the steel actually used was wasted in making the tank?
28. In the given figure, AB and AC are two equal chords of a circle with centre O. Show that O lies on the bisectors of $\angle BAC$.



29. Construct a triangle XYZ in which angle Y is 30° , angle Z is 90° and $XY + YZ + ZX = 11$ cm.
30. Draw the graph of the line $x - 2y = 4$. From the graph, find the co-ordinates of the point when $x = -1$.

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Class IX Mathematics
Solution

Time: 3 hrs

Total Marks: 80

Section A

1. $\frac{56}{1000} = 0.056$

2. Linear equation in two variables:

An equation in the form $ax + by + c = 0$, where a , b and c are real numbers, and a and b are not both zero is called a linear equation in two variables x and y .

3. If a triangle and a parallelogram are on the same base and between the same parallels, then the area of the triangle is equal to half the area of the Parallelogram.

$$\therefore \text{Area of triangle} = \frac{1}{2} \times \text{Area of a Parallelogram}$$

$$\therefore \frac{\text{Area of triangle}}{\text{Area of a Parallelogram}} = \frac{1}{2}$$

Hence, the ratio is 1 : 2.

4. $p(x) = x^3 + 10x^2 + px$
 $(x - 1)$ is a factor of $p(x)$.

$$\therefore x - 1 = 0$$

$$\therefore x = 1$$

Substituting $x = 1$ in $p(x) = 0$

$$\therefore 1 + 10 + p = 0$$

$$\therefore p = -11$$

5. Arranging the data in the ascending order: 35, 36, 39, 40, 41, 55, 61, 62, 65, 70, 71,
 $\therefore \text{Range} = \text{Maximum value} - \text{Minimum value} = 75 - 35 = 40$

6. Let the fourth angle of the quadrilateral be x .

Sum of all angles of a quadrilateral = 360°

$$\therefore 60^\circ + 110^\circ + 86^\circ + x = 360^\circ$$

$$\therefore 256^\circ + x = 360^\circ$$

$$\therefore x = 104^\circ$$

Section B

7.

$$\frac{12^{\frac{1}{2}}}{27^{\frac{1}{2}}}$$
$$= \frac{3^{\frac{1}{2}} \times 4^{\frac{1}{2}}}{3^{\frac{1}{2}} \times 9^{\frac{1}{2}}}$$
$$= \frac{2}{3}$$

8. Let $f(z) = 3z^3 + 8z^2 - 1$

The possible integral zeros of $f(z)$ are -1 and 1 .

$$f(z) = 3z^3 + 8z^2 - 1$$

$$f(-1) = 3(-1)^3 + 8(-1)^2 - 1 \neq 0$$

$\Rightarrow -1$ is not a zero of $f(z)$

$$f(1) = 3(1)^3 + 8(1)^2 - 1 \neq 0$$

$\Rightarrow 1$ is not a zero of $f(z)$

Therefore, $f(z)$ has no integral zero.

9. (i) Coordinates of A are $(-7, 3)$

(ii) Abscissa of point D is 4.

(iii) Point is B.

(iv) Coordinates of C are $(-3, -2)$

10. In parallelogram ABCD, $CD = AB = 16$ cm

[Opposite sides of a parallelogram are equal]

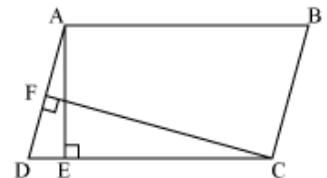
We know that,

Area of parallelogram = Base \times corresponding altitude

Area of parallelogram ABCD = $CD \times AE = AD \times CF$

$$16 \text{ cm} \times 8 \text{ cm} = AD \times 10 \text{ cm}$$

Thus, the length of AD is 12.8 cm.



11. Area of cross section of pipe = 5 cm^2

Speed of water flowing out of the pipe = 30 cm/sec

Volume of water that flows out in 1 sec = $5 \times 30 = 150 \text{ cm}^3$

Volume of water that flows out in 1 minute = $150 \times 60 = 9000 \text{ cm}^3 = 9 \text{ litres}$.

12. Let the measure of the smaller angle be x and that of the larger angle be y .
The larger angle is 3° less than twice the measure of the smaller angle, so

$$y = 2x - 3^\circ \quad \dots(1)$$

Given, that the two angles are complementary,

$$x + y = 90^\circ$$

$$\Rightarrow x + (2x - 3^\circ) = 90^\circ$$

$$\Rightarrow x + 2x - 3^\circ = 90^\circ$$

$$\Rightarrow 3x = 93^\circ$$

$$\Rightarrow x = 31^\circ$$

Substitute value of x in equation (1)

$$y = 2(31) - 3^\circ$$

$$\Rightarrow y = 59^\circ$$

So, the measures of the two angles are 31° and 59° .

Section C

13. Let $x = 0.\overline{001}$

Then, $x = 0.001001001\dots\dots\dots$ (i)

Therefore, $1000x = 1.001001001\dots\dots\dots$ (ii)

Subtracting (i) from (ii), we get

$$999x = 1 \Rightarrow x = \frac{1}{999}$$

$$\text{Hence, } 0.\overline{001} = \frac{1}{999}$$

- 14.

i. No.

$$\frac{x^2 + 3x + 2}{x} = x + 3 + 2x^{-1} \text{ has negative power of } x$$

ii. Yes

$$t^2(t^2 - 3)$$

$$t^4 - 3t^2$$

iii. Yes

$$\frac{(x^2 + 4x + 5)}{2} = \frac{x^2}{2} + \frac{4x}{2} + \frac{5}{2} = \frac{x^2}{2} + 2x + \frac{5}{2}$$

iv. No

$$\sqrt{3}x^2 + 6\sqrt{x} = \sqrt{3}x^2 + 6(x)^{1/2} \text{ has fractional power of } x$$

v. No

$$z + \frac{1}{z} \text{ i.e. } z + z^{-1} \text{ has negative power of } x$$

15. When $p(x) = ax^3 + 3x^2 - 3$ is divided by $(x - 4)$, the remainder is given by

$$R_1 = a(4)^3 + 3(4)^2 - 3 = 64a + 45$$

When $q(x) = 2x^3 - 5x + a$ is divided by $(x - 4)$, the remainder is given by

$$R_2 = 2(4)^3 - 5(4) + a = 108 + a$$

$$\text{Given: } R_1 + R_2 = 0$$

$$\Rightarrow 65a + 153 = 0$$

$$\Rightarrow a = \frac{-153}{65}$$

16. In $\triangle DCB$, $\angle DBC = \angle DCB$ (given)

$$DC = DB \text{ [Side opp. To equal } \angle\text{'s are equal].....(i)}$$

In $\triangle ABD$ and $\triangle ACD$

$$AB = AC \quad (\text{given})$$

$$BD = CD \quad [\text{from (i) }]$$

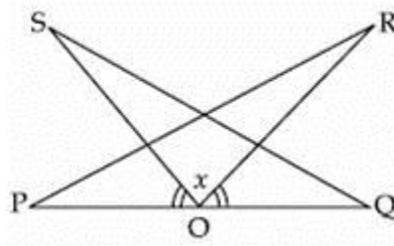
$$AD = AD \text{ common}$$

$$\triangle ABD \cong \triangle ACD \quad [\text{SSS Rule}]$$

$$\angle BAD = \angle CAD \quad (\text{CPCT})$$

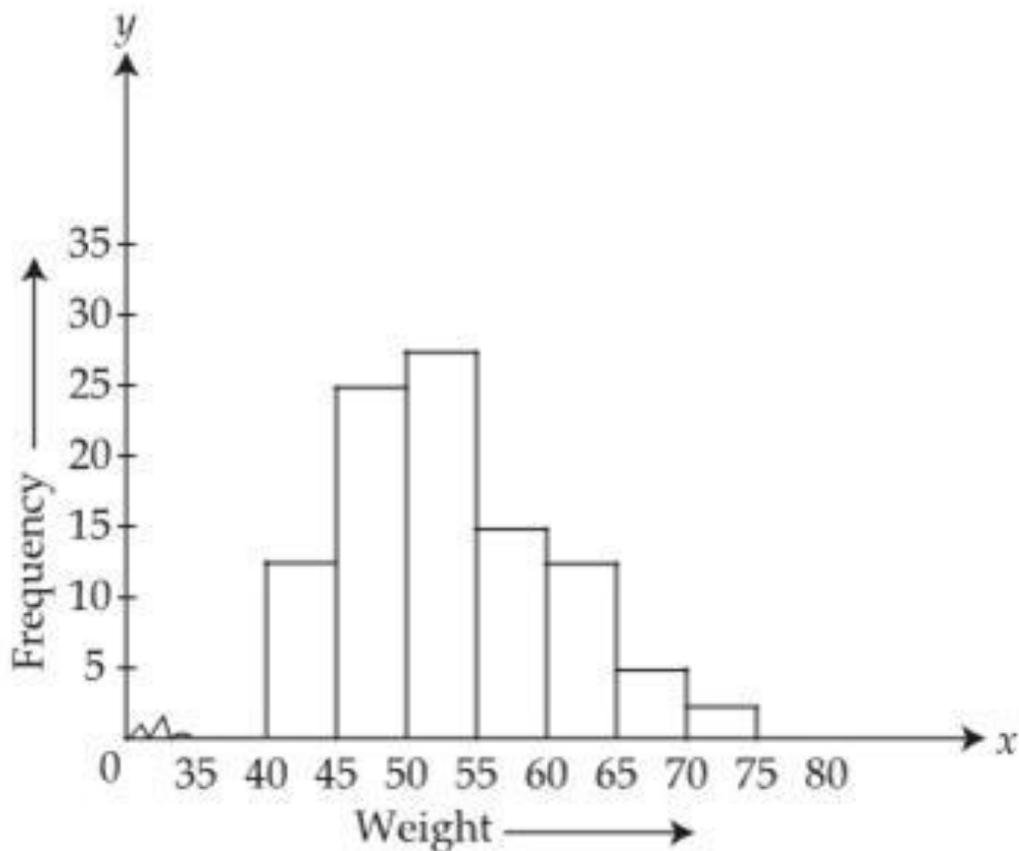
Hence, AD is bisector of $\angle BAC$.

17. In ΔPOR and ΔQOS
 $\angle QPR = \angle PQS$ (given)
 $OP = OQ$ (O is the mid-point of PQ)
 $\angle POS = \angle QOR$ (given)
 $\angle POS + x^\circ = \angle QOR + x^\circ$
 $\angle POR = \angle QOS$
 By ASA congruence rule,
 $\Delta PQR \cong \Delta QOS$
 $\Rightarrow PR = QS$ (By CPCT)
 Hence, proved.

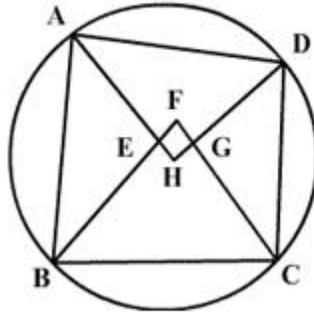


18. Steps of construction:

- i. We represent the weights on the horizontal axis. We choose the scale on the horizontal axis as 1 cm = 5 kg. Also, since the first class interval is starting from 35 and not zero, we show it on the graph by marking a kink or a break on the axis.
- ii. We represent the number of people (frequency) on the vertical axis. Since the maximum frequency is 28, we choose the scale as 1 cm = 5 people.
- iii. We now draw rectangles (or rectangular bars) of width equal to the class size and lengths according to the frequencies of the corresponding class intervals.



19.



In the given figure, ABCD is a cyclic quadrilateral.

AH, BF, CF and DH are the angle bisectors of $\angle A$, $\angle B$, $\angle C$ and $\angle D$

To prove EFGH is also a cyclic quadrilateral.

Proof :

$$\angle FEH \cong \angle AEB \dots\dots\dots (i) \quad (\text{vertically opposite angles})$$

$$\angle FGH \cong \angle DGC \dots\dots\dots (ii) \quad (\text{vertically opposite angles})$$

$$\Rightarrow \angle FEH + \angle FGH = \angle AEB + \angle DGC \quad (\text{Adding (i) and (ii)})$$

Now, by angle sum property

$$\angle AEB = 180^\circ - \left(\frac{1}{2} \angle A + \frac{1}{2} \angle B \right)$$

$$\text{and } \angle DGC = 180^\circ - \left(\frac{1}{2} \angle D + \frac{1}{2} \angle C \right)$$

$$\Rightarrow \angle FEH + \angle FGH = 180^\circ - \left(\frac{1}{2} \angle A + \frac{1}{2} \angle B \right) + 180^\circ - \left(\frac{1}{2} \angle D + \frac{1}{2} \angle C \right)$$

$$\Rightarrow \angle FEH + \angle FGH = 360^\circ - \frac{1}{2} (\angle A + \angle B + \angle C + \angle D)$$

$$\Rightarrow \angle FEH + \angle FGH = 360^\circ - \frac{1}{2} \times 360^\circ$$

$$\Rightarrow \angle FEH + \angle FGH = 360^\circ - 180^\circ = 180^\circ$$

$$\Rightarrow \angle EFG + \angle EFG = 360^\circ - 180^\circ = 180^\circ \quad \left[\begin{array}{l} \because \text{Sum of the angles} \\ \text{in a quadrilateral is } 360^\circ \end{array} \right]$$

Thus the sum of opposite angles of quadrilateral EFGH is 180° .

Therefore, EFGH is a cyclic quadrilateral.

20. Given: ABCD is a parallelogram such that angle bisector of adjacent angles A and B intersect at point P.

To prove: $\angle APB = 90^\circ$.

AD \parallel BC

$$\therefore \angle A + \angle B = 180^\circ \quad [\text{Consecutive interior angles}]$$

$$\therefore \frac{1}{2}\angle A + \frac{1}{2}\angle B = 90^\circ$$

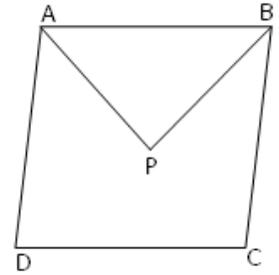
But,

$$\frac{1}{2}\angle A + \frac{1}{2}\angle B + \angle APB = 180^\circ \quad \dots(\text{Angle sum property of a triangle})$$

$$\therefore 90^\circ + \angle APB = 180^\circ$$

$$\therefore \angle APB = 90^\circ$$

Thus, the angle bisectors of two adjacent angles intersect at right angles.



21. If a dice is rolled once, the total number of possible outcomes = 6.

(i) The number of multiples of 1 = 6

$$\therefore \text{The probability of a multiple of 1} = \frac{6}{6} = 1$$

(ii) The number of multiples of 7 = 0

$$\therefore \text{The probability of a multiple of 7} = \frac{0}{6} = 0$$

22. The roller is cylindrical in shape.

Height (h) of cylindrical roller = Length of roller = 120 cm

Radius (r) of the circular end of roller = $\left(\frac{84}{2}\right)$ cm = 42 cm

$$\text{C.S.A. of roller} = 2\pi rh = 2 \times \frac{22}{7} \times 42 \times 120 = 31680 \text{ cm}^2$$

$$\text{Area of field} = 500 \times \text{C.S.A. of roller} = (500 \times 31680) \text{ cm}^2 = 15840000 \text{ cm}^2$$

$$\text{Area of field} = 1584 \text{ m}^2.$$

Section D

23.

$$\begin{aligned}
 m &= \frac{1}{2-\sqrt{3}} \text{ and } n = \frac{1}{2+\sqrt{3}} \\
 \therefore 7m^2 + 11mn - 7n^2 &= 7 \times \left(\frac{1}{2-\sqrt{3}} \right)^2 + 11 \left(\frac{1}{2-\sqrt{3}} \right) \left(\frac{1}{2+\sqrt{3}} \right) - 7 \left(\frac{1}{2+\sqrt{3}} \right)^2 \\
 &= \frac{7}{4+3-4\sqrt{3}} + \frac{11}{4-3} - \frac{7}{4+3+4\sqrt{3}} \\
 &= \frac{7}{7-4\sqrt{3}} + 11 - \frac{7}{7+4\sqrt{3}} \\
 &= \frac{7}{7-4\sqrt{3}} \times \frac{7+4\sqrt{3}}{7+4\sqrt{3}} + 11 - \frac{7}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}} \\
 &= \frac{7(7+4\sqrt{3})}{(7)^2 - (4\sqrt{3})^2} + 11 - \frac{7(7-4\sqrt{3})}{(7)^2 - (4\sqrt{3})^2} \\
 &= \frac{7(7+4\sqrt{3})}{49-48} + 11 - \frac{7(7-4\sqrt{3})}{49-48} \\
 &= 49 + 28\sqrt{3} + 11 - 49 + 28\sqrt{3} \\
 &= 11 + 56\sqrt{3}
 \end{aligned}$$

24. $2x = z$ (Alternate angles, as $l_1 \parallel l_2$)

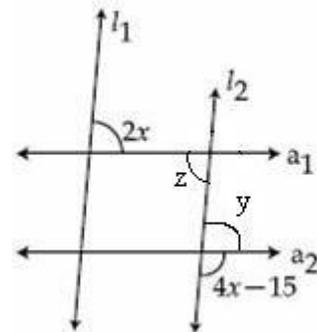
$y = z$ (Alternate angles, as $a_1 \parallel a_2$)

So, $2x = y$

Now, $y + 4x - 15 = 180^\circ$ (linear pair)

$$2x + 4x - 15 = 180^\circ$$

$$6x = 195^\circ \Rightarrow x = 32.5$$



25. Let $p(x) = 2x^4 + x^3 - 14x^2 - 19x - 6$ and $q(x) = x^2 + 3x + 2$

$$q(x) = x^2 + 3x + 2 = (x + 1)(x + 2)$$

$$\text{Now, } p(-1) = 2(-1)^4 + (-1)^3 - 14(-1)^2 - 19(-1) - 6 = 2 - 1 - 14 + 19 - 6 = 21 - 21 = 0$$

$$\text{And, } p(-2) = 2(-2)^4 + (-2)^3 - 14(-2)^2 - 19(-2) - 6 = 32 - 8 - 56 + 38 - 6 = 70 - 70 = 0$$

Therefore, $(x + 1)$ and $(x + 2)$ are the factors of $p(x)$, so $p(x)$ is divisible by $(x + 1)$ and $(x + 2)$.

Hence, $p(x)$ is divisible by $x^2 + 3x + 2$.

26. Given: Three friends Anuja, Nikita and Raghav daily go on morning walks and walk along PQ, QR and PR of triangular field PQR, respectively.

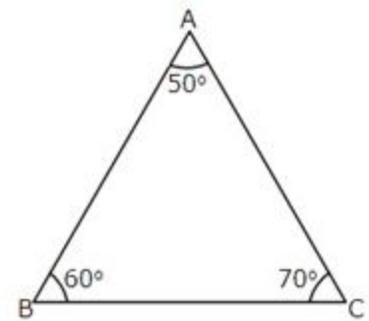
Here, $50^\circ < 60^\circ < 70^\circ$
 i.e. $\angle A < \angle B < \angle C$

We know that side opposite to greater angle is longer.

Hence, $BC < CA < AB$

So, distance AB is the maximum. Thus, Anuja walks the maximum distance. Further, distance QR is the least. Thus, Nikita walks the least distance.

Value: Awareness for a daily walk for better health without gender and religion bias.



27. Height (h) of the cylindrical tank = 4.5 m

Radius (r) of circular end of the cylindrical tank = $\left(\frac{4.2}{2}\right)$ m = 2.1 m

- i. Lateral or curved surface area of tank = $2\pi rh$

$$= \left(2 \times \frac{22}{7} \times 2.1 \times 4.5\right) \text{ m}^2$$

$$= 59.4 \text{ m}^2$$

- ii. Total surface area of tank = $2\pi r(r + h)$

$$= \left[2 \times \frac{22}{7} \times 2.1 \times (2.1 + 4.5)\right] \text{ m}^2$$

$$= 87.12 \text{ m}^2$$

Let A m² of the steel sheet be actually used to make the tank.

$$\therefore A \left(1 - \frac{1}{12}\right) = 87.12 \text{ m}^2 = \left(\frac{12}{11} \times 87.12\right) \text{ m}^2 = 95.04 \text{ m}^2$$

Thus, 95.04 m² steel was used in actually making the tank.

28. In $\triangle AOB$ and $\triangle AOC$,

OA = OA (common side)

OB = OC (radius of the circle)

AB = AC (given)

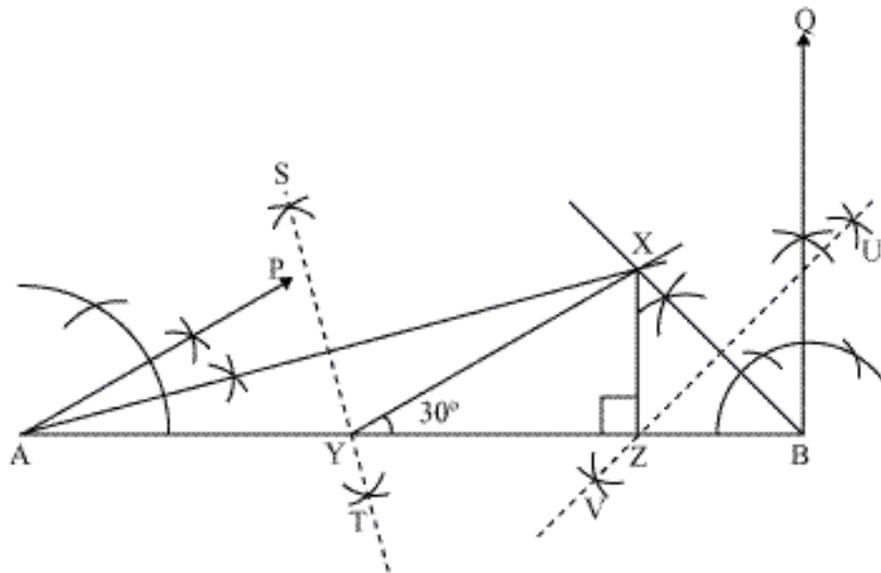
$\therefore \triangle AOB \cong \triangle AOC$

Hence, $\angle OAC = \angle OAB$

29. Steps of Construction:

1. Draw a line segment AB of 11 cm. (As $XY + YZ + ZX = 11\text{cm}$)
2. Construct an $\angle PAB$ of 30° at point A and $\angle QBA$ of 90° at point B.
3. Bisect $\angle PAB$ and $\angle QBA$. Let these bisectors intersect each other at point X.
4. Draw perpendicular bisector ST of AX and UV of BX.
5. Let ST intersect AB at Y and UV intersect AB at Z. Join XY, XZ.

XYZ is the required triangle.



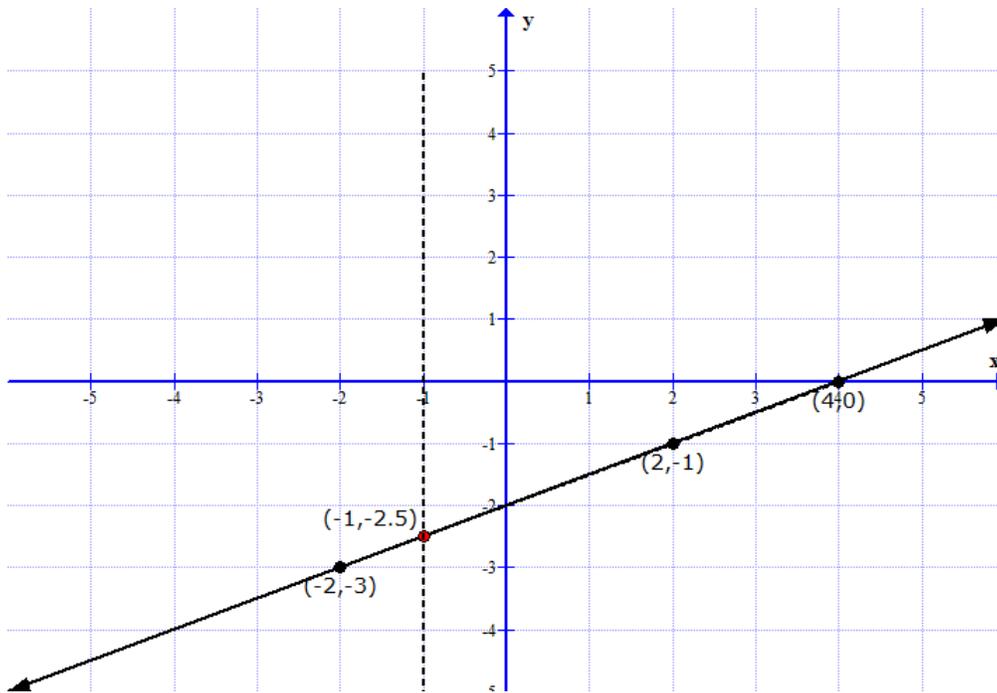
30. The equation of the given line is $x - 2y = 4$.

$$\Rightarrow y = \frac{x-4}{2}$$

We can plot a table of value of x and y as below:

x	0	2	-2	4
y	-2	-1	-3	0

We plot the points from the table on a graph paper and join them to get a straight line:



From the graph we can see that when $x = -1$, $y = -2.5$.