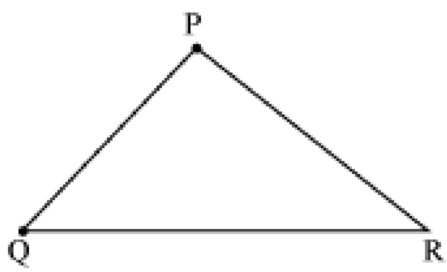
Triangles

- **Triangle:** A triangle is a three-sided polygon. It is the polygon with the least number of sides.



We denote this triangle as ΔPQR. Here, $\overline{PQ}$, $\overline{QR}$ and $\overline{RP}$ are the sides of ΔPQR. The points P, Q and R are the vertices of ΔPQR and the angles are ∠RPQ, ∠PQR and ∠QRP.

- A triangle can be classified on the basis of the measures of its angles and sides.

- Classification of triangles on the basis of the measures of its angles:

Name	**Nature of the angle**
Acute-angled triangle	Each angle is acute
obtuse-angled triangle	One angle is obtuse
Right-angled triangle	One angle is a right angle

- Classification of triangles on the basis of the lengths of its sides:

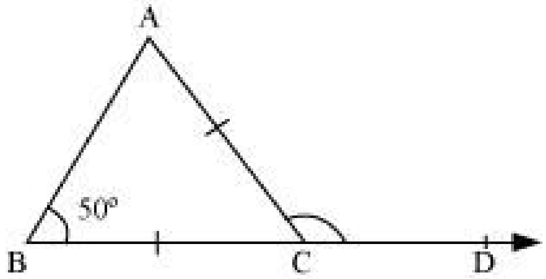
Name	**Nature of the angle**
Scalene triangle	All three sides are of unequal length
Isosceles triangle	Any two sides are of equal length

Equilateral triangle	All sides are of equal length

- Isosceles triangle

  In an isosceles triangle, (1) two sides are of equal length and (2) the base angles opposite to the equal sides are equal.

  Example:
  In the following figure, AC = BC. Find $m\angle ACD$.



  Solution: In ΔABC, AC = BC
  $\therefore \angle CBA = \angle CAB$
  $\therefore \angle ACD = \angle CAB + \angle CBA = 2 \times 50° = 100°$

- Equilateral triangle

  In an equilateral triangle, (1) all sides are equal and (2) each angle is of measure 60°.

- A triangle can be constructed if all its sides are known.

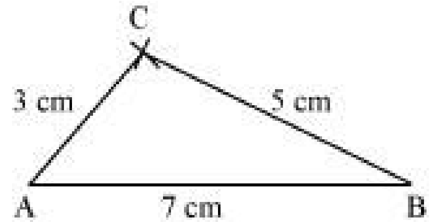
  Example:
  Construct a triangle whose sides are 3 cm, 5cm and 7 cm.

  Solution:

  1. Draw a line segment AB of length 7 cm. With A as centre and radius equal to 3 cm, draw an arc.

  2. With B as centre and radius 5 cm, draw another arc cutting the earlier drawn arc at C.

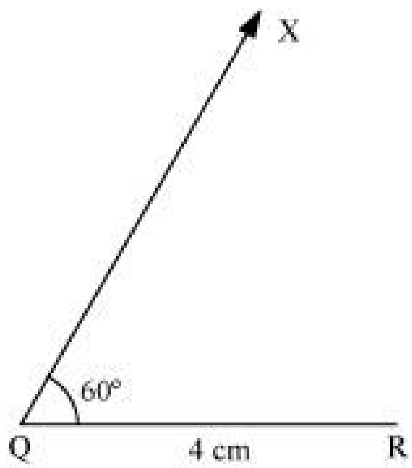
  3. Join AC and BC to get ΔABC.



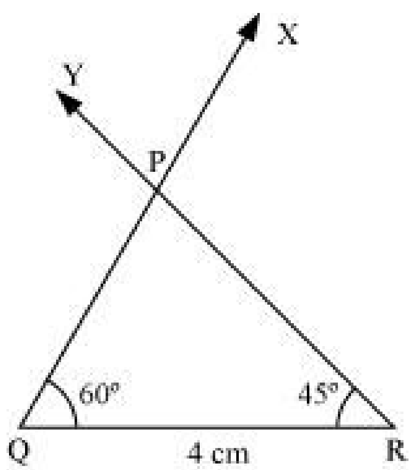
Example: Construct ΔPQR, where ∠PQR = 60°, ∠PRQ = 45° and QR = 4 cm.

Solution:

1. Draw a line segment QR of length 4 cm and draw a ray QX, making an angle of 60° with QR



2. Now, draw ray RY, making an angle of 45° with QR and intersecting QX at P. The resulting ΔPQR is the required triangle.



- A triangle can be constructed if the length of two sides and angle between them are given.

Example:

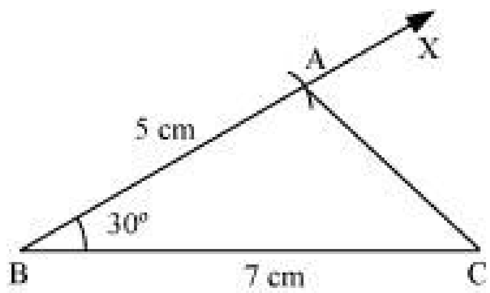
Construct ΔABC where BC = 7 cm, AB = 5 cm and $\angle ABC = 30°$

Solution:

1. Draw a line segment BC of length 7 cm and at B draw a ray BX, making an angle of 30° with BC.

2. With B as centre and radius equal to 5 cm, draw an arc cutting BX at A.

3. Join AC to get the required ΔABC.

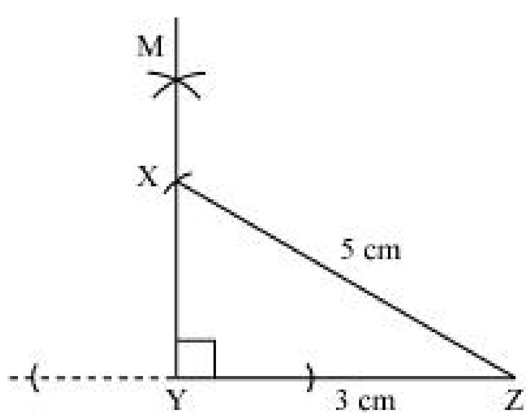


- A right-angled triangle can be constructed if the length of one of its sides or arms and the length of its hypotenuse are known.

Example:
Construct ΔXYZ, right-angled at Y, with XZ = 5 cm and YZ = 3 cm.
Solution:

1. Draw a line segment YZ of length 3 cm. At Y, draw $MY \perp YZ$.

2. With Z as centre and radius equal to 5 cm, draw an arc intersecting MY at X. Join XZ to get the required ΔXYZ.



- **Construction of circumcircle of given triangle:**

Example:

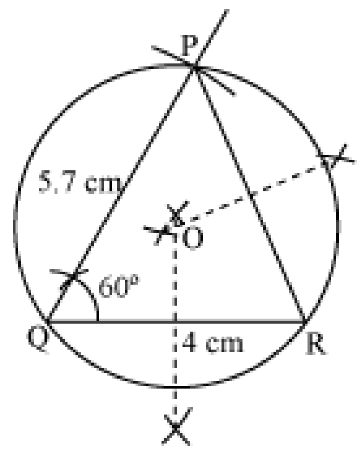
Construct the circumcircle of ΔPQR such that ∠Q = 60°, QR = 4 cm, and QP = 5.7 cm.

Solution:

Step 1: Draw a triangle PQR with ∠Q = 60°, QR = 4 cm, and QP = 5.7 cm

Step 2: Draw perpendicular bisector of any two sides, say QR and PR. Let these perpendicular bisectors meet at point O.

Step 3: With O as centre and radius equal to OP, draw a circle.



The circle so drawn passes through the points P, Q, and R, and is the required circumcircle of ΔPQR.

- **Construction of incircle of given triangle:**

  Example:

  Construct incircle of a right ΔPQR, right angled at Q, such that QR = 4 cm and PR = 6 cm.

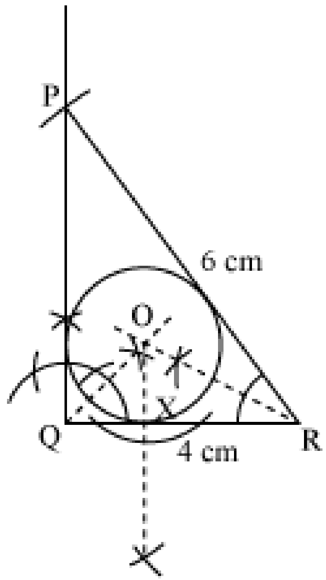
  Solution:

  Step 1: Draw a ΔPQR right-angled at Q with QR = 4 cm and PR = 6 cm.

  Step 2: Draw bisectors of ∠Q and ∠R. Let these bisectors meet at the point O.

  Step 3: From O, draw OX perpendicular to the side QR.

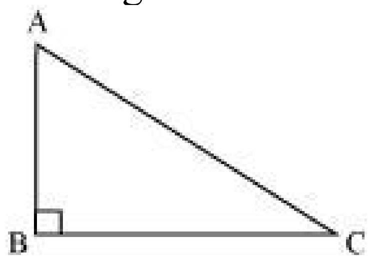
  Step 4: With O as centre and radius equal to OX, draw a circle.



The circle so drawn touches all the sides of ΔPQR and is the required incircle of ΔPQR.

- Right-angled triangle

  A triangle with one of the angles as 90°, is called a right-angled triangle.

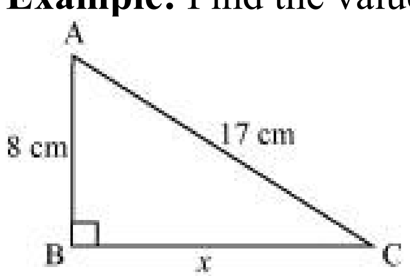
  

  ΔABC is a right-angled triangle with ∠B = 90°

- The side opposite to the right angle is called its hypotenuse. The other two sides are called the legs of the right-angled triangle, i.e., in the given figure, AC is the hypotenuse and AB, BC are the legs.
- In a right-angled triangle, the square of the hypotenuse is equal to the sum of squares of the other two sides, i.e., $AC^2 = AB^2 + BC^2$.

  This property is called Pythagoras Theorem.

  Example: Find the value of *x* in the following figure.

Solution: ΔABC is a right-angled triangle with $\angle B = 90°$.

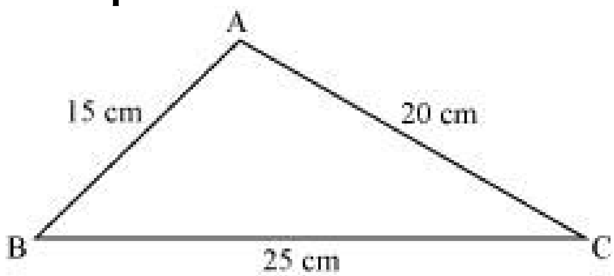
$AC^2 = AB^2 + BC^2$ [By Pythagoras Theorem]

$\Rightarrow (17 \text{ cm})^2 = (8 \text{ cm})^2 + x^2$

$\Rightarrow x = \sqrt{289 - 64}\text{cm} = 15\text{cm}$

- In a triangle, if Pythagoras property holds, then it is a right-angled triangle.

Example:



Prove that ΔABC is right-angled.

Solution: In ΔABC,

$AB = 15 \text{ cm} \Rightarrow AB^2 = 225 \text{ cm}^2$

$AC = 20 \text{ cm} \Rightarrow AC^2 = 400 \text{ cm}^2$

$BC = 25 \text{ cm} \Rightarrow BC^2 = 625 \text{ cm}^2$

$BC^2 = AB^2 + AC^2$

Therefore, Pythagoras property holds in ΔABC. Thus, it is a right-angled triangle.