

# Chapter 10. Quadratic And Exponential Functions

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## Ex. 10.6

### Answer 1CU.

Exponential growth	Exponential Decay
<p>* The general equation for exponential growth is <math>y = c(1+r)^t</math> where <math>y</math> represent final amount <math>c</math> represent initial amount, <math>r</math> represent the rate of change expressed as decimal and <math>t</math> represent the time.</p> <p>* Exponential growth is an increase by the same percent over a period of time.</p>	<p>* The general equation for exponential decay is <math>y = c(1-r)^t</math> where <math>y</math> represent final amount <math>c</math> represent the initial amount <math>r</math> represent the rate of change expressed as decimal and <math>t</math> represent the time.</p> <p>* Exponential Decay is a decrease by the same percent over a period of time.</p>

### Answer 1RM.

The generalized formula is  $y = c \cdot r^t$ ,  $r$  is the rate and  $t$  is time.

The general equation for exponential decay is,

$$\underbrace{\text{The final amount}}_y \quad \underbrace{\text{equals}}_= \quad \underbrace{\text{an initial amount}}_c \quad \underbrace{\text{times}}_t \quad \underbrace{\text{The quantity one min's a rate raised to the power of time}}_{(1-r)^t}$$

$$y = c \cdot (1-r)^t$$

The only difference between the generalized formula and the exponential decay equation is that rate equals  $(1-r)$  in exponential decay whereas  $r$  is the rate in generalized formula.

In exponential decay the amount decreases as the time increases.

So the final amount is the initial amount mines the decrease.

Thus the rate of the exponential decay is equals to  $(1-r)$ .

## Answer 2CU.

Consider the equation  $A = 500 \left[ 1 + \frac{0.07}{4} \right]^{4(6)}$

Claim: Solve the equation  $A = 500 \left[ 1 + \frac{0.07}{4} \right]^{4(6)}$

$$A = 500 \left[ 1 + \frac{0.07}{4} \right]^{4(6)} \quad (\text{Original equation})$$

$$= 500 \left[ \frac{1 + 0.07}{4} \right]^{4(6)}$$

$$= 500 \left[ \frac{4 + 0.07}{4} \right]^{24}$$

$$= 500 \left[ \frac{4.07}{4} \right]^{24}$$

$$= 50 [1.0175]^{24}$$

$$= 50 [1.5165]$$

$$= 758.2$$

$$\boxed{A = 758.2}$$

## Answer 2RM.

The generalized formula is  $y = c \cdot r^t$ ,  $r$  is the rate and  $t$  is time.

One special application of exponential growth is compound interest.

The equation for the compound interest is  $A = P \cdot \left( 1 + \frac{r}{n} \right)^{nt}$ .

Where  $A$  represents the amount of the investment.

$P$  is the principal (initial amount of the investment)

$r$  represent the annual rate of interest.

$n$  represent the number of times that the is compounded each year and  $t$  represents the number of years that the money is invested.

The only difference from the generalized formula is that rate equals  $\left( 1 + \frac{r}{n} \right)$  raised to the power of  $nt$  because one represents 100%.

If you multiply  $c$  by 100%, the final amount are the same as the initial amount we add 1 to the rate  $\frac{r}{n}$  so that the final amount is the initial amount plus the increase.

Thus, the rate of interest is  $\left( 1 + \frac{r}{n} \right)$ .

In compound interest,  $n$  represent the number of times that the is compounded each year and  $t$  represents the number of years that the money is invested.

Thus,  $nt$  equals the time in compound interest formula.

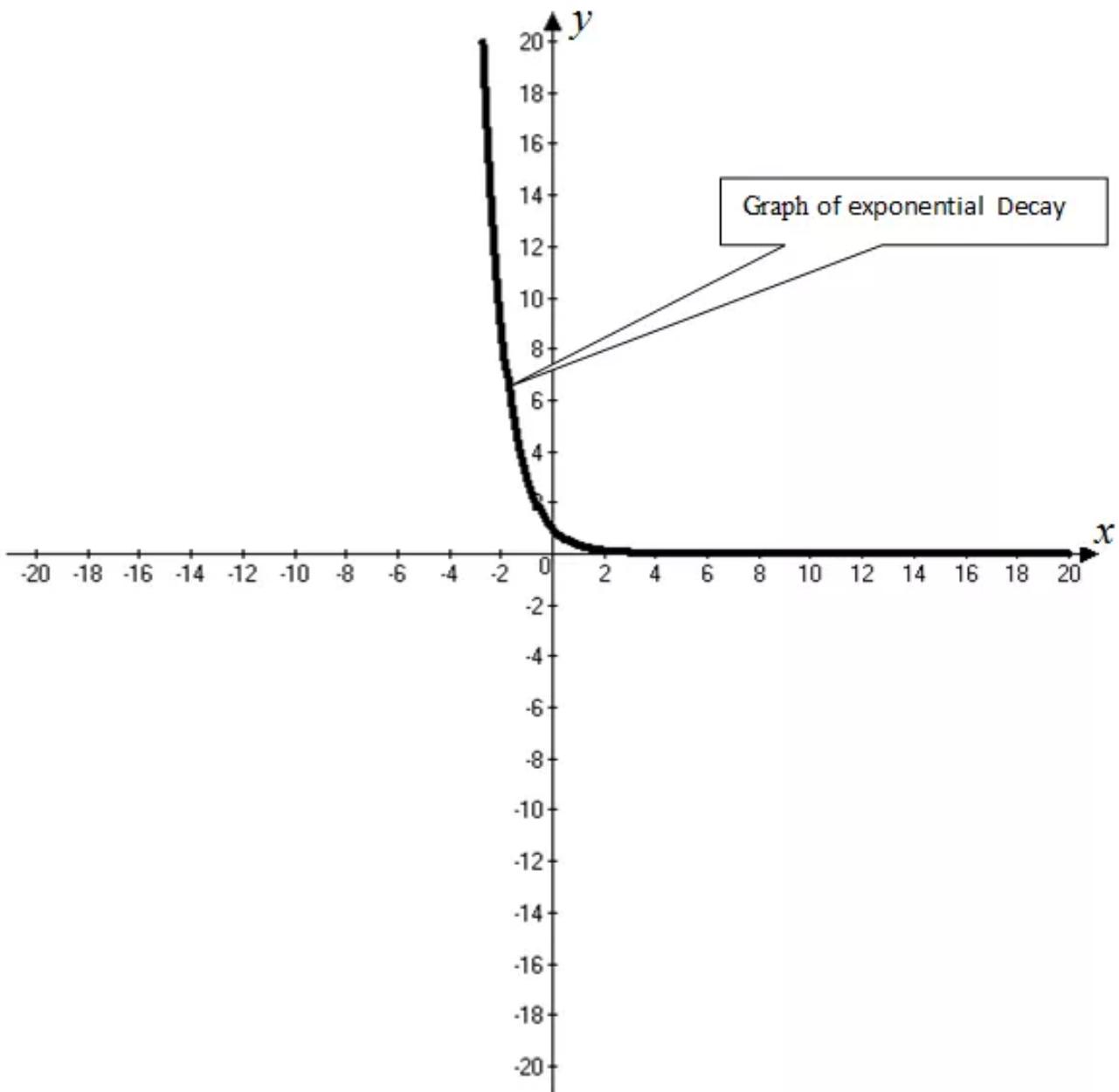
**Answer 3CU.**

The general equation for exponential decay is  $y = c(1-r)^t$ .

Where  $y$  represents the final amount,  $c$  represents the initial amount,  $r$  represents the rate and  $t$  represents the time.

In exponential decay the original amount decreased by the same percent over a period of time.

Sketch the graph of exponential decay:



### Answer 3RM.

(a)

Consider the initial amount  $P = \underline{\$2500}$

The annual rate  $r = 6\%$

$$= \frac{6}{100}$$

$$= 0.03$$

$$r = \underline{0.06}$$

It needs to find the value of the amount after 5 years and compounded quarterly.

So  $t = \underline{5}, n = \underline{4}$ .

(b)

Use the rule "The formula for compounded interest is  $A = P \cdot \left(1 + \frac{r}{n}\right)^{nt}$ ,

Where  $A$  represents the amount of the investment

$P$  is the principal (initial amount of the investment)

$R$  represents the annual rate of interest

$N$  represent the number of times that the is compounded each year and  $t$  represents the number of years that the money is invested.

$$A = P \cdot \left(1 + \frac{r}{n}\right)^{nt} \quad (\text{Compound interest formula})$$

$$= 2500 \left(1 + \frac{0.06}{4}\right)^{4 \cdot 5} \quad (\text{Replace } P = 2500, r = 0.06, n = 4 \text{ and } t = 5)$$

$$= 2500 \left(\frac{4 + 0.06}{4}\right)^{20}$$

$$= 2500 \left(\frac{4.06}{4}\right)^{20}$$

$$= 2500(1.015)^{20}$$

$$= 2500 \cdot (1.347)$$

$$A = \$3,367.14$$

Therefore, the value of the amount after 5 years is  $\boxed{\$3,367.14}$ .

### Answer 4CU.

Consider the median household income in the United states increased an average of 0.5% each year between 1979 and 1999.

Let  $y(t)$  be represent the income at a time  $t$  years according to problem.

Initial income in 1979 is \$37,060

$$y = 37,060 \text{ when } t = 1979 \text{ or } y(1979) = 37,060$$

\* Initial income for 10 year after 1979 is \$38,836

$$y = 38,836 \quad \text{when } t = \text{initial year} + 10 \text{years}$$

$$y = 38,836 \quad \text{when } t = 1979 + 10 = 1989 \text{ (or) } y(1989) = 38,836$$

\* The income for 20 years after 1979 is \$40,816

$$y = 40,816 \quad \text{when } t = \text{initial year} + 20 \text{years}$$

$$y = 40,816 \quad \text{when } t = 1979 + 20 = 1999 \text{ (or) } y(1999) = 40,816$$

Claim: To find the equation for the median house hold income after " $t$ " years after 1979.

Use the formula,

"The general equation for the exponential growth is  $y(t) = c(1+r)^t$  where  $y(t)$  represents the final amount at a time  $t$ ,  $c$  represents the initial amount,  $r$  represents the rate of growth and  $t$  represents the time"

$$y(t) = c(1+r)^t \text{ The general equation for exponential growth}$$

Substitute  $c = 37,060$  and  $r = 0.5\% = 0.005$

$$y(t) = 37,060(1+0.005)^t$$

Hence, the equation for the median house holds income for " $t$ " years after 1979 is

$$\boxed{y(t) = 37,060(1+0.005)^t}$$

### Answer 4RM.

(a)

Consider the initial amount of the car is,  $c = \underline{\$18,500}$ .

The rate of depreciation is 11%.

It needs to find the value of the car in 4 years.

Use the rule "The general equation for exponential decay is  $y = c(1-r)^t$

Where  $y$  represents the final amount,  $c$  represents the initial amount,  $r$  represents the rate of decay and  $t$  represents the time"

$$\begin{aligned}y &= c(1-r)^t && \text{(General equation for decay)} \\&= 18,500(1-0.11)^4 && \text{(Replace } c \text{ by 18,500, } r \text{ by 11\% or 0.11 and } t \text{ by 4)} \\&= 18,500(0.89)^4 \\&= 18,500(0.63) \\y &= 11,607.31\end{aligned}$$

Therefore, the value of the car in 4 years is  $\boxed{\$11,607.31}$ .

### Answer 5CU.

Consider the median household income in the United states increased an average of 0.5% each year between 1979 and 1999.

Let  $y(t)$  be represent the income at a time  $t$  years according to problem.

Initial income in 1979 is \$37,060

$$y = \$37,060 \text{ when } i = 1979 \text{ or } y(1979) = 37,060$$

\* Initial income for 10 year after 1979 is \$38,836

$$\begin{aligned}y &= 38,836 && \text{when } t = \text{initial year} + 10\text{years} \\y &= 38,836 && \text{when } t = 1979 + 10 = 1989 \text{ (or) } y(1989) = 38,836\end{aligned}$$

\* The income for 20 years after 1979 is \$40,816

$$\begin{aligned}y &= 40,816 && \text{when } t = \text{initial year} + 20\text{years} \\y &= 40,816 && \text{when } t = 1979 + 20 = 1999 \text{ (or) } y(1999) = 40,816\end{aligned}$$

Claim: To find the equation for the median house hold income after " $t$ " years after 1979.

Use the formula,

"The general equation for the exponential growth is  $y(t) = c(1+r)^t$  where  $y(t)$  represents the final amount at a time  $t$ ,  $c$  represents the initial amount,  $r$  represents the rate of growth and  $t$  represents the time"

$$y(t) = c(1+r)^t \text{ The general equation for exponential growth}$$

Substitute  $c = 37,060$  and  $r = 0.5\% = 0.005$

$$y(t) = 37,060(1+0.005)^t$$

Therefore, the equation for the median house holds income for " $t$ " years after 1979 is

$$y(t) = 37,060(1+0.005)^t$$

Step2: To find the median household income in 2009

First to find the year + After  $t$  years

After  $t$  years = 2009 – initial year

$$= 2009 - 1979$$

$$\boxed{t = 30}$$

To find the median house hold income for 30 years after 1979.

Now substitute  $t = 30$  in the original equation

$$y(t) = 37,060(1.005)^t \quad (\text{equation for after } t \text{ years})$$

$$y(30) = 37,060(1.005)^{30}$$

$$= 37,06091.1614$$

$$y(30) = 43,041$$

The income for 30 years after 1979 is 43,041

$$y = 43,041 \text{ when } t = \text{initial year} = 30 \text{ year}$$

$$y = 43,041 \text{ when } t = 1979 + 30 = 2009$$

$$(\text{or}) y(2009) = 43,041$$

Hence, the median household income in 2009 is  $\boxed{\$43,041}$

### **Answer 5RM.**

(a)

Consider the initial population  $c = 12.500$ .

The population increasing at annual rate  $r = 3.5\%$

$$= \underline{0.035}$$

It needs to predict the population in 5 years.

So  $\underline{t = 5}$  .

(b)

Use the rule "The general equation for exponential growth is  $y = c(1+r)^t$ "

Where  $y$  represents the final amount,  $c$  represents the initial amount,  $r$  represents the rate and  $t$  represents the year"

$$\begin{aligned}y &= c(1+r)^t && \text{(General equation for growth)} \\&= 12,500(1+0.035)^5 \\&= 12,500(1.035)^5 \\&= 12,500(1.19) \\&= 14,846.08\end{aligned}$$

Therefore, the population in 5 year is 14,846.08.

### Answer 6CU.

Now substitute the values of  $p = 400, r = 0.0725, n = 4$  and  $t = 7$  is the  $A = p\left(1 + \frac{r}{n}\right)^{nt}$

$$\begin{aligned}A &= p\left(1 + \frac{r}{n}\right)^{nt} && \text{(The equation for compound interest)} \\&= 400\left(1 + \frac{0.0725}{4}\right)^{4(7)} \\&= 400\left(\frac{4 + 0.0725}{4}\right)^{28} \\&= 400\left(\frac{4.0725}{4}\right)^{28} \\&= 400(1.018125)^{28} \\&= 400(1.6536) \\A &= 661.4\end{aligned}$$

Hence, \$661.4 the amount of an investment if \$400 is invested at an interest rate of 7.25% compounded quarterly for 7 years.

## Answer 7CU.

Let  $y(t)$  is the population of West Virginia at a year " $t$ ".

The initial population of West Virginia in 1995 is 1,821,000.

$$y = 1,821,000 \text{ when } t = 1995 \text{ (or) } y(1995) = 1,821,000.$$

Let the population of West Virginia is decreased in each year.

$$\begin{aligned} r &= 0.2\% \\ &= 0.002 \end{aligned}$$

Step1: To find the population of West Virginia in 5 years after 1995.

Use the formula is shown below:

"The general equation for exponential decay is  $c(1-r)^t$  where  $y$  – represents the final amount,  $c$  – represents the initial amount,  $r$  – represents the rate of decay and  $t$  – represents the time.

Now, substitute  $c = 1,821,000$ ,  $r = 0.002$  and  $t = 5$  in  $c(1-r)^t$ .

$$\begin{aligned} y &= c(1-r)^t && \text{(The equation for exponential decay)} \\ &= 1,821,000(1-0.002)^5 \\ &= 1,821,000(0.998)^5 \\ &= 1,821,000(0.99) \\ &= 1,802,862.69 \end{aligned}$$

Therefore, the population of West Virginia reached in 5 years after 1995 is 1,802,862.69.

Hence, population of West Virginia reached in  $(1995 + 5) = 2000$  years is 1,802,862.69 people.

Step2: To find the population of West Virginia reached in 15 years after 1995.

Use the formula is shown below:

"The general equation for exponential decay is  $c(1-r)^t$  where  $y$  – represents the final amount,  $c$  – represents the initial amount,  $r$  – represents the rate of decay and  $t$  – represents the time.

Now, substitute  $c = 1,821,000$ ,  $r = 0.002$  and  $t = 15$  in  $c(1-r)^t$ .

$$\begin{aligned} y &= c(1-r)^t && \text{(The equation for exponential decay)} \\ &= 1,821,000(1-0.002)^{15} \\ &= 1,821,000(0.998)^{15} \\ &= 1,821,000(0.9704) \\ &= 1,767,128 \end{aligned}$$

Therefore, the population of West Virginia reached in 15 years after 1995 is 1,767,128.

Hence population of West Virginia reached in  $(1995 + 15) = 2010$  years is 1,767,128 people.

## Answer 8CU.

A car sells for \$16,000 and the rate of depreciation is 18% .

Claim: To find the car sells after 8 years.

Initially a car sells is  $c = \$16,000$  .

The rate of depreciation is  $r = 18\% = \$16,000$  .

The number of years is  $t = 8$  .

Use the formula is shown below:

"The general equation for exponential growth is  $c(1+r)^t$  where  $y$  – represents the final amount,  $c$  – represents the initial amount,  $r$  – represents the rate of decay and  $t$  – represents the time.

Now, substitute  $c = 16,000$  ,  $r = 0.18$  and  $t = 8$  in  $y = c(1+r)^t$  .

$$\begin{aligned}y &= c(1+r)^t && \text{(The equation for exponential growth)} \\&= 16,000(1+0.18)^8 \\&= 16,000(1.18)^8 \\&= 16,000(3.7588) \\&= 60,141.75\end{aligned}$$

Therefore, a car sells for \$16,000 if the rate of depreciation is 18%, the value of the car after 8 years \$60,141.75

Hence, after 8 years the car sell is \$60,141.75 .

### Answer 9CU.

Computer use around the world has risen 19% annually since 1980.

If 18.9 million computers in use for  $t$  years in 1980

Claim: To write an equation for the number of computers in use for  $t$  years after 1980.

Initially the computers use around the world is  $c = 18.9$  million .

The computer use around the world has risen  $r = 19\% = 0.19$  .

Use the formula is shown below:

“The general equation for exponential growth is  $c(1+r)^t$  where  $y$  – represents the final amount,  $c$  – represents the initial amount,  $r$  – represents the rate of decay and  $t$  – represents the time.

Now, substitute  $c = 18.9$ ,  $r = 19\% = 0.19$  in  $y = c(1+r)^t$ :

$$y = c(1+r)^t \quad (\text{The equation for exponential growth})$$

$$y = 18.9(1+0.19)^t$$

$$y = 18.9(1.19)^t$$

Therefore, the number of computers in use for  $t$  years after 1980 is  $y = 18.9(1.19)^t$  millions.

### Answer 10PA.

Computer use around the world has risen 19% annually since 1980.

If 18.9 million computers were in use for  $t$  years in 1980,

Claim: Find the number of computers in 2015.

Find the number of computers  $(2015 - 1980) = 35$  years after 1980.

Step1: First to write an equation for the number of computers in use for  $t$  years in 1980.

Initially the computers use around the world is  $c = 18.9$  million .

The computer use around the world has risen  $r = 19\% = 0.19$  .

Use the formula is shown below:

“The general equation for exponential growth is  $c(1+r)^t$  where  $y$  – represents the final amount,  $c$  – represents the initial amount,  $r$  – represents the rate of decay and  $t$  – represents the time.

Now, substitute  $c = 18.9$ ,  $r = 19\% = 0.19$  in  $y = c(1+r)^t$  .

$$y = c(1+r)^t \quad \text{(The equation for exponential growth)}$$

$$y = 18.9(1+0.19)^t$$

$$y = 18.9(1.19)^t$$

Therefore, the number of computers in use for  $t$  years after 1980 is  $y = 18.9(1.19)^t$  millions.

Find the number of computers in use for 35 years after 1980.

Substitute  $t = 35$  in the equation  $y = 18.9(1.19)^t$  .

$$y = 18.9(1.19)^t \quad \left( \begin{array}{l} \text{equation for number of computers} \\ \text{in use for } t \text{ years after 1980} \end{array} \right)$$

$$= 18.9(1.19)^{35}$$

$$= 18.9(440.7)$$

$$= 8,329.24$$

Therefore, the number of computers in use for 35 years after 1980 is 8,329.24 millions.

Hence, the number of computers in 2015 is 8,329.24 millions

### Answer 11PA.

In 1997, there were 43.2 million people who use free weights.

Assuming the use of free weights increase 6% annually

Claim: Write an equation for the number of people using free weights “ $t$ ” years from 1997.

According to the problem,

Initially the people use free weights in 1997 is  $c = 43.2$  million.

The use of free weights increase annually is  $r = 6\% = 0.06$

Use the formula is shown below:

“The general equation for exponential growth is  $c(1+r)^t$  where  $y$  – represents the final amount,  $c$  – represents the initial amount,  $r$  – represents the rate of decay and  $t$  – represents the year”

$$y = c(1+r)^t \quad (\text{The equation for exponential growth})$$

$$y = 43.2(1+0.06)^t \quad (\text{Replace } c \text{ by } 43.2 \text{ and } r \text{ by } 0.06)$$

$$y = 43.2(1.06)^t$$

Therefore, the number of people using free weights  $t$  years from 1997 is  $y = 43.2[1.06]^t$ .

### Answer 12PA.

In 1997, there were 43.2 million people who use free weights.

Assume the use of free weights increases 6% annually.

Claim: Find the number of people using free weights in 2007.

According to the problem,

Initially the people use free weights in 1997 is 43.2 millions.

The use of free weights increase annually is  $r = 6\% = 0.06$

Write an equation for the number of people using free weights “ $t$ ” years from 1997.

Use the formula is shown below.

“The general equation for exponential growth is  $c(1+r)^t$  where  $y$  – represents the final amount,  $c$  – represents the initial amount,  $r$  – represents the rate of decay and  $t$  – represents the time.

Now, substitute  $c = 43.2$ ,  $r = 0.06$  in the equation  $y = c(1+r)^t$ .

$$y = c(1+r)^t \quad (\text{The equation for exponential growth})$$

$$y = 43.2(1+0.06)^t \quad (\text{Replace } c \text{ by } 43.2 \text{ and } r \text{ by } 0.06)$$

$$y = 43.2(1.06)^t$$

Therefore, an equation for the number of people using free weights  $t$  years from 1997 is

$$y = 43.2[1.06]^t \text{ million}$$

Step2: Find the number of people using free weights in 2007.

Find the number of people using free weights  $(2007 - 1997) = 10$  years from 1997.

Now substitute  $t = 10$  in the equation  $y = 43.2(1.06)^t$ .

Replace  $t$  by 10

$$\begin{aligned} y &= 43.2(1.06)^{10} \\ &= 43.2(1.06)^{10} \\ &= 43.2(1.7908) \\ y &= 77.36 \end{aligned}$$

Therefore, the number of people using free weights 10 years from 1997 is 77.36 million.

Hence, the number of people using free weights in  $(1997 + 10) = 2007$  years is 77.36 millions.

### Answer 13PA.

Consider the population of Mexico has been increasing at an annual rate of 1.7%.

The population of Mexico was 100,350,000 in the year 2000.

Claim: Find population of Mexico in 2012.

Find the population of Mexico of  $(2012 - 2000) = 12$  years for after 2000.

According to the problem,

Initially, the population of Mexico in 1990 is  $c = 100,350,000$ .

The rate of increasing population of Mexico is  $r = 1.7\%$

$$= 0.017$$

The number of years  $t = 12$  years.

Use the formula is shown below:

"The general equation for exponential growth is  $c(1+r)^t$  where  $y$  – represents the final amount,  $c$  – represents the initial amount,  $r$  – represents the rate of decay and  $t$  – represents the time.

$$y = c(1+r)^t \quad (\text{The equation for exponential growth})$$

$$y = 100,350,000(1+0.017)^{12}$$

$$y = 100,350,000(1.017)^{12}$$

$$y = 100,350,000(1.224)$$

$$y = 122,848,204$$

Therefore, the population of Mexico for 12 years after 2000 is 122,848,204 people.

Hence, the population of Mexico in  $(2000 + 12) = 2012$  years is about 122,848,204 people.

### Answer 14PA.

Consider Initial amount of an investment is  $P = \$500$ .

The rate of interest is  $r = 5.75\% = 0.0575$ .

The compounded monthly for 25 years ( $t = 25$ ).

Use the formula is shown below:

"The equation of compound interest is  $A = p\left(1 + \frac{r}{n}\right)^{nt}$ , where  $A$  represents amount of investment,  $p$  represents principal (initial amount of the investment),  $r$  represents the annual rate of interest,  $n$  represents number of times that interest is compounded each year,  $t$  – represents the number of years that the money is invested".

$$\begin{aligned}A &= p\left(1 + \frac{r}{n}\right)^{nt} \text{ (The equation for the compounded interest)} \\&= 500\left(1 + \frac{0.0575}{12}\right)^{12(25)} \\&= 500\left(1 + \frac{0.0575}{12}\right)^{300} \\&= 500\left(\frac{12 + 0.0575}{12}\right)^{300} \\&= 500\left(\frac{12.0575}{12}\right)^{300} \\&= 500(1.0048)^{300} \\&= 500(4.1957) \\A &= 2,097.864\end{aligned}$$

Therefore, the amount of an investment if \$500 is invested at an interest rate of 5.75% compounded monthly for 25 years is about  $\boxed{\$2,097.864}$ .

### Answer 15PA.

Consider initial amount of an investment is  $P = \$250$ .

The rate of interest is  $r = 10.3\% = 0.103$ .

The interest compounded monthly  $n = 4$ .

The number of years that the money invested is  $t = 40$  years.

Claim: To find the amount of an investment if \$250 is invested at interest of 10.3 compounded quarterly 40 years

Use the formula is shown below:

"The equation of compound interest is  $A = p \left(1 + \frac{r}{n}\right)^{nt}$ , where  $A$  represents amount of investment,  $p$  represents principal (initial amount of the investment),  $r$  represents the annual rate of interest,  $n$  represents number of times that interest is compounded each year,  $t$  – represents the number of years that the money is invested".

$$\begin{aligned}A &= p \left(1 + \frac{r}{n}\right)^{nt} \text{ (The equation for the compounded interest)} \\&= 250 \left(1 + \frac{0.103}{4}\right)^{4(40)} \\&= 250 \left(\frac{4 + 0.103}{4}\right)^{160} \\&= 250 \left(\frac{4.103}{4}\right)^{160} \\&= 250(1.0258)^{160} \\&= 250(58.434) \\&= 14,607.78\end{aligned}$$

Hence, after the amount of an investment if \$250 is invested at an interest rate of 10.3% compounded quarterly for 40 years is about  $\boxed{\$14,607.78}$ .

### **Answer 16PA.**

Consider the country of Latvia has been experiencing a 1.1% annual decrease in population.

In 2000, its population was 2,405,000.

The objective is to find Latvia population in 2015.

To find the Latvia population for  $(2015 - 2000) = 15$  years for after 2000

Let  $y(t)$  be the population at  $t$  year.

According to the problem,

Initial Latvia population is 2,405,000.

$$y = 2,405,000 \text{ when } t = 2000$$

The rate of decrease in the Latvia population annually is,

$$\begin{aligned} r &= 1.1\% \\ &= 0.011 \end{aligned}$$

Use the rule,

“The general equation for exponential decay is  $c(1-r)^t$ .”

Where  $y$  – represents the final amount,  $c$  – represents the initial amount,  $r$  – represents the rate of decay and  $t$  – represents year”.

$$y = c(1-r)^t \quad (\text{The equation for exponential growth})$$

$$\begin{aligned} y &= 2,405,000(1-0.011)^{15} \\ &= 2,405,000(0.8471) \\ &= 2,037,320 \end{aligned}$$

Therefore, Latvia population for 15 years after 2000 is 2,037,320

Hence, Latvia population in  $(2000+15) = 2015$  years is 2,037,320 people.

### **Answer 17PA.**

Consider the following data:

In 1994 the sales of music cassettes reached its peak at \$2,976,400,000.

Since, those cassettes sales have been declining. If the amount of annual percent of decrease in sales is 18.6%.

The objective is to find the cassettes sales for  $(2009-1994) = 15$  years after 1994.

Let  $y(t)$  be the sales of music cassettes at  $t$  years.

From the data,

Initially, the sales music cassette is \$2,976,400,000. That is,

$$y = 2,976,400,000 \text{ when } t = 1994 \text{ or } y(1994) = 2,976,400,000$$

The annual percent decrease in sales of music cassettes is,

$$\begin{aligned} r &= 18.6\% \\ &= 0.186 \end{aligned}$$

### **Answer 18PA.**

Consider the following data:

The increase in the number of visitors to Grand Lanyon National park is similarly to an exponential function.

If the Average visitation has increased to 5.63% annually, since 1920.

And the number of visitors to the park in 2020.

The objective is to find the number of visitors to the park for  $(2020 - 1920) = 100$  years after 1920.

Let  $y(t)$  be the number of visitors to the park at  $t$  years.

From the data,

Initially, the number of visitors in 1920 is 71,601. That is,

$$y = 71,601 \text{ when } t = 1920 \text{ or } y(1920) = 71,601$$

The visitation has increased annually is,

$$\begin{aligned} r &= 5.63\% \\ &= 0.0563 \end{aligned}$$

Use the rule,

“The general equation for exponential growth is  $c(1+r)^t$  .

Where  $y$  – represents the final amount,  $c$  – represents the initial amount,  $r$  – represents the rate of decay and  $t$  – represents year” .

$$y = c(1+r)^t \quad (\text{The equation for exponential growth})$$

$$= 71,601(1+0.0563)^{100} \quad \left( \begin{array}{l} \text{Replace } c \text{ by } 71,601 \\ r \text{ by } 0.0563 \text{ and } t \text{ by } 100 \end{array} \right)$$

$$= 71,601(1.0563)^{100}$$

$$= 71,601(239.18)$$

$$y = 17,125,650$$

Therefore, the number of visitors to the park is 100 years after 1920 is 17,125,650 million.

Hence, the number of visitors to the park in  $(2020+100) = 2020$  years is,

$$\boxed{17,125,650 \text{ million}}.$$

### **Answer 19PA.**

Consider a piece of office equipment valued at \$25,000 depreciates at a steady rate of 10% per year.

The objective is to find the value of the equipment in 8 years.

To find after 8 years the value of the equipment

Let  $y(t)$  be represent the value of equipment at a year.

According to the problem,

Initially the value of equipment is  $c = \$25,000$ .

The equipment valued depreciate at a steady rate of per year is,

$$\begin{aligned} r &= 10\% \\ &= 0.1 \end{aligned}$$

Use the rule,

“The general equation for exponential decay is  $c(1-r)^t$ .

Where  $y$  – represents the final amount,  $c$  – represents the initial amount,  $r$  – represents the rate of decay and  $t$  – represents year”.

$$\begin{aligned}y &= c(1-r)^t && \text{(The equation for exponential growth)} \\&= 25,000(1-0.1)^8 && \left( \begin{array}{l} \text{Replace } c \text{ by } 25,000 \\ r \text{ by } 0.1 \text{ and } t \text{ by } 8 \end{array} \right) \\&= 25,000(0.9)^8 \\&= 25,000(0.43) \\&= 10,761.168\end{aligned}$$

Therefore, the value of the equipment in 8 years is  $\boxed{\$10,761.168}$ .

### Answer 20PA.

Consider the following data:

A new car costs \$23,000. It is expected to depreciate 12% each year.

The objective is to find the value of the car in 5 years.

Let  $y(t)$  be represent the cost of car at  $t$  years.

From the data,

Initially the car cost is \$23,000.

The car cost is depreciate in each year is,  $r = 12\%$   
 $= 0.12$

Use the rule,

“The general equation for exponential decay is  $c(1-r)^t$ .

Where  $y$  – represents the final amount,  $c$  – represents the initial amount,  $r$  – represents the rate of decay and  $t$  – represents year”.

$$\begin{aligned}y &= c(1-r)^t && \text{(The equation for exponential growth)} \\&= 23,000(1-0.12)^5 && \left( \begin{array}{l} \text{Replace } c \text{ by } 23,000 \\ r \text{ by } 0.12 \text{ and } t \text{ by } 5 \end{array} \right) \\&= 23,000(0.88)^5 \\&= 23,000(0.53) \\&= 12,138\end{aligned}$$

Therefore, after 5 years the cost of car is \$12,138.

Hence, the cost of car in 5 years is  $\boxed{\$12,138}$ .

### Answer 21PA.

Consider the following data:

The percent of population those 65 years old or older continues to rise.

And the percent of the US population 'p' that is least 65 years old can be approximated by

$$p = 3.86(1.013)^t.$$

Where  $t$  represents number of year since 1900.

The objective is to find the percent of population will be 65 years of age or order for  $(2010 - 1900) = 110$  years after 1900.

Now, substitute  $t = 110$  in the equation  $p = 3.86(1.013)^t$ .

Hence, obtain the population that 65 years after 110 years is,

$$\begin{aligned} p &= 3.86(1.013)^t && \text{(Original equation)} \\ &= 3.86(1.013)^{110} && \text{(Replace } t \text{ by 110)} \\ &= 3.86(4.1403) \\ &= 15.98 \end{aligned}$$

Therefore, the population will be 65 years age older for  $(1900 + 110) = 2010$  year is 15.98 percent.

Hence, the population will be 65 years of age or older in year 2010 is 15.98%.

### Answer 22PA.

Consider the following data:

The percent of population those 65 years old or older continues to rise. The percent of the US population 'p' that is least 65 years old can be approximated by  $p = 3.86(1.013)^t$ .

Where  $t$  represents number of year since 1990.

The objective is to find the percent of population will be 65 years of age or order for  $(2010 - 1900) = 110$  years after 1900.

Now, substitute  $t = 110$  in the equation  $p = 3.86(1.013)^t$ .

The population those 65 years after 110 years is,

$$\begin{aligned} p &= 3.86(1.013)^t && \text{(Original equation)} \\ &= 3.86(1.013)^{110} && \text{(Replace } t \text{ by 110)} \\ &= 3.86(4.1403) \\ &= 15.98 \end{aligned}$$

Therefore, the population will be 65 years age older for  $(1900 + 110) = 2010$  year is 15.98 percent.

Hence, the population will be 65 years of age or older in year 2010 is 15.98%.

### Answer 23PA.

The objective is to determine whether the following equation represents growth or decay.

Consider the equation is  $y = 500(1.026)^t$ .

Rewrite the equation  $y = 500(1.026)^t$  in the standard form of growth decay of

$$y = c(1+r)^t \text{ or } y = c(1-r)^t \text{ Respectively}$$

$$y = 500(1.026)^t \quad \text{original equation}$$

$$y = 500(1+1.026)^t \quad \text{write 1.026 as } 1+0.026$$

Therefore,  $y = 500(1+1.026)^t$  is in the form of  $y = c(1+r)^t$ .

Use the rule,

"If the value of  $(1+r)$  is greater than 1, then the value of exponential  $(1+r)^t$  is an increasing and  $y = c(1+r)^t$  is an exponential growth.

If the value of  $(1-r)$  is less is than "1" then the value of  $(1-r)^t$  is a decreasing and  $y = c(1-r)^t$  is an exponential decay".

Therefore, the equation  $y = 500(1+1.026)^t$  is an exponential growth.

And annual rate is,

$$r = 0.026$$

$$= 2.6\%$$

Hence, the equation  $y = 500(1+1.026)^t$  is an exponential growth.

The initial value  $c = 500$ .

And the exponential rate  $r = 2.6\%$ .

### Answer 24PA.

The objective is to determine whether the following equation represents growth or decay.

Consider the equation is  $y = 500(0.761)^t$ .

Rewrite the equation  $y = 500(0.761)^t$  in the standard form of growth decay of

$y = c(1+r)^t$  or  $y = c(1-r)^t$  respectively.

$y = 500(0.761)^t$       original equation

$y = 500(1-0.239)^t$       write 0.761 as  $1-0.239$

Therefore,  $y = 500(1-0.239)^t$  is in the form of  $y = c(1-r)^t$ .

Use the rule,

"If the value of  $(1+r)$  is greater than 1, then the value of exponential  $(1+r)^t$  is an increasing and  $y = c(1+r)^t$  is an exponential growth.

If the value of  $(1-r)$  is less is than "1" then the value of  $(1-r)^t$  is a decreasing and  $y = c(1-r)^t$  is an exponential decay".

Therefore, the equation  $y = 500(1-0.239)^t$  is an exponential decay.

The initial value  $c = 500$  and exponential rate is  $r = 0.239$ .

Hence, the equation  $y = 500(1-0.239)^t$  is an exponential decay.

The initial value  $c = 500$ .

The exponential rate  $r = 23.9\%$ .

### Answer 25PA.

Consider the following data:

The half-life of a radioactive elements is the that it takes for one – half a quantity of element to decay carbon – 14 is found in all living organisms and has a half-life of 5730 years.

Archaeologists use this fact to estimate the age of fossils.

Consider organisms with an original carbon -14 content of 256 grams.

The number of grams remaining in the organisms' fossil after  $t$  years is  $256 \cdot (0.5)^{\frac{t}{5730}}$ .

Let  $y(t)$  be represents the number of grams remaining in organisms fossils.

The objective is to find amount of carbon – 14 today, of the organism diet 5730 years.

Substitute  $t = 5730$  years in original equation  $y = 256 \cdot (0.5)^{\frac{t}{5730}}$ .

$$\begin{aligned}y &= 256 \cdot (0.5)^{\frac{t}{5730}} && \left( \begin{array}{l} \text{Equation for the number of grams} \\ \text{remaining in carbon -14} \end{array} \right) \\&= 256 \cdot (0.5)^{\frac{5730}{5730}} && \text{Replace } t \text{ by } 5730 \\&= 256 \cdot (0.5)^1 \\&= 128\end{aligned}$$

Therefore, if the organism died 5730 years ago, the amount of carbon – 14 today is 128 grams.

Hence, the amount of carbon – 14 today is 128 grams if the organism diet 5730 years ago.

### Answer 26PA.

Find the amount of carbon – 14 today, of the organism diet 1000 years.

Put  $t = 1000$  years into original equation  $y = 256 \cdot (0.5)^{\frac{t}{5730}}$

$$\begin{aligned}y &= 256 \cdot (0.5)^{\frac{t}{5730}} && \left( \begin{array}{l} \text{Equation for the number of grams} \\ \text{remaining in carbon -14} \end{array} \right) \\&= 256 \cdot (0.5)^{\frac{1000}{5730}} && \text{(Replace } t \text{ by } 1000) \\&= 256 \cdot (0.5)^{0.1746} \\&= 256(0.89) \\&= 226.8\end{aligned}$$

Therefore, if the organism died 1000 years ago, the amount of carbon – 14 today is

226.8 grams.

**Answer 27PA.**

Find the amount of carbon – 14 today, of the organism diet 10,000 years.

Put  $t = 10,000$  years into original equation  $y = 256 \cdot (0.5)^{\frac{t}{5730}}$

$$y = 256 \cdot (0.5)^{\frac{t}{5730}} \quad \left( \begin{array}{l} \text{Equation for the number of grams} \\ \text{remaining in carbon -14} \end{array} \right)$$

$$= 256 \cdot (0.5)^{\frac{10000}{5730}} \quad (\text{Replace } t \text{ by 1000})$$

$$= 256 \cdot (0.5)^{1.75}$$

$$= 256(0.29)$$

$$= 76.109$$

Hence, the amount of carbon – 14 today is 76.109 grams if the organism diet 10,000 years ago.

**Answer 28PA.**

Put  $y(t) = 32$  grams in the original equation  $y = 256 \cdot (0.5)^{\frac{t}{5730}}$

$$y = 256 \cdot (0.5)^{\frac{t}{5730}} \quad \left( \begin{array}{l} \text{Equation for the number of grams remaining} \\ \text{in the organisms fassil after 't' years} \end{array} \right)$$

$$32 = 256 \cdot (0.5)^{\frac{t}{5730}} \quad (\text{Replace } y(t) \text{ by 32})$$

$$\frac{32}{256} = \frac{256}{256} \cdot (0.5)^{\frac{t}{5730}} \quad (\text{Divide by 256 each side})$$

$$\frac{32}{256} = (0.5)^{\frac{t}{5730}}$$

$$\log\left(\frac{32}{256}\right) = \log(0.5)^{\frac{t}{5730}} \quad (\text{taking log on both sides})$$

$$\log\left(\frac{32}{256}\right) = \frac{t}{5730} \log(0.5) \quad (\text{Use the rule } \log a^m = m \log a)$$

$$\frac{t}{5730} = \frac{\log\left(\frac{32}{256}\right)}{\log(0.5)}$$

$$\frac{t}{5730} = 3$$

$$t = (5730)(3)$$

$$t = 17,190$$

Hence the organism is 17,190 years old.

### Answer 30PA.

Use the given equation  $y = 1698(1 + 0.046)^t$  to find the average family spending for restaurant meals in 2010 which is in the form of standard exponential growth  $y = c(1+r)^t$

Here  $y$  represents the final amount at  $t$  years

$c$  represents the initial amount at 1698

$r$  represents the rate of growth is 0.046 (or) 4.6% and  $t$  represents the years.

Put  $t = 2010$  into the original equation  $y = 1698(1 + 0.046)^t$

$$y = 1698(1 + 0.046)^t \quad (\text{original equation})$$

$$y = 1698(1 + 0.046)^{2010} \quad (\text{Replace } t \text{ by } 2010)$$

$$= 1698(1.046)^{2010}$$

$$= 1698(1.814 \times 10^{39})$$

$$= 3.08 \times 10^{42}$$

Therefore the average family's spending for restaurant meals in 2010 is  $\boxed{3.08 \times 10^{42}}$ .

### Answer 31PA.

We observe the choices (A) & (B) which are  $y = 50x^3$  and  $y = 30x^2 + 10$  respectively. These two are polynomial in  $x$ .

So eliminate the choices (A) & (B). Now we have the choice (C) & (D).

Consider the choice (D) is  $y = 80(0.92)^t$  in the form of

$$y = c(1+r)^t \text{ or } y = c(1-r)^t$$

$$y = 80(0.92)^t \quad (\text{original equation})$$

$$y = 80(1 - 0.08)^t$$

Therefore  $y = 80(1 - 0.08)^t$  is in the form of  $y = c(1-r)^t$

Use the rule

" $y = c(1-r)^t$  is the general equation for exponential decay".

Therefore  $y = 80(1 - 0.08)^t$  is exponential decay.

So eliminate the choice  $\boxed{D}$  also

The equation for exponential growth is  $y = 35(1.05)^x$

Hence, Answer is  $\boxed{C}$

**Answer 32PA.**

Put  $P = \$5000$ ,  $r = 0.0825$ ,  $n = 4$  and  $t = 4$  into  $A = P\left(1 + \frac{r}{n}\right)^{nt}$  we obtain the balance of account after 4 years

$$\begin{aligned}
 A &= P\left(1 + \frac{r}{n}\right)^{nt} && \text{(equation for compounded interest)} \\
 &= 5000\left(1 + \frac{0.0825}{4}\right)^{4 \cdot 4} && \text{(Replace } P \text{ by } 5,000, r \text{ by } 0.0825, n \text{ by } 4, t \text{ by } 4) \\
 &= 5000\left(\frac{4 + 0.0825}{4}\right)^{16} \\
 &= 5000\left(\frac{4.0825}{4}\right)^{16} \\
 &= 5000(1.020625)^{16} \\
 &= 5000(1.39) \\
 &= 6,931.53
 \end{aligned}$$

Therefore, the balance of account after 4 years is \$6,931.53

Answer: D

**Answer 33MYS.**

Graph the function  $y = \left(\frac{1}{8}\right)^x$

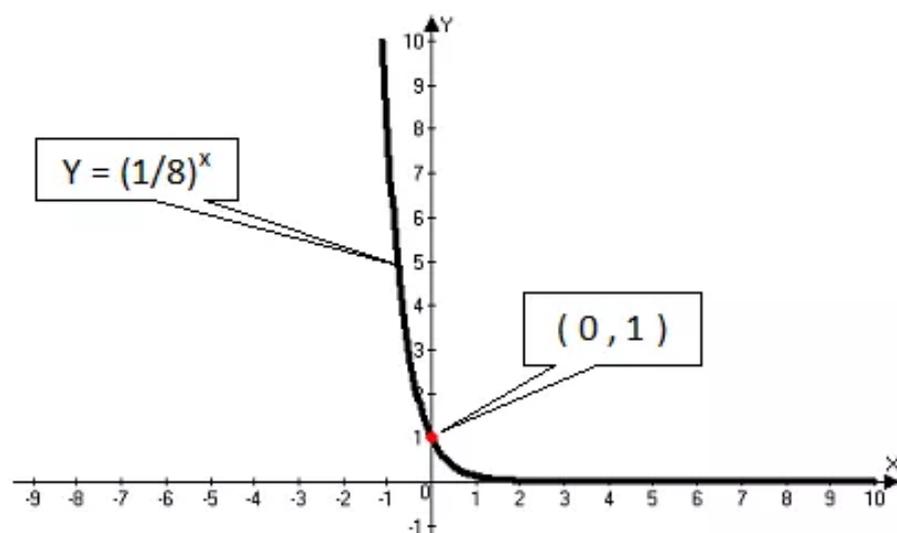
Construct the table for  $y = \left(\frac{1}{8}\right)^x$

Substitute the different values of 'x' in original function  $y = \left(\frac{1}{8}\right)^x$  we obtain values of y. plotting these all ordered pairs and connected them we obtain smooth curve.

Table  $y = \left(\frac{1}{8}\right)^x$ :

$x$	$\left(\frac{1}{8}\right)^x$	$y$	$(x, y)$
-3	$\left(\frac{1}{8}\right)^{-3} = 512$	512	$(-3, 512)$
-2	$\left(\frac{1}{8}\right)^{-2} = 64$	64	$(-2, 64)$
-1	$\left(\frac{1}{8}\right)^{-1} = 8$	8	$(-1, 8)$
0	$\left(\frac{1}{8}\right)^0 = 1$	1	$(0, 1)$
1	$\left(\frac{1}{8}\right)^1 = 0.125$	0.125	$(1, 0.125)$
2	$\left(\frac{1}{8}\right)^2 = 0.0156$	0.0156	$(2, 0.0156)$
3	$\left(\frac{1}{8}\right)^3 = 0.00195$	0.00195	$(3, 0.00195)$

Connect these all ordered pairs we obtain smooth curve



To find the  $y$  – intercept of function  $y = \left(\frac{1}{8}\right)^x$

By the graph  $y = \left(\frac{1}{8}\right)^x$  the curve  $y = \left(\frac{1}{8}\right)^x$  cuts  $y$  – axis at  $(0, 1)$

Hence, the  $y$  intercept of the function  $y = \left(\frac{1}{8}\right)^x$  is  $\boxed{1}$ .

**Answer 34MYS.**

Graph the function  $y = 2^x - 5$

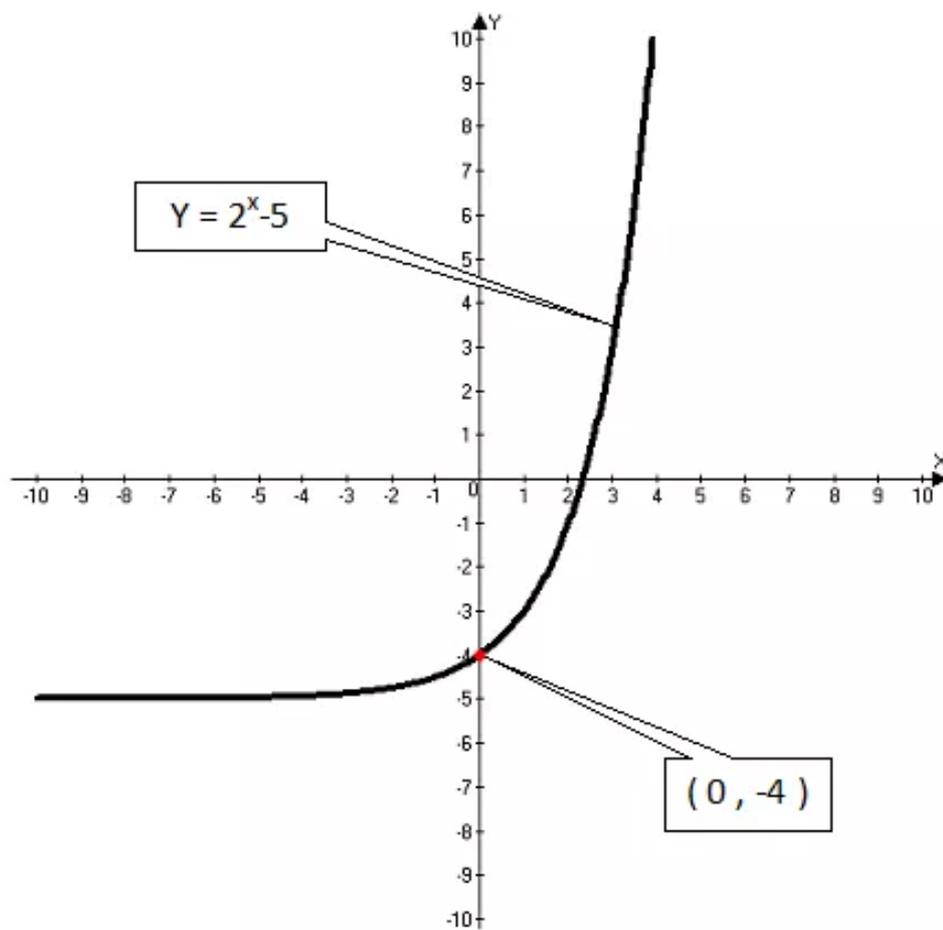
Construct the table for  $y = 2^x - 5$ .

Substitute the different values of 'x' in original function  $y = 2^x - 5$  we obtain values of  $y$ . plotting these all ordered pairs and connected them we obtain smooth curve.

Table  $y = 2^x - 5$ :

$x$	$2^x - 5$	$y$	$(x, y)$
-3	$2^{-3} - 5 = -4.875$	-4.875	$(-3, -4.875)$
-2	$2^{-2} - 5 = -4.75$	-4.75	$(-2, -4.75)$
-1	$2^{-1} - 5 = -4.5$	-4.5	$(-1, -4.5)$
0	$2^0 - 5 = -4$	-4	$(0, -4)$
1	$2^1 - 5 = -3$	-3	$(1, -3)$
2	$2^2 - 5 = -1$	-1	$(2, -1)$
3	$2^3 - 5 = 3$	3	$(3, 3)$

Connect these all ordered pairs we obtain smooth curve



Find the  $y$  – intercept of function  $y = 2^x - 5$

By the graph  $y = 2^x - 5$  the curve  $y = 2^x - 5$  cuts  $y$  – axis at  $(0, -4)$

Hence, the  $y$  intercept of the function  $y = 2^x - 5$  is  $\boxed{-4}$ .

### Answer 35MYS.

Graph the function  $y = 4(3^x - 6)$

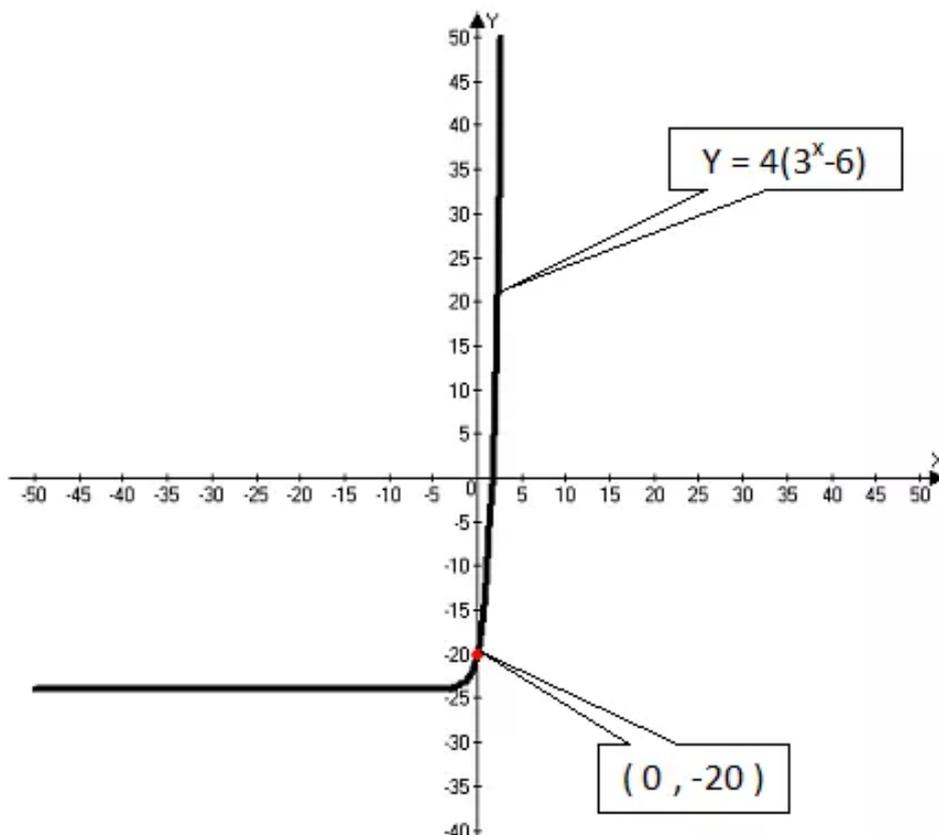
Construct the table for  $y = 4(3^x - 6)$ .

Substitute the different values of 'x' in original function  $y = 4(3^x - 6)$  we obtain values of y. plotting these all ordered pairs and connected them we obtain smooth curve.

Table  $y = 4(3^x - 6)$

x	$4(3^x - 6)$	y	(x,y)
-3	$4(3^{-3} - 6) = -23.85$	-23.85	(-3, -23.85)
-2	$4(3^{-2} - 6) = -23.56$	-23.56	(-2, -23.56)
-1	$4(3^{-1} - 6) = -22.67$	-22.67	(-1, -22.67)
0	$4(3^0 - 6) = -20$	-20	(0, -20)
1	$4(3^1 - 6) = -12$	-12	(1, -12)
2	$4(3^2 - 6) = 12$	12	(2, 12)

Connect these all ordered pairs we obtain smooth curve



Find  $y$  – intercept of function  $y = 4(3^x - 6)$ .

By the graph of  $y = 4(3^x - 6)$ , the curve  $y = 4(3^x - 6)$  at  $y$  – axis is  $(0, -20)$

Therefore,  $y$  – intercept of function  $y = 4(3^x - 6)$  is  $\boxed{-20}$ .

**Answer 36MYS.**

Compare the equation  $m^2 - 9m - 10 = 0$  with the standard form of quadratic equation

$$ax^2 + bx + c = 0$$

$a = 1, b = -9, c = -10$  and  $x = m$

Use the formula “The solution of quadratic equation  $ax^2 + bx + c = 0$  where  $a \neq 0$  is given by the quadratic formula

$$m^2 - 9m - 10 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(equation for quadratic formula)

$$m = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(1)(-10)}}{2 \cdot 1}$$

(Replace  $a$  by 1,  $b$  by  $-9$   
 $c$  by  $-10$  and  $x$  by  $m$ )

$$= \frac{9 \pm \sqrt{81 + 40}}{2}$$

$$= \frac{9 \pm \sqrt{121}}{2}$$

$$= \frac{9 \pm 11}{2}$$

$$m = \frac{9 - 11}{2} \text{ or } m = \frac{9 + 11}{2}$$

$$= \frac{2}{2} \quad \text{or} \quad = \frac{20}{2}$$

$$= -1 \quad \text{or} \quad = 10$$

Therefore, the solutions to the given quadratic equations are  $m = \boxed{-1}$  or  $m = \boxed{10}$ .

Check: Put each value of  $m$  in original equation  $m^2 - 9m - 10 = 0$

$$m^2 - 9m - 10 = 0 \quad (\text{original equation})$$

$$(-1) - 9(-1) - 10 = 0 \quad (\text{Replace } m \text{ by } -1)$$

$$1 + 9 - 10 = 0$$

$$10 - 10 = 0$$

$$0 = 0 \quad \text{True}$$

$$m^2 - 9m - 10 = 0 \quad (\text{original equation})$$

$$(10) - 9(10) - 10 = 0 \quad (\text{Replace } m \text{ by } 10)$$

$$10 - 90 - 10 = 0$$

$$100 - 100 = 0$$

$$0 = 0 \quad \text{True}$$

Therefore, each value of  $m = -1$  and  $m = 10$  satisfies the equation  $m^2 - 9m - 10 = 0$

Hence, solution set is  $\{-1, 10\}$ .

### Answer 37MYS.

Consider the equation:

$$2t^2 - 4t = 3$$

The objective is to solve the equation  $2t^2 - 4t = 3$  by using quadratic formula

Rewrite the equation  $2t^2 - 4t = 3$  in the standard form of quadratic equation  $ax^2 + bx + c = 0$

$$2t^2 - 4t = 3 \quad (\text{original equation})$$

$$2t^2 - 4t - 3 = 3 - 3 \quad (\text{Subtract 3 on both side})$$

$$2t^2 - 4t - 3 = 0$$

Now solve the equation  $2t^2 - 4t - 3 = 0$  by using quadratic formula

Now compare the equation  $2t^2 - 4t - 3 = 0$  with the standard form of quadratic equation

$$ax^2 + bx + c = 0$$

We obtain  $a = 2$ ,  $b = -4$ ,  $c = -3$  and  $x = t$

Use the formula "The solution of quadratic equation  $ax^2 + bx + c = 0$  where  $a \neq 0$  is given by the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (\text{equation for quadratic formula})$$

$$t = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-3)}}{2 \cdot (2)} \quad \left( \begin{array}{l} \text{Replace } a \text{ by } 2, b \text{ by } -4 \\ c \text{ by } -3 \text{ and } x \text{ by } t \end{array} \right)$$

$$t = \frac{4 \pm \sqrt{16 + 24}}{4}$$

$$t = \frac{4 \pm \sqrt{40}}{4}$$

$$t = \frac{4 + \sqrt{40}}{4} \quad \text{or} \quad t = \frac{4 - \sqrt{40}}{4}$$

$$t \approx 2.6 \quad \text{or} \quad t \approx -0.6$$

Therefore  $t \approx -0.6$  or  $t \approx 2.6$ .

Check:

Substitute each value  $t$  in the original equation  $2t^2 - 4t = 3$

$$2t^2 - 4t = 3 \quad (\text{original equation})$$

$$2(-0.6)^2 - 4(-0.6) \stackrel{?}{=} 3 \quad (\text{Replace } t \text{ by } -0.6)$$

$$0.72 - 2.4 \stackrel{?}{=} 3$$

$$3 = 3 \quad \text{True}$$

$$2t^2 - 4t = 3 \quad (\text{original equation})$$

$$2(2.6)^2 - 4(2.6) \stackrel{?}{=} 3 \quad (\text{Replace } t \text{ by } 2.6)$$

$$2(6.76) - 4(10.4) \stackrel{?}{=} 3$$

$$13.52 - 41.6 \stackrel{?}{=} 3$$

$$3 = 3 \quad \text{True}$$

Thus, each value of ' $t$ ' satisfies the equation  $2t^2 - 4t = 3$

Therefore, the solution set is  $\{-0.6, 2.6\}$ .

## Answer 38MYS.

Consider the equation:

$$7x^2 + 3x + 1 = 0$$

The objective is to solve the equation  $7x^2 + 3x + 1 = 0$  by using quadratic formula.

Now compare the equation  $7x^2 + 3x + 1 = 0$  in the standard form of quadratic equation

$$ax^2 + bx + c = 0 \text{ where } a \neq 0. \text{ We obtain } a = 7, b = 3, c = 1$$

Use the formula "The solution of quadratic equation  $ax^2 + bx + c = 0$  where  $a \neq 0$  is given by the quadratic formula"

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (\text{Equation for quadratic formula})$$

$$= \frac{-(3) \pm \sqrt{3^2 - 4 \cdot 7 \cdot 1}}{2 \cdot 1} \quad (\text{Replace } a \text{ by } 7, b \text{ by } 3, c \text{ by } 1)$$

$$= \frac{-3 \pm \sqrt{9 - 28}}{2}$$

$$= \frac{-3 \pm \sqrt{-19}}{2}$$

$$= \frac{-3 \pm \sqrt{-1} \sqrt{19}}{2}$$

$$= \frac{-3 \pm i \sqrt{19}}{2}$$

Therefore, the equation  $7x^2 + 3x + 1 = 0$  has no real roots

Because the equation  $7x^2 + 3x + 1 = 0$  has complex roots.

oots

Consider the equation:

$$7x^2 + 3x + 1 = 0$$

The objective is to solve the equation  $7x^2 + 3x + 1 = 0$  by using quadratic formula.

Now compare the equation  $7x^2 + 3x + 1 = 0$  in the standard form of quadratic equation

$$ax^2 + bx + c = 0 \text{ where } a \neq 0. \text{ We obtain } a = 7, b = 3, c = 1$$

Use the formula "The solution of quadratic equation  $ax^2 + bx + c = 0$  where  $a \neq 0$  is given by the quadratic formula"

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (\text{Equation for quadratic formula})$$

$$= \frac{-(3) \pm \sqrt{3^2 - 4 \cdot 7 \cdot 1}}{2 \cdot 1} \quad (\text{Replace } a \text{ by } 7, b \text{ by } 3, c \text{ by } 1)$$

$$= \frac{-3 \pm \sqrt{9 - 28}}{2}$$

$$= \frac{-3 \pm \sqrt{-19}}{2}$$

$$= \frac{-3 \pm \sqrt{-1} \sqrt{19}}{2}$$

$$= \frac{-3 \pm i\sqrt{19}}{2}$$

Therefore, the equation  $7x^2 + 3x + 1 = 0$  has no real roots

Because the equation  $7x^2 + 3x + 1 = 0$  has complex roots.

oots

### Answer 39MYS.

Consider the expression  $m^7 (m^3 b^2)$

$$\begin{aligned} m^7 (m^3 b^2) &= (m^7 m^3) b^2 && \left( \begin{array}{l} \text{Use the associate rule} \\ a \cdot (b \cdot c) = (a \cdot b) \cdot c \end{array} \right) \\ &= (m^{7+3}) b^2 && \left( \text{Use the rule } a^m a^n = a^{m+n} \right) \\ &= m^{10} b^2 \end{aligned}$$

Therefore,  $m^7 (m^3 b^2) = m^{10} b^2$

**Answer 40MYS.**

Consider the expression  $-3(ax^3y)^2$

$$\begin{aligned}
 -3(ax^3y)^2 &= -3(a^2(x^3)^2 \cdot y^2) && \left( \begin{array}{l} \text{Use the associate rule} \\ (abc)^m = a^m b^m c^m \end{array} \right) \\
 &= -3a^2 x^{3 \cdot 2} y^2 && \left( \text{Use the rule } (a^m)^n = a^{mn} \right) \\
 &= -3a^2 x^6 y^2
 \end{aligned}$$

Therefore,  $\boxed{-3(ax^3y)^2 = -3a^2 x^6 y^2}$

**Answer 41MYS.**

Consider the expression  $(0.3x^3y^2)^2$

$$\begin{aligned}
 (0.3x^3y^2)^2 &= (0.3)^2 (x^3)^2 (y^2)^2 && \left( \begin{array}{l} \text{Use the associate rule} \\ (abc)^m = a^m b^m c^m \end{array} \right) \\
 &= 0.09 x^{3 \cdot 2} y^{2 \cdot 2} && \left( \text{Use the rule } (a^m)^n = a^{mn} \right) \\
 &= 0.09 x^6 y^4
 \end{aligned}$$

Therefore,  $\boxed{(0.3x^3y^2)^2 = 0.09x^6y^4}$

**Answer 42MYS.**

Consider the equation  $|7x + 2| = 2$

Claim: Solve for  $|7x + 2| = 2$

$$|7x + 2| = -2 \quad (\text{original equation})$$

$$7x + 2 = \pm(-2) \quad (\text{use the rule if } |x| = a \text{ then } x = \pm a)$$

$$7x + 2 - 2 = -2 \pm(-2) \quad (\text{Subtract 2 on both sides})$$

$$7x = -2 \pm(-2)$$

$$\frac{7x}{7} = \frac{-2 \pm(-2)}{7} \quad (\text{Divide 7 on both sides})$$

$$x = \frac{-2 \pm(-2)}{7}$$

$$x = \frac{-2 + (-2)}{7} \quad \text{or} \quad x = \frac{-2 - (-2)}{7}$$

$$x = \frac{-2 - 2}{7} \quad \text{or} \quad x = \frac{-2 + 2}{7}$$

$$x = \frac{-4}{7} \quad \text{or} \quad x = \frac{0}{7}$$

$$x = 0.57 \quad \text{or} \quad x = 0$$

Therefore, the solution set is  $\boxed{\{0, 0.57\}}$

**Answer 43MYS.**

Consider the equation  $|3 - 3x| = 0$

Claim: Solve for  $|3 - 3x| = 0$

$$|3 - 3x| = 0 \quad (\text{original equation})$$

$$3 - 3x = \pm(0) \quad (\text{use the rule if } |x| = a \text{ then } x = \pm a)$$

$$3 - 3x + 3x = 0 + 3x \quad (\text{Add } 3x \text{ on both sides})$$

$$3 = 3x$$

$$3x = 3$$

$$\frac{3x}{3} = \frac{3}{3} \quad (\text{Divide by 3 on both sides})$$

$$x = 1$$

Therefore, the solution set is  $\boxed{\{1\}}$

**Answer 44MYS.**

Consider the equality  $|t+4| \geq 3$

Claim: Solve for  $|t+4| \geq 3$

$$|t+4| \geq 3 \quad (\text{original equation})$$

$$t+4 \geq \pm 3 \quad (\text{use the rule if } |x| \geq a \text{ then } x \geq \pm a)$$

$$t+4-4 \geq \pm 3-4 \quad (\text{Subtract 4 on both sides})$$

$$t \geq \pm 3-4$$

$$t \geq 3-4 \quad \text{or} \quad t \geq -3-4$$

$$t \geq -1 \quad \text{or} \quad t \geq -7$$

Hence, solution set is  $\{t | -1 < t\}$  or  $\{t | -7 < t\}$

**Answer 45MYS.**

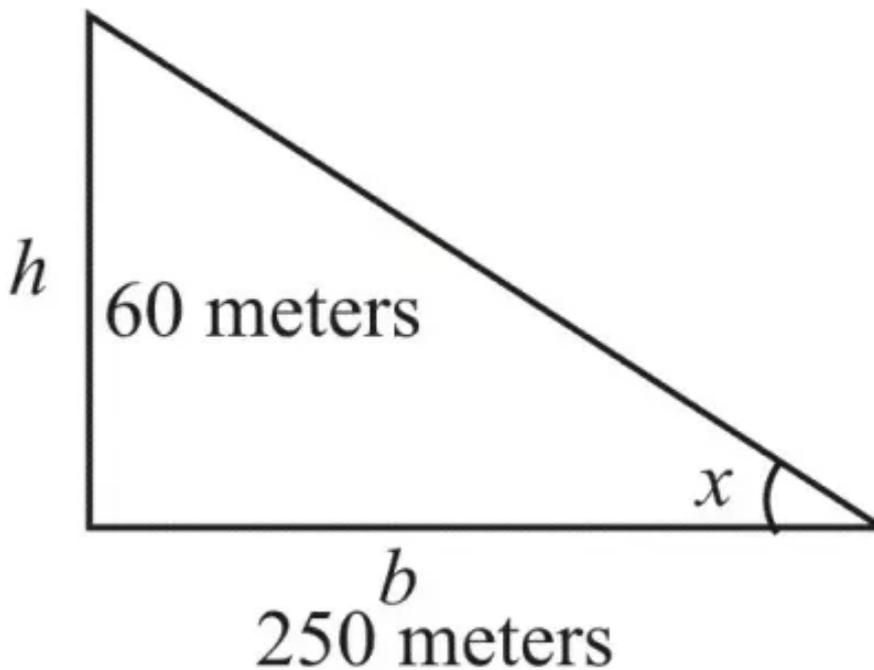
Consider the slope of any hill in the country cannot be greater than 0.33

We have the rule  $\boxed{\text{slope} = \text{Tan}\theta}$

According to the parabola a hill rises 60 meters over a horizontal distance

The height of the hill  $h = 60$  meters

The horizontal distance  $b = 150$  meters



Slope the hill =  $\tan \theta$

$$= \frac{\text{opposite side of } x}{\text{Adjacent side of } x}$$

$$= \frac{60}{250}$$

$$= \frac{6}{25}$$

$$= 0.24$$

Slope of the hill = 0.24

Therefore, the slope of the hill 0.24 is not greater than 0.33

$$0.24 < 0.33$$

Hence, The hill meet the requirements.

### Answer 46MYS.

Consider the sequence 8, 11, 14, 17, .....

Step1: Look at the pattern



$$\begin{array}{cccc} 8 & 11 & 14 & 17 \\ & +3 & +3 & +3 \end{array}$$

In this is sequence, each term is found by adding 3 to previous terms. This sequence is arithmetic.

Step2: The common difference is 3. Use this information to find next three terms.



$$\begin{array}{ccccccc} 8, & 11, & 14, & 17, & 20, & 23, & 26, \\ & & & & +3 & +3 & +3 \end{array}$$

The next three terms are 20, 23 and 26

**Answer 47MYS.**

Consider the sequence 7, 4, 1, -2, .....

Step1: Look at the pattern



7      4      1      -2    ....  
      -3     -3     -3

In this is sequence, each term is found by adding  $-3$  to previous terms. This sequence is arithmetic.

Step2: The common difference is  $-3$ . Use this information to find next three terms.



7    4    1    -2    -5    -8    -11  
              -3    -3    -3

The next three terms are  $-5$ ,  $-8$  and  $-11$

**Answer 48MYS.**

Consider the sequence 1.5, 2.6, 3.7, 4.8, .....

Step1: Look at the pattern



1.5      2.6      3.7      4.8  
      1.1      1.1      1.1

In this is sequence, each term is found by adding 1.1 to previous terms. This sequence is arithmetic.

Step2: The common difference is 1.1. Use this information to find next three terms.



1.5    2.6    3.7    4.8    5.9    7.0    8.1  
              1.1    1.1    1.1

The next three terms are 5.9, 7.0 and 8.1