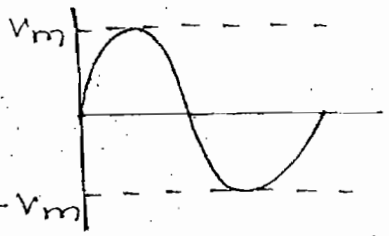
Lecture - 4

Ques:- Find RMS value of following waveforms

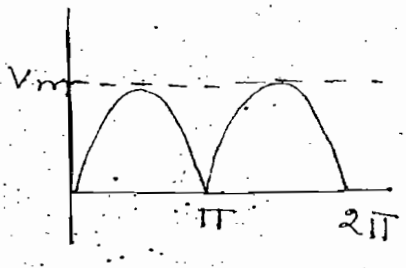
(I) $V_{RMS} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V^2 \, d\omega t}$



$$V_{RMS} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (V_m \sin\omega t)^2 \, dt}$$

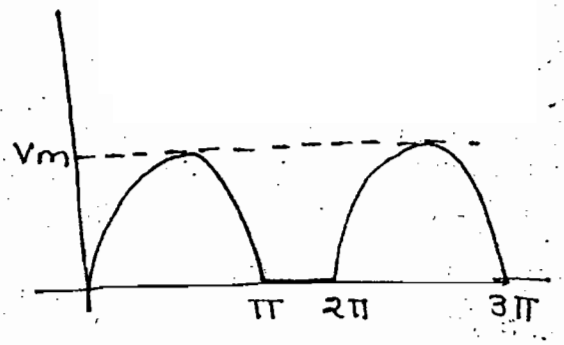
$$V_{RMS} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_m^2 \left(\frac{1-\cos 2\omega t}{2}\right) dt}$$

$$\Rightarrow \boxed{V_{RMS} = \frac{V_m}{\sqrt{2}}}$$



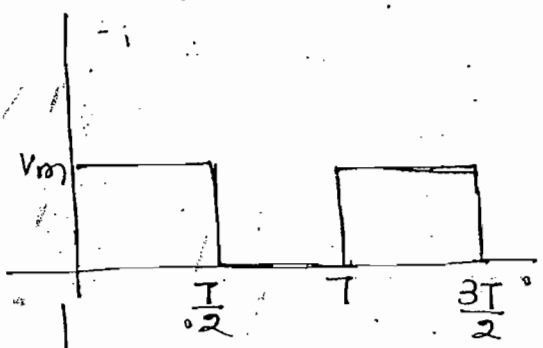
(II) $\boxed{V_{RMS} = \frac{V_m}{\sqrt{2}}}$

→ $V_{RMS} = \frac{V_m}{2}$



→ $V_{RMS} = \frac{V_m}{\sqrt{2}}$

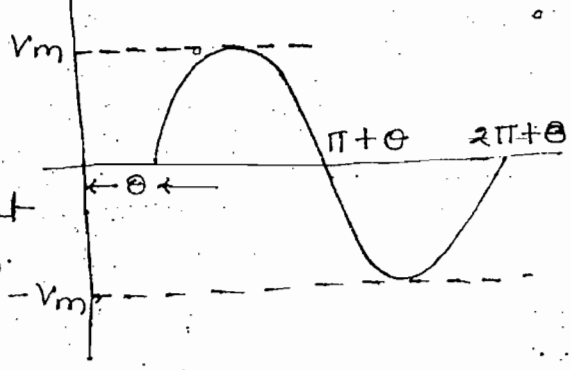
$$\left[V_{RMS} = \sqrt{\frac{1}{T}\left[\int_0^{T/2} V_m^2\, dt + \int_{T/2}^{T} 0\, dt\right]} \right]$$



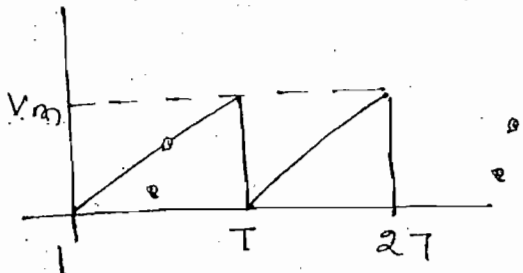
→ $V_{RMS} = \frac{V_m}{\sqrt{2}}$

Note:

RMS value is independent on the position of starting of waveform. But it depends on the shape of waveform



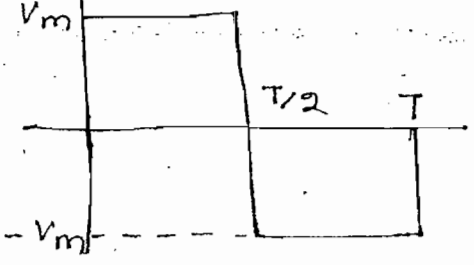
→ $V_{RMS} = \frac{V_m}{\sqrt{3}}$



$0 < t < T \quad y = mx$

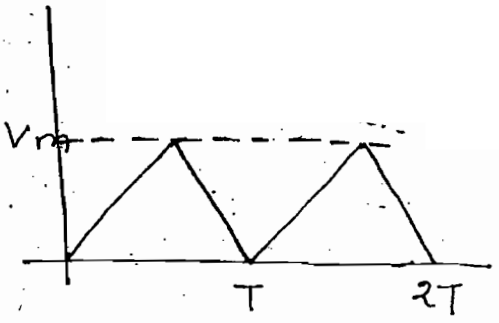
$\Rightarrow V = \frac{V_m}{T} t$

→ $V_{RMS} = V_m$



→ $V_{RMS} = \sqrt{\frac{1}{T}\int_0^T V^2 dt}$

$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T \left(\frac{V_m}{T}t\right)^2 dt}$



$\Rightarrow \boxed{V_{RMS} = \frac{V_m}{\sqrt{3}}}$

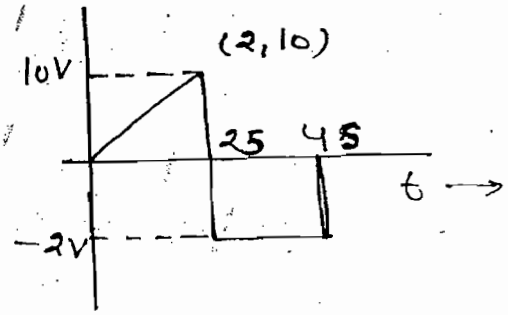
→ $0 < t < 2$

$y = mx$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10-0}{2-0}$

$= 5$

$V = 5t$

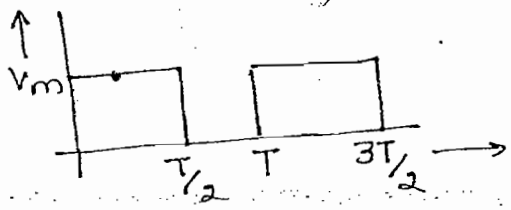


$V_{RMS} = \sqrt{\frac{1}{4}\left[\int_0^2 (5t)^2 dt + \int_2^4 (-2)^2 dt\right.}$

$\Rightarrow V_{RMS} = \sqrt{\frac{1}{4}\left[25\left(t^3/3\right)^2 + (4t)_2^4\right.}$

$\Rightarrow V_{RMS} = 4$ ------

Ques:- Find power dissipation in the resistor for the given voltage waveform



(a) $P_{av} = \frac{P_{peak}}{\sqrt{2}}$

(b) $P_{av} = \frac{P_{peak}}{2}$

(c) $P_{av} = P_{peak}$

(d) $P_{av} = \frac{P_{peak}}{\sqrt{3}}$

Soln:- B. $P_{peak} = \frac{V_m^2}{R}$

$P_{av} = \frac{V_{RMS}^2}{R} = \frac{\left(V_m/\sqrt{2}\right)^2}{R} = \frac{V_m^2}{2R} = \frac{P_{peak}}{2}$

Note:-

→ $P_{RMS} = \frac{V_{RMS}^2}{R}$

** → $\boxed{\frac{P_{DC}}{P_{AC}} = \frac{I_{av}^2 R}{I_{RMS}^2 R}}$

ques:- Find RMS value for the following function

$V(t) = 3 + \sin 3t + \cos t$

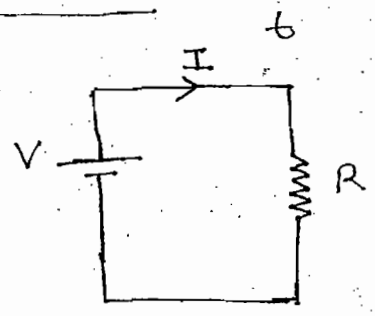
Soln:- $V_{RMS} = \sqrt{V_{RMS_1}^2 + V_{RMS_2}^2 + V_{RMS_3}^2 + \cdots + V_{RMS_n}^2}$

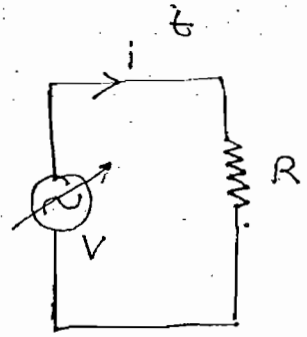
[Used when different ω are present]

$\Rightarrow V_{RMS} = \sqrt{(3)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}$

$\Rightarrow \boxed{V_{RMS} = \sqrt{10}}$

Average Value:-





$I = \frac{V}{R}$ | $i = \frac{V}{R}$

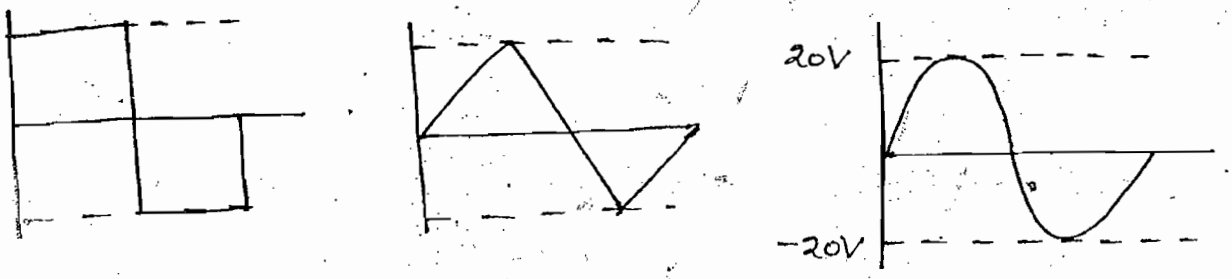
$Q = It \longrightarrow DC$ | $Q = it \longrightarrow AC$

$\boxed{Q_{A.C} = Q_{DC}}$

→ Average value is defined based on charge transfer in the circuit

→ The voltage at which charge transfer in AC circuit is equal to charge transfer in DC circuit is called as V_{avg} provided both AC and DC circuit having equal value of resistance and operated for same time.

Symmetrical Wave :-



$$\text{Form factor} = \frac{V_{RMS}}{V_{av}}$$

→ Avg Value for complete symmetrical wave = 0

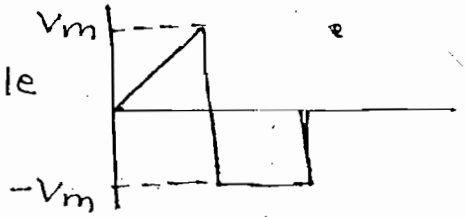
→ Hence we can find Average value only for half cycles for symmetrical waveform

$$V_{av} = \frac{1}{\pi}\int_0^{\pi} V\,d\omega t$$

[For current and voltage waveform

Unsymmetrical Wave :-

→ Finding average value of unsymmetrical wave angle of complete cycle is considered

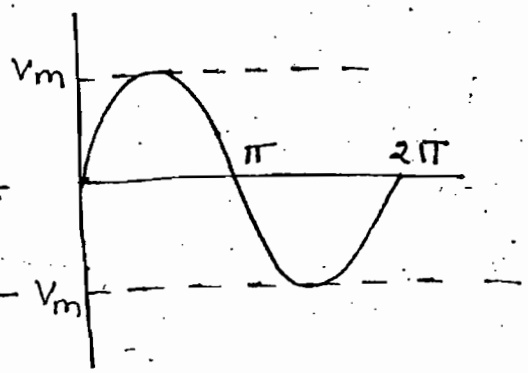


$$V_{av} = \frac{1}{2\pi}\left[\int_0^{\pi} V\,d\omega t + \int_{\pi}^{2\pi} V\,d\omega t\right]$$

Ques:- Find Avg. value of following waveforms

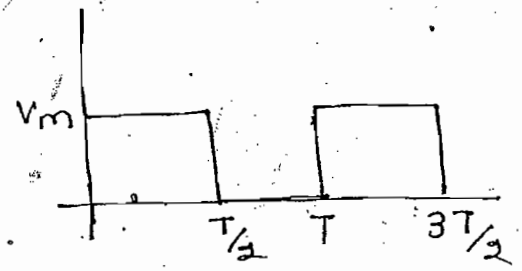
→ $V_{av} = \frac{1}{\pi}\int_0^{\pi} V\,d\omega t$

$V_{av} = \frac{1}{\pi}\int_0^{\pi} V_m \sin\omega t\,d\omega t$

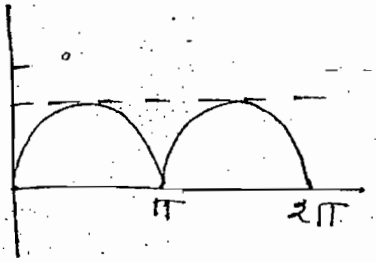


$\Rightarrow \boxed{V_{av} = \frac{2V_m}{\pi}}$

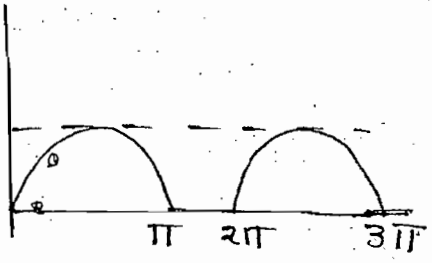
→ $V_{av} = \frac{V_m}{2}$



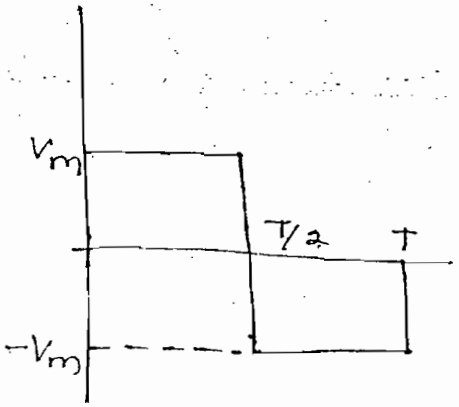
→ $V_{av} = \frac{2V_m}{\pi}$



→ $V_{av} = \frac{V_m}{\pi}$

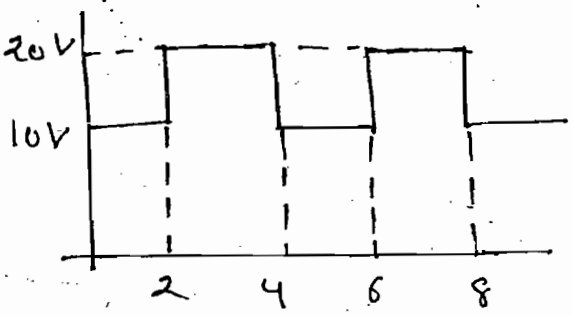


→ $V_{RMS} = V_{av} = V_m$

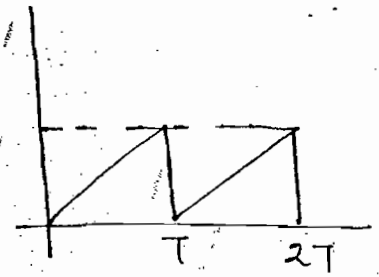


$$\rightarrow V_{av} = \frac{1}{4}\left[\int_0^2 10\,dt + \int_2^4 20\,dt\right]$$

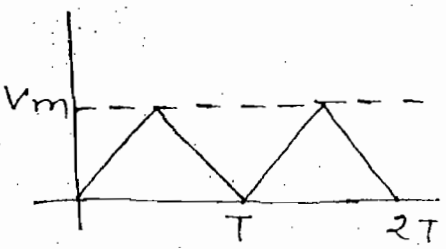
$$\boxed{V_{av} = 15}$$



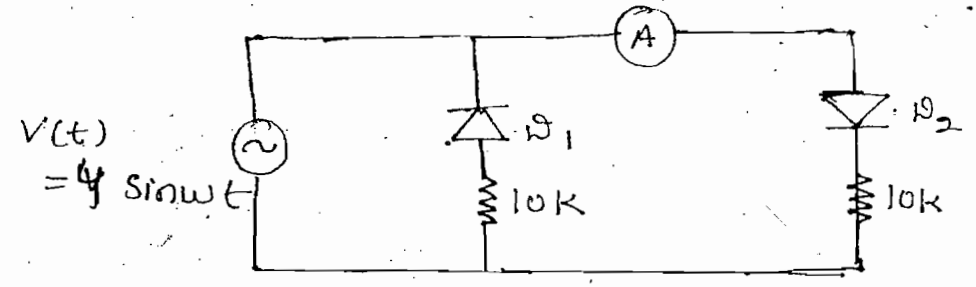
$$\rightarrow \boxed{V_{av} = \frac{V_m}{2}}$$



$$\rightarrow \boxed{V_{av} = \frac{V_m}{2}}$$



Ques! – When the circuit is having ideal diodes and avg. value of indicating ammeter. Find reading of ammeter



Sol^n:– $V_{av} = \frac{V_m}{\pi} = \frac{4}{\pi}$

$$\therefore I_{av} = \frac{V_{av}}{10 \times 10^3} = \frac{4/\pi}{10 \times 10^3}$$

$$\Rightarrow I_{av} = \frac{0.4}{\pi}\ mA$$

Note:-

$$\text{Form factor} = \frac{V_{RMS}}{V_{av}} = \frac{V_m/\sqrt{2}}{2V_m/\pi} = 1.11$$ → For sine wave

$$\text{Peak factor} = \frac{V_m}{V_{RMS}} = \frac{V_m}{V_m/\sqrt{2}} = \sqrt{2}$$

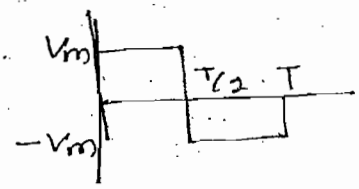
Power System:-

11 KV, 33KV, 66 KV, 132 KV, 220 KV] → basis of form factor

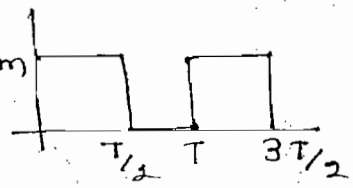
→ To justify above shape of waveform form factor and peak factor concepts are introduced

Query:- Which of the following waveforms have form factor = peak factor ?

(a)

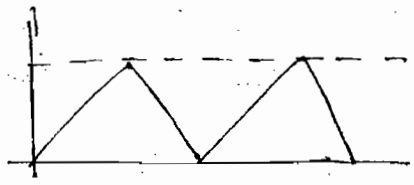


(b)



(c)

(d)



Soln:-

(a) $V_{RMS} = V_{av} = V_m$

Form factor = 1

Peak factor = 1

(b) $V_{RMS} = \frac{V_m}{\sqrt{2}}$, $V_{av} = \frac{V_m}{2}$

$$\text{Form factor} = \frac{V_{RMS}}{V_{av}} = \sqrt{2}$$

$$\text{Peak factor} = \frac{V_m}{V_{RMS}} = \sqrt{2}$$

(c) $V_{RMS} = V_m = V_{av} = V$

Form factor = 1

Peak factor = 1

(d) $V_{rms} = \frac{V_m}{\sqrt{3}}$

$V_{av} = V_m/2$

Form factor $= 2/\sqrt{3}$

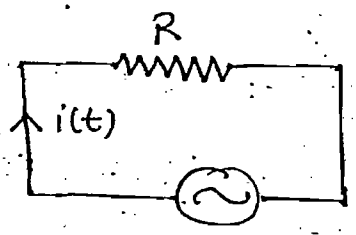
Peak factor $= \sqrt{3}$

AC Source Across Resistor :-

$$i(t) = \frac{V(t)}{R}$$

$$i(t) = \frac{V_m \sin\omega t}{R}$$

$$\Rightarrow \boxed{i(t) = I_m \sin\omega t}$$

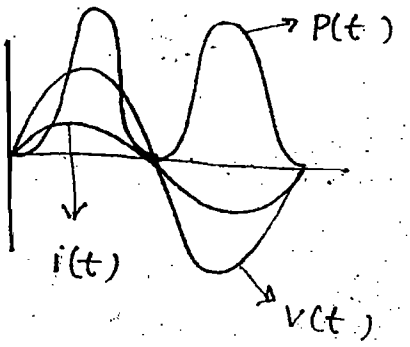


$V(t) = V_m \sin\omega t$

$$P(t) = V(t)\, i(t)$$

$$P(t) = V_m \sin\omega t \; I_m \sin\omega t$$

$$\boxed{P(t) = \frac{V_m I_m}{2}(1 - \cos 2\omega t)}$$



$$P_{av} = \frac{1}{2\pi}\int_0^{2\pi} P(t)\, d\omega t$$

$$\Rightarrow \boxed{P_{av} = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}} = V_{RMS} I_{RMS}}$$

Eg :- $f = 50$ Hz or C/sec

$\Rightarrow f_p = 100$ Hz (Power of frequency)

→ When voltage or current completes one cycle then power completes two cycle.

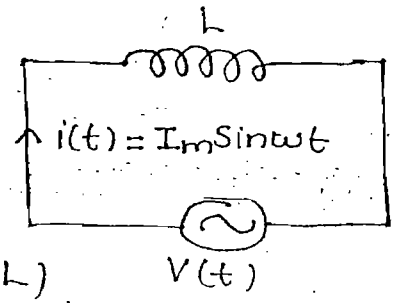
AC Source across Inductor :-

$$V = L\frac{di}{dt}$$

$$\therefore V(t) = L\frac{d}{dt}(I_m \sin\omega t)$$

$$V(t) = \omega L I_m \cos\omega t \quad (X_L = \omega L)$$

$$V(t) = V_m \sin(\omega t + 90)$$



$$P = I^2R = VI\cos\theta$$
$$Q_L = X^2X_L = VI\sin\theta$$
$$S = I^2Z = VI^* \rightarrow \text{conjugate}$$

eg:- $V = 10\angle 40^\circ$ $\quad i = 5\angle 15^\circ$

$S = Vi$

$\Rightarrow S = 10\angle 40 \; 5\angle 15$

$\Rightarrow S = 50\angle 55$

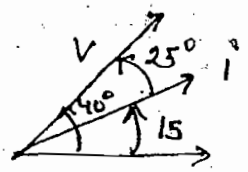
Wrong

$S = Vi^*$

$\Rightarrow S = 10\angle 40 \; 5\angle -15$

$\Rightarrow S = 50\angle 25$

Correct



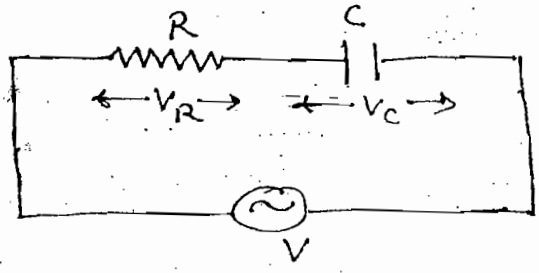
R-C Series Circuit :-

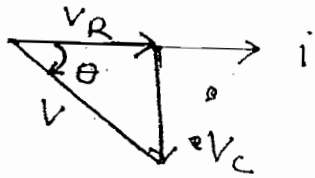
By KVL

$V = V_R\angle 0 + V_c\angle -90^\circ$

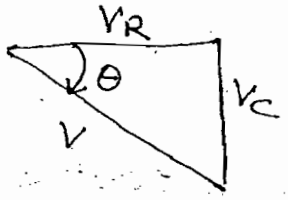
$\Rightarrow IZ = IR - jIX_C$

$\Rightarrow \boxed{Z = R - jX_c}$

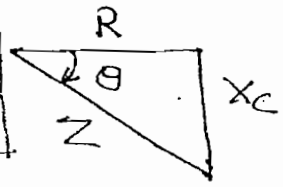




$$\boxed{V = \sqrt{V_R^2 + V_c^2}} \qquad \boxed{\theta = \tan^{-1}\left(\frac{-V_c}{R}\right)}$$

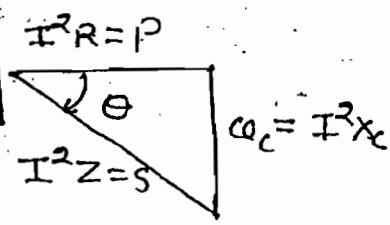


$$\boxed{Z = \sqrt{R^2 + X_c^2}} \qquad \boxed{\theta = \tan^{-1}\left(\frac{-X_c}{R}\right)}$$



$$S = \sqrt{P^2 + Q^2}$$

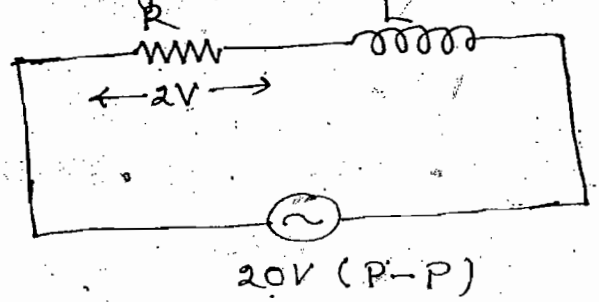
$$\theta = \tan^{-1}\left(\frac{-Q_c}{P}\right)$$



Power Factor :-

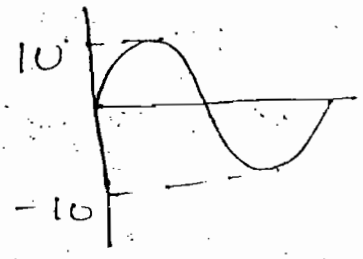
$$\text{Power Factor} = \cos\theta = \frac{V_R}{V} = \frac{R}{Z} = \frac{P}{S} \rightarrow \text{leading}$$

Ques:- Find voltage across inductor



Soln:- $V_m = 10$

$$V_{RMS} = \frac{10}{\sqrt{2}} = V$$



$$V = \sqrt{V_R^2 + V_L^2}$$

$$\Rightarrow \frac{10}{\sqrt{2}} = \sqrt{2^2 + V_L^2} \Rightarrow \boxed{V_L = \sqrt{46}\ V}$$

Ques:- Find circuit element for given voltage and current equations

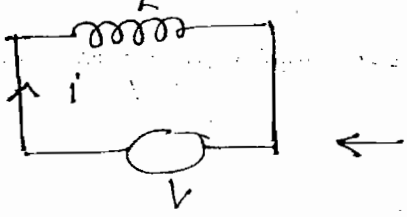
$$V(t) = 9\sin(t+45)$$

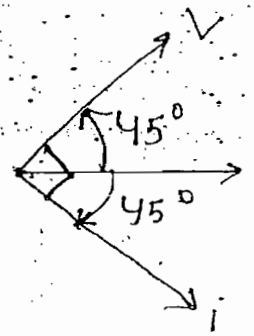
$$i(t) = 3\sin(t-45^\circ)$$

Soln:-

$$X_L = \frac{V}{I}$$

$$X_L = \frac{9/\sqrt{2}}{3/\sqrt{2}}$$





$$\Rightarrow X_L = 3 = \omega L \Rightarrow \boxed{L = 3} \quad (\because \omega = 1)$$

Ques:- Find circuit element for given voltage and current equations

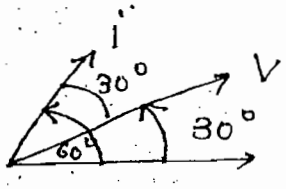
$$V(t) = 9\sin(t+30^\circ)$$
$$i(t) = 3\sin(2t+60^\circ)$$

Note:-

By using above equations it is not possible to design the network since frequency of voltage and current are unequal.

Ques:- Find active power, reactive power and apparent power by using following equations:-

$$V(t) = 9\sin(t+30^\circ)$$
$$i(t) = 3\sin(t+60)$$



Soln:- $P = VI\cos\theta$

$$\Rightarrow P = \frac{9}{\sqrt{2}} \cdot \frac{3}{\sqrt{2}} \cos 30$$

$$\Rightarrow P = 27 \times \frac{\sqrt{3}}{2}$$

→ R-C circuit (I leading)

$$Q_c = VI\sin\theta$$

$$\Rightarrow Q_c = \frac{9}{\sqrt{2}} \frac{3}{\sqrt{2}} \sin 30^\circ$$

$$\Rightarrow Q_c = \frac{27}{4}$$

$$S = \sqrt{P^2 + Q_c^2} =$$

Alternate Way:-

$$Z = V/I$$

$$Z = \frac{9/\sqrt{2}}{3/\sqrt{2}}$$

$$Z = 3$$

$$\cos\theta = \frac{R}{Z}$$

$$\Rightarrow \cos 30^\circ = R/3 \Rightarrow R = 3\cos 30^\circ$$

$$X_c = \sqrt{Z^2 - R^2}$$

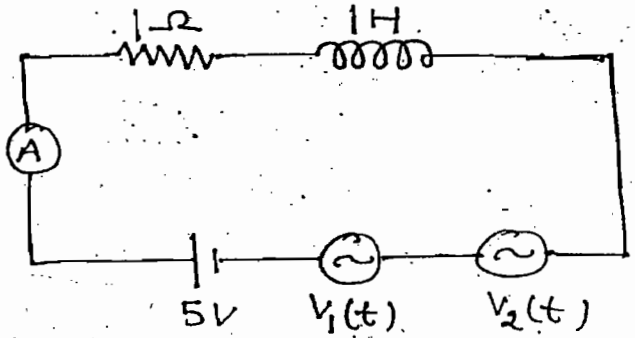
$$P = i^2R = (3/\sqrt{2})^2 R$$

$$Q_c = I^2X_c = (3/\sqrt{2})^2 X_c$$

Ques:- Find ammeter reading and power factor of the ckt shown

$V_1(t) = 10 \sin t$

$V_2(t) = 10\sqrt{5} \sin 2t$



Soln:- When multiple sources are present then at one time only one source is activated

For V_1 $\quad X_{L_1} = \omega_1 L = 1$

$$Z_1 = \sqrt{R^2 + X_{L_1}^2} = \sqrt{1^2+1^2} = \sqrt{2}$$

$$i_1 = \frac{V_1}{Z_1} = \frac{10/\sqrt{2}}{\sqrt{2}} = 5$$

For V_2 $\quad X_{L_2} = \omega_2 L = 2$

$$Z_2 = \sqrt{1^2+2^2} = \sqrt{5}$$

$$i_2 = \frac{V_2}{Z_2} = \frac{10\sqrt{5}/\sqrt{2}}{\sqrt{5}} = \frac{10}{\sqrt{2}}$$

For V_3. $\quad DC \rightarrow \omega = 0$

$$i_3 = \frac{V_3}{R} = \frac{5}{1} \Rightarrow i_3 = 5$$

When same frequency are present then directly add them But above question different frequency are present Hence use general formula. $(i = \sqrt{i_1^2 + i_2^2 + i_3^2} = 10A)$ Ans

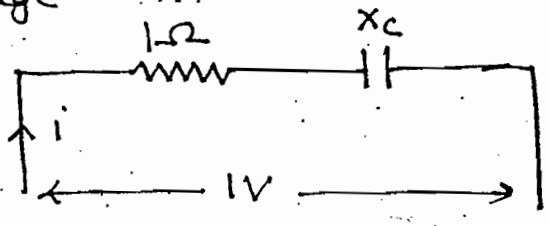
For power factor use power triangle not impedance triangle (same reason)

$$P.F = \cos\theta = P/S \Rightarrow \cos\theta = i^2R/V_i = \frac{iR}{V} = 0.55$$

$$V = \sqrt{V_1^2 + V_2^2 + V_3^2} = \sqrt{\left(\frac{10}{\sqrt{2}}\right)^2 + \left(\frac{10\sqrt{5}}{\sqrt{2}}\right)^2 + 5^2}$$

$=$

Ques:- In the circuit shown power dissipation in the resistor is 500 mW. Find angle of current w.r.t source voltage



Soln:- $P = i^2R \Rightarrow P = \left(\frac{V}{Z}\right)^2 R$

$$\Rightarrow P = \frac{V^2}{R^2 + X_C^2} R$$

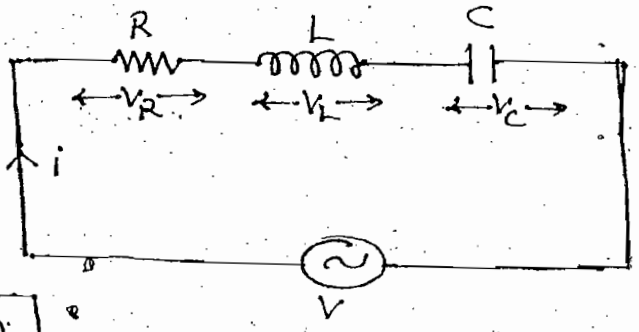
$$\Rightarrow \frac{500}{1000} = \frac{1^2}{1^2 + X_C^2} \quad (1) \Rightarrow X_C = 1$$

$$Z = R - jX_C$$

$$Z = 1 - j1 \Rightarrow \theta = \tan^{-1}\left(\frac{-1}{1}\right) = -45^\circ$$

$$i = \frac{V\angle 0}{Z\angle -45} = \frac{V\angle +45}{Z} \Rightarrow \theta = 45^\circ \text{ Ans}$$

R-L-C Series Circuit :-

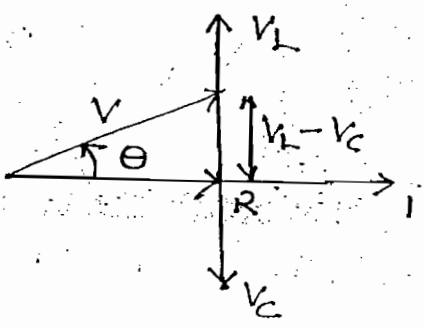


By KVL

$$V = V_R\angle 0 + V_L\angle 90 + V_C\angle -90$$

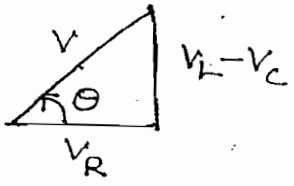
$$\Rightarrow IZ = IR + jIX_L - jX_C$$

$$\Rightarrow \boxed{Z = R + j(X_L - X_C)}$$



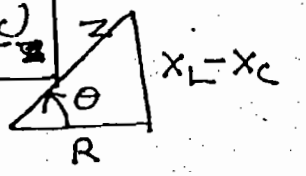
$$\boxed{V = \sqrt{V_R^2 + (V_L - V_C)^2}}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{V_L - V_C}{V_R}\right)}$$



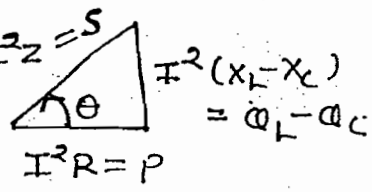
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\theta = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$



$$S = \sqrt{P^2 + (Q_L - Q_C)^2}$$

$$\theta = \tan^{-1}\left(\frac{Q_L - Q_C}{P}\right)$$



Power factor :–

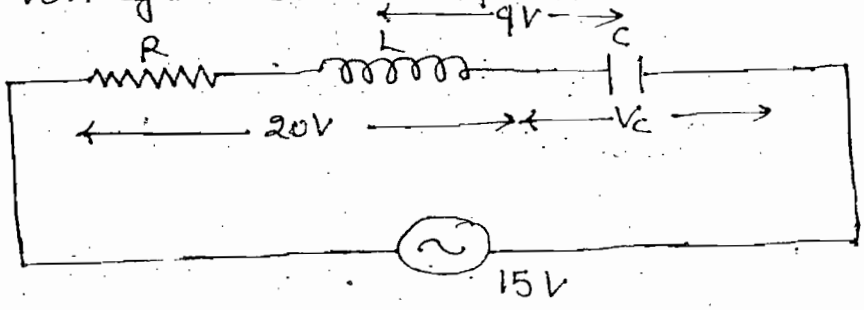
$$\cos\theta = \frac{V_R}{V} = \frac{R}{Z} = \frac{P}{S}$$

(i) If $V_L > V_C$ ⟶ lagging power factor

(ii) If $V_L < V_C$ ⟶ leading power factor

(iii) If $V_L = V_C$ ⟶ Unity power factor

Ques :– Find voltage across capacitor of circuit shown

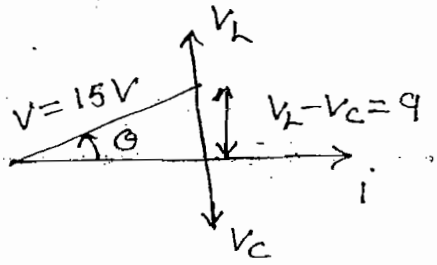


(a) 7 (b) 25 (c) 7 or 25 (d) 20

Soln :– $V_L - V_C = 9V$

$V = 15V$

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$



$$\Rightarrow 15^2 = V_R^2 + 9^2$$

$$V_R = \sqrt{225 - 81} = 12V$$

$$20 = \sqrt{V_R^2 + V_L^2} = \sqrt{12^2 + V_L^2} \Rightarrow V_L = 16$$

$$V_L - V_C = 9 \Rightarrow V_C = 7V, \text{ Ans}$$

dl