Class XII Session 2024-25
Subject - Mathematics
Sample Question Paper - 9

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If f(x) = x^2 + 4x - 5 and A = $\begin{vmatrix} 1 & 2 \\ 4 & -3 \end{vmatrix}$, then f(A) is equal to **[1]**

a) $\begin{vmatrix} 8 & 4 \\ 8 & 0 \end{vmatrix}$

b) $\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}$

c) $\begin{vmatrix} 0 & -4 \\ 8 & 8 \end{vmatrix}$

d) $\begin{vmatrix} 2 & 1 \\ 2 & 0 \end{vmatrix}$

2. If A is a 3-rowed square matrix and IAI = 4 then adj (adj A) = ? **[1]**

a) 128A

b) 64A

c) 4A

d) 16A

3. For any 2 $\times$ 2 matrix, If A(adj A) = $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, then |A| is equal to **[1]**

a) 20

b) 10

c) 0

d) 100

4. If f(x) = |x| + |x - 2|, then **[1]**

a) f(x) is continuous at x = 0 but not at x = 2

b) f(x) is continuous at x = 0 and at x = 2

c) f(x) is continuous at x = -2 but not at x = 0

d) f(x) is continuous at x = 2 but not at x = 0

5. If a line makes angles $\frac{\pi}{4}$, $\frac{3\pi}{4}$ with X-axis and Y-axis respectively, then the angle which it makes with Z-axis is **[1]**

a) π

b) $\frac{\pi}{2}$

c) 0^o

d) both 0^o and π

6. Which of the following is the integrating factor of (x log x) $\frac{dy}{dx}$ + y = 2 logx? **[1]**

a) e^x

b) x

c) log x

d) log (log x)

7. Which of the following statements is correct? **[1]**

a. Every LPP admits an optimal selection.

b. A LPP admits unique optimal solution.

c. If a LPP admits two optimal solutions it has an infinite solution.

d. The set of all feasible solutions of a LPP is not a convex set.

a) Option (d)

b) Option (a)

c) Option (b)

d) Option (c)

8. $\vec{a}+\vec{b}+\vec{c}=0$ such that $|\vec{a}| = 3$, $|\vec{b}| = 5$ and $|\vec{c}| = 7$. **[1]**

What is the angle between $\vec{a}$ and $\vec{b}$?

a) $\frac{\pi}{3}$

b) $\frac{\pi}{2}$

c) $\frac{\pi}{4}$

d) $\frac{\pi}{6}$

9. $\int \frac{\sin x}{(1+\sin x)} dx = ?$ **[1]**

a) x + tan x – sec x + C

b) $x+\frac{2}{\tan\frac{x}{2}+1}+c$

c) x – tan x – sec x + C

d) x - tan x + sec x + C

10. For what value of x, the matrix $A=\begin{bmatrix}0 & 1 & -2\\ -1 & 0 & 3\\ x & -3 & 0\end{bmatrix}$ is skew-symmetric matrix? **[1]**

a) x = 2

b) x = -2

c) x = 1

d) x = 3

11. The linear programming problem minimize Z = 3x + 2y subject to constraints x + y $\geq$ 8, 3x + 5y $\leq$15, x $\geq$ 0 and y $\geq$ 0, has **[1]**

a) no feasible solution

b) one solution

c) infinitely many solutions

d) two solutions

12. If $\vec{a}=(\hat{i}+2\hat{j}-3\hat{k})$ and $\vec{b}=(3\hat{i}-\hat{j}+2\hat{k})$ then the angle between $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ is **[1]**

a) $\frac{\pi}{2}$

b) $\frac{2\pi}{3}$

c) $\frac{\pi}{4}$

d) $\frac{\pi}{3}$

13. The existence of the unique solution of the system of equations: **[1]**

x + y + z= λ

5x - y + μ z = 10

2x+ 3y - z = 6 depends on

a) λ and μ both

b) λ only

c) neither λ nor μ

d) μ only

14. In a certain town, 40% persons have brown hair, 25% have brown eyes, and 15% have both. If a person selected at random has brown hair, the chance that a person selected at random with brown hair is with brown eyes **[1]**

a) $\frac{1}{3}$

b) $\frac{3}{20}$

c) $\frac{3}{8}$

d) $\frac{2}{3}$

15. The solution of the differential equation = x dx + y dy = x^2y dy - y^2x dx is **[1]**

a) $x^3 + 1 = C(1 - y^3)$

b) $x^3 - 1 = C(1 + y^3)$

c) $x^2 + 1 = C(1 - y^2)$

d) $x^2 - 1 = C\left(1 + y^2\right)$

16. If β is perpendicular to both α and γ, where $\alpha = \hat{k}$ and $\gamma = \gamma = 2\hat{i} + 3\hat{j} + 4\hat{k}$, then what is β equal to? **[1]**

a) $-2\hat{i} + 3\hat{j}$

b) $3\hat{i} + 2\hat{j}$

c) $2\hat{i} - 3\hat{j}$

d) $-3\hat{i} + 2\hat{j}$

17. If y = $\sin^{-1}x$, then $(1 - x^2)y_2$ is equal to **[1]**

a) xy_2

b) xy_1

c) xy

d) x^2

18. A line passes through the point A (5, -2. 4) and it is parallel to the vector $\left(2\hat{i} - \hat{j} + 3\hat{k}\right)$. The vector equation of the line is **[1]**

a) $\vec{r} \cdot (5\hat{i} - 2\hat{j} + 4\hat{k}) = \sqrt{14}$

b) $\vec{r} \cdot (5\hat{i} + 2\hat{j} - 4\hat{k}) = \sqrt{12}$

c) $\vec{r} = (5\hat{i} - 2\hat{j} + 4\hat{k}) + \lambda(2\hat{i} - \hat{j} + 3\hat{k})$

d) $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(5\hat{i} - 2\hat{j} + 4\hat{k})$

19. **Assertion (A):** If x is real, then the minimum value of $x^2 - 8x + 17$ is 1. **[1]**

Reason (R): If f''(x) > 0 at a critical point, then the value of the function at the critical point will be the minimum value of the function.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** If $A = \{x \in z : 0 \leq x \leq 12\}$ and R is the relation in A given by R = {(a, b) : a = b. Then, the set of all elements related to 1 is {1, 2}. **[1]**

Reason (R): If R_1 and R_2 are equivalence relation in a set A, then $R_1 \cap R_2$ is an equivalence relation.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Write the interval for the principal value of function and draw its graph: $\sec^{-1}x$. **[2]**

OR

$\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

22. Find the values of a for which the functionf(x) = sin x - ax + 4 is increasing function on R. **[2]**

23. The volume of a cube increases at a constant rate. Prove that the increase in its surface area varies inversely as the length of the side. **[2]**

OR

Find two positive numbers whose sum is 14 and the sum of whose squares is minimum.

24. Evaluate: $\int \frac{\sin 2x}{\sin 5x \sin 3x} dx$ **[2]**

25. Find the maximum or minimum values, if any, without using derivatives, of the function: f(x) = $|\sin 4x + 3|$ **[2]**

Section C

26. Evaluate $I = \int \frac{\log\left(1+\frac{1}{x}\right)}{x(1+x)} dx$ **[3]**

27. For A, B and C the chances of being selected as the manager of a firm are in the ratio 4 : 1 : 2 respectively. The respective probabilities for them to introduce a radical change in marketing strategy are 0.3, 0.8 and 0.5. If the change does take place, find the probability that it is due to the appointment of B or C. **[3]**

28. Find $\int e^{-x}$ sin 2x dx. Hence show that $\int_{-\pi/4}^{\pi/4} e^{-x} |\sin 2x| dx = \frac{1}{5}\left(4 + e^{\pi/4} - e^{-\pi/4}\right)$ **[3]**

OR

Evaluate: $\int_0^a \sqrt{a^2 - x^2} dx$

29. Show that the differential equation $\left(x \cos \frac{y}{x}\right)$ (y dx + x dy) = $\left(y \sin \frac{y}{x}\right)$ (x dy - y dx) is homogeneous and solve it. **[3]**

OR

In the differential equation show that it is homogeneous and solve it: $y^2 + (x^2 - xy)\frac{dy}{dx} = 0$.

30. Solve the Linear Programming Problem graphically: **[3]**

Maximize Z = 7x + 10y Subject to

$x + y \leq 30000$

$y \leq 12000$

$x \geq 6000$

$x \geq y$

$x, y \geq 0$

OR

Solve the Linear Programming Problem graphically:

Minimize Z = 30x + 20y Subject to

$x + y \leq 8$

$x + 4y \geq 12$

5x + 8y = 20

$x, y \geq 0$

31. If y = $\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$, $x^2 \leq 1$, then find $\frac{dy}{dx}$. **[3]**

Section D

32. Find the area of the region enclosed by the parabola $x^2 = y$ the line y = x + 2 and x - axis. **[5]**

33. Let L be the set of all lines in xy plane and R be the relation in L define as R = $\{(L_1, L_2) : L_1 \parallel L_2\}$. Show that R is an equivalence relation. Find the set of all lines related to the line y = 2x + 4. **[5]**

OR

Let A and B be two sets. Show that f: $A \times B \to B \times A$ such that f(a, b) = (b, a) is

(i) injective

(ii) bijective

34. The cost of 4kg onion, 3kg wheat and 2kg rice is Rs. 60. The cost of 2kg onion, 4kg wheat and 6kg rice is Rs. 90. The cost of 6kg onion 2kg wheat and 3kg rice is Rs. 70. Find the cost of each item per kg by matrix method. **[5]**

35. Find the perpendicular distance of the point (1, 0, 0) from the line $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$. Also, find the coordinates of the foot of the perpendicular and the equation of the perpendicular. **[5]**

OR

$\overrightarrow{AB} = 3\hat{i} - \hat{j} + \hat{k}$ and $\overrightarrow{CD} = -3\hat{i} + 2\hat{j} + 4\hat{k}$ are two vectors. The position vectors of the points A and C are $6\hat{i} + 7\hat{j} + 4\hat{k}$ and $-9\hat{j} + 2\hat{k}$, respectively. Find the position vector of a point P on the line AB and a point Q on the line CD such that $\overrightarrow{PQ}$ is perpendicular to $\overrightarrow{AB}$ and $\overrightarrow{CD}$ both.

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

To teach the application of probability a maths teacher arranged a surprise game for 5 of his students namely Govind, Girish, Vinod, Abhishek and Ankit. He took a bowl containing tickets numbered 1 to 50 and told the students go one by one and draw two tickets simultaneously from the bowl and replace it after noting the numbers.

i. Teacher ask Govind, what is the probability that tickets are drawn by Abhishek, shows a prime number on one ticket and a multiple of 4 on other ticket? (1)

ii. Teacher ask Girish, what is the probability that tickets drawn by Ankit, shows an even number on first ticket and an odd number on second ticket? (1)

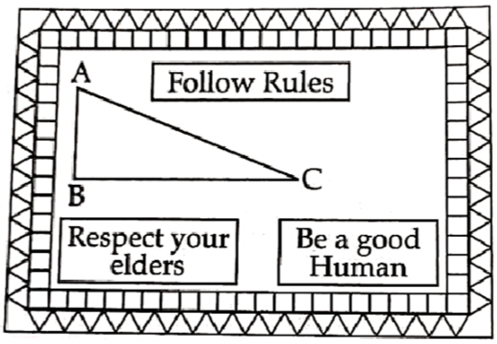
iii. Teacher asks Abhishek, what is the probability that tickets drawn by Vinod, shows a multiple of 4 on one ticket and a multiple 5 on other ticket? (2)

OR

Teacher asks Vinod, what is the probability that both tickets drawn by Girish shows odd number? (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

The slogans on chart papers are to be placed on a school bulletin board at the points A, B and C displaying A (follow Rules), B (Respect your elders) and C (Be a good human). The coordinates of these points are (1, 4, 2), (3, -3, -2) and (-2, 2, 6), respectively.



i. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ be the position vectors of points A, B, C, respectively, then find $|\vec{a} + \vec{b} + \vec{c}|$. (1)

ii. If $\vec{a} = 4\hat{i} + 6\hat{j} + 12\hat{k}$, then find the unit vector in direction of $\vec{a}$. (1)

iii. Find area of $\triangle$ABC. (2)

OR

Write the triangle law of addition for $\triangle$ABC. Suppose, if the given slogans are to be placed on a straight line, then the value of $|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$. (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

The relation between the height of the plant (y in cm) with respect to exposure to sunlight is governed by the following equation $y = 4x - \frac{1}{2}x^2$ where x is the number of days exposed to sunlight.

i. Find the rate of growth of the plant with respect to sunlight. (1)
ii. What is the number of days it will take for the plant to grow to the maximum height? (1)
iii. Verify that height of the plant is maximum after four days by second derivative test and find the maximum height of plant. (2)

OR

What will be the height of the plant after 2 days? (2)

Solution

Section A

1. **(a)** $\begin{vmatrix} 8 & 4 \\ 8 & 0 \end{vmatrix}$

Explanation: A = $\begin{vmatrix} 1 & 2 \\ 4 & -3 \end{vmatrix}$, A^2 = $\begin{vmatrix} 1 & 2 \\ 4 & -3 \end{vmatrix} \begin{vmatrix} 1 & 2 \\ 4 & -3 \end{vmatrix} = \begin{vmatrix} 9 & -4 \\ -8 & 17 \end{vmatrix}$

f(x) = x^2 + 4x - 5

$\because$ f(A) = A^2 + 4A - 5I = $\begin{vmatrix} 9 & -4 \\ -8 & 17 \end{vmatrix} + \begin{vmatrix} 4 & 8 \\ 16 & -12 \end{vmatrix} + \begin{vmatrix} -5 & 0 \\ 0 & -5 \end{vmatrix}$ = $\begin{vmatrix} 8 & 4 \\ 8 & 0 \end{vmatrix}$

2.

(c) 4A

Explanation: The property states that

adj(adj A) = $|A|^{n-2}$.A

Here n = 2

adj(adj A) = $|4|^{3-2}$.A

= 4A

3.

(b) 10

Explanation: We know that

A $\times$ adjA = |A| I_{nxn} , where I is the unit matrix of order nxn.--------------------[1]

$A(\operatorname{adj} A)=\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$ Using the above property of matrices (1), we get

$A(\operatorname{adj} A)=10\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

A(adj A) = (10) I_{2x2}

|A| I_{2x2} = 10 I_{2x2}

|A| = 10

4.

(b) f(x) is continuous at x = 0 and at x = 2

Explanation: f(x) is continuous at x = 0 and at x = 2

5.

(b) $\frac{\pi}{2}$

Explanation: We have,

$\cos^2\frac{\pi}{4}+\cos^2\frac{3\pi}{4}+\cos^2\gamma=1$

$\Rightarrow \frac{1}{2}+\frac{1}{2}+\cos^2\gamma=1$

$\Rightarrow \cos\gamma=0$

$\Rightarrow \gamma=\frac{\pi}{2}$

6.

(c) log x

Explanation: We have,

$(x\log x)\frac{dy}{dx}+y=2\log x$

$\Rightarrow \frac{dy}{dx}+\frac{1}{x\log x}y=\frac{2}{x}$

Comparing with $\frac{dy}{dx}$ = Py = Q

$P=\frac{1}{x\log x}, Q=\frac{2}{x}$

I.F. = $\int\frac{1}{x\log x}dx$ = $e^{\log(\log x)}$ = log x

7.

(d) Option (c)

Explanation: If a LPP admits two optimal solutions it has an infinite solution.

8. **(a)** $\frac{\pi}{3}$

Explanation: $\frac{\pi}{3}$

9.

(b) $x+\frac{2}{\tan\frac{x}{2}+1}+c$

Explanation: Given

$\int\frac{\sin x}{1+\sin x}dx$

$=\int dx-\int\frac{dx}{1+\sin x}$

$=x-\int\frac{dx}{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}$

$=x-\int\frac{dx}{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2}$

$=x-\int\frac{\sec^2\frac{x}{2}dx}{\left(\tan\frac{x}{2}+1\right)^2}$

Let, $\tan\frac{x}{2}+1=z$

$\Rightarrow\frac{1}{2}\sec^2\frac{x}{2}dx=dz$

So,

$x-\int\frac{2dz}{z^2}$

$=x+\frac{2}{z}+c$

$=x+\frac{2}{\tan\frac{x}{2}+1}+c$

where c is the integrating constant.

10. **(a)** x = 2

Explanation: Given, $A=\begin{bmatrix}0&1&-2\\-1&0&3\\x&-3&0\end{bmatrix}$

We know that, if A is a skew-symmetric matrix, then

$A=-A^T$ ...(i)

From Eq. (i) We, get

$$\begin{bmatrix}0&1&-2\\-1&0&3\\x&-3&0\end{bmatrix}=-\begin{bmatrix}0&-1&x\\1&0&-3\\-2&3&0\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}0&1&-2\\-1&0&3\\x&-3&0\end{bmatrix}=\begin{bmatrix}0&1&-x\\-1&0&3\\2&-3&0\end{bmatrix}$$

On comparing the corresponding element, we get

-2 = -x ⇒ x = 2

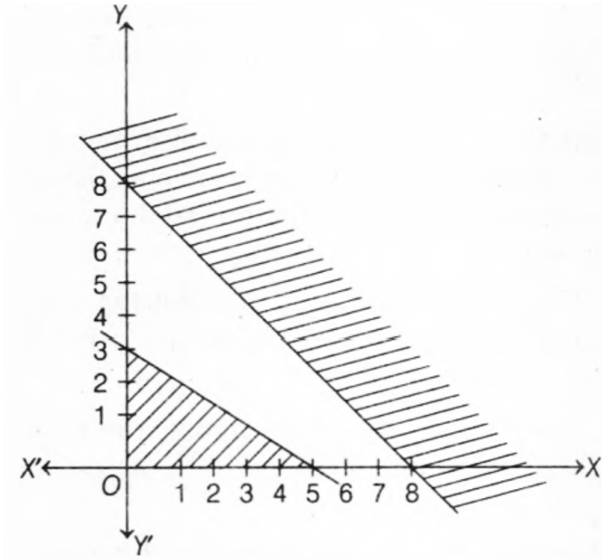
11. **(a)** no feasible solution

Explanation: Table for equation x + y = 8 is

x	0	8
y = 8 - x	8	0

Table for equation 3x + 5y = 15 is

x	0	5
y = $\frac{15-3x}{5}$	3	0



It can be concluded from the graph, that there is no point, which can satisfy all the constraints simultaneously. Therefore, the problem has no feasible solution.

12. **(a)** $\frac{\pi}{2}$

Explanation: Given vectors $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}\, and\, \vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$

Now, $\vec{a} + \vec{b} = 4\hat{i} + \hat{j} - \hat{k}$ and $\vec{a} - \vec{b} = -2\hat{i} + 3\hat{j} - 5\hat{k}$

let θ be the angle between the vectors $\vec{a} + \vec{b}\, and\, \vec{a} - \vec{b}$

$\Rightarrow cos\theta = \frac{-8+3+5}{\sqrt{16+1++}\times\sqrt{4+9+25}}$ = 0 = $\frac{\pi}{2}$

13.

(d) μ only

Explanation: The given system of linear equation :-

x + y + z = λ

5x - y + μz = 10

2x + 3y – z = 6

The matrix equation corresponding to the above system is :

$$\begin{bmatrix} 1 & 1 & 1 \\ 5 & -1 & \mu \\ 2 & 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \lambda \\ 10 \\ 6 \end{bmatrix}$$

Suppose A = $\begin{bmatrix} 1 & 1 & 1 \\ 5 & -1 & \mu \\ 2 & 3 & -1 \end{bmatrix}$

$\therefore$ |A| = $\begin{vmatrix} 1 & 1 & 1 \\ 5 & -1 & \mu \\ 2 & 3 & -1 \end{vmatrix}$ = 1(1-3μ) -1(-5-2μ) +1(15 + 2)

= 1 - 3μ +5 +2μ +17 = 23-μ

For the existence of the unique solution, the value of |A| must not be equal to 0.

Therefore, the existence of the unique solution merely depends on the value of μ.Which is the required solution.

14.

(c) $\frac{3}{8}$

Explanation: Let A be the event that a person has brown hair, B be the event that a person has brown eyes. Then,

P(A) = $\frac{40}{100}$, P(B) = $\frac{25}{100}$, $P(A \cap B) = \frac{15}{100}$

Required probability = $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{15}{100}}{\frac{40}{100}} = \frac{3}{8}$

15.

(d) $x^2 - 1 = C\left(1 + y^2\right)$

Explanation: We have,

xdx + ydy = x^2y dy - y^2x dx

x dx + y^2x dx = x^2y dy - y dy

$x\left(1+y^2\right)dx = y\left(x^2-1\right)dy$

$\frac{xdx}{x^2-1} = \frac{ydy}{1+y^2}$

$\int \frac{xdx}{x^2-1} = \int \frac{ydy}{1+y^2}$

$\frac{1}{2}\int \frac{2xdx}{x^2-1} = \frac{1}{2}\int \frac{2ydy}{1+y^2}$

$\frac{1}{2}\log\left(x^2-1\right) = \frac{1}{2}\log\left(1+y^2\right) + \log c$

$\log\left(x^2-1\right) = \log\left(1+y^2\right) + \log c$

$x^2 - 1 = \left(1+y^2\right)c$

16.

(d) $-3\hat{i} + 2\hat{j}$

Explanation: Given that, $\alpha = \hat{k}$

and $\gamma = 2\hat{i} + 3\hat{j} + 4\hat{k}$

Since, β is perpendicular to both α and γ.

i.e., $\beta = \pm(\alpha \times \gamma) = \pm \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ 2 & 3 & 4 \end{vmatrix}$

$= \pm \hat{i}(0-3) - \hat{j}(0-2) + \hat{k}(0-0)$

$= \pm(-3\hat{i} + 2\hat{j})$

17.

(b) xy_1

Explanation: $y = \sin^{-1}x$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} \Rightarrow \sqrt{1-x^2} \cdot \frac{dy}{dx} = 1$

Again, differentiating both sides w.r.to x, we get

$\sqrt{1-x^2} \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \left(\frac{-2x}{2\sqrt{1-x^2}}\right) = 0$

Simplifying, we get $(1 - x^2)y_2 = xy_1$

18.

(c) $\vec{r} = (5\hat{i} - 2\hat{j} + 4\hat{k}) + \lambda(2\hat{i} - \hat{j} + 3\hat{k})$

Explanation: Fixed point is $5\hat{i} - 2\hat{j} + 4\hat{k}$ and parallel vector is $2\hat{\imath} - \hat{\jmath} + 3\hat{k}$

Equation $\vec{r} = 5\hat{\imath} - 2\hat{\jmath} + 4\hat{k} + \lambda(2\hat{\imath} - \hat{\jmath} + 3\hat{k})$

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Let $f(x) = x^2 - 8x + 17$

$\therefore$ f'(x) = 2x - 8

So, f'(x) = 0, gives x = 4

Here x = 4 is the critical number

Now, f''(x) = 2 > 0 , $\forall$ x

So, x = 4 is the point of local minima.

$\therefore$ Minimum value of f(x) at x = 4,

f(4) = 4 $\times$ 4 - 8 $\times$ 4 + 17 = 1

Hence, we can say that both Assertion and Reason are true and Reason is the correct explanation of the Assertion.

20.

(d) A is false but R is true.

Explanation: Assertion: The elements that are related to 1 will be those elements from set A which are equal to 1. Hence, the set of elements related to 1 is {1}.

Reason: Since, R_1 and R_2 are equivalence relations, therefore (a, a) $\in R_1$, $(a,a) \in R_2,\ \forall a \in A$.

This implies that $(a,a) \in R_1 \cap R_2, \forall a$.

Hence, $R_1 \cap R_2$ is reflexive.

Further, (a, b) $\in R_1 \cap R_2 \Rightarrow$ (a, b) $\in R_1$ and (a, b) $\in R_2$ and (b, a) $\in R_2$

$\Rightarrow$ (b, a) $\in R_1 \cap R_2$

Hence, $R_1 \cap R_2$ is symmetric.

Similarly, (a, b) $\in R_1 \cap R_2$ and (b, c) $\in R_1 \cap R_2$

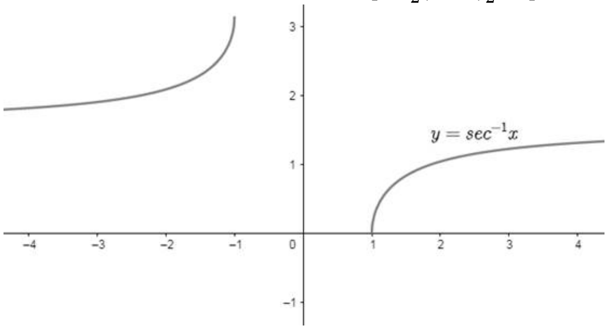
$\Rightarrow$ (a, c) $\in R_1$ and (a, c) $\in R_2 \Rightarrow$(a, c) $\in R_1 \cap R_2$.

This implies that $R_1 \cap R_2$ is transitive.

Hence, $R_1 \cap R_2$ is an equivalence relation.

Section B

21. Principal value branch of sec^1 x is $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$ and its graph is shown below.



OR

Let $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = y$

$\Rightarrow \sec y = \frac{2}{\sqrt{3}}$

$\Rightarrow \sec y = \sec\frac{\pi}{6}$

Since, the principal value branch of sec^{-1} is $[0, \pi]$.

Therefore, Principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ is $\frac{\pi}{6}$.

22. Given: f(x) = sinx - ax + 4

f (x) = cos x - a

Given : F(x) is increasing on R

$\Rightarrow f'(x) > 0$

$\Rightarrow$ cosx - a > 0

$\Rightarrow$ cos x > a

We know

Cos x > -1 , $\forall x \in R$

$\therefore$ a < -1

$\Rightarrow a \in (-\infty, -1)$

23. Let the side of a cube be x unit.

$\therefore$ Volume of cube (V) $= x^3$

On differentiating both side w.r.t. t, we get

$\frac{dV}{dt} = 3x^2\frac{dx}{dt} = k$ [constant]

$\Rightarrow \frac{dx}{dt} = \frac{k}{3x^2}$ ...(i)

Also, surface area of cube, $S = 6x^2$

On differentiating w.r.t. t, we get

$\frac{dS}{dt} = 12x.\frac{dx}{dt}$

$\Rightarrow \frac{dS}{dt} = 12x.\frac{k}{3x^2}$ [using Eq. (i)]

$\Rightarrow \frac{dS}{dt} = \frac{12k}{3x} = 4\left(\frac{k}{x}\right)$

$\Rightarrow \frac{dS}{dt}\alpha\frac{1}{x}$

Hence, the surface area of the cube varies inversely as the length of the side.

OR

Let the numbers be x and y. Then,

x + y = 14...(i)

Let S be the sum of the squares of x and y. Then,

$S = x^2 + y^2$

$\Rightarrow$ S = $x^2 + (14 - x)^2$

$\Rightarrow$ S = $2x^2 - 28x + 196$

$\Rightarrow \frac{dS}{dx}$ = 4x -28 and $\frac{d^2S}{dx^2}$ = 4

The critical points of S are given by $\frac{dS}{dx}$ = 0.

$\because \frac{dS}{dx} = 0 \Rightarrow 4x - 28 = 0 \Rightarrow x = 7$

Clearly $\frac{d^2S}{dx^2} = 4 > 0$

Thus, S is minimum when x = 7. Putting x = 7 in equation (i), we obtain y = 7.

Hence. the required numbers are both equal to 7.

24. Let I = $\int \frac{\sin 2x}{\sin 5x \sin 3x} dx$. Then, we have

$I = \int \frac{\sin(5x-3x)}{\sin 5x \sin 3x} dx$

$= \int \frac{\sin 5x \cos 3x - \cos 5x \sin 3x}{\sin 5x \sin 3x} dx$

$= \int \frac{\sin 5x \cos 3x}{\sin 5x \sin 3x} dx - \int \frac{\cos 5x \sin 3x}{\sin 5x \sin 3x} dx$

$= \int \frac{\cos 3x}{\sin 3x} dx - \int \frac{\cos 5x}{\sin 5x} dx$

$= \int \cot 3x dx - \int \cot 5x dx$

$= \frac{1}{3} \log|\sin 3x| - \frac{1}{5} \log|\sin 5x| + c$

$\therefore I = \frac{1}{3} \log|\sin 3x| - \frac{1}{5} \log|\sin 5x| + c$

25. Maximum value = 4, Minimum value = 2

We know that

$-1 \leq \sin \theta \leq 1$

$\therefore \ -1 \leq \sin 4x \leq 1$

Adding 3, on both sides, of above

We get

$-1 + 3 \leq \sin 4x + 3 \leq 1 + 3$

$2 \leq |\sin 4x + 3| \leq 4$

Hence min.Value is 2 and max value is 4.

Section C

26. Let $I = \int \frac{\log\left(1+\frac{1}{x}\right)}{x(1+x)} dx$ ...(i)

Let $\log\left(1 + \frac{1}{x}\right)$ = t then,

$d\left[\log\left(1 + \frac{1}{x}\right)\right]$ = dt

$\Rightarrow \quad \frac{1}{1+\frac{1}{x}} \times \frac{-1}{x^2}$ dx = dt

$\Rightarrow \quad \frac{1}{\frac{x+1}{x}} \times \frac{-1}{x^2}$ dx = dt

$\Rightarrow \quad \frac{-x}{x^2(x+1)}$ dx = dt

$\Rightarrow \quad \frac{dx}{x(x+1)}$ = -dt

Putting log $\left(1 + \frac{1}{x}\right)$ = t and $\frac{dx}{x(x+1)}$ = - dt in equation (i), we get

$I = -\int t dt$

$= -\frac{t^2}{2} + c$

$= -\frac{1}{2}\left[\log\left(1 + \frac{1}{x}\right)\right]^2 + c$

$\therefore \quad I = -\frac{1}{2}\left[\log\left(1 + \frac{1}{x}\right)\right]^2 + c$

27. Let A, E_1, E_2 and E_3 denote the events that the change takes place, A is selected, B is selected and C is selected, respectively.

Therefore, we have,

P(E_1) = $\frac{4}{7}$

P(E_2) = $\frac{1}{7}$

P(E_3) = $\frac{2}{7}$

Now,we have,

P($\frac{A}{E_1}$) = 0.3

P($\frac{A}{E_2}$) = 0.8

P($\frac{A}{E_3}$) = 0.5

Using Bayes' theorem, we have,

$= P(\frac{E_1}{A}) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$

$= \frac{\frac{4}{7}\times 0.3}{\frac{4}{7}\times 0.3+\frac{1}{7}\times 0.8+\frac{2}{7}\times 0.5}$

$= \frac{1.2}{1.2+0.8+1} = \frac{1.2}{3} = \frac{12}{30} = \frac{2}{5}$

$\therefore$ Required probability = 1 - P($\frac{A}{E_1}$) $= 1 - \frac{2}{5} = \frac{3}{5}$

28. Let the given integral be, $I = \int e^{-x}$ sin 2x dx . Then, using integration by parts we have..

$I = \frac{1}{2}e^{-x}\cos 2x - \int(-1)e^{-x} \times -\frac{1}{2}\cos 2x dx$

$\Rightarrow \quad I = -\frac{1}{2}e^{-x}\cos 2x - \frac{1}{2}\int e^{-x}\cos 2x dx$

$\Rightarrow \quad I = -\frac{1}{2}e^{-x}\cos 2x - \frac{1}{2}\left\{\frac{1}{2}e^{-x}\sin 2x - \int(-1)e^{-x} \times \frac{1}{2}\sin 2x dx\right\}$

$\Rightarrow \quad I = -\frac{1}{2}e^{-x}\cos 2x - \frac{1}{4}e^{-x}\sin 2x - \frac{1}{4}\int e^{-x}\sin 2x dx$

$\Rightarrow \quad I = -\frac{1}{2}e^{-x}\cos 2x - \frac{1}{4}e^{-x}\sin 2x - \frac{1}{4}I$

$\Rightarrow \quad \frac{5}{4}I = -\frac{1}{4}e^{-x}(2\cos 2x + \sin 2x)$

$\Rightarrow \quad I = -\frac{1}{5}e^{-x}(\sin 2x + 2\cos 2x) + C$

Now we have,

$I = \int_{-\pi/4}^{\pi/4} e^{-x}|\sin 2x|dx = \int_{-\pi/4}^{0} e^{-x}|\sin 2x|dx + \int_{0}^{\pi/4} e^{-x}|\sin 2x|dx$

$\Rightarrow \quad I = -\int_{-\pi/4}^{0} e^{-x}\sin 2x dx + \int_{0}^{\pi/4} e^{-x}\sin 2x dx$

$\Rightarrow \quad I = -\left[-\frac{1}{5}e^{-x}(\sin 2x + 2\cos 2x)\right]_{-\pi/4}^{0} + \left[-\frac{1}{5}e^{-x}(\sin 2x + 2\cos 2x\right]_{0}^{\pi/4}$

$\Rightarrow \quad I = -\left[-\frac{2}{5} + \frac{1}{5}e^{\pi/4}(-1)\right] + \left[-\frac{1}{5}e^{-\pi/4} + \frac{2}{5}\right]$

$\Rightarrow \quad I = \frac{4}{5} + \frac{1}{5}\left(e^{\pi/4} - e^{-\pi/4}\right) = \frac{1}{5}\left(4 + e^{\pi/4} - e^{-\pi/4}\right)$

OR

Let $x = a\sin\theta$

Differentiating w.r.t. x, we get

$dx = a\cos\theta d\theta$

Now,

$x = 0 \Rightarrow \theta = 0$

$x = a \Rightarrow \theta = \frac{\pi}{2}$

$\therefore \int_0^2 \sqrt{a^2 - x^2}dx$

$= \int_0^{\frac{\pi}{2}} \sqrt{a^2\left(1 - \sin^2\theta\right)}\, a\cos\theta d\theta$

$= a^2 \int_0^{\frac{x}{2}} \cos^2\theta d\theta$

$= \frac{a^2}{2}\int_0^{\frac{\pi}{2}}(1 + \cos 2\theta)d\theta$ [using $cos^2\theta = \frac{(1+cos 2\theta)}{2}$]

$= \frac{a^2}{2}\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{2}}$

$= \frac{a^2}{2}\left[\frac{\pi}{2} + 0 - 0 - 0\right]$

$= \frac{\pi a^2}{4}$

$\therefore \int_0^2 \sqrt{a^2 - x^2}dx = \frac{\pi a^2}{4}$

29. We can write the given differential equation as,

($\frac{y}{x}$ + y^2 sin $\frac{y}{x}$) dx = (xy sin $\frac{y}{x}$ - x^2 cos$\frac{y}{x}$)dy

$\Rightarrow \frac{dy}{dx} = \frac{\left\{xy\cos\frac{y}{x} + y^2\sin\frac{y}{x}\right\}}{\left\{xy\sin\frac{y}{x} - x^2\cos\frac{y}{x}\right\}}$

$\Rightarrow \frac{dy}{dx} = \frac{(\frac{y}{x})\cos(\frac{y}{x}) + (\frac{y}{x})^2\sin(\frac{y}{x})}{(\frac{y}{x})\sin(\frac{y}{x}) - \cos(\frac{y}{x})} = f\left(\frac{y}{x}\right)$ ...(i)

Therefore, the given differential equation is homogeneous.

Put y = vx and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ in (i),

$\Rightarrow \quad x\frac{dv}{dx} = \left\{\frac{(v\cos v + v^2\sin v)}{(v\sin v - \cos v)} - v\right\}$

$\Rightarrow \quad x\frac{dv}{dx} = \frac{2v\cos v}{(v\sin v - \cos v)}$

$\Rightarrow \int \frac{(v\sin v - \cos v)}{v\cos v}dv = \int \frac{2}{x}dx$

$\Rightarrow \int$ tan v dv - $\int \frac{dv}{v} = \int \frac{2}{x} dx$

$\Rightarrow$ -log |cos v| - log |v| - 2 log |x| = constant

$\Rightarrow$ log |cos v| + log |v| + 2 log |x| = log $|C_1|$ where C_1 is an arbitrary constant

$\Rightarrow$ log $|x^2$ v cos v| = log $|C_1|$

$\Rightarrow x^2$ v cos v = $\pm$ C_1 = C(say)

$\Rightarrow$ x y cos $\frac{y}{x}$ = C, which is the required solution [$\because$ v = $\frac{y}{x}$]

OR

The given differential equation is,

$y^2 + (x^2 - xy)\frac{dy}{dx} = 0$

$\frac{dx}{dy} = \frac{xy-x^2}{y^2} = \frac{x}{y} - (\frac{x}{y})^2$

$\Rightarrow \frac{dx}{dy} = f\left(\frac{x}{y}\right)$

$\Rightarrow$ the given differential equation is a homogenous equation.

The solution of the given differential equation is:

Put x = vy

$\Rightarrow \frac{\mathrm{dx}}{\mathrm{dy}} = \mathrm{v} + \mathrm{y}\frac{\mathrm{dv}}{\mathrm{dy}}$

$\Rightarrow v + y\frac{dv}{dy} = \frac{vy}{y} - \left(\frac{vy}{y}\right)^2$

$\Rightarrow y\frac{dv}{dy}$ = v - v^2 - v

$\Rightarrow y\frac{dv}{dy}$ = -v^2

$\Rightarrow \frac{\mathrm{dv}}{\mathrm{v}^2} = -\frac{\mathrm{dy}}{\mathrm{y}}$

Integratin both the sides we get

$\Rightarrow \int \frac{\mathrm{dv}}{\mathrm{v}^2} = -\int \frac{\mathrm{dy}}{\mathrm{y}} + \mathrm{c}$

$\Rightarrow \frac{-1}{v}$ = -ln|y| + c

$\Rightarrow \frac{y}{x}$ = -(ln|y| + c)

$\Rightarrow$ y = -x(ln |y| + c)

30. We have to maximize Z = 7x + 10y

First, we will convert the given inequations into equations, we obtain the following equations:

x + y = 30000, y = 12000, x = 6000, x = y, x = 0 andy = 0

Region represented by x + y $\leq$ 30000 :

The line x + y = 30000 meets the coordinate axes at A(30000, 0) and B(0, 30000) respectively.

By joining these points we obtain the line x + y = 30000 Clearly (0, 0) satisfies the inequation x + y $\leq$ 30000.

So, the region containing the origin represents the solution set of the inequationx + y $\leq$ 30000

The line y = 12000 is the line that passes through C(0, 12000) and parallel to x-axis.

The line x = 6000 is the line that passes through (6000, 0) and parallel to y-axis.

Region represented by x $\geq$ y:

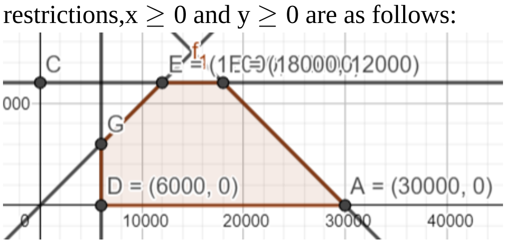
The line x = y is the line that passes through the origin. The points to the right of the line x = y satisfy the inequation x $\geq$ y Like by taking the point (-12000, 6000).

Here, 6000 > -12000 which implies y > x. Hence, the points to the left of line x = y will not satisfy the given in equation x $\geq$ y

Region represented by x $\geq$ 0 and y $\geq$ 0 :

since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the in equations x $\geq$ 0 andy $\geq$ 0

The feasible region determined by subject to the constraints are, x + y $\leq$ 30000, y $\leq$ 12000, x $\geq$ 6000, x$\geq$ y, and non-negative restrictions,x $\geq$ 0 and y $\geq$ 0 are as follows:



The corner points of the feasible region are D(6000, 0), A(3000, 0), F(18000, 12000) and E(12000, 12000).

The values of objective function at the corner points are as follows:

Corner point	Z = 7x + 10y
D(6000, 0)	$7 \times 6000 + 10 \times 0 = 42000$
A(3000, 0)	$7 \times 3000 + 10 \times 0 = 21000$
F(18000, 12000)	$7 \times 18000 + 10 \times 12000 = 246000$
$E(12000, 12000)$	$7 \times 12000 + 10 \times 12000 = 204000$

We see that the maximum value of the objective function Z is 246000 which is at F(18000,12000)

that means at x = 18000 and y = 12000

Thus, the optimal value of objective function z is 246000.

OR

First, we will convert the given inequations into equations, we obtain the following equations:

x + y = 8, x + 4 y = 12, x = 0 and y = 0

5 x + 8 y = 20 is already an equation.

Region represented by x + y $\leq$ 8 The line x + y = 8 meets the coordinate axes at A(8,0) and B(0,8) respectively. By joining these points we obtain the line x + y = 8 . Clearly (0,0) satisfies the inequation x + y $\leq$ 8.50, the region in x y plane which contain the origin represents the solution set of the inequation x + y $\leq$ 8.

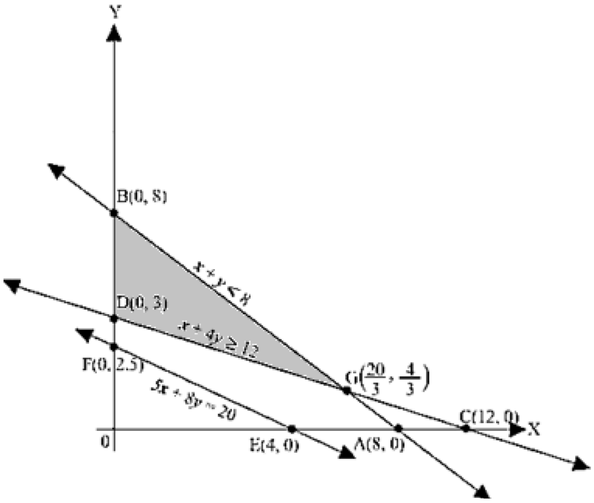
Region represented by x + 4 y $\geq$ 12 :

The line x + 4 y = 12 meets the coordinate axes at C(12,0) and D(0,3) respectively. By joining these points we obtain the line x + 4 y = 12 . Clearly (0,0) satisfies the inequation x + 4 y $\geq$ 12 . So, the region in x y plane which does not contain the origin represents the solution set of the inequation x + 4 y $\geq$ 12.

The line 5 x + 8 y = 20 is the line that passes through E(4,0) and $F\left(0, \frac{5}{2}\right)$ Region represented by x $\geq$ 0 and y $\geq$ 0 :

since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations x $\geq$ 0 and y $\geq$ 0.

The feasible region determined by subject to the constraints arex + y $\leq$ 8, x + 4 y $\geq$ 12,5 x + 8 y = 20 and the non-negative restrictions , x $\geq$ 0 and y $\geq$ 0 are as follows.



The corner points of the feasible region are B(0,8), D(0,3), $G\left(\frac{20}{3}, \frac{4}{3}\right)$

The values of objective function at corner points are as follows:

Corner point: Z = 30x +20y

B(0,8): 160

D(0,3): 60

$G\left(\frac{20}{3}, \frac{4}{3}\right)$: 266.66

Therefore, the minimum value of objective function Z is 60 at the point D(0,3) . Hence, x = 0 and y = 3 is the optimal solution of the given LPP.

Thus, the optimal value of objective function Z is 60.

31. Given, y= $\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$

Put $x^2 = \sin\theta \Rightarrow \theta = \sin^{-1} x^2$

$\therefore \quad y = \tan^{-1}\left(\frac{\sqrt{1+\sin\theta}+\sqrt{1-\sin\theta}}{\sqrt{1+\sin\theta}-\sqrt{1-\sin\theta}}\right)$

$= \tan^{-1}\left(\frac{\sqrt{\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}+\sqrt{\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}}{\sqrt{\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}-\sqrt{\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}}\right)$

$= \tan^{-1}\left[\frac{\sqrt{\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)^2}+\sqrt{\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)^2}}{\sqrt{\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)^2}-\sqrt{\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)^2}}\right]$

$= \tan^{-1}\left[\frac{\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)+\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)}{\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)-\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)}\right]$

$= \tan^{-1}\left(\frac{2\cos\frac{\theta}{2}}{2\sin\frac{\theta}{2}}\right)$

$= \tan^{-1}\left(\cot\frac{\theta}{2}\right)$

$= \tan^{-1}\left[\tan\left(\frac{\pi}{2}-\frac{\theta}{2}\right)\right]$

$= \frac{\pi}{2}-\frac{\theta}{2}$

$\Rightarrow y = \frac{\pi}{2}-\frac{1}{2}\sin^{-1} x^2$

Therefore, on differentiating both sides w.r.t x, we get,

$\frac{dy}{dx} = -\frac{1}{2}\frac{1}{\sqrt{1-\left(x^2\right)^2}}(2x)$

$= \frac{-x}{\sqrt{1-x^4}}$

Section D

32. Equation of parabola is x^2 = y …(i)

Equation of line is y = x + 2 …(ii)

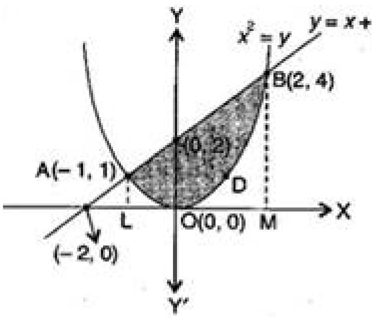
Here the two points of intersections of parabola (i) and line (ii) are A (-1, 1) and B (2, 4).

Area ALODBM = Area bounded by parabola (i) and x - axis

$= \left|\int_{-1}^{2} x^2 dx\right| = \left(\frac{x^3}{3}\right)_{-1}^{2}$

$= \frac{8}{3}-\left(\frac{-1}{3}\right)$

$= \frac{8}{3}+\frac{1}{3} = \frac{9}{3} = 3$ sq units



Also Area of trapezium ALMB = Area bounded by line (ii) and x - axis

$= \left|\int_{-1}^{2}(x+2)\,dx\right| = \left(\frac{x^2}{2}+2x\right)_{-1}^{2}$

$= 2+4-\left(\frac{1}{2}-2\right)$

$= 6-\frac{1}{2}+2$

$= \frac{15}{2}$ sq. units

Now Required area = Area of trapezium ALMB – Area ALODBM

$= \frac{15}{2}-3 = \frac{9}{2}$ sq. units

33. $L_1 \| L_1$ i.e $(L_1, L_1) \in$ R Hence reflexive

Let $(L_1, L_2) \in R$, then

$L_1 \| L_2$ which implies $L_2 \| L_1$

$\Rightarrow (L_2, L_1) \in R$ Hence symmetric

We know the

$L_1 \| L_2$ and $L_2 \| L_3$

Then $L_1 \| L_3$

Therefore, $(L_1, L_2) \in R$ and $(L_2, L_3) \in R$ implies $(L_1, L_3) \in R$

Hence Transitive

Hence, R is an equivalence relation.

Any line parallel to y = 2x + 4 is of the form y = 2x + K, where k is a real number.

Therefore, set of all lines parallel to y = 2x + 4 is {y : y = 2x + k, k is a real number}

OR

i. Let $(a_1\ b_1)$ and $(a_2, b_2) \in A \times B$ such that

$f(a_{1,}\ b_1) = f(a_2, b_2)$

$\Rightarrow (a_{1,}\ b_1) = (a_2, b_2)$

$\Rightarrow a_1 = a_2$ and $b_1 = b_2$

$\Rightarrow (a_1, b_1) = (a_2, b_2)$

Therefore, f is injective.

ii. Let (b, a) be an arbitrary

Element of $B \times A$. then $b \in B$ and $a \in A$

$\Rightarrow (a, b)\ ) \in (A \times B)$

Thus for all $(b, a) \in B \times A$ their exists $(a, b)\ ) \in (A \times B)$

such that

f (a, b) = (b, a)

So $f: A \times B \rightarrow B \times A$

is an onto function.

Hence f is bijective.

34. Let cost of 1kg onion = x

cost of 1kg wheat = y

cost of 1kg rise = z

By the question ,we have,

4x + 3y + 2z = 60

2x + 4y + 6z = 90

6x + 2y + 3z = 70

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix} B = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix} X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$|A| = \begin{vmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{vmatrix} = 50 \neq 0$$

$Now, A_{11} = 0, A_{12} = 30, A_{13} = -20$

$A_{21} = -5, A_{22} = 0, A_{23} = 10$

$A_{31} = 10, A_{32} = -20, A_{33} = 10$

$$\therefore adjA = \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

$X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$$

x = 5, y = 8, z = 8

35. Suppose the point (1, 0, 0) be P and the point through which the line passes be Q(1,-1,-10). The line is parallel to the vector $\vec{b} = 2\hat{i} - 3\hat{j} + 8\hat{k}$

Now,

$\overrightarrow{PQ} = 0\hat{i} - \hat{j} - 10\hat{k}$

$\therefore \vec{b} \times \overrightarrow{PQ}$ = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 8 \\ 0 & -1 & -10 \end{vmatrix}$

= $38\hat{i} + 20\hat{j} - 2\hat{k}$

$\Rightarrow |\vec{b} \times \overrightarrow{PQ}| = \sqrt{38^2 + 20^2 + 2^2}$

= $\sqrt{1444 + 400 + 4}$

= $\sqrt{1848}$

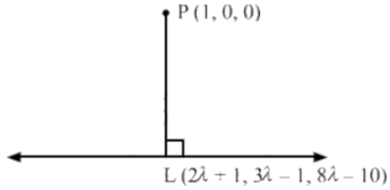
d = $\frac{|\vec{b} \times \overrightarrow{PQ}|}{|\vec{b}|}$

= $\frac{\sqrt{1848}}{\sqrt{77}}$

= $\sqrt{24}$

= $2\sqrt{6}$

Suppose L be the foot of the perpendicular drawn from the point P(1,0,0) to the given line-



The coordinates of a general point on the line

$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ are given by

$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8} = \lambda$

$\Rightarrow x = 2\lambda + 1$

$y = -3\lambda - 1$

$z = 8\lambda - 10$

Suppose the coordinates of L be

$(2\lambda + 1, -3\lambda - 1, 8\lambda - 10)$

Since,The direction ratios of PL are proportional to,

$2\lambda + 1 - 1, -3\lambda - 1 - 0, 8\lambda - 10 - 0$, i.e., $2\lambda, -3\lambda - 1, 8\lambda - 10$

Since,The direction ratios of the given line are proportional to 2, -3, 8, but PL is perpendicular to the given line.

$\therefore 2(2\lambda) - 3(-3\lambda - 1) + 8(8\lambda - 10)$ = 0

$\Rightarrow \lambda$ = 1 Substituting λ = 1 in $(2\lambda + 1, -3\lambda - 1, 8\lambda - 10)$ we get the coordinates of L as (3, -4, -2). Equation of the line PL is given by

$\frac{x-1}{3-1} = \frac{y-0}{-4-0} = \frac{z-0}{-2-0}$

= $\frac{x-1}{1} = \frac{y}{-2} = \frac{z}{-1}$

$\Rightarrow \vec{r} = \hat{i} + \lambda(\hat{i} - 2\hat{j} - \hat{k})$

OR

We have, $\overrightarrow{AB} = 3\hat{i} - \hat{j} + \hat{k}$ and $\overrightarrow{CD} = -3\hat{i} + 2\hat{j} + 4\hat{k}$

Also, the position vectors of A and C are $6\hat{i} + 7\hat{j} + 4\hat{k}$ and $-9\hat{j} + 2\hat{k}$, respectively. Since, $\overrightarrow{PQ}$ is perpendicular to both $\overrightarrow{AB}$ and $\overrightarrow{CD}$.

So, P and Q will be foot of perpendicular to both the lines through A and C.

Now, equation of the line through A and parallel to the vector $\overrightarrow{AB}$ is,

$\overrightarrow{r} = (6\hat{i} + 7\hat{j} + 4\hat{k}) + \lambda(3\hat{i} - \hat{j} + \hat{k})$

And the line through C and parallel to the vector $\overrightarrow{CD}$ is given by

$\overrightarrow{r} = -9\hat{j} + 2\hat{k} + \mu(-3\hat{i} + 2\hat{j} + 4\hat{k})$ ..... (i)

Let $\overrightarrow{r} = (6i + 7j + 4\hat{k}) + \lambda(3\hat{i} - \hat{j} + \hat{k})$

and $\overrightarrow{r} = -9\hat{j} + 2\hat{k} + \mu(-3\hat{i} + 2\hat{j} + 4\hat{k})$ .... (ii)

Let $P(6 + 3\lambda, 7 - \lambda, 4 + \lambda)$ is any point on the first line and Q be any point on second line is given by $(-3\mu, -9 + 2\mu, 2 + 4\mu)$.

$\therefore \overrightarrow{PQ} = (-3\mu - 6 - 3\lambda)\hat{i} + (-9 + 2\mu - 7 + \lambda)\hat{j} + (2 + 4\mu - 4 - \lambda)\hat{k}$

$= (-3\mu - 6 - 3\lambda)\hat{i} + (2\mu + \lambda - 16)\hat{j} + (4\mu - \lambda - 2)\hat{k}$

If $\overrightarrow{PQ}$ is perpendicular to the first line, then

$3(-3\mu - 6 - 3\lambda) - (2\mu + \lambda - 16) + (4\mu - \lambda - 2) = 0$

$\Rightarrow -9\mu - 18 - 9\lambda - 2\mu - \lambda + 16 + 4\mu - \lambda - 2 = 0$

$\Rightarrow -7\mu - 11\lambda - 4 = 0$ ..... (iii)

If $\overrightarrow{PQ}$ is perpendicular to the second line, then

$-3(-3\mu - 6 - 3\lambda) + (2\mu + \lambda - 16) + (4\mu - \lambda + 2) = 0$

$\Rightarrow 9\mu + 18 + 9\lambda + 4\mu + 2\lambda - 32 + 16\mu - 4\lambda - 8 = 0$

$\Rightarrow 29\mu + 7\lambda - 22 = 0$ ..... (iv)

On solving Eqs. (iii) and (iv), we get

$-49\mu - 77\lambda - 28 = 0$

$\Rightarrow 319\mu + 77\lambda - 242 = 0$

$\Rightarrow 270\mu - 270 = 0$

$\Rightarrow \mu = 1$

Using μ in Eq. (iii), we get

$-7(1) = -11\lambda - 4 = 0$

$\Rightarrow -7 - 11\lambda - 4 = 0$

$\Rightarrow -11 - 11\lambda = 0$

$\Rightarrow \lambda = -1$

$\therefore \overrightarrow{PQ} = [-3(1) - 6 - 3(-1)]\,\hat{i} + [2(1) + (-1) - 16]\,\hat{j} + [4(1) - (-1) - 2]\,\hat{k}$

$= -6\hat{i} - 15\hat{j} + 3\hat{k}$

Section E

36. i. Required probability = P(one ticket with prime number and other ticket with a multiple of 4)

$= 2\left(\frac{15}{50} \times \frac{12}{49}\right) = \frac{36}{245}$

ii. P(First ticket shows an even number and second ticket shows an odd number)

= $\frac{25}{50} \times \frac{25}{49} = \frac{25}{98}$

iii. Required probability = P(one number is a multiple of 4 and other is a multiple of 5)

= P(multiple of 5 on first ticket and multiple of 4 on second ticket) + P(multiple of 4 on first ticket and multiple of 5 on second ticket)

= $\frac{10}{50} \times \frac{12}{49} + \frac{12}{50} \times \frac{10}{49}$

= $\frac{12}{245} + \frac{12}{245}$

= $\frac{25}{245}$

= $\frac{5}{49}$

OR

Probability that both tickets drawn by Girish shows odd number

= $\frac{25}{50} \times \frac{24}{49}$

= $\frac{12}{49}$

37. i. Here,

Position vector of A is $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$

Position vector of B is $\vec{b} = 3\hat{i} - 3\hat{j} - 2\hat{k}$

Position vector of C is $\vec{c} = -2\hat{i} + 2\hat{j} + 6\hat{k}$

$\therefore \vec{a} + \vec{b} + \vec{c}$ = (1 + 3 - 2)$\hat{i}$ + (4 - 3 + 2)$\hat{j}$ + (2 - 2 + 6)$\hat{k}$

= 2$\hat{i}$ + 3$\hat{j}$ + 6$\hat{k}$

Thus, $|\vec{a} + \vec{b} + \vec{c}|$ = $\left|\sqrt{(2)^2 + (3)^2 + (6)^2}\right|$

= $|\sqrt{4+9+16}|$
= $\sqrt{29}$

ii. Given, $\vec{a} = 4\hat{i} + 6\hat{j} + 12\hat{k}$,
$|\vec{a}| = \sqrt{4^2 + 6^2 + 12^2}$ = 14
Therefore, the unit vector in direction of $\vec{a}$ is given by
$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{4\hat{i}+6\hat{j}+12\hat{k}}{14}$
= $\frac{4}{14}\hat{i} + \frac{6}{14}\hat{j} + \frac{12}{14}\hat{k}$
= $\frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$

iii. We have, A(1, 4, 2), B(3, -3, -2) and C(-2, 2, 6)
Now, $\overrightarrow{AB} = \vec{b} - \vec{a} = 2\hat{i} - 7\hat{j} - 4\hat{k}$
and $\overrightarrow{AC} = \vec{c} - \vec{a} = -3\hat{i} - 2\hat{j} + 4\hat{k}$
$\therefore \overrightarrow{AB} \times \overrightarrow{AC}$ = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -7 & -4 \\ -3 & -2 & 4 \end{vmatrix}$
= $\hat{i}(-28-8) - \hat{j}(8-12) + \hat{k}(-4-21)$
= - $36\hat{i} + 4\hat{j} - 25\hat{k}$
Now, $|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{(-36)^2 + 4^2 + (-25)^2}$
= $|\sqrt{1296+16+625}| = \sqrt{1937}$
$\therefore$ Area of $\triangle$ABC = $\frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$
= $\frac{1}{2}\sqrt{1937}$ sq. units

OR

Triangle law of addition for $\triangle$ABC is given by
$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}$ = $\vec{0}$
If the given points lie on the straight line, then the points will be collinear and so area of $\triangle$ABC = 0
Then, $|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$ = 0.
Also, if a, b, c are the position vector of the three vertices A, B and C of $\triangle$ABC, then area of triangle is
$\frac{1}{2}|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$.

38. i. The rate of growth = $\frac{dy}{dx}$
= $\frac{d(4x - \frac{1}{2}x^2)}{dx}$
= 4 - x

ii. For the height to be maximum or minimum
$\frac{dy}{dx}$ = 0
$\Rightarrow \frac{dy}{dx} = \frac{d\left(4x - \frac{1}{2}x^2\right)}{dx} = 4 - \frac{1}{2} \cdot 2x$ = 0
$\frac{dy}{dx}$ = 4 - x = 0
$\Rightarrow$ x = 4
$\therefore$ Number of required days = 4

iii. $\frac{dy}{dx}$ = 4 - x
$\Rightarrow \frac{d^2y}{dx^2}$ = -1 < 0
$\Rightarrow$ Function attains maximum value at x = 4
We have
y = 4x - $\frac{1}{2}x^2$
$\therefore$ when x = 4 the height of the plant will be maximum which is y = $4 \times 4 - \frac{1}{2} \times (4)^2$ = 16 - 8 = 8 cm

OR

We have, y = 4x - $\frac{1}{2}x^2$
$\therefore$ When x = 4 the height of the plant will be maximum which is
y = $4 \times 4 - \frac{1}{2} \times (4)^2$
= 8 - 2 = 6 cm