Long Answer Type Questions

[4 MARKS]

Que 1. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60°, respectively. Find the height of the tower.

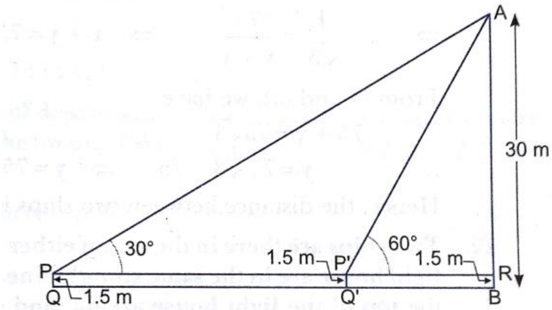

Fig. 11.26

Sol. Let AB be a building of height 20 m and BC be the transmission tower of height x m and D be any point on the ground (**Fig. 11.34**).

Here, $\angle BDA = 45° \ and \ \angle ADC = 60°$

Now, in ΔADC, we have

$$\tan 60° = \frac{AC}{AD} \quad \Rightarrow \quad \sqrt{3} = \frac{x+20}{AD}$$

$$\Rightarrow \quad AD = \frac{x+20}{\sqrt{3}} \qquad \ldots(i)$$

Again, in $\Delta ADB, we\ have\ \tan 45° = \frac{AB}{AD}$

$$\Rightarrow \quad 1 = \frac{20}{AD} \quad \Rightarrow \quad AD = 20\ m \qquad \ldots(ii)$$

Putting the value of AD in equation (i), we have

$$20 = \frac{x+20}{\sqrt{3}} \quad \Rightarrow \quad 20\sqrt{3} = x + 20$$

$$\Rightarrow \quad x = 20\sqrt{3} - 20 = 20(\sqrt{3} - 1) = 20(1.732 - 1) = 20 \times 0.732 = 14.64\ m$$

Hence, the height of tower is 14.64 m.

Que 2. A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point, the angle of elevation of the bottom of the pedestal is 45°. Find the height of the pedestal.

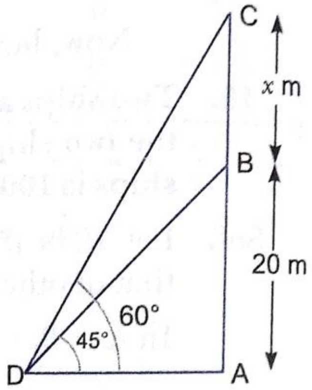

Fig. 11.34

Sol. Let AB be a building of height 20 m and BC be the transmission tower of height x m and D be any point on the ground (**Fig. 11.34**).
Here, $\angle BDA = 45° \; and \; \angle ADC = 60°$
Now, in ΔADC, we have

$$\tan 60° = \frac{AC}{AD} \quad \Rightarrow \quad \sqrt{3} = \frac{x+20}{AD}$$

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Again, in ΔADB, we have $\tan 45° = \frac{AB}{AD}$

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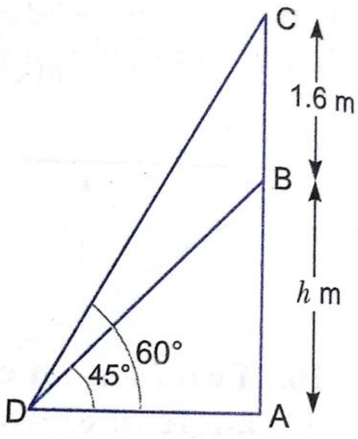

Fig. 11.35

Sol. Let AB be the pedestal of height h metres and BC be the statue of height 1.6 m **(Fig. 11.35).**

Let D be any point on the ground such that,

$$\angle BDA = 45° \text{ } and \text{ } \angle CDA = 60°$$

Now, in $\Delta BDA, we\ have$

$$\tan 45° = \frac{AB}{DA} = \frac{h}{DA} \quad \Rightarrow \quad 1 = \frac{h}{DA}$$

$\therefore \quad DA = h$ …(i)

Again in $\Delta ADC, we\ have$

$$\tan\ 60° = \frac{AC}{AD} = \frac{AB+BC}{AD}$$

$\Rightarrow \quad \sqrt{3} = \frac{h+16}{h}$ [From equation (i)]

$\Rightarrow \quad \sqrt{3}h = h + 16 \quad \Rightarrow \quad (\sqrt{3} - 1)\,h = 1.6$

$\therefore \quad h = \frac{1.6}{\sqrt{3}-1} = \frac{1.6}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{1.6(\sqrt{3}+1)}{3-1} = \frac{1.6(\sqrt{3}+1)}{2} = 0.8 \times (\sqrt{3} + 1)\,m$

Hence, height of the pedestal is 0.8 $(\sqrt{3} + 1)\ m.$

Que 3. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

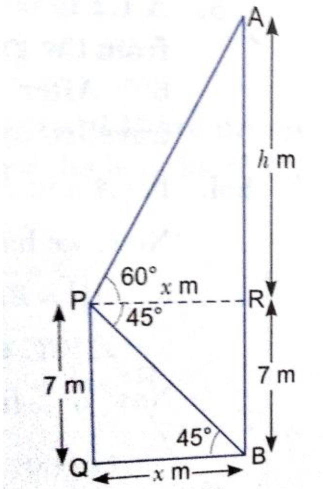

Fig. 11.36

Sol. Let PQ be the building of height 7 metres and AB be the cable tower. Now it is given that the angle of elevation of the top A of the tower observed from the top P of building is 60° and the angle of depression of the base B of the tower observed from P is 45° **(Fig. 11.36).**
So, $\angle APR = 60° \; and \; \angle QBP = 45°$
Let QB = x m, AR = h m then, PR = x m
Now, in $\Delta APR,$ we have

$$\tan 60° = \frac{AR}{PR} \quad \Rightarrow \quad \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow \quad \sqrt{3x} = h \quad \Rightarrow \quad h = \sqrt{3x} \qquad \ldots(i)$$

Again, in $\Delta PBQ,$ we have

$$\tan 45° = \frac{PQ}{QB} \quad \Rightarrow \quad 1 = \frac{7}{x} \quad \Rightarrow \quad x = 7 \ldots(ii)$$

Putting the value of x in equation (i), we have

$$h = \sqrt{3} \times 7 = 7\sqrt{3}$$

i.e., AR = 7 $\sqrt{3}$ meters
So, the height of tower = AB = AR + RB = $7\sqrt{3} + 7 = 7(\sqrt{3} + 3)\, m.$

Que 4. At a point, the angle of elevation of a tower is such that its tangent is $\frac{5}{12}$. On walking 240 m nearer to the tower, the tangent of the angle of elevation becomes $\frac{3}{4}$. Find the height of the tower.

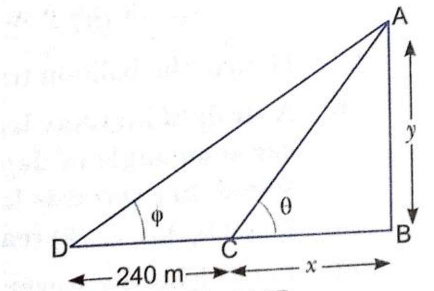

Fig. 11.37

Sol. In the **Fig. 11.37,** Let AB be the tower, C and D be the positions of observation from where given that

$$\tan\phi = \frac{5}{12} \qquad \ldots(i)$$

And $$\tan\phi = \frac{3}{4} \qquad \ldots(ii)$$

Let BC = x m. AB = y m

Now in right-angled triangle ABC

$$\tan\phi = \frac{y}{x} \qquad \ldots(iii)$$

From (ii) and (iii), we get $\frac{3}{4} = \frac{y}{x}$

$$\Rightarrow \quad x = \frac{4}{3}y \qquad \ldots(iv)$$

Also in right – angled triangle ABD, we get

$$\tan\phi = \frac{y}{x+240} \qquad \ldots(v)$$

From (i) and (v), we get

$$\frac{5}{12} = \frac{y}{x+240} \quad \Rightarrow \quad 12y = 5x + 1200 \qquad \ldots(vi)$$

$$\Rightarrow \quad 12y = 5 \times \frac{4}{3}y + 1200 \quad (Using\ (iv))$$

$$\Rightarrow \quad 12y - \frac{20}{3}y = 1200 \quad \Rightarrow \quad \frac{36y - 20y}{3} = 1200$$

$$\Rightarrow \quad 16y = 3600 \quad \Rightarrow \quad y = \frac{3600}{16} = 225$$

Hence, the height of the tower is 225 metres.

Que 5. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of elevation reduces to 30° (Fig. 11.38). Find the distance travelled by the balloon during the interval.

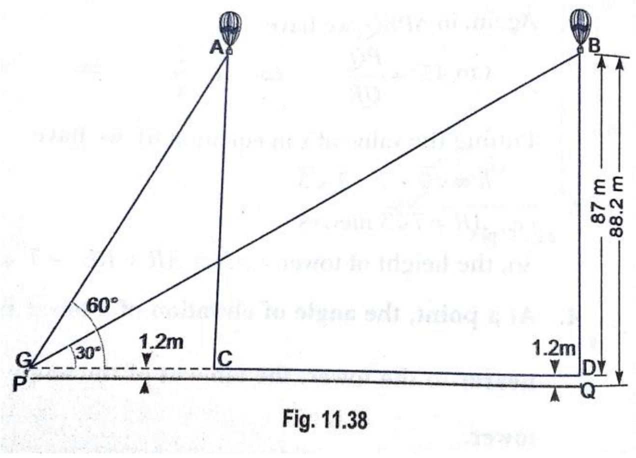

Fig. 11.38

Sol. Let A and be two positions of the balloon and G be the point of observation. (Eyes of the girl)

Now, we have

$AC = BD = BQ - DQ = 88.2\,m - 1.2\,m = 87\,m$

$\angle AGC = 60°, \ \angle BGD = 30°$

Now, in $\Delta AGC, we\ have$

$\tan 60° = \frac{AC}{GC} \quad \Rightarrow \quad \sqrt{3} = \frac{87}{GC}$

$\Rightarrow \ GC = \dfrac{87}{\sqrt{3}} = \dfrac{87}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{87 \times \sqrt{3}}{3}$

$\Rightarrow \ GC = 29 \times \sqrt{3}$ …(i)

Again, in $\Delta BGD, we\ have$

$\tan 30° = \frac{BD}{GD} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{87}{GD}$

$GD = 87 \times \sqrt{3}$ …(ii)

From (i) and (ii), we have

$CD = 87 \times \sqrt{3} - 29 \times \sqrt{3}$

$= \sqrt{3}(87 - 29) = 58\sqrt{3}$

Hence, the balloon travels $58\sqrt{3}$ metres.

Que 6. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

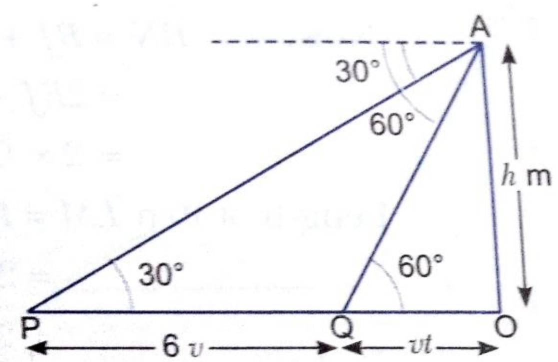

Fig. 11.39

Sol. Let OA be the tower of height h, and P be the initial position of the car when the angle of depression is 30°.
After 6 seconds, the car reaches to Q such that the angle of depression at Q is 60°. Let the speed of the car be v metre per second. Then,

PQ = 6v (∵ Distance = speed × time)

And let the car take t seconds to reach the tower OA from Q **(Fig. 11.39).** Then OQ = vt metres.
Now, in ΔAQO we have

$$\tan\ 60° = \frac{OA}{QO}$$

$$\Rightarrow \qquad \sqrt{3} = \frac{h}{vt} \qquad \Rightarrow \quad h = \sqrt{3}\,vt \qquad \ldots\text{(i)}$$

Now, in $\Delta APO, we\ have$

$$\tan 30° = \frac{OA}{PO}$$

$$\Rightarrow \qquad \frac{1}{\sqrt{3}} = \frac{h}{6v+vt} \qquad \Rightarrow \quad \sqrt{3}h = 6v + vt \qquad \ldots\text{(ii)}$$

Now, substituting the value of h from (i) into (ii), we have

$$\sqrt{3} \times \sqrt{3}\,vt = 6v + vt$$

$$\Rightarrow\ 3vt = 6v + vt \quad \Rightarrow\ 2vt = 6v \quad \Rightarrow\ t = \frac{6v}{2v} = 3$$

Hence, the car will reach the tower from Q in 3 seconds.

Que 7. In Fig. 11.40, ABDC is a trapezium in which AB || CD. Line segments RN and LM are drawn parallel to AB such that AJ = JK = KP. If AB = 0.5 m and AP = BQ = 1.8 m, find the lengths of AC, BD, RN and LM.

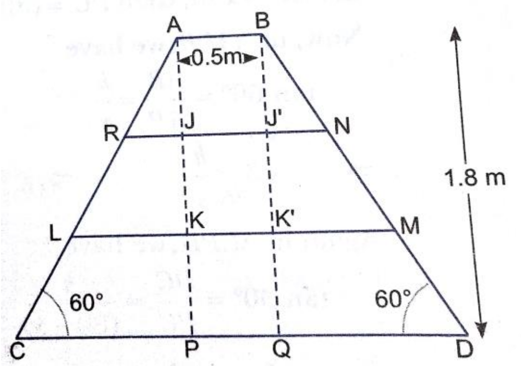

Fig. 11.40

Sol. we have,

AP = 1.8m

∴ AJ = JK = KP = 0.6 m

⇒ AK = 2AJ = 1.2 m

In ΔARJ and $\Delta BNJ'$, we have

AJ = BJ’, ∠ARJ = ∠BNJ’ = 60°

and ∠AJR = ∠BJ’ N = 90°

∴ $\Delta ARJ \cong \Delta BNJ'$

⇒ RJ = NJ’ (By AAS congruence criterion)

Similarly, $\Delta ALK \cong \Delta BMK'$ ⇒ LK = MK’

In ΔARJ, $\tan 60° = \frac{AJ}{RJ}$

$$\Rightarrow \quad \sqrt{3} = \frac{0.6}{RJ} \quad \Rightarrow \quad RJ = \frac{0.6}{\sqrt{3}} = \frac{0.6\sqrt{3}}{3} = 0.2 \times 1.732 = 0.3464\, m$$

In ΔALK,

$$\tan 60° = \frac{AK}{LK} \quad \Rightarrow \quad \sqrt{3} = \frac{1.2}{LK}$$

$$\Rightarrow \quad LK = \frac{1.2}{\sqrt{3}} = \frac{1.2\times\sqrt{3}}{3} = 0.4 \times 1.732\, m = 0.6928\, m$$

In ΔACP, $\sin 60° = \frac{AP}{AC}$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{1.8}{AC} \quad \Rightarrow \quad AC = \frac{3.6}{\sqrt{3}} = \frac{3.6 \times \sqrt{3}}{3} = 1.2 \times 1.732 = 2.0784\, m$$

Since $\Delta ACP \cong \Delta BDQ$

So, BD = AC = 2.0784 m

Now, RN = RJ + JJ’ +J, N

= 2RJ + AB [$\because RJ = J'N$ and $JJ' = AB$]

= 2 ×0.3464 + 0.5 = 1.1928 m

Length of step LM = LK + KK’ + K’ M

= 2LK + AB [$\because LK = K'M$ and $KK' = AB$]

= 2× 0.3464 + 0.5 = 1.1928 m

Length of step LM = LK + KK’ + K’ M

$= 2LK + AB \qquad [\because LK = K'M \text{ and } KK' = AB]$

$= 2 \times 0.6928 + 0.5 = 1.8856$ m

Thus, length of each leg = 2.0784 m = 2.1 m

Length of step RN = 1.1928 m = 1.2 m

and, length of step LM = 1.8856 m = 1.9 m

Que 8. Two poles of equal heights are standing opposite to each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60 ° and 30 °, respectively. Find the height of the poles the distances of the point from the poles.

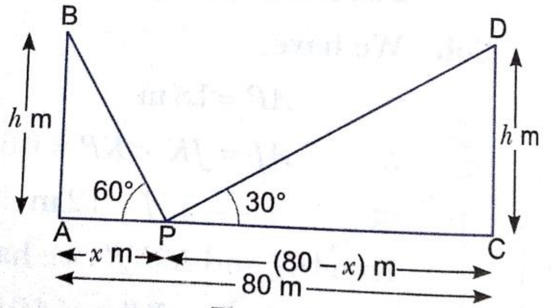

Fig. 11.41

Sol. Let AB and CD be two poles of equal height h metre and let P be any point between the poles, such that

$$\angle APB = 60° \text{ and } \angle DPC = 30°.$$

The distance between two poles is 80 m. (Given)

Let AP = x m, then PC = (80 – x) m.

Now, in ΔAPB, we have

$$\tan 60° = \frac{AB}{AP} = \frac{h}{x}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{x} \quad \Rightarrow \quad h = \sqrt{3}x \qquad \ldots(i)$$

Again in ΔCPD, we have

$$\tan 30° = \frac{DC}{PC} = \frac{h}{(80 - x)}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{80-x} \quad \Rightarrow \quad h = \frac{80-x}{\sqrt{3}} \qquad \ldots(ii)$$

From (i) and (ii), we have

$$\sqrt{3}x = \frac{80-x}{\sqrt{3}} \quad \Rightarrow \quad mx = 80 - x \quad \Rightarrow \quad 4x = 80 \quad \Rightarrow \quad x = \frac{80}{4} = 20\,m$$

Now, putting the value of x in equation (i), we have

$$h = \sqrt{3} \times 20 = 20\sqrt{3}$$

Hence, the height of the pole is $20\sqrt{3}m$ and the distance of the point first pole is 20 m and that of the second pole is 60 m.