STATISTICS

Que.1.   **[Marks :(4)]**

The mean and standard deviation of marks obtained by 50 students of a class in three subjects, mathematics, Physics and Chemistry are given below:

Subject	mathematics	Physics	Chemistry
Mean	42	32	40.9
Standard deviation	12	15	20

Which of the three subjects shows the highest variability in marks and which shows the lowest?

Ans.

For Mathematics:

$\bar{x} = 42, \sigma = 12$

$\therefore$ CV of Mathematics $= \dfrac{12}{42} \times 100 = 28.57\%$

For Physics:

$\bar{x} = 32, \sigma = 15$

$\therefore$ CV of Physics $= \dfrac{15}{32} \times 100 = 46.88\%$

For Chemistry:

$\bar{x} = 40.9, \sigma = 20$

$\therefore$ CV of Chemistry $= \dfrac{20}{40.9} \times 100 = 48.9\%$

Thus Chemistry with highest CV shows highest variability and Mathematics with lowest CV shows lowest variability.

Que.2.   **[Marks :(4)]**

Find the mean deviation about the mean for the following data:

x_i	10	30	50	70	90
f_i	4	24	28	16	8

Ans.

x_i	f_i	$f_i x_i$	$\|x_i - 50\|$	$\|x_i - 50\| f_i$
10	4	40	40	160
30	24	720	20	480
50	28	1400	0	0
70	16	1120	20	320
90	8	720	40	320
	80	4000		1280

$$\text{Mean} = \bar{x} = \frac{\sum_{i=1}^{n} x_i f_i}{\sum_{i=1}^{n} f_i} = \frac{4000}{80} = 50$$

$$\text{M.D}(\bar{x}) = \frac{\sum_{i=1}^{n} f_i |x_i - \bar{x}|}{\sum_{i=1}^{n} f_i} = \frac{1280}{80} = 16$$

Que.3.   **[Marks :(4)]**

Find the mean deviation about the median for the following data:

x_i	15	21	27	30	35
f_i	3	5	6	7	8

Ans.

x_i	f_i	c.f	$\|x_i - 30\|$	$\|x_i - 30\| f_i$
15	3	3	15	45
21	5	8	9	45
27	6	14	3	18
30	7	21	0	0
35	8	29	5	40
	29			148

$$\frac{\sum_{i=1}^{n} f_i}{2} = \frac{29}{2} = 14.5$$

The c . f just greater than 14.5 is 21 and corresponding value of x is 30

Therefore; median, M = 30

Hence; M.D about median

$$= \frac{\sum_{i=1}^{n} f_i |x_i - M|}{\sum_{i=1}^{n} f_i} = \frac{148}{29} = 5.1$$

Que.4.   **[Marks :(6)]**

Find the mean deviation about the mean for the following data:

Height	95-105	105-115	115-125	125-135	135-145	145-155
Number of Boys	9	13	26	30	12	10

Ans.

Height	x_i	f_i	$x_i f_i$	$\|x_i - 125.3\|$	$\|x_i - 125.3\| f_i$
95-105	100	9	900	25.3	227.7
105-115	110	13	1430	15.3	198.9
115-125	120	26	3120	5.3	137.8
125-135	130	30	3900	4.7	141
135-145	140	12	1680	14.7	176.4
145-155	150	10	1500	24.7	247
		100	12530		1128.8

$$\text{Mean} = \bar{x} = \frac{\sum_{i=1}^{n} x_i f_i}{\sum_{i=1}^{n} f_i} = \frac{12530}{100} = 125.3$$

$$\text{M.D}(\bar{x}) = \frac{\sum_{i=1}^{n} f_i |x_i - \bar{x}|}{\sum_{i=1}^{n} f_i} = \frac{1128.8}{100} = 11.28$$

Que.5. **[Marks :(6)]**

Find the mean deviation about the median for the following data:

Marks	0-10	10-20,	20-30	30-40	40-50	50-60
Number of Girls	6	8	14	16	4	2

Ans.

marks	x_i	f_i	c . f	$|x_i - 27.86|$	$|x_i - 27.86|f_i$
0-10	5	6	6	22.86	137.16
10-20.	15	8	14	12.86	102.88
20-30	25	14	28	2.86	40.04
30-40	35	16	44	7.14	114.24
40-50	45	4	48	17.14	68.56
50-60	55	2	50	27.14	54.28
		50			517.16

Median class is the class in which the $\left(\dfrac{N}{2}\right)^{th}$ observation lies.

$$\frac{N}{2} = \frac{50}{2} = 25$$

Median class = 20-30

$$M = \text{Median} = l + \frac{C\left(\dfrac{N}{2} - f_0\right)}{f_1}$$

$$= 20 + \frac{25 - 14}{14} \times 10 = 27.86$$

$$M.D\ (M) = \frac{\sum\limits_{i=1}^{n} f_i |x_i - M|}{\sum\limits_{i=1}^{n} f_i} = \frac{517.16}{50} = 10.34$$

Que.6. **[Marks :(4)]**

13. Find the variance and standard deviation of the following data:

x_i	2	4	6	8	10	12	14	16
f_i	4	4	5	15	8	5	4	5

Ans.

x_i	x_i^2	f_i	$x_i f_i$	$x_i^2 f_i$
2	4	4	8	16
4	16	4	16	64
6	36	5	30	180
8	64	15	120	960
10	100	8	80	800
12	144	5	60	720
14	196	4	56	784
16	256	5	80	1280
		50	450	4804

$$\text{Mean} = \bar{x} = \frac{\sum_{i=1}^{n} x_i f_i}{\sum_{i=1}^{n} f_i} = \frac{450}{50} = 9$$

$$\text{Variance} (\sigma^2) = \frac{\sum_{i=1}^{n} f_i x_i^2}{\sum_{i=1}^{n} f_i} - (\bar{x})^2$$

$$= \frac{4804}{50} - (9)^2 = 15.08$$

$$\text{Standard Deviation} (\sigma) = \sqrt{Variance}$$
$$= \sqrt{15.08} = 3.88$$

Que.7.   **[Marks :(6)]**

Find the variance and standard deviation of the following data

Marks	20-30	30-40	40-50	50-60	60-70	70-80	80-90
number of students	3	6	13	15	14	5	4

Ans.

Marks	x_i	x_i^2	f_i	$x_i f_i$	$x_i^2 f_i$
20-30	25	625	3	75	1875
30-40	35	1225	6	210	7350
40-50	45	2025	13	585	26325
50-60	55	3025	15	825	45375
60-70	65	4225	14	910	59150
70-80	75	5625	5	375	28125
80-90	85	7225	4	340	28900
			60	3320	197100

$$\text{Mean} = \bar{x} = \frac{\sum_{i=1}^{n} x_i f_i}{\sum_{i=1}^{n} f_i} = \frac{3320}{60} = 55.33$$

$$\text{Variance } (\sigma^2) = \frac{\sum_{i=1}^{n} f_i x_i^2}{\sum_{i=1}^{n} f_i} - \left(\bar{x}\right)^2$$

$$= \frac{197100}{60} - (55.33)^2 = 223.22$$

$$\text{Standard Deviation } (\sigma) = \sqrt{Variance}$$

$$= \sqrt{223.22} = 14.94$$

Que.8.   **[Marks :(6)]**

The scores of two batsmen A and B in 5 innings during a certain match are as follows:

A	10	15	80	70	25
B	8	9	7	10	6

Find:
(i) Mean score of each batsman.   (2)
(ii) Standard deviation of the scores of each batsman.   (2)
(iii)Which of the batsman is more consistent?   (2)

Ans.

A	B	A×A	B×B
10	8	100	64
15	9	225	81
80	7	6400	49
70	10	4900	100
25	6	625	36
200	40	12250	330

(i) Mean (A) $= \dfrac{\sum\limits_{i=1}^{n} x_i}{n} = \dfrac{200}{5} = 40$

Mean (B) $= \dfrac{\sum\limits_{i=1}^{n} x_i}{n} = \dfrac{40}{5} = 8$

(ii) Variance $(\sigma^2)\,[A] = \dfrac{\sum\limits_{i=1}^{n} x_i^2}{n} - \left(\bar{x}\right)^2$

$= \dfrac{12250}{5} - 40^2 = 2450 - 1600 = 850$

Standard deviation $(\sigma)\,[A] = \sqrt{850} = 29.15$

Variance $(\sigma^2)\,[B] = \dfrac{\sum\limits_{i=1}^{n} x_i^2}{n} - \left(\bar{x}\right)^2$

$= \dfrac{330}{5} - 8^2 = 66 - 64 = 2$

Standard deviation $(\sigma)\,[B] = \sqrt{2} = 1.414$

(iii)   CV (A) $= \dfrac{\sigma}{\bar{x}} \times 100 = \dfrac{29.15}{40} \times 100 = 72.87$

CV (B) $= \dfrac{\sigma}{\bar{x}} \times 100 = \dfrac{1.414}{8} \times 100 = 17.67$

Less the CV more consistent, therefore B is consistent.

Que.9.   **[Marks :(4)]**

An analysis of monthly wages paid to workers in two firms A and B, belongs to the same industry, gives the following results

	Firm – A	Firm – B
No of wage earners	586	648
Mean	Rs. 5253	Rs. 5253
variance	100	121

(i) Which Firm A or B pays larger amount as monthly wage?

(1)

(ii) Which Firm A or B, show greater variability in individual wages?

(2)

Ans.

(i) Mean is same for Firm A and B. But number of earners is larger in B. Therefore B spends the maximum amount as wages.

(ii) CV (A) < CV (B) since there mean are same and Variance if B is more than A.

Que.10.   [Marks :(6)]

Consider the sequence 2,4,6,...,112

(i) Find the number of terms in the sequence.

(1)

(ii) Find the mean of the sequence.

(1)

(iii) Find the sum of the squares of each term in the sequence.

(2)

(iv) Hence evaluate the variance of the sequence.

(2)

Ans.

(i) No of terms = 56

(ii) Mean $= \dfrac{\sum x_i}{n} = \dfrac{2+4+...+112}{56}$

$= \dfrac{2(1+2+...+56)}{56}$

$= \dfrac{2 \times 56 \times 57}{2 \times 56} = 57$

(iii) $\sum x_i^2 = 2^2 + 4^2 + ... + 112^2$

$= 4(1^2 + 2^2 + ... + 56^2)$

$= \dfrac{4 \times 56(56+1)(112+1)}{6}$

$= 112 \times 19 \times 113$

(iv) Variance $= \dfrac{\sum x_i^2}{n} - \left(\bar{x}\right)^2$

$= \dfrac{112 \times 19 \times 113}{56} - (57)^2$

$= 1045$

Que.11.   **[Marks :(6)]**

Consider the observations 6,7,10,12,13,4,8,12

(i) Find the mean of the observations.
$\qquad$ (1)

(ii) Find the variance of the observations.
$\qquad$ (2)

(iii) If every observations are multiplied by 2, then
 (a) Find the new mean.
$\qquad$ (1)

 (b) Find the new variance.
$\qquad$ (1)

Ans.

(i) Mean $= \dfrac{\sum x_i}{n} = \dfrac{72}{8} = 9$

(ii) $\sum x_i^2 = 722$

$\qquad$ Variance $= \dfrac{\sum x_i^2}{n} - \left(\overline{x}\right)^2$

$\qquad\qquad = \dfrac{722}{8} - (9)^2 = 9.25$

(iii) (a) New mean $= 2 \times \overline{x} = 18$
$\qquad$ (b) New Variance
$\qquad\qquad = 2^2 \times$ Variance $= 4 \times 9.25 = 37$

Que.12.   **[Marks :(5)]**

Consider the following observations
6,8,10,12,14,16,18,20,22,24

(i) Find the mean of the observations.
$\qquad$ (1)

(ii) Find the variance of the observations.
$\qquad$ (2)

(iii) If 10 is subtracted from each observation, then
 (a) find the new mean.
$\qquad$ (1)

 (b) find the new variance.
$\qquad$ (1)

Ans. **[Marks :(5)]**

(i) Mean $= \dfrac{\sum x_i}{n} = \dfrac{150}{10} = 15$

(ii) $\sum x_i^2 = 2580$

Variance $= \dfrac{\sum x_i^2}{n} - \left(\overline{x}\right)^2$

$= \dfrac{2580}{10} - (15)^2 = 33$

(iii) (a) New mean $= \overline{x} - 10 = 15 - 10 = 5$

(b) New Variance $= 33$

Que.13. **[Marks :(4)]**

If the mean and variance of the data 1, 2, 6, x, y are respectively 4.4 and 8.24, then find x and y.

(4)

Ans.

Mean $= \dfrac{1+2+6+x+y}{5} = 4.4$

$x+y = 13 ....(1)$

Variance $= 8.24 = \dfrac{1+4+36+x^2+y^2}{5} - (4.4)^2$

$8.24 = \dfrac{41+x^2+y^2}{5} - 19.36$

$x^2 + y^2 = 97...(2)$

$(1) \Rightarrow (x+y)^2 = x^2 + y^2 + 2xy = 169$

$xy = 36 ....(3)$

Using (1) and (3)

$x = 4$ and $y = 9$ or $x = 9$ and $y = 4$

Que.14. **[Marks :(4)]**

(i) Find the mean and variance of the first 10 natural numbers.

(2)

(ii) If each number is multiplied by -1 and then add 1 to each number, then estimate the new mean and variance.

(2)

Ans.

(i) Mean $= \dfrac{\Sigma x_i}{n} = \dfrac{1+2+....+10}{10} = 5.5$

Variance $= \dfrac{\Sigma x_i^2}{n} - \left(\overline{x}\right)^2$

$= \dfrac{10(10+1)(20+1)}{6\times10} - \left(5.5\right)^2 = 8.25$

(ii) New mean $= (-1)5.5+1 = -5.5+1 = -4.5$

New variance $= (-1)^2 \times 8.25 = 8.25$

Que.15. [Marks :(4)]

Consider the following data of some temperature in degree centigrade (C)

10,23,32,39,29,27,28,15,18,29

  (i) Find the mean and variance of the data.

(2)

  (ii) If the temperature is converted to Fahrenheit (F) using the function

$F = f(C) = \dfrac{9}{5}C + 32$, then estimate the

mean and variance of the data converted to Fahrenheit (F).

(2)

Ans.

(i) Mean $= \dfrac{\Sigma x_i}{n} = \dfrac{250}{10} = 25$

Variance $= \dfrac{\Sigma x_i^2}{n} - \left(\overline{x}\right)^2$

$= \dfrac{6918}{10} - \left(25\right)^2 = 66.8$

(ii) Mean of Fahrenheit (F)

$= \overline{x} \times \dfrac{9}{5} + 32 = 45 + 32 = 77$

Variance of Fahrenheit (F)

$= \left(\dfrac{9}{5}\right)^2 \times 66.8 = 216.4$