Chapter 13. Statistics

Ex. 13.1

Answer 1CU.

The three types of sampling techniques are:

- Simple random sample
- Stratified random sample
- Systematic random sample

We need to find how these techniques are similar and different.

These three are similar in the respect that they all are probability or random sampling techniques. Each element in the population has a known, non-zero chance of being selected.

They are different in the respect that:

In Simple random sample, each element has the same probability of selection.

In Stratified random sample, the population is divided into similar non-overlapping subpopulations.

In Systematic random sample, the selection is done according to a specified time or item interval.

Answer 2CU.

We need to find the difference between a convenience sample and a voluntary response sample.

Convenience sample involves choosing individuals who are easiest to reach.

For example,

Suppose we are interested in learning how much people in our town enjoy reading books. Going to a local library will certainly enable us to find a lot of citizens whom we might ask this question. However, on average these individuals probably enjoy reading more than somebody met on a street. So, this is not reliable method.

Voluntary response sample consists of people who choose themselves by responding to a general appeal and they are biased.

For example,

A radio call in show records that 80% of its 50 callers voiced negative opinions about a local game. These 50 callers are unlikely to represent the entire local population. Volunteer callers are more likely to have strong opinions and are typically more negative than the entire population.

Answer 3CU.

We need to give an example of biased sample.

The example is as follows:

Telephone sampling is common in marketing surveys. A simple random sample may be chosen from the sampling frame consisting of a list of telephone numbers of people in the area being surveyed. This method involves taking a simple random sample, but it is not a simple random sample of the target population. It will miss people who don't have a phone. It may also miss people who only have a cell phone that has an area code not in the region being surveyed. It will also miss people who don't wish to be surveyed, including those who monitor calls on an answering machine and don't answer those from telephone surveyors. Thus the method systematically excludes certain types of consumers in the area.

Answer 4CU.

Sample: Local newspaper reader

Population: All the newspaper reader

This is a simple random sample. The mayor is equally likely to have been chosen. Hence this is an unbiased sample.

Answer 5CU.

Sample: Work from 4 students in the first period math class

Population: All the students in the first period math class

The first 4 students voluntarily raise their hands. Hence this is a voluntary response sample of biased sample.

Answer 6CU.

Sample: 25 boxes containing nails

Population: All the boxes containing nails

One nail is selected from each of the 25 boxes. Hence this is stratified sampling of unbiased techniques.

Answer 7CU.

Sample: 12 pencils of the closest box

Population: All the boxes containing pencils

The report of 12 pencils is not likely to be representative of all the pencils in the boxes. S this is a convenience sampling of biased techniques.

Answer 8PA.

Sample: Piece of paper with the names of 3 sophomores

Population: Piece of paper with all the sophomores' name

The pieces of papers are equally likely to get the sophomores name. So this is a simple sampling of unbiased techniques.

Answer 9PA.

Sample: 20 shoppers outside a fast-food restaurant

Population: All the shoppers outside the fast-food restaurant

The preferred cola is not equally likely to be chosen. So this is a convenience sampling of biased techniques.

Answer 10PA.

Sample: No of people recycled

Population: All the people recycled

People recycle voluntarily. So this is a voluntary response sampling of biased techniques.

Answer 11PA.

Sample: 10 people from each country

Population: All the people from 86 countries

People are chosen randomly hence it is unbiased. The population is divided into some groups hence it is stratified sampling.

Answer 12PA.

Sample: First 5 and last 5 scooters off the line

Population: All the scooters off the line

The manufacturer checks those scooters which can be easily accessed. Hence this is a convenience sampling of biased sample.

Answer 13PA.

Sample: 3 students of the class

Population: All the students of the class

The pieces of papers are randomly chosen and all the pieces are divided into 3 pieces parts. Hence this is a stratified sampling of unbiased sample.

Answer 14PA.

Sample: The corn in the 8-ounce jar

Population: The pile of corn in the storage silo

The corn in the jar is unlikely to represent the corn in the storage silo for moisture content. Hence this is convenience sampling of type biased.

Answer 15PA.

Sample: The gender makeup of district court judges.

Population: All the judges of Federal Reserve districts.

The number is selected randomly and the number is selected after 20th item interval. Hence this is systematic sampling of type unbiased.

Answer 16PA.

Sample: Some of the viewers of the television station

Population: All the viewers of the television station..

Viewers can call voluntarily to give the vote. Hence this is voluntary response sampling of type biased.

Answer 17PA.

Sample: 4 closest colleagues in the Senate

Population: All the closest colleagues in the Senate

The 4 colleagues discussion is unlikely to represent the discussion of all the colleagues. Hence this is convenience sampling of type biased.

Answer 18PA.

Sample: A handful of cherries

Population: All the cherries in the case

The handful of cherries is unlikely to represent all the cherries in the case. Hence this is convenience sampling of type biased.

Answer 19PA.

Sample: One high definition television

Population: All the high definition television

The units are checked after a time interval of 10 minutes of a work shift. For the rest of the shift every 15th item is checked. Hence this is systematic sampling of type unbiased.

Answer 20PA.

Sample: One person from each department

Population: All the person from all the departments

One person is randomly picked to get reactions about the benefit package. Hence this is simple random sampling of type unbiased.

Answer 21PA.

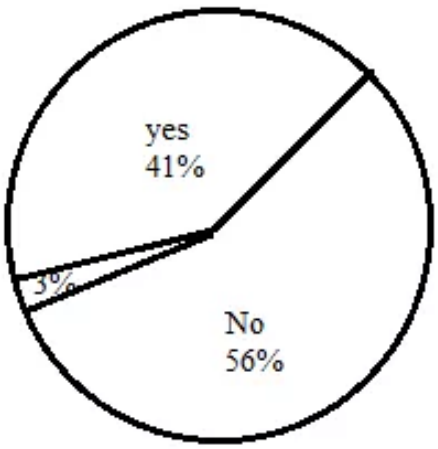
Sample: Some readers of a magazine

Population: All the readers of the magazine

The readers voluntarily mail the name of their favourite actor. Hence this is voluntary response sampling of type biased.

Answer 23PA.

Consider the graph:



The response of the parents is divided into three non-overlapping groups as yes, no and not sure. So the sample is unbiased and it is a stratified sampling.

Answer 34PA.

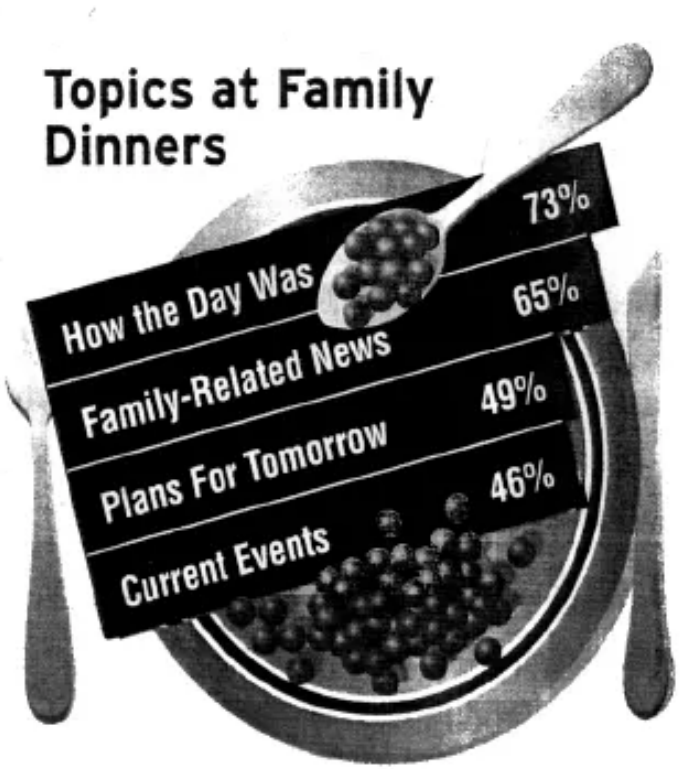
Take the opinion of every 10th student in your school to conduct the survey about the new dress code. This survey is done in an item interval. So this is an unbiased way by systematic sampling.

Answer 25PA.

The voters are selected randomly to poll to know whether he/she is likely to be elected. This is an unbiased way by simple sampling.

Answer 26PA.

Consider the figure:



In the family dinners, 73% people discuss about how that day was passed. 65% people discuss about family related news. 49% people discuss about the plans for next day. 46% people discuss about current events. This is a biased technique of voluntary response sample because all the people are voluntarily keeping their opinion.

Answer 27PA.

Some of the tomato crops are selected randomly to likely represent all the tomato crops whether these crops are ready to harvest. This is simple random sample of unbiased techniques.

Answer 28PA.

We check every 10th infant car seat to determine whether the seats meet the government standards. This is systematic sampling of unbiased techniques.

Answer 29PA.

The proposal should be:

Divide the student body as basketball team, student in the band and drama club team. Then we take a simple random sample from each of the three groups. This is stratified random sample.

Answer 30PA.

• An unbiased way to check the CDs:

Some CDs are picked randomly to check the defect by simple random sample.

• A biased way to check the CDs:

Take 10 CDs from the top to check the defects. These 10 CDs are unlikely to represent all the CDs. This is a convenience sampling.

Answer 31PA.

To predict the candidate who will win the seat in city council, the method

$\boxed{(C)\text{Publish a survey and ask readers to reply}}$ would give the newspaper the most accurate result.

Answer 32PA.

The cookie manufacturer should use the method

$\boxed{(D)\text{Divide the United States into 6 regions. Then pick 3 cities in each region at random, and conduct a taste test in each of the 18 cities}}$

to get the most accurate result.

Answer 33MYS.

Consider the equation

$$\frac{10}{3y}-\frac{5}{2y}=\frac{1}{4}$$

Now

$$\frac{10}{3y}-\frac{5}{2y}=\frac{1}{4}$$

$$\frac{20-15}{6y}=\frac{1}{4} \qquad [\text{Taking L.C.M.}]$$

$$\frac{5}{6y}=\frac{1}{4}$$

$$6y=20 \qquad \left[\begin{array}{l}\text{Since}\ \frac{a}{b}=\frac{c}{d}\\ \Rightarrow\ ad=bc\end{array}\right]$$

$$y=\frac{20}{6}$$

$$y=\frac{10}{3}$$

Thus the value of y is $\boxed{\frac{10}{3}}$.

Answer 34MYS.

Consider the equation

$$\frac{3}{r+4}-\frac{1}{r}=\frac{1}{r}$$

Now,

$$\frac{3}{r+4}-\frac{1}{r}=\frac{1}{r}$$

$$\frac{3}{r+4}=\frac{2}{r} \qquad \left[\begin{array}{l}\text{Since}\,\frac{a}{b}=\frac{c}{d}\\ \Rightarrow\quad ad=bc\end{array}\right]$$

$$3r=2(r+4)$$

$$3r=2r+8$$

$$3r-2r=8$$

$$r=8$$

Thus the value of r is $\boxed{8}$.

Answer 35MYS.

Consider the equation

$$\frac{1}{4m}-\frac{2m}{m-3}=2$$

Now,

$$\frac{1}{4m}-\frac{2m}{m-3}=2$$

$$\frac{m-3-8m^2}{4m(m-3)}=2 \qquad [\text{Taking L.C.M.}]$$

$$2\{4m(m-3)\}=m-3-8m^2 \qquad \left[\begin{array}{l}\text{Since}\,\frac{a}{b}=\frac{c}{d}\\ \Rightarrow\quad ad=bc\end{array}\right]$$

$$8m^2-24m=m-3-8m^2$$

$$16m^2-25m+3=0$$

Therefore,

$$m=\frac{25\pm\sqrt{(-25)^2-4\times16\times3}}{32}$$

$$=\frac{25\pm\sqrt{625-192}}{32}$$

$$=\frac{25\pm\sqrt{433}}{32}$$

Answer 36MYS.

Consider the term

$$\frac{2+\frac{5}{x}}{\frac{x}{3}+\frac{5}{6}}$$

Now,

$$\frac{2+\frac{5}{x}}{\frac{x}{3}+\frac{5}{6}}=\frac{\frac{2x+5}{x}}{\frac{2x+5}{6}} \qquad \left[\begin{array}{l}\text{Taking LCM in numerator}\\ \text{and denominator}\end{array}\right]$$

$$=\frac{\cancel{(2x+5)}}{x}\times\frac{6}{\cancel{(2x+5)}} \qquad \left[\text{Since } \frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}\right]$$

$$=\boxed{\frac{6}{x}}$$

Answer 37MYS.

Consider the term

$$\frac{a+\frac{35}{a+12}}{a+7}$$

Now,

$$\frac{a+\frac{35}{a+12}}{a+7}=\frac{\frac{a(a+12)+35}{a+12}}{a+7} \qquad [\text{Taking LCM in numerator}]$$

$$=\frac{\frac{a^2+12a+35}{a+12}}{a+7}$$

$$=\frac{\frac{a^2+7a+5a+35}{a+12}}{a+7} \qquad [\text{Since } 35=7\times5]$$

$$=\frac{\frac{a(a+7)+5(a+7)}{a+12}}{a+7} \qquad [\text{Taking common}]$$

$$=\frac{\cancel{(a+7)}(a+5)}{a+12}\times\frac{1}{\cancel{(a+7)}} \qquad \left[\text{Since } \frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}\right]$$

$$=\boxed{\frac{a+5}{a+12}} \qquad [\text{By simplifying}]$$

Answer 38MYS.

Consider the expression

$$\frac{\frac{t^2-4}{t^2+5t+6}}{t-2}$$

Now,

$$\frac{\frac{t^2-4}{t^2+5t+6}}{t-2} = \frac{\frac{t^2-2^2}{t^2+2t+3t+6}}{t-2} \qquad \left[\text{Since } 4=2^2 \text{ and } 6=2\times 3\right]$$

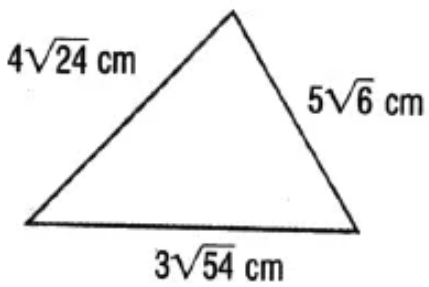
$$= \frac{\frac{(t+2)(t-2)}{t(t+2)+3(t+2)}}{t-2} \qquad \left[\text{Since } a^2-b^2=(a+b)(a-b)\right]$$

$$= \frac{\cancel{(t+2)}\,\cancel{(t-2)}}{\cancel{(t+2)}\,(t+3)} \times \frac{1}{\cancel{(t-2)}} \qquad \left[\text{Since } \frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}\right]$$

$$= \boxed{\frac{1}{t+3}}$$

Answer 39MYS.

Consider the figure



We know that, the perimeter of a triangle is the sum of the lengths of its sides.

Thus, the perimeter, P of the given triangle is

$$P = 4\sqrt{24}+5\sqrt{6}+3\sqrt{54} \ \ cm$$

$$= 4\sqrt{4}\sqrt{6}+5\sqrt{6}+3\sqrt{6}\sqrt{9} \ \ cm$$

$$= 4\times 2\sqrt{6}+5\sqrt{6}+3\times 3\sqrt{6} \ \ cm \qquad \left[\begin{array}{l}\text{Square root of 4 and 9 are 2}\\ \text{and 3 respectively}\end{array}\right]$$

$$= 8\sqrt{6}+5\sqrt{6}+9\sqrt{6} \ \ cm$$

$$= \boxed{22\sqrt{6} \ \ cm}$$

Answer 40MYS.

Consider the equation:

$$x^2-6x-40=0$$

To solve the equation, we have

$$\begin{aligned} x^2-6x-40&=0\\ x^2-10x+4x-40&=0 \quad [\text{Since } 40=4\times10]\\ x(x-10)+4(x-10)&=0\\ (x-10)(x+4)&=0 \end{aligned}$$

Either Or

$$\begin{aligned} x-10&=0 \quad x+4=0\\ x&=10 \quad x=-4 \end{aligned}$$

Thus, the solutions are $x=\boxed{10,-4}$

Answer 42MYS.

Consider the equation:

$$6b^2+15=-19b$$

To solve the equation, we have

$$\begin{aligned} 6b^2+15&=-19b\\ 6b^2+19b+15&=0 \quad [\text{Add } 19b]\\ 6b^2+10b+9b+15&=0 \quad [\text{Since } 6\times15=90=9\times10]\\ 2b(3b+5)+3(3b+5)&=0\\ (3b+5)(2b+3)&=0 \end{aligned}$$

Either Or

$$\begin{aligned} 3b+5&=0 \quad 2b+3=0\\ 3b&=-5 \quad 2b=-3\\ b&=-\frac{5}{3} \quad b=-\frac{3}{2} \end{aligned}$$

Thus, the solutions are $b=\boxed{-\frac{5}{3},-\frac{3}{2}}$

Consider the equation:

$$2d^2 = 9d + 3$$

To solve the equation, we have

$$\begin{aligned} 2d^2 &= 9d + 3 \\ 2d^2 - 9d - 3 &= 0 \qquad [\text{Subtract } 9d + 3] \end{aligned}$$

Here, $a = 2, b = -9, c = -3$

Therefore

$$\begin{aligned} d &= \frac{-(-9) \pm \sqrt{(-9)^2 - 4 \times 2(-3)}}{2 \times 2} \\ &= \frac{9 \pm \sqrt{81 + 24}}{4} \\ &= \frac{9 \pm \sqrt{105}}{4} \\ &= \frac{9 \pm 10.25}{4} \end{aligned}$$

Either Or

$$\begin{aligned} d &= \frac{9 + 10.25}{4} & d &= \frac{9 - 10.25}{4} \\ &= \frac{19.25}{4} & &= \frac{-1.25}{4} \\ &= 4.81 & &= -0.31 \end{aligned}$$

Thus, the solutions are $d = \boxed{4.81, -0.31}$

Answer 43MYS.

Consider the expression:

$$(y + 5)(y + 7)$$

Now

$$\begin{aligned} (y + 5)(y + 7) &= y(y + 7) + 5(y + 7) \\ &= y^2 + 7y + 5y + 35 \qquad [\text{Multiply}] \\ &= \boxed{y^2 + 12y + 35} \qquad [\text{Add like terms}] \end{aligned}$$

Answer 44MYS.

Consider the expression:

$$(c-3)(c-7)$$

Now

$$\begin{aligned}(c-3)(c-7) &= c(c-7)-3(c-7)\\ &= c^2-7c-3c+21 \quad [\text{Multiply}]\\ &= \boxed{c^2-10c+21} \quad [\text{Add like terms}]\end{aligned}$$

Answer 45MYS.

Consider the expression:

$$(x+4)(x-8)$$

Now

$$\begin{aligned}(x+4)(x-8) &= x(x-8)+4(x-8)\\ &= x^2-8x+4x-32 \quad [\text{Multiply}]\\ &= \boxed{x^2-4x-32} \quad [\text{Add like terms}]\end{aligned}$$

Answer 46MYS.

Consider the expression:

$$4.5+3.8$$

Now

$$0.5+0.8=1.3 \quad \left[\begin{array}{l}\text{First add the sum in}\\ \text{after decimal part}\end{array}\right]$$

$$4+3=7.0 \quad \left[\begin{array}{l}\text{Second add the sum in}\\ \text{before decimal part}\end{array}\right]$$

Combining both, we have

$$\begin{aligned}4.5+3.8 &= 7.0+1.3\\ &= \boxed{8.3} \quad [\text{Add like terms}]\end{aligned}$$

Answer 47MYS.

Consider the expression:

$16.9+7.21$

Now

$0.9+0.21=1.11$ [First add the sum in after decimal part]

$16+7=23$ [Second add the sum in before decimal part]

Combining both, we have

$16.9+7.21=1.11+23.00$

$=\boxed{24.11}$ [Add like terms]

Answer 48MYS.

Consider the expression:

$3.6+18.5$

Now

$0.6+0.5=1.1$ [First add the sum in after decimal part]

$3+18=21$ [Second add the sum in before decimal part]

Combining both, we have

$3.6+18.5=1.1+21.0$

$=\boxed{22.1}$ [Add like terms]

Answer 49MYS.

Consider the expression:

$7.6-3.8$

Now

$7.6-3.8=\boxed{3.8}$ [Borrow 1 from 7 and make 1.6 and subtract 0.8. Again subtract 3 from remaining 6 before decimal part]

Consider the expression:

$7.6-3.8$

Now

$7.6-3.8=\boxed{3.8}$ [Borrow 1 from 7 and make 1.6 and subtract 0.8. Again subtract 3 from remaining 6 before decimal part]

Answer 50MYS.

Consider the expression:

$18-4.7$

Now

$18-4.7=18.0-4.7$ [Write 18 as 18.0]

$=\boxed{13.3}$ [Borrow 1 from 8 and make 1.0 and subtract 0.7. Again subtract 4 from remaining 17 before decimal part]

Answer 51MYS.

Consider the expression:

$13.2-0.75$

Now

$13.2-0.75=13.20-0.75$ [Write 13.2 as 13.20]

$=\boxed{12.45}$ [Borrow 1 from 2 and make .10 and subtract .05 after decimal. Again borrow 1 from 3 and make 1.1 and subtract 0.7 after decimal. Again subtract 0 from remaining 12 before decimal part]