Areas Related to a Circle

Exercise-13.1

Question 1:

Find the circumference and the area of the circle whose radius is 8.4 cm.

Solution :

Radius of the circle r = 8.4 cm

Circumference of the circle = $2\pi r$

$$= 2 \times \frac{22}{7} \times 8.4$$

$$= \frac{2 \times 22 \times 84}{7 \times 10}$$

$$= 52.8 \text{ cm}$$

Area of the circle = πr^2

$$= \frac{22}{7} \times 8.4 \times 8.4$$

$$= \frac{22 \times 84 \times 84}{7 \times 10 \times 10}$$

$$= 221.76 \text{ cm}^2$$

Question 2:

Find the circumference of the circle whose area is 38.5 m^2.

Solution :

Here area of the circle = 38.5 m^2

Area of a circle = πr^2

$$\therefore 38.5 = \frac{22}{7} \times r^2$$

$$\therefore \frac{385}{10} \times \frac{7}{22} = r^2$$

$$\therefore \frac{49}{4} = r^2$$

$$\therefore r = \frac{7}{2} \text{ m}$$

Circumference of the circle = $2\pi r$

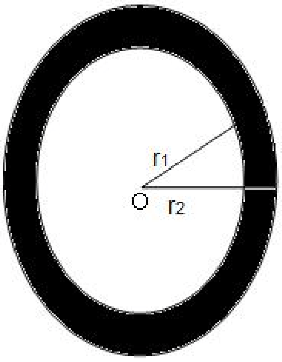
$$= 2 \times \frac{22}{7} \times \frac{7}{2}$$

$$= 22 \text{ m}$$

Question 3:

The inner circumference of a circular race track is 44 m less than the outer circumference. If the outer circumference is 396 m, then find the width of the track.

Solution :



Let the radius of the inner circle be r_1 m and the radius of the outer circle be r_2 m.

It is given that,

Circumference of inner circle = Circumference of outer circle − 44

$\therefore 2\pi r_1 = 2\pi r_2 - 44$

$\therefore 44 = 2\pi r_2 - 2\pi r_1$

$\therefore 44 = 2\pi(r_2 - r_1)$

$\therefore 44 = 2 \times \frac{22}{7} \times (r_2 - r_1)$

$\therefore (r_2 - r_1) = \frac{44 \times 7}{44}$

$\therefore r_2 - r_1 = 7$

Now,

Width of the track = Difference of two radii

$= r_2 - r_1$

$= 7$ m

Thus, the distance between two tracks is 7 metres.

Question 4:

The radius of the wheel of a truck is 7 cm. It takes 250 revolution per minute. Find the speed of the truck in km/hr.

Solution :

Radius of the wheel of the truck = 70 cm

∴ Distance covered in 1 revolution

= Circumference of the wheel

$= 2\pi r$

$= 2 \times \frac{22}{7} \times 70$

= 440 cm

∴ Distance covered in 250 revolutions

$= 440 \times 250$ cm

= 110000 cm

∴ Distance coverd in 1 hour = distance covered in 60 minutes

$= 60 \times$ distance covered in 1 minute

$= 60 \times$ distance covered in 250 revolutions

$= 60 \times 110000$

$= (60 \times 110000)$ cm

$= \left(60 \times \frac{110000}{100}\right)$ m

$= \left(60 \times \frac{110000}{100} \times \frac{1}{1000}\right)$ km

$= \frac{60 \times 11}{10}$ km

$= \frac{660}{10}$ km

= 66 km

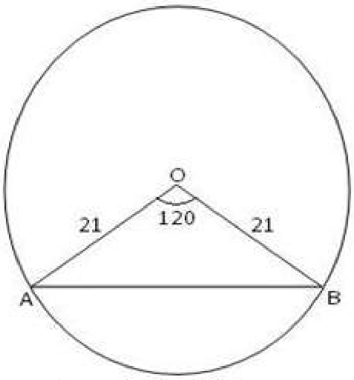
Hence, the speed of the truck is 66 km/h.

Exercise-13.2

Question 1:

An arc of a circle whose radius is 21 cm subtends an angle of measure 120 at the centre. Find the length of the arc and area of the sector.

Solution :



Here radius r = 21 cm and measure of the central angle $\theta = 120°$.

Length of minor arc $= \frac{\pi r \theta}{180}$

$= \frac{22}{7} \times \frac{21 \times 120}{180}$

= 44 cm

Next,

Area of minor sector $= \frac{\pi r^2 \theta}{360}$

$= \frac{22}{7} \times \frac{21 \times 21 \times 120}{360}$

$= 462 \text{ cm}^2$

Question 2:

The radius of a circular ground is 63 m. There is 7 m wide road inside the ground as shown in figure 13.10. The blue coloured portion of the road, shown in figure 13.10 is to be repaired. If the rate of repair work of the road costs ₹ 25 per m^2, find the total cost of repair.

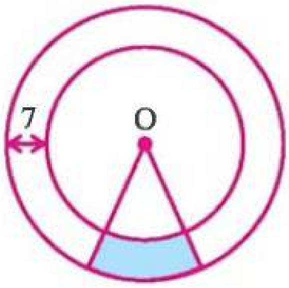
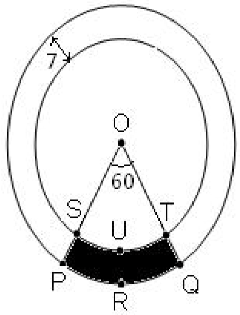


Figure 13.10

Solution :



Here radius of the outer circle r_1 = OP = OQ = 63 m

Distance between the inner and outer circle = 7 m

∴ Radius of the inner circle r_2 = OS = OT = 63 − 7 = 56 m

Now, Area of the shaded portion

= Area of minor sector OPRQ − Area of minor sector OSUT

$$= \frac{\pi r_1^2\theta}{360} - \frac{\pi r_2^2\theta}{360}$$

$$= \frac{\pi\theta}{360}\left(r_1^2 - r_2^2\right)$$

$$= \frac{\pi\theta}{360}\left(r_1 + r_2\right)\left(r_1 - r_2\right)$$

$$= \frac{22}{7} \times \frac{60}{360}(63 + 56)(63 - 56)$$

$$= \frac{22}{7 \times 6} \times 119 \times 7$$

$$= \frac{1309}{3}\ m^2$$

Cost of repairing 1 m^2 of road = Rs. 25

∴ Cost of repairing $\frac{1309}{3}$ m^2 of road

$$= \text{Rs.}\left(\frac{1309}{3} \times 25\right)$$

= Rs. 10,908.33

∴ The total cost of repair is Rs. 10,908.33

Question 3:

A regular hexagon of side 10 cm is cut from a plane circular sheet of radius 10 cm as shown in the figure 13.11. Find the area of the remaining part of the sheet. ($\sqrt{3}$ = 1.73) (π = 3.14).

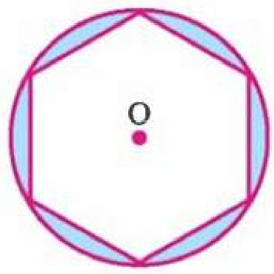
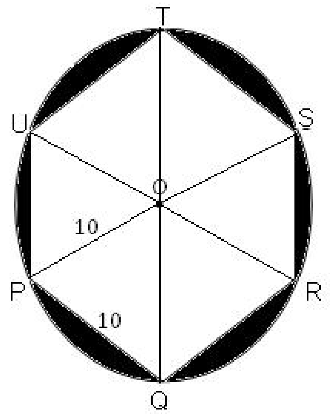


Figure 13.11

Solution :



From the figure above we can say that,
The hexagon comprises of 6 equilateral triangles.
∴ Area of a regular hexagon
= 6 x Area of the equilateral triangle.

$= 6 \times \frac{\sqrt{3}}{4} \times a^2$ (where a = side of equilateral triangle)

$= \frac{3}{2} \times \sqrt{3} \times a^2$

Now,
Area of the remaining part of the sheet
= Area of the cirle − Area of the regular hexagon

$= \pi r^2 - \frac{3}{2} \times \sqrt{3} \times a^2$

$= 3.14 \times 10 \times 10 - \frac{3}{2} \times 1.73 \times 10 \times 10$

$= 314 - \frac{3}{2} \times \frac{173}{10 \times 10} \times 10 \times 10$

$= 314 - 259.5$

$= 54.5 \text{ cm}^2$

*The answer is obtained using value of π = 3.14 and $\sqrt{3}$ = 1.73.
The answer given at the end of the textbook is obtained with different values.

Question 4:

The length of a minute hand of a circular dial is 10 cm. Find the area of the sector formed by the present position and position five minute of the minute hand. (π = 3.14)

Solution :

In 5 minutes, the minute hand revolves through an angle of measure

$\theta = \frac{360}{60} \times 5 = 30°$.

Now, the area of a minor sector $= \frac{\pi r^2\theta}{360}$

$\therefore$ The area covered by the minute hand in an interval of 5 minutes

$= \frac{3.14 \times 10 \times 10 \times 30}{360}$

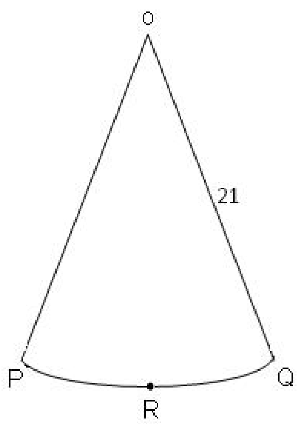
$= 26.17\ cm^2$

$\therefore$ The area swept by the minute hand from the present position and the position after five minutes is $26.17\ cm^2$.

Question 5:

The radius of a field in the form of a sector is 21 m. The cost of constructing a wall around the field is ₹ 1875 at the rate of ₹ 25 per meter. If it costs ₹ 10 per m^2 to the field. what will be the cost of tilling the whole field ?

Solution :



Length of the wall constructed at the cost of Rs. 25 = 1 m

$\therefore$ Length of the wall constructed at the cost of Rs. 1875 =

$\frac{1875}{25} = 75$ m

We have, $l + 2r = 75$, where l is the length of minor $\widehat{PRQ}$ and r = radius

$\therefore l + 2(21) = 75$

$\therefore l + 42 = 75$

$\therefore l = 33$ m

Area of the field in the form of a minor sector $= \frac{\pi r^2\theta}{360}$

$= \frac{r}{2} \times \frac{\pi r\theta}{180}$

$= \frac{r}{2} \times l \quad \left(\because \text{length of arc} = \frac{\pi r\theta}{180}\right)$

$= \frac{21}{2} \times 33$

$= 346.5\ m^2$

Cost of tilling $1\ m^2$ region = Rs. 10

$\therefore$ Cost of tilling $346.5\ m^2$ region

= Rs. (346.5×10)

= Rs. 3465

Question 6:

The length of a side of a square field is 20 m. A cow is tied at the corner by means of a 6 m long rope. Find the area of the field which the cow can graze. Also find the increase in grazing area, if the length of the rope is increased by 2 m.(π = 3.14)

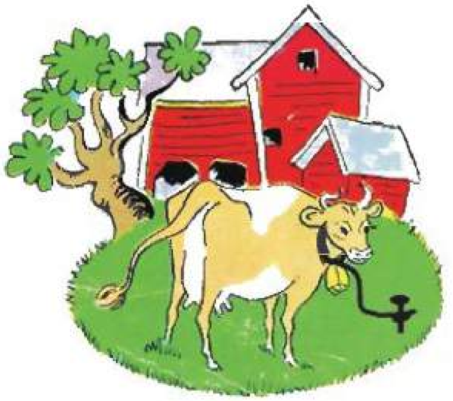
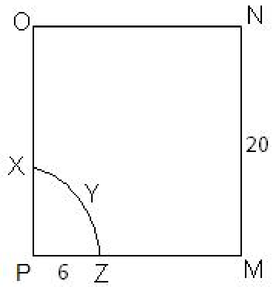


Figure 13.12

Solution :



We assume that the cow is tied by a 6 m long rope at the vertex P of the square field MNOP.

∴ The cow can graze in the region covered by the minor sector PXYZ.

Here, radius r = 6 m and $\theta = 90^\circ$ (angle of square).

Area of minor sector PXYZ $= \dfrac{\pi r^2\theta}{360}$

$= \dfrac{3.14 \times 6 \times 6 \times 90}{360}$

$= 28.26\ m^2$

If the length of the rope is increased by 2 m, then for the new minor sector radius r_1 = 8 m and $\theta = 90^\circ$.

Area of the new minor sector

$= \dfrac{\pi r_1^2\theta}{360}$

$= \dfrac{3.14 \times 8 \times 8 \times 90}{360}$

$= 50.24\ m^2$

∴ Increase in the grazing area

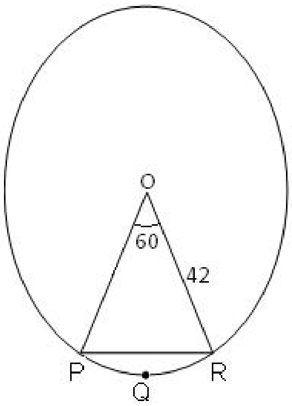
= 50.24 – 28.26

$= 21.98\ m^2$

Question 7:

A chord of a circle of radius 42 cm subtends an angle of measure 60 at the centre. Find the area of the minor segment of the circle. ($\sqrt{3}$ = 1.73)

Solution :



For radius r = 42 cm and $\theta = 60°$,

Area of minor sector OPQR

$= \frac{\pi r^2 \theta}{360}$

$= \frac{22}{7} \times \frac{42 \times 42 \times 60}{360}$

$= 924 \text{ cm}^2$

In ΔOPR, m$\angle$O = 60° and OP = OR = 42 cm.

$\therefore$ ΔOPR is an equilateral triangle.

Area of equilateral $\Delta OPR = \frac{\sqrt{3}}{4}(\text{side})^2$

$= \frac{1.73}{4} \times (42)^2$

$= 762.93 \text{ cm}^2$

Area of minor segment $\overline{PR} \cup \widehat{PQR}$

= Area of minor sector OPQR − Area of ΔOPR

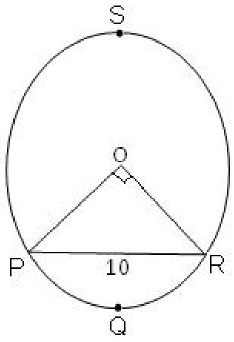
$= 924 - 762.93$

$= 161.07 \text{ cm}^2$

Question 8:

A chord of a circle, of length 10 cm, subtends a right angle at the centre. Find the areas of the minor segment and the major segment formed by the chord. (π = 3.14)

Solution :



$\overline{PR}$ is a chord of the circle with centre O,

PR = 10 cm, $\theta = 90°$, radius = OP = OR = r

In ΔOPR, $m\angle O = 90°$.

$\therefore OP^2 + OR^2 = PR^2$ (By Pythagoras Theorem)

$\therefore r^2 + r^2 = 10^2$

$\therefore 2r^2 = 100$

$\therefore r^2 = 50$

$\therefore r^2 = 5\sqrt{2}$ cm

Area of minor segment $\overline{PR} \cup \widehat{PQR}$

= Area of minor sector OPQR − Area of ΔPQR

$= \frac{\pi r^2 \theta}{360} - \frac{1}{2} \times OP \times OR$

$= \frac{3.14 \times 5\sqrt{2} \times 5\sqrt{2} \times 90}{360} - \frac{1}{2} \times 5\sqrt{2} \times 5\sqrt{2}$

= 39.25 − 25

= 14.25 cm^2

Area of major segment $\overline{PR} \cup \widehat{PSR}$

=Area of circle − Area of minor segment $\overline{PR} \cup \widehat{PQR}$

$= \pi r^2 - 14.25$

$= 3.14 \times 5\sqrt{2} \times 5\sqrt{2} - 14.25$

= 157 − 14.25

= 142.75 cm^2

Exercise-13.3

Question 1:

A rectangle whose length and breadth are 12 cm and 5 cm respectively is inscribed in a circle. Find the area of the blue coloured region, as shown in the figure 13.19.

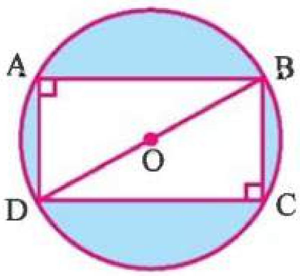
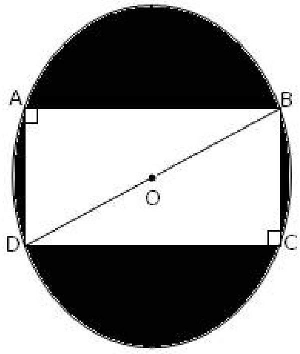


Figure 13.19

Solution :



In rectangle ABCD, length = 12 cm and breadth = 5 cm.

Area of rectangle ABCD = length x breadth

$= 12 \times 5$

$= 60\ cm^2$

In ΔBCD, $m\angle C = 90^\circ$

$\therefore BD^2 = BC^2 + CD^2$ (By Pythagoras Theorem)

$= 5^2 + 12^2 = 25 + 144 = 169 = 13^2$

$\therefore BD = 13$ cm

$\therefore$ Radius of the circle $r = \frac{\text{Diameter}}{2} = \frac{13}{2}$ cm

Now,

Area of a circle $= \pi r^2$

$= 3.14 \times \frac{13}{2} \times \frac{13}{2}$

$= 132.665$

$= 132.67\ cm^2$

Area of the coloured region

= Area of the circle – Area of rectangle ABCD

$= 132.67 - 60$

$= 72.67\ cm^2$

Question 2:

ABCD, square park, has each side of length 80 m. There is a flower bed at each corner in the form of a sector of radius 7 m, as shown in figure 13.20. Find the area of the remaining part of the park.

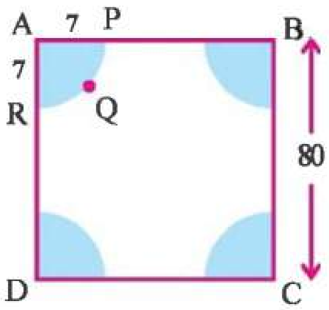
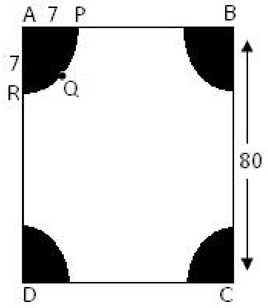


Figure 13.20

Solution :



Length of the square park = 80 cm

Area of the square park = 80 x 80 = 6400 cm^2

Also, radius of each flower bed r = 7 m and θ = 90°.

Total area of four flower beds = $4\times\frac{\pi r^2\theta}{360}$

$= 4\times\frac{22}{7}\times\frac{7\times7\times90}{360} = 154\ cm^2$

Then,

Area of the remaining part of the park

= Area of square park ABCD – Area of four flower beds

= 6400 - 154

= 6246 m^2

Question 3:

What will be the cost of covering the in figure 13.21 with a silver foil if the ₹ 100 per m^2?

(π = 3.14)

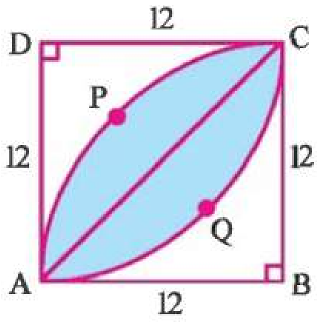
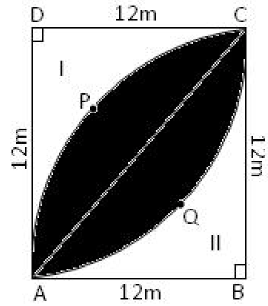


Figure 13.21

Solution :



Let the white portion be divided into two parts of equal area by I and II.

Length of the square = 12 m

$\therefore$ Radius of each sector r = 12m and $\theta = 90^\circ$

Area of part I

= Area of square ABCD – Area of sector BAPC

$= (\text{Length})^2 - \frac{\pi r^2\theta}{360}$

$= (12)^2 - \frac{3.14 \times 12 \times 12 \times 90}{360}$

$= 144 - 113.04$

$= 30.96\ m^2$

But, area of both the parts is equal,

$\therefore$ Area of part I = Area of part II = 30.96 m^2

$\therefore$ Area of the white protion = 30.96 + 30.96

= 61.92 m^2

Cost of covering 1 m^2 area by silver foil = Rs. 100

$\therefore$ Cost of covering 61.92 m^2 area by silver foil

= Rs. (61.92×100)

= Rs. 6192

Question 4:

ABCD is a square plate of 1 m length. As shown in figure circles are drawn with their center at A, B, C, D respectively, each with radius equal to 42 cm. The blue coloured part at each corner, as shown in the figure 13.22 is cut. What is the area of the remaining portion of the plate ?

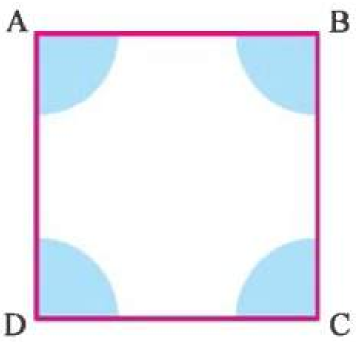
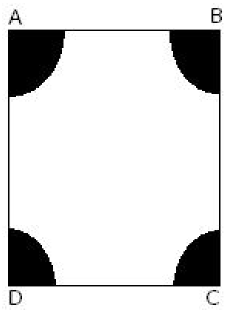


Figure 13.22

Solution :



Length of square plate ABCD = 1 m = 100 cm
The four sectors drawn with centres A, B, C and D are congruent and equal in area.
For each sector, radius r = 42 cm and the measure of angle $\theta = 90^\circ$.

Area of each sector = $\dfrac{\pi r^2 \theta}{360}$

$= \dfrac{42 \times 42 \times 90}{360}$

$= 1386 \text{ cm}^2$

Area of the remaining portion of the plate
= Area of square plate ABCD – Area of 4 sectors
$= (\text{Length})^2 - 4 \times \text{area of each sector}$
$= (100)^2 - 4 \times 1386$
$= 10000 - 5544$
$= 4456 \text{ cm}^2$

Question 5:

$\overline{OA}$ and $\overline{OB}$ are two mutually perpendicular radii of a circle of radius 10.5 cm. D $\in \overline{OB}$ and OD = 6 cm. Find the area of blue coloured region shown in figure 13.23.

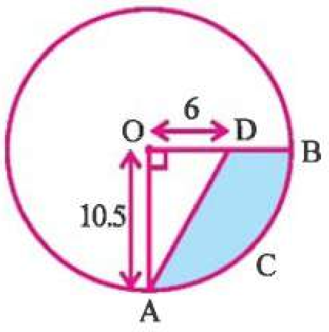
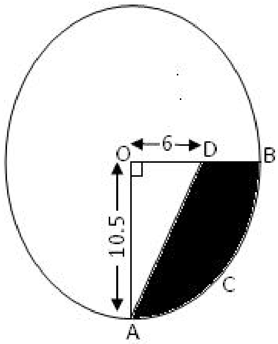


Figure 13.23

Solution :



For sector OACB, radius r = 10.5 cm = $\frac{21}{2}$ cm and the measure of angle $\theta = 90$.

In ΔAOD, OA = 10.5 cm = $\frac{21}{2}$ cm and OD = 6 cm.

Area of the coloured region

= Area of sector OACB − Area of ΔAOD

$= \frac{\pi r^2 \theta}{360} - \frac{1}{2} \times OA \times OD$

$= \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times \frac{90}{360} - \frac{1}{2} \times \frac{21}{2} \times 6$

$= 86.625 - 31.5$

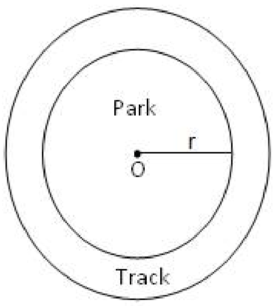
$= 55.125 \text{ cm}^2$

Exercise-13

Question 1:

The area of a circular park is 616 m^2. There is a 3.5 m wide track around the park running parallel to the boundary. Calculate the cost of fencing on the outer circle at the rate of ₹ 5 per meter.

Solution :



Area of the circular park = πr^2

$\therefore 616 = \frac{22}{7} \times r^2$

$\therefore 616 \times \frac{7}{22} = r^2$

$\therefore r^2 = 196$

$\therefore r = 14$

Radius of the park alongwith the track R = 14 + 3.5

= 17.5 m

Circumference of the park alongwith the track = $2\pi R$

$= 2 \times \frac{22}{7} \times 17.5$

= 110 m

Cost of fencing 1 m = Rs. 5

$\therefore$ Cost of fencing 110 m = Rs. (110×5)

= Rs. 550

Question 2:

A man is cycling in such a way that the wheels of the cycle are making 140 revolutions per minute. If the diameter of the wheel is 60 cm, then how much distance will he cover in 2 hours ?

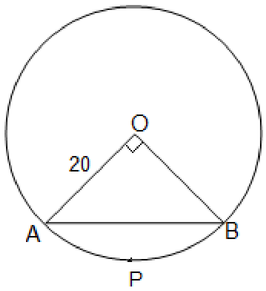
Solution :

Diameter of the wheel d = 60 cm
Number of revolutions made per minute = 140
$\therefore$ Number of revolutions made in 2 hours
(i.e. 120 minutes) = 120×140
Then,
Distance covered in 1 revolution
= Circumference of the wheel
= πd
$= \left(\frac{22}{7} \times 60\right)$ cm
$\therefore$ Total distance covered in (120×140) revolutions
$= \left(120 \times 140 \times \frac{22}{7} \times 60\right)$ cm
= 3168000 cm
$= \frac{3168000}{100}$ m
$= \frac{3168000}{100 \times 1000}$ km
= 31.68 km
$\therefore$ The distance covered in 2 hours is 31.68 km

Question 3:

If a chord $\overline{AB}$ of ⊙(O, 20) subtends right angle at O, find the area of the minor segrnent.

Solution :



Radius r = 20 cm and $\theta = 90°$.
Area of minor segment $\overline{AB} \cup \widehat{APB}$
= Area of minor sector OAPB − Area of ΔOAB
$= \frac{\pi r^2\theta}{360} - \frac{1}{2} \times OA \times OB$
$= \frac{3.14 \times 20 \times 20 \times 90}{360} - \frac{1}{2} \times 20 \times 20$
= 314 − 200
= 114 cm^2

Question 4:

There are two arcs $\widehat{APB}$ of ⊙(O, OA) and $\widehat{AQB}$ of (M, MA) as shown in figure 13.24. Find the area enclosed by two arcs.

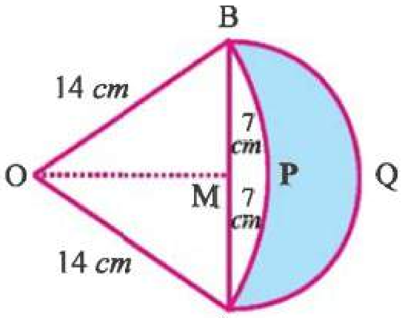
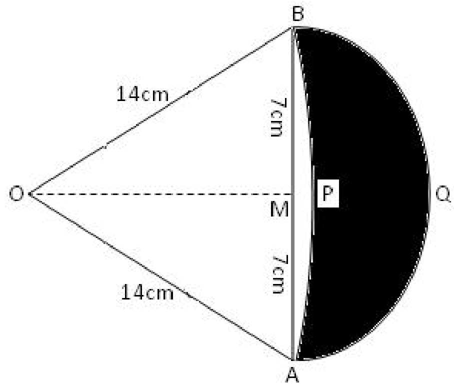


Figure 13.24

Solution :



For ΔOAB, OA = OB = 14 cm and

AB = AM + BM = 7 + 7 = 14 cm

$\therefore$ ΔOAB is an equilatrel triangle.

$\therefore$ $m\angle AOB = 60$

For minor sector OAPB, radius r = 14cm and $\theta = m\angle AOB = 60$.

Area of minor segment $\overline{AB} \cup \widehat{APB}$

= Area of minor sector OAPB − Area of equilateral ΔOAB

$= \frac{\pi r^2\theta}{360} - \frac{\sqrt{3}}{4} \times OA^2$

$= \frac{22}{7} \times \frac{14 \times 14 \times 60}{360} - \frac{1.73}{4} \times 14 \times 14$

$= 102.67 - 84.77$

$= 17.90\ cm^2$

Radius of the semicirde with centre

M = r = 7cm

Area of the semicircle with centre M

$= \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$

$= 77\ cm^2$

Area of the region enclosed by two arcs

= Area of the semicirde − Area of minor segment $\overline{AB} \cup \widehat{APB}$

$= 77 - 17.90$

$= 59.10\ cm^2$

Question 5:

The length of a side of a square garden ABCD is 70 m. A minor segment of ⊙(O, OA) is drawn on each of two opposite sides for developing lawn, as shown in figure 13.24. Find the area of the lawn.

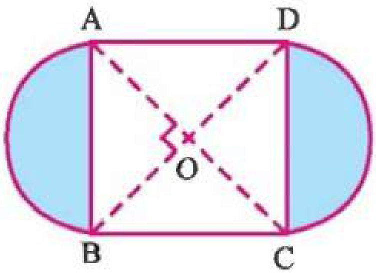
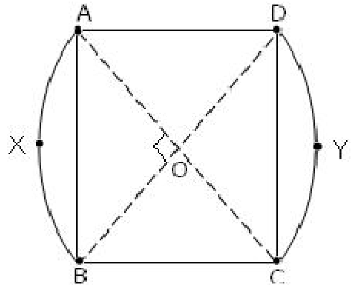


Figure 13.25

Solution :



In a square, diagonals are congruent and bisect each other at right angle.

$\therefore$ OA = OB = r and m$\angle$AOB = 90

In ΔAOB, m$\angle$O = 90

$\therefore OA^2 + OB^2 = AB^2$

$\therefore r^2 + r^2 = 70^2$

$\therefore 2r^2 = 4900$

$\therefore r^2 = 2450$

$\therefore r = 35\sqrt{2}$ m

In ΔAOB, m$\angle$O = 90

Area of minor segment $\overline{AB} \cup \widehat{AXB}$

= Area of minor sector OAXB – Area of ΔAOB

$= \dfrac{\pi r^2 \theta}{360} - \dfrac{1}{2} \times OA \times OB$

$= \dfrac{22}{7} \dfrac{\left(35\sqrt{2}\right)^2 \times 90}{360} - \dfrac{1}{2} \times r \times r$

$= \dfrac{22}{7} \times \dfrac{2450 \times 90}{360} - \dfrac{1}{2} \times 35\sqrt{2} \times 35\sqrt{2}$

= 1925 – 1225

= 700 m^2

Minor segments $\overline{AB} \cup \widehat{AXB}$ and $\overline{DC} \cup \widehat{DYC}$ being congruent have equal areas.

$\therefore$ Area of the lawn

= 2 x Area of minor segment $\overline{AB} \cup \widehat{AXB}$

= 2 x 700

= 1400 m^2

Question 6:

ABCD is a square of side 20 cm. Find the area of blue coloured region formed by the semi-circles drawn on each side as shown in figure 13.26. (π = 3.14)

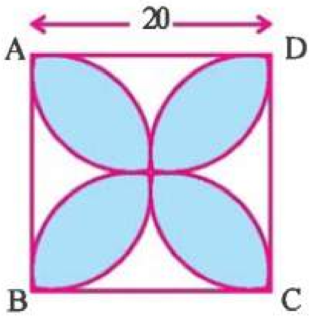


Figure 13.26

Solution :

Let l be the length of the square.

Length of square ABCD = l = 20 cm

∴ Radius of each semi-circle drawn on each side of the square $r = \frac{l}{2} = \frac{20}{2} = 10$ cm

Area of the coloured region

= Area of 4 semicircle – Area of the square

$= 4 \times \frac{1}{2}\pi r^2 - l^2$

$= 2 \times 3.14 \times 10 \times 10 - 20 \times 20$

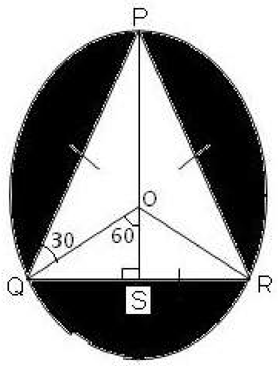
$= 628 - 400$

$= 228\ cm^2$

Question 7:

On a circular table top of radius 30 cm a design is formed leaving an equilateral triangle inscribed in a circle. Find the area of the design. (π = 3.14)

Solution :



Radius of $\odot(O, OQ) = r = 30cm$

Area of $\odot(O, OQ) = \pi r^2$

$= 3.14 \times 30 \times 30$

$= 2826\, cm^2$

ΔPQR is an equilateral triangle.

$\therefore m\angle QPR = 60^\circ$

$\therefore m\angle QOR = 2 \times m\angle QPR = 120^\circ$

In ΔOQR, let $\overline{OS} \perp \overline{QR}$, $S \in \overline{OR}$.

In ΔOQR, as $OQ = OR$,

$m\angle QOS = \frac{1}{2} m\angle QOR = 60^\circ$

In ΔOSQ, $m\angle S = 90^\circ$

$\therefore \sin 60^\circ = \frac{QS}{OQ}$

$\therefore \frac{\sqrt{3}}{2} = \frac{QS}{OQ}$

$\therefore QS = 15\sqrt{3}$ cm

Now, $QR = 2QS = 2 \times 15\sqrt{3} = 30\sqrt{3}$ cm

$\therefore$ For equilateral ΔPQR,

$PQ = QR = RS = 30\sqrt{3}$ cm

Now, area of equilateral ΔPQR

$= \frac{\sqrt{3}}{4}(\text{Length of side})^2$

$= \frac{\sqrt{3}}{4}\left(30\sqrt{3}\right)^2$

$= \frac{\sqrt{3} \times (30)^2 \times 3}{4}$

$= \frac{1.73 \times 900 \times 3}{4}$

$= 1167.75\ cm^2$

Area of the design

$=$ Area of $\odot(O, OR) -$ Area of ΔPQR

$= 2826 - 1167.75$

$= 1658.25\, cm^2$

Question 8:

Figure 13.27 shows a kite formed by a square PQRS and an iscosceles right triangle ARB whose congruent sides are 5 cm long. $\overset{\frown}{APB}$ is an arc of a $\odot$ (R, 42). Find the area of the blue coloured region.

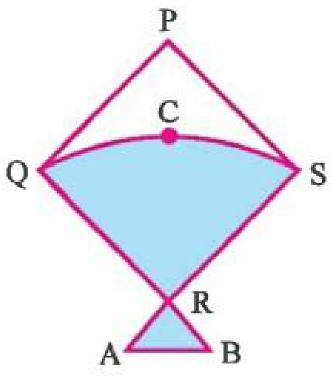
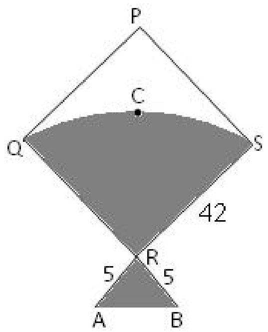


Figure 13.27

Solution :



For minor sector RQCS, radius r = 42 cm
and $\theta = 90°$ (For square PQRS, $m\angle R = 90°$)
In isosceles right angled ΔARB, $m\angle R = 90°$
and RA = RB = 5cm.
Area of the coloured region
= Area of minor sector RQCS + Area of ΔARB

$$= \frac{\pi r^2\theta}{360} + \frac{1}{2} \times RA \times RB$$

$$= \frac{22}{7} \times \frac{42 \times 42 \times 90}{360} + \frac{1}{2} \times 5 \times 5$$

$$= 1386 + 12.5$$

$$= 1398.5 \text{ cm}^2$$

Question 9:

In figure 13.28, ABCD is a square with sides having length 8 cm. Find the area of the blue coloured region. (π = 3.14)

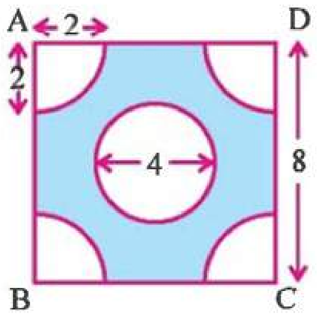
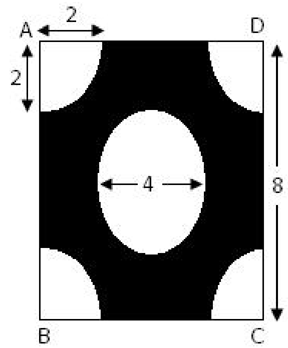


Figure 13.28

Solution :



Length of square ABCD = l = 8 cm

Area of square ABCD = l^2 = $(8)^2$ = 64 cm^2

Here, four congruent and equal minor sectors are formed at the vertices of the square with radius r = 2 cm and $\theta = 90°$ (angle of the square)

$\therefore$ Area of four minor sectors $= 4 \times \frac{\pi r^2 \theta}{360}$

$= 4 \times \frac{3.14 \times 2 \times 2 \times 90}{360}$

$= 12.56 cm^2$

For the circle in the middle of the square, diameter d = 4cm

$\therefore$ Radius of the circle $r = \frac{4}{2} = 2 cm$

Area of the circle in the middle

$= \pi r^2$

$= 3.14 \times 2 \times 2$

$= 12.56 cm^2$

Area of the coloured region

= Area of square ABCD – Area of four minor sectors – Area of the circle

= 64 – 12.56 – 12.56

= 64 – 25.12

= 38.88 cm^2

Question 10:

A circle is inscribed in ΔPQR where m∠Q = 90, PQ = 8 cm and QR = 6 cm. Find the area of the blue coloured region shown in figure 13.29. (π = 3.14)

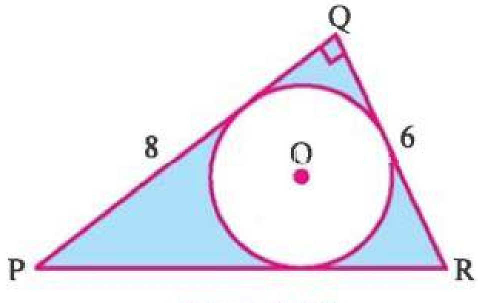
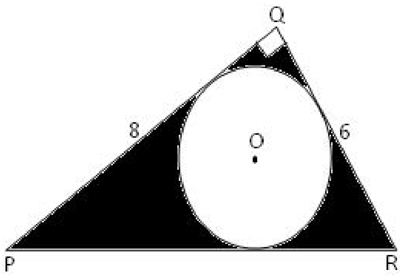


Figure 13.29

Solution :



In ΔPQR, $m\angle Q = 90°$, PQ = 8 cm and QR = 6 cm.

$\therefore PR^2 = PQ^2 + QR^2$ (By Pythagoras Theorem)

$= (8)^2 + (6)^2 = 64 + 36 = 100 = 10^2$

$\therefore PR = 10 cm$

$\odot(O, r)$ is the incircle of the right angled ΔPQR in which $\overline{PR}$ is the hypotenuse.

$\therefore$ Radius of incircle $= r = \dfrac{PQ + QR - PR}{2} = \dfrac{8 + 6 - 10}{2} = \dfrac{4}{2} = 2$ cm

Now,

Area of the coloured region

= Area of ΔPQR – Area of $\odot(O, r)$

$= \frac{1}{2} \times PQ \times QR - \pi r^2$

$= \frac{1}{2} \times 8 \times 6 - 3.14 \times 2 \times 2$

$= 24 - 12.56$

$= 11.44 \text{ cm}^2$

Question 11:

Select a proper option (a), (b), (c) or (d) from given options :

Question 11(1):

If an arc of a circle subtends an angle of measure θ at the centre, then the area of the minor sector is ……..

Solution :

d. $\dfrac{\pi r^2\theta}{360}$

According to the formula,

Area of a minor sector $= \dfrac{\pi r^2\theta}{360}$

Question 11(2):

The area of a sector is given by the formula ……

(r is the radius and l is the length of an arc.)

Solution :

a. $\frac{1}{2}rl$

Area of a minor sector $= \frac{\pi r^2\theta}{360}$

$= \frac{r}{2} \times \frac{\pi r\theta}{180}$

$= \frac{r}{2} \times l \left(\because \text{length of minor arc } l = \frac{\pi r\theta}{180}\right)$

$= \frac{1}{2}rl$

Question 11(3):

$\overline{OA}$ and $\overline{OB}$ are the two mutually perpendicular radii of a circle having radius 9 cm. The area of the minor sector corresponding to ∠AOB is ……… cm^2. (π = 3.14)

Solution :

b. 63.585

For the minor sector, radius r = 9 cm θ = 90°.

Area of a minor sector $= \frac{\pi r^2\theta}{360}$

$= \frac{3.14 \times 9 \times 9 \times 90}{360}$

$= 63.585\ cm^2$

Question 11(4):

A sector subtends an angle of measure 120 at the centre of a circle having radius of 21 cm. The area of the sector is ……. cm^2

Solution :

a. 462

For the given sector, radius r = 21cm and θ = 120.

Area of a minor sector $= \frac{\pi r^2\theta}{360}$

$= \frac{22}{7} \times \frac{21 \times 21 \times 120}{360}$

$= 462\ cm^2$

Question 11(5):

If the area and the circumference of a circle are numerically equal, then r ……….

Solution :

d. 2

Area of a circle = Circumference of a circle

$\therefore \pi r^2 = 2\pi r$

$\therefore \frac{\pi r^2}{\pi r} = 2$

$\therefore r = 2$

Question 11(6):

The length of an arc subtending an angle of measure 60 at the centre of a circle whose area is 616 is ………….

Solution :

c. $\frac{44}{3}$

Area of a circle $= \pi r^2$

$\therefore 616 = \frac{22}{7} \times r^2$

$\therefore r^2 = 616 \times \frac{7}{22}$

$\therefore r^2 = 196$

$\therefore r = 14$

For the the given arc, $r = 14$ and $\theta = 60$.

Length of a minor arc $= \frac{\pi r \theta}{180}$

$= \frac{22}{7} \times \frac{14 \times 60}{180}$

$= \frac{44}{3}$

Question 11(7):

The area of a minor sector of ⊙(O, 15) is 150. The length of the corresponding arc is …… . ($\pi = 3.14$)

Solution :

b. 60

Area of a minor sector $= \frac{1}{2} rl$

$\therefore 150 = \frac{1}{2} \times 5 \times l$

$\therefore l = \frac{150 \times 2}{5}$

$\therefore l = 60$

Question 11(8):

If the radius of a circle is increased by 10 %, then corresponding increase in the area of the circle is ………. ($\pi = 3.14$)

Solution :

c. 21%

Let the radius of original circle = r

∴ Area of original circle = πr^2

But, the radius of the circle is increased by 10%

∴ Radius of new circle R = $\frac{10r}{100} + r = 1.1r$

Area of new circle = πR^2

$= \pi (1.1r)^2$

$= 1.21\pi r^2$

Increase in area = $1.21\pi r^2 - \pi r^2$

$= 0.21\pi r^2$

Percentage increase in area = $\frac{0.21\pi r^2}{\pi r^2} \times 100 = 21\%$

Question 11(9):

If the ratio of the area of two circles is 1 : 4, then the ratio of their circumference ………..

Solution :

b. 1 : 2

For the first circle,

radius = r_1, circumference = C_1, Area = A_1.

For the second circle,

radius = r_2, circumference = C_2 and Area = A_2.

Given that, $A_1 : A_2 = 1 : 4$

$\therefore \frac{A_1}{A_2} = \frac{1}{4}$

$\therefore \frac{\pi r_1^2}{\pi r_2^2} = \frac{1}{4}$

$\therefore \frac{r_1^2}{r_2^2} = \frac{1}{4} \quad \therefore \frac{r_1}{r_2} = \frac{1}{2}$

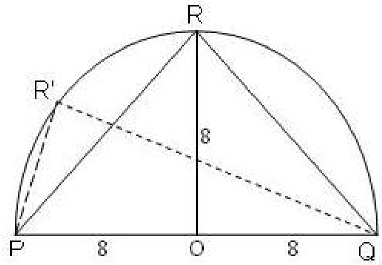
Now, $\frac{C_1}{C_2} = \frac{2\pi r_1}{2\pi r_2} = \frac{r_1}{r_2} = \frac{1}{2}$

∴ Ratio of the circumferences of the circles = 1 : 2

Question 11(10):

The area of the largest triangle inscribed in a semi-circle of radius 8 is ………..

Solution :



For ΔPQR, inscribed in a semicircle with radius 8,
base PQ = 8+8 = 16 and the maximum altitude OR = 8.
For any other $\Delta PQR'$, altitude is less than 8 always.
$\therefore$ The area of the largest triangle

$= \frac{1}{2} \times AB \times OC$

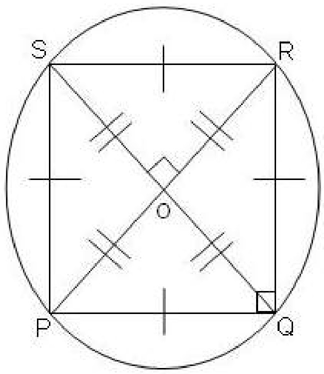
$= \frac{1}{2} \times 16 \times 8$

$= 64$

Question 11(11):

If the circumference of a circle is 44 then the length of a side of a square inscribed in the circle is ……..

Solution :



b. $7\sqrt{2}$

Circumference of a circle = $2\pi r$

$\therefore 44 = 2 \times \frac{22}{7} \times r$

$\therefore r = \frac{44 \times 7}{2 \times 22}$

$\therefore r = 7$

$\therefore OP = OQ = OR = OS = 7$

As diagonals of a square bisect each other at right angles,
$m\angle O = 90^\circ$ in ΔROS.

$SR^2 = OS^2 + OR^2$

$= 7^2 + 7^2 = 2(49)$

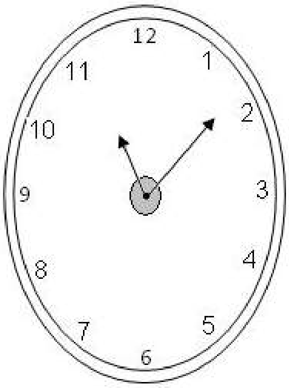
$\therefore SR = 7\sqrt{2}$

$\therefore$ The length of a side of the square is $7\sqrt{2}$.

Question 11(12):

The length of minute hand of a clock is 14 cm. If the minute hand moves from 2 to 11 on the circular dial, then area covered by it is ……… cm^2.

Solution :



c. 462

The measure of the angle of sector formed when the minute hand moves from 2 to 11

$$\theta = \frac{45}{60} \times 360 = 270^\circ$$

Radius of the circle r = Length of minute hand = 14 cm

Area of the sector formed when the minute hand moves from 2 to 11

$$= \frac{\pi r^2 \theta}{360}$$

$$= \frac{22}{7} \times 14 \times 14 \times \frac{270}{360}$$

$$= 616 \times \frac{3}{4}$$

$$= 462 \text{ cm}^2$$

Question 11(13):

The length of minute hand of a clock is 15 cm. If the minute hand moves for 20 minutes on a circular dial of a clock, the area covered by it is …….. cm^2. (π = 3.14)

Solution :

a. 235.5

For the minor sector covered by the minute hand in 20 minutes, radius r = 15 cm and

$$\theta = \frac{20}{60} \times 360 = 120^\circ$$

Area of a minor sector

$$= \frac{\pi r^2 \theta}{360}$$

$$= \frac{3.14 \times 15 \times 15 \times 120}{360}$$

$$= 235.5 \text{ cm}^2$$