Angles, Linesand Triangles

Exercise 4A

Question 1:

(i) Angle: Two rays having a common end point form an angle.

(ii) Interior of an angle: The interior of ∠AOB is the set of all points in its plane, which lie on the same side of OA as B and also on same side of OB as A.

(iii) Obtuse angle: An angle whose measure is more than 90° but less than 180°, is called an obtuse angle.

(iv) Reflex angle: An angle whose measure is more than 180° but less than 360° is called a reflex angle.

(v) Complementary angles: Two angles are said to be complementary, if the sum of their measures is 90o.

(vi) Supplementary angles: Two angles are said to be supplementary, if the sum of their measures is 180°.

Question 2:

∠A = 36° 27′ 46″ and ∠B = 28° 43′ 39″

∴ Their sum = (36° 27′ 46″) + (28° 43′ 39″)

	Deg	Min	Sec
	36°	27′	46″
+	28°	43′	39″
	65°	11′	25″

[1° = 60′; 1′ = 60″]

Therefore, the sum ∠A + ∠B = 65° 11′ 25″

Question 3:

Let ∠A = 36° and ∠B = 24° 28′ 30″

Their difference = 36° – 24° 28′ 30″

	Deg	Min	Sec
	36°	0′	0″
−	24°	28′	30″
	11°	31′	30″

[1°=60′; 1′=60″]

Thus the difference between two angles is ∠A – ∠B = 11° 31′ 30″

Question 4:

(i) Complement of $58^\circ = 90^\circ - 58^\circ = 32^\circ$

(ii) Complement of $16^\circ = 90 - 16^\circ = 74^\circ$

(iii) $\frac{2}{3}$ of a right angle $= \frac{2}{3} \times 90^\circ = 60^\circ$

Complement of $60^\circ = 90^\circ - 60^\circ = 30^\circ$

(iv) $1^\circ = 60'$

$\Rightarrow 90^\circ = 89^\circ\, 60'$

	Deg	Min
	89°	60′
	~~90°~~	~~0′~~
−	46°	30′
	43°	30′

Complement of $46^\circ\, 30' = 90^\circ - 46^\circ\, 30' = 43^\circ\, 30'$

(v) $90^\circ = 89^\circ\, 59'\, 60''$

	Deg	Min	Sec
	89°	59′	60″
	~~90°~~	~~0′~~	~~0″~~
−	52°	43′	20″
	37°	16′	40″

Complement of $52^\circ\, 43'\, 20'' = 90^\circ - 52^\circ\, 43'\, 20''$

$= 37^\circ\, 16'\, 40''$

(vi) $90^\circ = 89^\circ\, 59'\, 60''$

	Deg	Min	Sec
	89°	59′	60″
	~~90°~~	~~0′~~	~~0″~~
−	68°	35′	45″
	21°	24′	15″

$\therefore$ Complement of $(68^\circ\, 35'\, 45'')$

$= 90^\circ - (68^\circ\, 35'\, 45'')$

$= 89^\circ\, 59'\, 60'' - (68^\circ\, 35'\, 45'')$

$= 21^\circ\, 24'\, 15''$

Question 5:

(i) Supplement of $63^\circ = 180^\circ - 63^\circ = 117^\circ$

(ii) Supplement of $138^\circ = 180^\circ - 138^\circ = 42^\circ$

(iii) $\frac{3}{5}$ of a right angle $= \frac{3}{5} \times 90^\circ = 54^\circ$

$\therefore$ Supplement of $54^\circ = 180^\circ - 54^\circ = 126^\circ$

(iv) $1^\circ = 60'$

$\Rightarrow 180^\circ = 179^\circ\, 60'$

	Deg	Min
	179°	60′
	~~180°~~	~~0′~~
−	75°	36′
	104°	24′

Supplement of $75^{o}\,36'$ = $180^{o} - 75^{o}\,36'$ = $104^{o}\,24'$

(v) $1^{o} = 60'$, $1' = 60''$

⇒ $180^{o} = 179^{o}\,59'\,60''$

	Deg	Min	Sec
	179°	59′	60″
	~~180°~~	~~0′~~	~~0″~~
−	124°	20′	40″
	55°	39′	20″

Supplement of $124^{o}\,20'\,40''$ = $180^{o} - 124^{o}\,20'\,40''$

= $55^{o}\,39'\,20''$

(vi) $1^{o} = 60'$, $1' = 60''$

⇒ $180^{o} = 179^{o}\,59'\,60''$

	Deg	Min	Sec
	179°	59′	60″
	~~180°~~	~~0′~~	~~0″~~
−	108°	48′	32″
	71°	11′	28″

∴ Supplement of $108^{o}\,48'\,32''$ = $180^{o} - 108^{o}\,48'\,32''$

= $71^{o}\,11'\,28''$.

Question 6:

(i) Let the required angle be x^{o}

Then, its complement = $90^{o} - x^{o}$

$$\therefore \quad x^{o} = 90^{o} - x^{o}$$
$$\Rightarrow \quad x + x = 90$$
$$\Rightarrow \quad 2x = 90$$
$$\Rightarrow \quad x = \frac{90}{2} = 45$$

∴ The measure of an angle which is equal to its complement is 45^{o}.

(ii) Let the required angle be x^{o}

Then, its supplement = $180^{o} - x^{o}$

$$\therefore \quad x^{o} = 180^{o} - x^{o}$$
$$\Rightarrow \quad x + x = 180$$
$$\Rightarrow \quad 2x = 180$$
$$\Rightarrow \quad x = \frac{180}{2} = 90$$

∴ The measure of an angle which is equal to its supplement is 90^{o}.

Question 7:

Let the required angle be x^{o}

Then its complement is $90^{o} - x^{o}$

$$\Rightarrow \quad x^{o} = \left(90^{o} - x^{o}\right) + 36^{o}$$
$$\Rightarrow \quad x^{o} + x^{o} = 90^{o} + 36^{o}$$
$$\Rightarrow \quad 2x^{o} = 126^{o}$$
$$\Rightarrow \quad x = \frac{126}{2} = 63$$

∴ The measure of an angle which is 36^{o} more than its complement is 63^{o}.

Question 8:

Let the required angle be x^{o}

Then its supplement is $180^{o} - x^{o}$

$$\Rightarrow \quad x^{o} = \left(180^{o} - x^{o}\right) - 25^{o}$$
$$\Rightarrow \quad x^{o} + x^{o} = 180^{o} - 25^{o}$$
$$\Rightarrow \quad 2x = 155$$
$$\Rightarrow \quad x = \frac{155}{2} = 77\frac{1}{2}$$

∴ The measure of an angle which is 25° less than its supplement is $77\frac{1}{2}^\circ = 77.5^\circ$.

Question 9:

Let the required angle be x°

Then, its complement = 90° – x°

$$\Rightarrow \quad x^\circ = 4(90^\circ - x^\circ)$$
$$\Rightarrow \quad x^\circ = 360^\circ - 4x^\circ$$
$$\Rightarrow \quad 5x = 360$$
$$\Rightarrow \quad x = \frac{360}{5} = 72$$

∴ The required angle is 72°.

Question 10:

Let the required angle be x°

Then, its supplement is 180° – x°

$$\Rightarrow \quad x^\circ = 5(180^\circ - x^\circ)$$
$$\Rightarrow \quad x^\circ = 900^\circ - 5x^\circ$$
$$\Rightarrow \quad x + 5x = 900$$
$$\Rightarrow \quad 6x = 900$$
$$\Rightarrow \quad x = \frac{900}{6} = 150.$$

∴ The required angle is 150°.

Question 11:

Let the required angle be x°

Then, its complement is 90° – x° and its supplement is 180° – x°

That is we have,

$$180^\circ - x^\circ = 4(90^\circ - x^\circ)$$
$$180^\circ - x^\circ = 360^\circ - 4x^\circ$$
$$4x^\circ - x^\circ = 360^\circ - 180^\circ$$
$$3x = 180$$
$$x = \frac{180}{3} = 60^\circ$$

∴ The required angle is 60°.

Question 12:

Let the required angle be x°

Then, its complement is 90° – x° and its supplement is 180° – x°

$$\therefore \quad 90^\circ - x^\circ = \frac{1}{3}(180^\circ - x^\circ)$$
$$\Rightarrow \quad 90 - x = 60 - \frac{1}{3}x$$
$$\Rightarrow \quad x - \frac{1}{3}x = 90 - 60$$
$$\Rightarrow \quad \frac{2}{3}x = 30$$
$$\Rightarrow \quad x = \frac{30 \times 3}{2} = 45$$

∴ The required angle is 45°.

Question 13:

Let the two required angles be x° and 180° – x°.

Then,

$$\frac{x^\circ}{180^\circ - x^\circ} = \frac{3}{2}$$

⇒ 2x = 3(180 – x)

⇒ 2x = 540 – 3x

⇒ 3x + 2x = 540

⇒ 5x = 540

⇒ x = 108

Thus, the required angles are 108° and 180° – x° = 180° – 108° = 72°.

Question 14:

Let the two required angles be x° and 90° – x°.

Then

$$\frac{x°}{90° - x°} = \frac{4}{5}$$

⇒ 5x = 4(90 – x)

⇒ 5x = 360 – 4x

⇒ 5x + 4x = 360

⇒ 9x = 360

⇒ x = $\frac{360}{9}$ = 40

Thus, the required angles are 40° and 90° – x° = 90° – 40° = 50°.

Question 15:

Let the required angle be x°.

Then, its complementary and supplementary angles are (90° – x) and (180° – x) respectively.

Then, 7(90° – x) = 3 (180° – x) – 10°

⇒ 630° – 7x = 540° – 3x – 10°

⇒ 7x – 3x = 630° – 530°

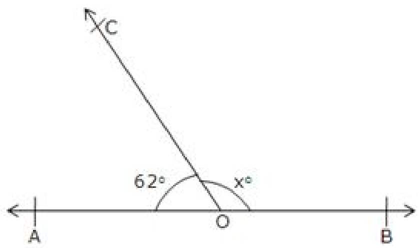
⇒ 4x = 100°

⇒ x = 25°

Thus, the required angle is 25°.

Exercise 4B

Question 1:



Since ∠BOC and ∠COA form a linear pair of angles, we have

∠BOC + ∠COA = 180°

⇒ x° + 62° = 180°

⇒ x = 180 – 62

∴ x = 118°

Question 2:

Since, ∠BOD and ∠DOA form a linear pair.

∠BOD + ∠DOA = 180°

∴ ∠BOD + ∠DOC + ∠COA = 180°

⇒ (x + 20)° + 55° + (3x – 5)° = 180°

⇒ x + 20 + 55 + 3x – 5 = 180

⇒ 4x + 70 = 180

⇒ 4x = 180 – 70 = 110

⇒ x = $\frac{110}{4}$ = 27.5

∴ ∠AOC = (3 × 27.5 – 5)° = 82.5-5 = 77.5°

And, ∠BOD = (x + 20)° = 27.5° + 20° = 47.5°.

Question 3:

Since ∠BOD and ∠DOA from a linear pair of angles.

⇒ ∠BOD + ∠DOA = 180^o

⇒ ∠BOD + ∠DOC + ∠COA = 180^o

⇒ $x^o + (2x - 19)^o + (3x + 7)^o = 180^o$

⇒ $6x - 12 = 180$

⇒ $6x = 180 + 12 = 192$

⇒ $x = \frac{192}{6} = 32$

⇒ $x = 32$

⇒ ∠AOC = $(3x + 7)^o = (3\ 32 + 7)^o = 103^o$

⇒ ∠COD = $(2x - 19)^o = (2\ 32 - 19)^o = 45^o$

and ∠BOD = $x^o = 32^o$

Question 4:

x: y: z = 5: 4: 6

The sum of their ratios = 5 + 4 + 6 = 15

But $x + y + z = 180^o$

[Since, XOY is a straight line]

So, if the total sum of the measures is 15, then the measure of x is 5.

If the sum of angles is 180^o, then, measure of $x = \frac{5}{15} \times 180 = 60$

And, if the total sum of the measures is 15, then the measure of y is 4.

If the sum of the angles is 180^o, then, measure of $y = \frac{4}{15} \times 180 = 48$

And ∠z = 180^o – ∠x – ∠y

= $180^o - 60^o - 48^o$

= $180^o - 108^o = 72^o$

∴ x = 60, y = 48 and z = 72.

Question 5:

AOB will be a straight line, if two adjacent angles form a linear pair.

∴ ∠BOC + ∠AOC = 180^o

⇒ $(4x - 36)^o + (3x + 20)^o = 180^o$

⇒ $4x - 36 + 3x + 20 = 180$

⇒ $7x - 16 = 180^o$

⇒ $7x = 180 + 16 = 196$

⇒ $x = \frac{196}{7} = 28$

∴ The value of x = 28.

Question 6:

Since ∠AOC and ∠AOD form a linear pair.

∴ ∠AOC + ∠AOD = 180^o

⇒ 50^o + ∠AOD = 180^o

⇒ ∠AOD = $180^o - 50^o = 130^o$

∠AOD and ∠BOC are vertically opposite angles.

∠AOD = ∠BOC

⇒ ∠BOC = 130^o

∠BOD and ∠AOC are vertically opposite angles.

∴ ∠BOD = ∠AOC

⇒ ∠BOD = 50^o

Question 7:

Since ∠COE and ∠DOF are vertically opposite angles, we have,

∠COE = ∠DOF

⇒ ∠z = 50°
Also ∠BOD and ∠COA are vertically opposite angles.
So, ∠BOD = ∠COA
⇒ ∠t = 90°
As ∠COA and ∠AOD form a linear pair,
∠COA + ∠AOD = 180°
⇒ ∠COA + ∠AOF + ∠FOD = 180° [∠t = 90°]
⇒ t + x + 50° = 180°
⇒ 90° + x° + 50° = 180°
⇒ x + 140 = 180
⇒ x = 180 – 140 = 40
Since ∠EOB and ∠AOF are vertically opposite angles
So, ∠EOB = ∠AOF
⇒ y = x = 40
Thus, x = 40 = y = 40, z = 50 and t = 90

Question 8:

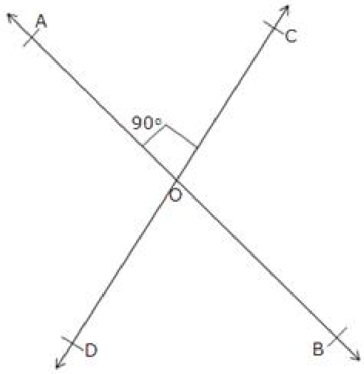
Since ∠COE and ∠EOD form a linear pair of angles.
⇒ ∠COE + ∠EOD = 180°
⇒ ∠COE + ∠EOA + ∠AOD = 180°
⇒ 5x + ∠EOA + 2x = 180
⇒ 5x + ∠BOF + 2x = 180
[∴ ∠EOA and BOF are vertically opposite angles so, ∠EOA = ∠BOF]
⇒ 5x + 3x + 2x = 180
⇒ 10x = 180
⇒ x = 18
Now ∠AOD = 2x° = 2 × 18° = 36°
∠COE = 5x° = 5 × 18° = 90°
and, ∠EOA = ∠BOF = 3x° = 3 × 18° = 54°

Question 9:

Let the two adjacent angles be 5x and 4x.
Now, since these angles form a linear pair.
So, 5x + 4x = 180°
⇒ 9x = 180°
⇒ x = $\frac{180}{9}$ = 20
∴ The required angles are 5x = 5x = 5 20° = 100°
and 4x = 4 × 20° = 80°

Question 10:

Let two straight lines AB and CD intersect at O and let ∠AOC = 90°.



Now, ∠AOC = ∠BOD [Vertically opposite angles]

⇒ ∠BOD = 90°
Also, as ∠AOC and ∠AOD form a linear pair.
⇒ 90° + ∠AOD = 180°
⇒ ∠AOD = 180° – 90° = 90°
Since, ∠BOC = ∠AOD [Verticallty opposite angles]
⇒ ∠BOC = 90°
Thus, each of the remaining angles is 90°.

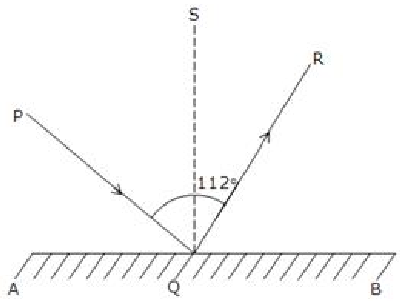
Question 11:

Since, ∠AOD and ∠BOC are vertically opposite angles.
∴ ∠AOD = ∠BOC
Now, ∠AOD + ∠BOC = 280° [Given]
⇒ ∠AOD + ∠AOD = 280°
⇒ 2∠AOD = 280°
⇒ ∠AOD = $\frac{280}{2}$ = 140°
⇒ ∠BOC = ∠AOD = 140°
As, ∠AOC and ∠AOD form a linear pair.
So, ∠AOC + ∠AOD = 180°
⇒ ∠AOC + 140° = 180°
⇒ ∠AOC = 180° – 140° = 40°
Since, ∠AOC and ∠BOD are vertically opposite angles.
∴ ∠AOC = ∠BOD
⇒ ∠BOD = 40°
∴ ∠BOC = 140°, ∠AOC = 40° , ∠AOD = 140° and ∠BOD = 40°.

Question 12:

Since ∠COB and ∠BOD form a linear pair
So, ∠COB + ∠BOD = 180°
⇒ ∠BOD = 180° – ∠COB .... (1)
Also, as ∠COA and ∠AOD form a linear pair.
So, ∠COA + ∠AOD = 180°
⇒ ∠AOD = 180° – ∠COA
⇒ ∠AOD = 180° – ∠COB .... (2)
[Since, OC is the bisector of ∠AOB, ∠BOC = ∠AOC]
From (1) and (2), we get,
∠AOD = ∠BOD (Proved)

Question 13:

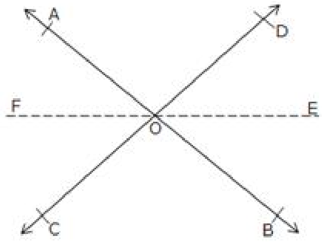


Let QS be a perpendicular to AB.
Now, ∠PQS = ∠SQR
Because angle of incident = angle of reflection
⇒ ∠PQS = ∠SQR = $\frac{112}{2}$ = 56°
Since QS is perpendicular to AB, ∠PQA and ∠PQS are complementary angles.
Thus, ∠PQA + ∠PQS = 90°

⇒ ∠PQA + 56° = 90°
⇒ ∠PQA = 90° – 56° = 34°

Question 14:

Given : AB and CD are two lines which are intersecting at O. OE is a ray bisecting the ∠BOD. OF is a ray opposite to ray OE.



To Prove: ∠AOF = ∠COF

Proof : Since $\overrightarrow{OE}$ and $\overrightarrow{OF}$ are two opposite rays, $\overleftrightarrow{EF}$ is a straight line passing through O.

∴ ∠AOF = ∠BOE

and ∠COF = ∠DOE

[Vertically opposite angles]

But ∠BOE = ∠DOE (Given)

∴ ∠AOF = ∠COF

Hence, proved.

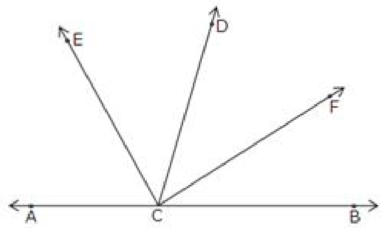
Question 15:

Given: $\overrightarrow{CF}$ is the bisector of ∠BCD and $\overrightarrow{CE}$ is the bisector of ∠ACD.

To Prove: ∠ECF = 90°

Proof: Since ∠ACD and ∠BCD forms a linear pair.

∠ACD + ∠BCD = 180°



∠ACE + ∠ECD + ∠DCF + ∠FCB = 180°

∠ECD + ∠ECD + ∠DCF + ∠DCF = 180°

because ∠ACE = ∠ECD

and ∠DCF = ∠FCB

2(∠ECD) + 2 (∠CDF) = 180°

2(∠ECD + ∠DCF) = 180°

∠ECD + ∠DCF = $\frac{180}{2}$ = 90°

∠ECF = 90° (Proved)

Exercise 4C

Question 1:

Since AB and CD are given to be parallel lines and t is a transversal.

So, ∠5 = ∠1 = 70° [Corresponding angles are equal]

∠3 = ∠1 = 70° [Vertically opp. Angles]

∠3 + ∠6 = 180° [Co-interior angles on same side]

∴ ∠6 = 180° – ∠3

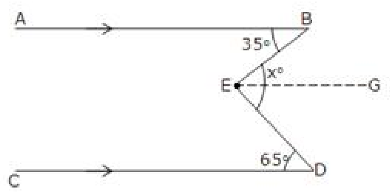
= 180° – 70° = 110°

$\angle 6 = \angle 8$ [Vertically opp. Angles]
$\Rightarrow \angle 8 = 110^o$
$\Rightarrow \angle 4 + \angle 5 = 180^o$ [Co-interior angles on same side]
$\angle 4 = 180^o - 70^o = 110^o$
$\angle 2 = \angle 4 = 110^o$ [Vertically opposite angles]
$\angle 5 = \angle 7$ [Vertically opposite angles]
So, $\angle 7 = 70^o$
$\therefore \angle 2 = 110^o, \angle 3 = 70^o , \angle 4 = 110^o, \angle 5 = 70^o, \angle 6 = 110^o, \angle 7 = 70^o$ and $\angle 8 = 110^o$.

Question 2:
Since $\angle 2 : \angle 1 = 5 : 4$.
Let $\angle 2$ and $\angle 1$ be 5x and 4x respectively.
Now, $\angle 2 + \angle 1 = 180^o$, because $\angle 2$ and $\angle 1$ form a linear pair.
So, $5x + 4x = 180^o$
$\Rightarrow 9x = 180^o$
$\Rightarrow x = 20^o$
$\therefore \angle 1 = 4x = 4 \times 20^o = 80^o$
And $\angle 2 = 5x = 5 \times 20^o = 100^o$
$\angle 3 = \angle 1 = 80^o$[Vertically opposite angles]
And $\angle 4 = \angle 2 = 100^o$ [Vertically opposite angles]
$\angle 1 = \angle 5$ and $\angle 2 = \angle 6$ [Corresponding angles]
So, $\angle 5 = 80^o$ and $\angle 6 = 100^o$
$\angle 8 = \angle 6 = 100^o$[Vertically opposite angles]
And $\angle 7 = \angle 5 = 80^o$ [Vertically opposite angles]
Thus, $\angle 1 = 80^o, \angle 2 = 100^o, \angle 3 = \angle 80^o, \angle 4 = 100^o, \angle 5 = 80^o, \angle 6 = 100^o, \angle 7 = 80^o$ and $\angle 8 = 100^o$.

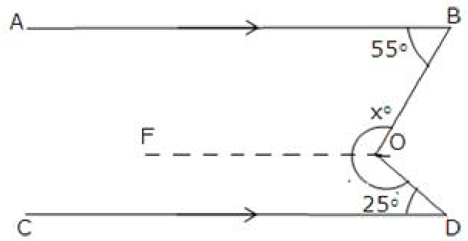
Question 3:
Given: AB || CD and AD || BC
To Prove: $\angle ADC = \angle ABC$
Proof: Since AB || CD and AD is a transversal. So sum of consecutive interior angles is 180^o.
$\Rightarrow \angle BAD + \angle ADC = 180^o$ ....(i)
Also, AD || BC and AB is transversal.
So, $\angle BAD + \angle ABC = 180^o$ ....(ii)
From (i) and (ii) we get:
$\angle BAD + \angle ADC = \angle BAD + \angle ABC$
$\Rightarrow \angle ADC = \angle ABC$ (Proved)

Question 4:
(i) Through E draw EG || CD. Now since EG||CD and ED is a transversal.

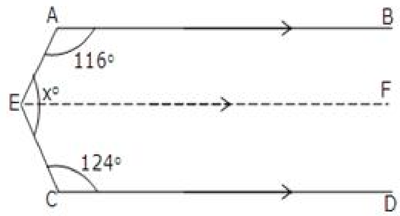


So, $\angle GED = \angle EDC = 65^o$ [Alternate interior angles]
Since EG || CD and AB || CD,
EG||AB and EB is transversal.
So, $\angle BEG = \angle ABE = 35^o$ [Alternate interior angles]
So, $\angle DEB = x^o$

⇒ ∠BEG + ∠GED = 35° + 65° = 100°.
Hence, x = 100.
(ii) Through O draw OF||CD.



Now since OF || CD and OD is transversal.
∠CDO + ∠FOD = 180°
[sum of consecutive interior angles is 180°]
⇒ 25° + ∠FOD = 180°
⇒ ∠FOD = 180° – 25° = 155°
As OF || CD and AB || CD [Given]
Thus, OF || AB and OB is a transversal.
So, ∠ABO + ∠FOB = 180° [sum of consecutive interior angles is 180°]
⇒ 55° + ∠FOB = 180°
⇒ ∠FOB = 180° – 55° = 125°
Now, x° = ∠FOB + ∠FOD = 125° + 155° = 280°.
Hence, x = 280.
(iii) Through E, draw EF || CD.



Now since EF || CD and EC is transversal.
∠FEC + ∠ECD = 180°
[sum of consecutive interior angles is 180°]
⇒ ∠FEC + 124° = 180°
⇒ ∠FEC = 180° – 124° = 56°
Since EF || CD and AB ||CD
So, EF || AB and AE is a trasveral.
So, ∠BAE + ∠FEA = 180°
[sum of consecutive interior angles is 180°]
∴ 116° + ∠FEA = 180°
⇒ ∠FEA = 180° – 116° = 64°
Thus, x° = ∠FEA + ∠FEC
= 64° + 56° = 120°.
Hence, x = 120.

Question 5:
Since AB || CD and BC is a transversal.
So, ∠ABC = ∠BCD [alternate interior angles]
⇒ 70° = x° + ∠ECD ....(i)
Now, CD || EF and CE is transversal.
So, ∠ECD + ∠CEF = 180° [sum of consecutive interior angles is 180°]
∴ ∠ECD + 130° = 180°
⇒ ∠ECD = 180° – 130° = 50°

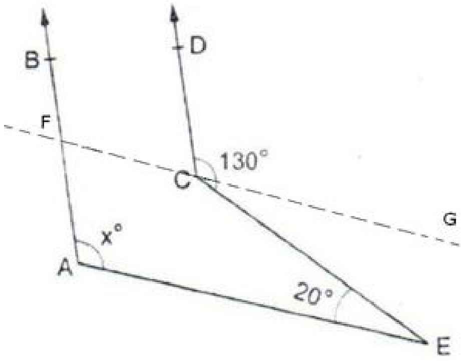
Putting ∠ECD = 50^o in (i) we get,

$70^o = x^o + 50^o$

⇒ x = 70 – 50 = 20

Question 6:

Through C draw FG || AE



Now, since CG || BE and CE is a transversal.

So, ∠GCE = ∠CEA = 20^o [Alternate angles]

∴ ∠DCG = 130^o – ∠GCE

= $130^o – 20^o = 110^o$

Also, we have AB || CD and FG is a transversal.

So, ∠BFC = ∠DCG = 110^o [Corresponding angles]

As, FG || AE, AF is a transversal.

∠BFG = ∠FAE [Corresponding angles]

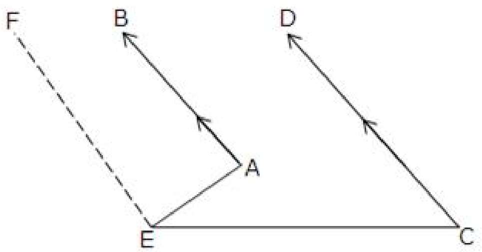
∴ x^o = ∠FAE = 110^o.

Hence, x = 110

Question 7:

Given: AB || CD

To Prove: ∠BAE – ∠DCE = ∠AEC



Construction : Through E draw EF || AB

Proof : Since EF || AB, AE is a transversal.

So, ∠BAE + ∠AEF = 180^O ....(i)

[sum of consecutive interior angles is 180^o]

As EF || AB and AB || CD [Given]

So, EF || CD and EC is a transversal.

So, ∠FEC + ∠DCE = 180^o ....(ii)

[sum of consecutive interior angles is 180^o]

From (i) and (ii) we get,

∠BAE + ∠AEF = ∠FEC + ∠DCE

⇒ ∠BAE – ∠DCE = ∠FEC – ∠AEF = ∠AEC [Proved]

Question 8:

Since AB || CD and BC is a transversal.

So, ∠BCD = ∠ABC = x^o [Alternate angles]

As BC || ED and CD is a transversal.

$\angle BCD + \angle EDC = 180^o$

$\Rightarrow \angle BCD + 75^o = 180^o$

$\Rightarrow \angle BCD = 180^o - 75^o = 105^o$

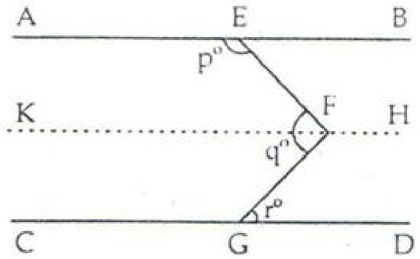
$\angle ABC = 105^o$ [since $\angle BCD = \angle ABC$]

$\therefore x^o = \angle ABC = 105^o$

Hence, x = 105.

Question 9:

Through F, draw KH || AB || CD



Now, KF || CD and FG is a transversal.

$\Rightarrow \angle KFG = \angle FGD = r^o$ .... (i)

[alternate angles]

Again AE || KF, and EF is a transversal.

So, $\angle AEF + \angle KFE = 180^o$

$\angle KFE = 180^o - p^o$ .... (ii)

Adding (i) and (ii) we get,

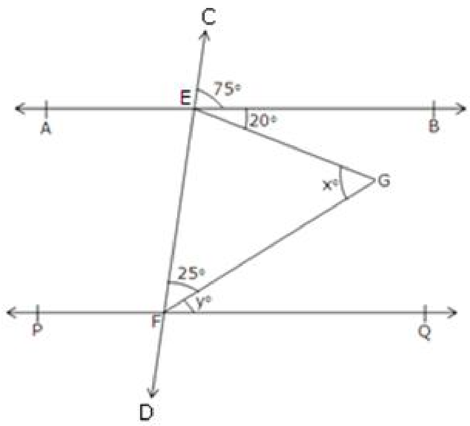
$\angle KFG + \angle KFE = 180 - p + r$

$\Rightarrow \angle EFG = 180 - p + r$

$\Rightarrow q = 180 - p + r$

i.e., $p + q - r = 180$

Question 10:



Since AB || PQ and EF is a transversal.

So, $\angle CEB = \angle EFQ$ [Corresponding angles]

$\Rightarrow \angle EFQ = 75^o$

$\Rightarrow \angle EFG + \angle GFQ = 75^o$

$\Rightarrow 25^o + y^o = 75^o$

$\Rightarrow y = 75 - 25 = 50$

Also, $\angle BEF + \angle EFQ = 180^o$ [sum of consecutive interior angles is 180^o]

$\angle BEF = 180^o - \angle EFQ$

$= 180^o - 75^o$

$\angle BEF = 105^o$

$\therefore \angle FEG + \angle GEB = \angle BEF = 105^o$

$\Rightarrow \angle FEG = 105^o - \angle GEB = 105^o - 20^o = 85^o$

In ΔEFG we have,

$x^o + 25^o + \angle FEG = 180^o$

$\Rightarrow x° + 25° + 85° = 180°$

$\Rightarrow x° + 110° = 180°$

$\Rightarrow x° = 180° - 110°$

$\Rightarrow x° = 70°$

Hence, x = 70.

Question 11:

Since AB || CD and AC is a transversal.

So, $\angle BAC + \angle ACD = 180^o$ [sum of consecutive interior angles is 180^o]

$\Rightarrow \angle ACD = 180^o - \angle BAC$

$= 180^o - 75^o = 105^o$

$\Rightarrow \angle ECF = \angle ACD$ [Vertically opposite angles]

$\angle ECF = 105^o$

Now in ΔCEF,

$\angle ECF + \angle CEF + \angle EFC = 180^o$

$\Rightarrow 105^o + x^o + 30^o = 180^o$

$\Rightarrow x = 180 - 30 - 105 = 45$

Hence, x = 45.

Question 12:

Since AB || CD and PQ a transversal.

So, $\angle PEF = \angle EGH$ [Corresponding angles]

$\Rightarrow \angle EGH = 85^o$

$\angle EGH$ and $\angle QGH$ form a linear pair.

So, $\angle EGH + \angle QGH = 180^o$

$\Rightarrow \angle QGH = 180^o - 85^o = 95^o$

Similarly, $\angle GHQ + 115^o = 180^o$

$\Rightarrow \angle GHQ = 180^o - 115^o = 65^o$

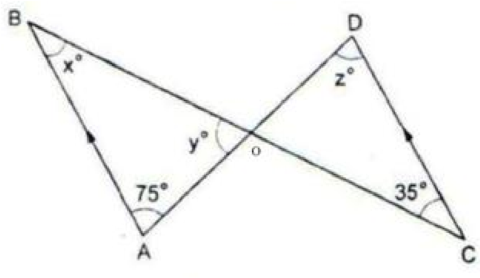
In ΔGHQ, we have,

$x^o + 65^o + 95^o = 180^o$

$\Rightarrow x = 180 - 65 - 95 = 180 - 160$

$\therefore x = 20$

Question 13:



Since AB || CD and BC is a transversal.

So, $\angle ABC = \angle BCD$

$\Rightarrow x = 35$

Also, AB || CD and AD is a transversal.

So, $\angle BAD = \angle ADC$

$\Rightarrow z = 75$

In ΔABO, we have,

$\angle AOB + \angle BAO + \angle BOA = 180^o$

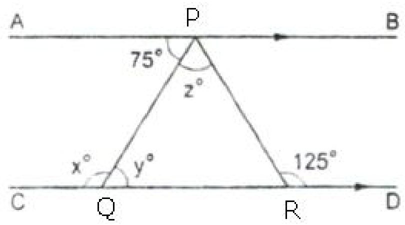
$\Rightarrow x^o + 75^o + y^o = 180^o$

$\Rightarrow 35 + 75 + y = 180$

$\Rightarrow y = 180 - 110 = 70$

∴ x = 35, y = 70 and z = 75.

Question 14:



Since AB || CD and PQ is a transversal.

So, y = 75 [Alternate angle]

Since PQ is a transversal and AB || CD, so x + APQ = 180^o

[Sum of consecutive interior angles]

⇒ $x^o = 180^o$ – APQ

⇒ x = 180 – 75 = 105

Also, AB || CD and PR is a transversal.

So, ∠APR = ∠PRD [Alternate angle]

⇒ ∠APQ + ∠QPR = ∠PRD [Since ∠APR = ∠APQ + ∠QPR]

⇒ $75^o + z^o = 125^o$

⇒ z = 125 – 75 = 50

∴ x = 105, y = 75 and z = 50.

Question 15:

∠PRQ = $x^o = 60^o$ [vertically opposite angles]

Since EF || GH, and RQ is a transversal.

So, ∠x = ∠y [Alternate angles]

⇒ y = 60

AB || CD and PR is a transversal.

So, ∠PRD = ∠APR [Alternate angles]

⇒ ∠PRQ + ∠QRD = ∠APR [since ∠PRD = ∠PRQ + ∠QRD]

⇒ x + ∠QRD = 110^o

⇒ ∠QRD = $110^o - 60^o = 50^o$

In ΔQRS, we have,

∠QRD + $t^o + y^o = 180^o$

⇒ 50 + t + 60 = 180

⇒ t = 180 – 110 = 70

Since, AB || CD and GH is a transversal

So, $z^o = t^o = 70^o$ [Alternate angles]

∴ x = 60 , y = 60, z = 70 and t = 70

Question 16:

(i) Lines l and m will be parallel if 3x – 20 = 2x + 10

[Since, if corresponding angles are equal, lines are parallel]

⇒ 3x – 2x = 10 + 20

⇒ x = 30

(ii) Lines will be parallel if $(3x + 5)^o + 4x^o = 180^o$

[if sum of pairs of consecutive interior angles is 180^o, the lines are parallel]

So, (3x + 5) + 4x = 180

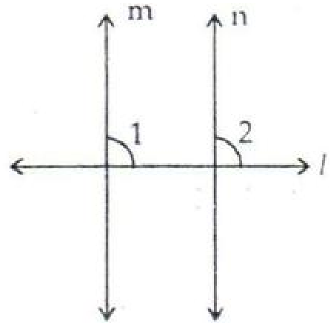
⇒ 3x + 5 + 4x = 180

⇒ 7x = 180 – 5 = 175

⇒ x = $\frac{175}{7}$ = 25

Question 17:

Given: Two lines m and n are perpendicular to a given line l.



To Prove: m || n

Proof : Since m ⊥ l

So, ∠1 = 90°

Again, since n ⊥ l

∠2 = 90°

∴ ∠1 = ∠2 = 90°

But ∠1 and ∠2 are the corresponding angles made by the transversal l with lines m and n and they are proved to be equal.

Thus, m || n.

Exercise 4D

Question 1:

Since, sum of the angles of a triangle is 180°

∠A + ∠B + ∠C = 180°

⇒ ∠A + 76° + 48° = 180°

⇒ ∠A = 180° – 124° = 56°

∴ ∠A = 56°

Question 2:

Let the measures of the angles of a triangle are (2x)°, (3x)° and (4x)°.

Then, 2x + 3x + 4x = 180 [sum of the angles of a triangle is 180°]

⇒ 9x = 180

⇒ x = $\frac{180}{9}$ = 20

∴ The measures of the required angles are:

2x = (2 × 20)° = 40°

3x = (3 × 20)° = 60°

4x = (4 × 20)° = 80°

Question 3:

Let 3∠A = 4∠B = 6∠C = x (say)

Then, 3∠A = x

⇒ ∠A = $\frac{x}{3}$

4∠B = x

⇒ ∠B = $\frac{x}{4}$

and 6∠C = x

⇒ ∠C = $\frac{x}{6}$

As ∠A + ∠B + ∠C = 180°

$$\Rightarrow \quad \frac{x}{3} + \frac{x}{4} + \frac{x}{6} = 180$$
$$\Rightarrow \quad \frac{4x + 3x + 2x}{12} = 180$$
$$\Rightarrow \quad 9x = 180 \times 12$$
$$\Rightarrow \quad x = \frac{180 \times 12}{9} = 240$$

$$\therefore \angle A = \frac{x}{3} = \frac{240}{3} = 80^{\circ}$$

$$\angle B = \frac{x}{4} = \frac{240}{4} = 60^{\circ}$$

$$\angle C = \frac{x}{6} = \frac{240}{6} = 40^{\circ}$$

Question 4:

$\angle A + \angle B = 108^{o}$ [Given]

But as $\angle A$, $\angle B$ and $\angle C$ are the angles of a triangle,

$\angle A + \angle B + \angle C = 180^{o}$

$\Rightarrow 108^{o} + \angle C = 180^{o}$

$\Rightarrow C = 180^{o} – 108^{o} = 72^{o}$

Also, $\angle B + \angle C = 130^{o}$ [Given]

$\Rightarrow \angle B + 72^{o} = 130^{o}$

$\Rightarrow \angle B = 130^{o} – 72^{o} = 58^{o}$

Now as, $\angle A + \angle B = 108^{o}$

$\Rightarrow \angle A + 58^{o} = 108^{o}$

$\Rightarrow \angle A = 108^{o} – 58^{o} = 50^{o}$

$\therefore \angle A = 50^{o}$, $\angle B = 58^{o}$ and $\angle C = 72^{o}$.

Question 5:

Since. $\angle A$, $\angle B$ and $\angle C$ are the angles of a triangle .

So, $\angle A + \angle B + \angle C = 180^{o}$

Now, $\angle A + \angle B = 125^{o}$ [Given]

$\therefore 125^{o} + \angle C = 180^{o}$

$\Rightarrow \angle C = 180^{o} – 125^{o} = 55^{o}$

Also, $\angle A + \angle C = 113^{o}$ [Given]

$\Rightarrow \angle A + 55^{o} = 113^{o}$

$\Rightarrow \angle A = 113^{o} – 55^{o} = 58^{o}$

Now as $\angle A + \angle B = 125^{o}$

$\Rightarrow 58^{o} + \angle B = 125^{o}$

$\Rightarrow \angle B = 125^{o} – 58^{o} = 67^{o}$

$\therefore \angle A = 58^{o}$, $\angle B = 67^{o}$ and $\angle C = 55^{o}$.

Question 6:

Since, $\angle P$, $\angle Q$ and $\angle R$ are the angles of a triangle.

So, $\angle P + \angle Q + \angle R = 180^{o}$ ....(i)

Now, $\angle P – \angle Q = 42^{o}$ [Given]

$\Rightarrow \angle P = 42^{o} + \angle Q$ ....(ii)

and $\angle Q – \angle R = 21^{o}$ [Given]

$\Rightarrow \angle R = \angle Q – 21^{o}$ ....(iii)

Substituting the value of $\angle P$ and $\angle R$ from (ii) and (iii) in (i), we get,

$\Rightarrow 42^{o} + \angle Q + \angle Q + \angle Q – 21^{o} = 180^{o}$

$\Rightarrow 3\angle Q + 21^{o} = 180^{o}$

$\Rightarrow 3\angle Q = 180^{o} – 21^{o} = 159^{o}$

$\angle Q = \frac{159}{3} = 53^{o}$

∴ $\angle P = 42^o + \angle Q$

$= 42^o + 53^o = 95^o$

$\angle R = \angle Q - 21^o$

$= 53^o - 21^o = 32^o$

∴ $\angle P = 95^o$, $\angle Q = 53^o$ and $\angle R = 32^o$.

Question 7:

Given that the sum of the angles A and B of a ABC is 116^o, i.e., $\angle A + \angle B = 116^o$.

Since, $\angle A + \angle B + \angle C = 180^o$

So, $116^o + \angle C = 180^o$

⇒ $\angle C = 180^o - 116^o = 64^o$

Also, it is given that:

$\angle A - \angle B = 24^o$

⇒ $\angle A = 24^o + \angle B$

Putting, $\angle A = 24^o + \angle B$ in $\angle A + \angle B = 116^o$, we get,

⇒ $24^o + \angle B + \angle B = 116^o$

⇒ $2\angle B + 24^o = 116^o$

⇒ $2\angle B = 116^o - 24^o = 92^o$

$\angle B = \frac{92}{2} = 46^o$

Therefore, $\angle A = 24^o + 46^o = 70^o$

∴ $\angle A = 70^o$, $\angle B = 46^o$ and $\angle C = 64^o$.

Question 8:

Let the two equal angles, A and B, of the triangle be x^o each.

We know,

$\angle A + \angle B + \angle C = 180^o$

⇒ $x^o + x^o + \angle C = 180^o$

⇒ $2x^o + \angle C = 180^o$ ....(i)

Also, it is given that,

$\angle C = x^o + 18^o$....(ii)

Substituting $\angle C$ from (ii) in (i), we get,

⇒ $2x^o + x^o + 18^o = 180^o$

⇒ $3x^o = 180^o - 18^o = 162^o$

$x = \frac{162}{3} = 54^o$

Thus, the required angles of the triangle are 54^o, 54^o and $x^o + 18^o = 54^o + 18^o = 72^o$.

Question 9:

Let $\angle C$ be the smallest angle of ABC.

Then, $\angle A = 2\angle C$ and $B = 3\angle C$

Also, $\angle A + \angle B + \angle C = 180^o$

⇒ $2\angle C + 3\angle C + \angle C = 180^o$

⇒ $6\angle C = 180^o$

⇒ $\angle C = 30^o$

So, $\angle A = 2\angle C = 2\,(30^o) = 60^o$

$\angle B = 3\angle C = 3\,(30^o) = 90^o$

∴ The required angles of the triangle are 60^o, 90^o, 30^o.

Question 10:

Let ABC be a right angled triangle and $\angle C = 90^o$

Since, $\angle A + \angle B + \angle C = 180^o$

⇒ $\angle A + \angle B = 180^o - \angle C = 180^o - 90^o = 90^o$

Suppose $\angle A = 53^o$

Then, $53^o + \angle B = 90^o$
$\Rightarrow \angle B = 90^o - 53^o = 37^o$
$\therefore$ The required angles are 53^o, 37^o and 90^o.

Question 11:
Let ABC be a triangle.
Given, $\angle A + \angle B = \angle C$
We know, $\angle A + \angle B + \angle C = 180^o$
$\Rightarrow \angle C + \angle C = 180^o$
$\Rightarrow 2\angle C = 180^o$
$\Rightarrow \angle C = \frac{180}{2} = 90^o$
So, we find that ABC is a right triangle, right angled at C.

Question 12:
Given : ΔABC in which $\angle A = 90^o$, $AL \perp BC$
To Prove: $\angle BAL = \angle ACB$
Proof :
In right triangle ΔABC,
$\Rightarrow \angle ABC + \angle BAC + \angle ACB = 180^o$
$\Rightarrow \angle ABC + 90^o + \angle ACB = 180^o$
$\Rightarrow \angle ABC + \angle ACB = 180^o - 90^o$
$\therefore \angle ABC + \angle ACB = 90^o$
$\Rightarrow \angle ACB = 90^o - \angle ABC$ ....(1)
Similarly since ΔABL is a right triangle, we find that,
$\angle BAL = 90^o - \angle ABC$ ...(2)
Thus from (1) and (2), we have
$\therefore \angle BAL = \angle ACB$ (Proved)

Question 13:
Let ABC be a triangle.
So, $\angle A < \angle B + \angle C$
Adding A to both sides of the inequality,
$\Rightarrow 2\angle A < \angle A + \angle B + \angle C$
$\Rightarrow 2\angle A < 180^o$ [Since $\angle A + \angle B + \angle C = 180^o$]
$\Rightarrow \angle A < \frac{180}{2} = 90^o$
Similarly, $\angle B < \angle A + \angle C$
$\Rightarrow \angle B < 90^o$
and $\angle C < \angle A + \angle B$
$\Rightarrow \angle C < 90^o$
ΔABC is an acute angled triangle.

Question 14:
Let ABC be a triangle and $\angle B > \angle A + \angle C$
Since, $\angle A + \angle B + \angle C = 180^o$
$\Rightarrow \angle A + \angle C = 180^o - \angle B$
Therefore, we get
$\angle B > 180^o - \angle B$
Adding $\angle B$ on both sides of the inequality, we get,
$\Rightarrow \angle B + \angle B > 180^o - \angle B + \angle B$
$\Rightarrow 2\angle B > 180^o$
$\Rightarrow \angle B > \frac{180}{2} = 90^o$
i.e., $\angle B > 90^o$ which means $\angle B$ is an obtuse angle.

ΔABC is an obtuse angled triangle.

Question 15:

Since ∠ACB and ∠ACD form a linear pair.

So, ∠ACB + ∠ACD = 180^o

⇒ ∠ACB + 128^o = 180^o

⇒ ∠ACB = 180^o – 128 = 52^o

Also, ∠ABC + ∠ACB + ∠BAC = 180^o

⇒ 43^o + 52^o + ∠BAC = 180^o

⇒ 95^o + ∠BAC = 180^o

⇒ ∠BAC = 180^o – 95^o = 85^o

∴ ∠ACB = 52^o and ∠BAC = 85^o.

Question 16:

As ∠DBA and ∠ABC form a linear pair.

So, ∠DBA + ∠ABC = 180^o

⇒ 106^o + ∠ABC = 180^o

⇒ ∠ABC = 180^o – 106^o = 74^o

Also, ∠ACB and ∠ACE form a linear pair.

So, ∠ACB + ∠ACE = 180^o

⇒ ∠ACB + 118^o = 180^o

⇒ ∠ACB = 180^o – 118^o = 62^o

In ∠ABC, we have,

∠ABC + ∠ACB + ∠BAC = 180^o

74^o + 62^o + ∠BAC = 180^o

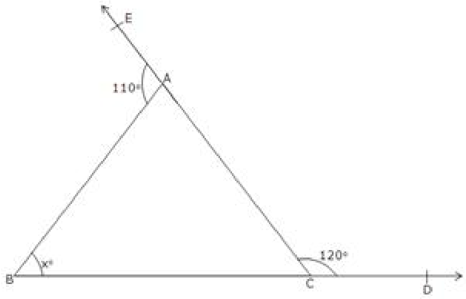
⇒ 136^o + ∠BAC = 180^o

⇒ ∠BAC = 180^o – 136^o = 44^o

∴ In triangle ABC, ∠A = 44^o, ∠B = 74^o and ∠C = 62^o

Question 17:

(i) ∠EAB + ∠BAC = 180^o [Linear pair angles]



110^o + ∠BAC = 180^o

⇒ ∠BAC = 180^o – 110^o = 70^o

Again, ∠BCA + ∠ACD = 180^o [Linear pair angles]

⇒ ∠BCA + 120^o = 180^o

⇒ ∠BCA = 180^o – 120^o = 60^o

Now, in ΔABC,

∠ABC + ∠BAC + ∠ACB = 180^o

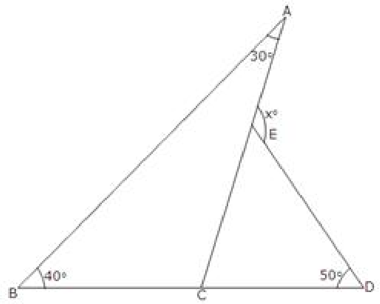
x^o + 70^o + 60^o = 180^o

⇒ x + 130^o = 180^o

⇒ x = 180^o – 130^o = 50^o

∴ x = 50

(ii)



In ΔABC,

$\angle A + \angle B + \angle C = 180^o$

$\Rightarrow 30^o + 40^o + \angle C = 180^o$

$\Rightarrow 70^o + \angle C = 180^o$

$\Rightarrow \angle C = 180^o - 70^o = 110^o$

Now $\angle BCA + \angle ACD = 180^o$ [Linear pair]

$\Rightarrow 110^o + \angle ACD = 180^o$

$\Rightarrow \angle ACD = 180^o - 110^o = 70^o$

In ΔECD,

$\Rightarrow \angle ECD + \angle CDE + \angle CED = 180^o$

$\Rightarrow 70^o + 50^o + \angle CED = 180^o$

$\Rightarrow 120^o + \angle CED = 180^o$

$\angle CED = 180^o - 120^o = 60^o$

Since $\angle$AED and $\angle$CED from a linear pair

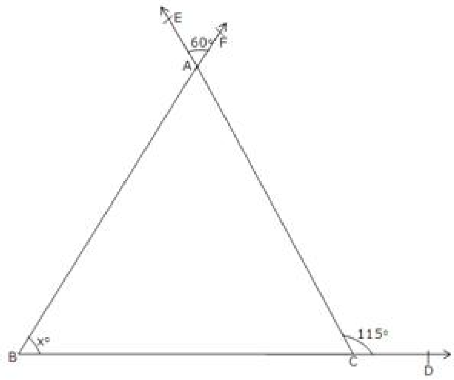
So, $\angle AED + \angle CED = 180^o$

$\Rightarrow x^o + 60^o = 180^o$

$\Rightarrow x^o = 180^o - 60^o = 120^o$

$\therefore x = 120$

(iii)



$\angle EAF = \angle BAC$ [Vertically opposite angles]

$\Rightarrow \angle BAC = 60^o$

In ΔABC, exterior $\angle$ACD is equal to the sum of two opposite interior angles.

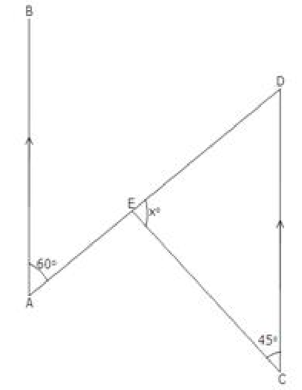
So, $\angle ACD = \angle BAC + \angle ABC$

$\Rightarrow 115^o = 60^o + x^o$

$\Rightarrow x^o = 115^o - 60^o = 55^o$

$\therefore x = 55$

(iv)



Since AB || CD and AD is a transversal.

So, ∠BAD = ∠ADC

⇒ ∠ADC = 60^o

In ∠ECD, we have,

∠E + ∠C + ∠D = 180^o

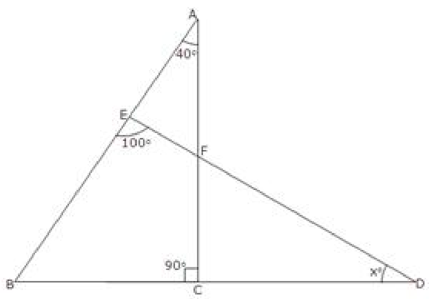
⇒ $x^o + 45^o + 60^o = 180^o$

⇒ $x^o + 105^o = 180^o$

⇒ $x^o = 180^o – 105^o = 75^o$

∴ x = 75

(v)



In ΔAEF,

Exterior ∠BED = ∠EAF + ∠EFA

⇒ $100^o = 40^o$ + ∠EFA

⇒ ∠EFA = $100^o – 40^o = 60^o$

Also, ∠CFD = ∠EFA [Vertically Opposite angles]

⇒ ∠CFD = 60^o

Now in ΔFCD,

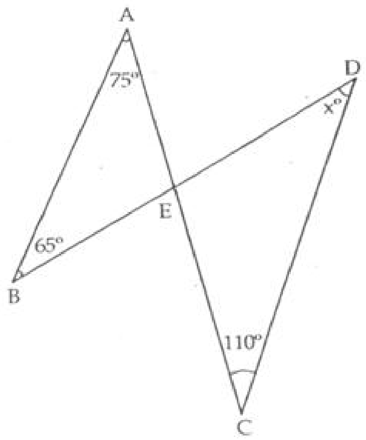
Exterior ∠BCF = ∠CFD + ∠CDF

⇒ $90^o = 60^o + x^o$

⇒ $x^o = 90^o – 60^o = 30^o$

∴ x = 30

(vi)



In ΔABE, we have,

$\angle A + \angle B + \angle E = 180^\circ$

$\Rightarrow 75^\circ + 65^\circ + \angle E = 180^\circ$

$\Rightarrow 140^\circ + \angle E = 180^\circ$

$\Rightarrow \angle E = 180^\circ - 140^\circ = 40^\circ$

Now, $\angle CED = \angle AEB$ [Vertically opposite angles]

$\Rightarrow \angle CED = 40^\circ$

Now, in ΔCED, we have,

$\angle C + \angle E + \angle D = 180^\circ$

$\Rightarrow 110^\circ + 40^\circ + x^\circ = 180^\circ$

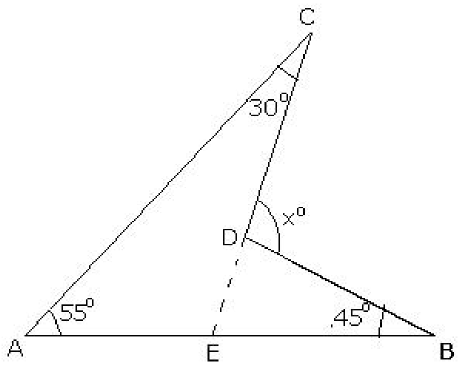
$\Rightarrow 150^\circ + x^\circ = 180^\circ$

$\Rightarrow x^\circ = 180^\circ - 150^\circ = 30^\circ$

$\therefore x = 30$

Question 18:

Produce CD to cut AB at E.



Now, in ΔBDE, we have,

Exterior $\angle CDB = \angle CEB + \angle DBE$

$\Rightarrow x^\circ = \angle CEB + 45^\circ$ .....(i)

In ΔAEC, we have,

Exterior $\angle CEB = \angle CAB + \angle ACE$

$= 55^\circ + 30^\circ = 85^\circ$

Putting $\angle CEB = 85^\circ$ in (i), we get,

$x^\circ = 85^\circ + 45^\circ = 130^\circ$

$\therefore x = 130$

Question 19:

The angle $\angle BAC$ is divided by AD in the ratio 1 : 3.

Let $\angle BAD$ and $\angle DAC$ be y and 3y, respectively.

As BAE is a straight line,

$\angle BAC + \angle CAE = 180^\circ$ [linear pair]

⇒ ∠BAD + ∠DAC + ∠CAE = 180^o

⇒ y + 3y + 108^o = 180^o

⇒ 4y = 180^o – 108^o = 72^o

⇒ y = $\frac{72}{4}$ = 18^o

Now, in ΔABC,

∠ABC + ∠BCA + ∠BAC = 180^o

y + x + 4y = 180^o

[Since, ∠ABC = ∠BAD (given AD = DB) and ∠BAC = y + 3y = 4y]

⇒ 5y + x = 180

⇒ 5 × 18 + x = 180

⇒ 90 + x = 180

∴ x = 180 – 90 = 90

Question 20:

Given : A ΔABC in which BC, CA and AB are produced to D, E and F respectively.

To prove : Exterior ∠DCA + Exterior ∠BAE + Exterior ∠FBD = 360^o

Proof : Exterior ∠DCA = ∠A + ∠B ....(i)

Exterior ∠FAE = ∠B + ∠C ....(ii)

Exterior ∠FBD = ∠A + ∠C ....(iii)

Adding (i), (ii) and (iii), we get,

Ext. ∠DCA + Ext. ∠FAE + Ext. ∠FBD

= ∠A + ∠B + ∠B + ∠C + ∠A + ∠C

= 2∠A + 2∠B + 2∠C

= 2 (∠A + ∠B + ∠C)

= 2 × 180^o

[Since, in triangle the sum of all three angle is 180^o]

= 360^o

Hence, proved.

Question 21:

In ΔACE, we have,

∠A + ∠C + ∠E = 180^o....(i)

In ΔBDF, we have,

∠B + ∠D + ∠F = 180^o....(ii)

Adding both sides of (i) and (ii), we get,

∠A + ∠C +∠E + ∠B + ∠D + ∠F = 180^o + 180^o

⇒ ∠A + ∠B + ∠C + ∠D + ∠E + ∠F = 360^o.

Question 22:

Given : In ΔABC, bisectors of ∠B and ∠C meet at O and ∠A = 70^o

In ΔBOC, we have,

$\Rightarrow \angle BOC + \frac{1}{2}\angle B + \frac{1}{2}\angle C = 180^o$

$\Rightarrow \angle BOC = 180^o - \frac{1}{2}\angle B - \frac{1}{2}\angle C$

$= 180^o - \frac{1}{2}(\angle B + \angle C)$

$= 180^o - \frac{1}{2}\left[180^o - \angle A\right]$

$\left[\because \angle A + \angle B + \angle C = 180^o\right]$

$= 180^o - \frac{1}{2}\left[180^o - 70^o\right]$
$= 180^o - \frac{1}{2} \times 110^o$

$\angle BOC + \angle OBC + \angle OCB = 180^o$
$= 180^o - 55^o = 125^o$
$\therefore \angle BOC = 125^o$.

Question 23:

We have a ΔABC whose sides AB and AC have been procued to D and E. $A = 40^o$ and bisectors of $\angle CBD$ and $\angle BCE$ meet at O.
In ΔABC, we have,
Exterior $\angle CBD = C + 40^o$

$$\Rightarrow \quad \angle CBO = \frac{1}{2}\text{ Ext. } \angle CBD = \frac{1}{2}\left(\angle C + 40^o\right) = \frac{1}{2}\angle C + 20^o$$

And exterior $\angle BCE = B + 40^o$

$$\Rightarrow \quad \angle BCO = \frac{1}{2}\text{ Ext. } \angle BCE = \frac{1}{2}\left(\angle B + 40^o\right) = \frac{1}{2}\angle B + 20^o.$$

Now, in ΔBCO, we have,
$\angle BOC = 180^o - \angle CBO - \angle BCO$
$= 180^o - \frac{1}{2}\angle C - 20^o - \frac{1}{2}\angle B - 20^o$
$= 180^o - \frac{1}{2}\angle C - \frac{1}{2}\angle B - 20^o - 20^o$
$= 180^o - \frac{1}{2}(\angle B + \angle C) - 40^o$
$= 140^o - \frac{1}{2}(\angle B + \angle C)$
$= 140^o - \frac{1}{2}\left[180^o - \angle A\right]$

$= 140^o - 90^o + \frac{1}{2}\angle A$

$= 50^o + \frac{1}{2}\angle A$

$= 50^o + \frac{1}{2} \times 40^o$

$= 50^o + 20^o$
$= 70^o$
Thus, $\angle BOC = 70^o$

Question 24:

In the given ΔABC, we have,
$\angle A : \angle B : \angle C = 3 : 2 : 1$
Let $\angle A = 3x$, $\angle B = 2x$, $\angle C = x$. Then,
$\angle A + \angle B + \angle C = 180^o$
$\Rightarrow 3x + 2x + x = 180^o$
$\Rightarrow 6x = 180^o$

⇒ $x = 30^\circ$

$\angle A = 3x = 3\ 30^\circ = 90^\circ$

$\angle B = 2x = 2\ 30^\circ = 60^\circ$

and, $\angle C = x = 30^\circ$

Now, in ΔABC, we have,

Ext $\angle ACE = \angle A + \angle B = 90^\circ + 60^\circ = 150^\circ$

$\angle ACD + \angle ECD = 150^\circ$

⇒ $\angle ECD = 150^\circ - \angle ACD$

⇒ $\angle ECD = 150^\circ - 90^\circ$ [since , AD ⊥ CD, $\angle ACD = 90^\circ$]

⇒ $\angle ECD = 60^\circ$

Question 25:

In ΔABC, AN is the bisector of ∠A and AM ⊥ BC.

Now in ΔABC we have;

$\angle A = 180^\circ - \angle B - \angle C$

⇒ $\angle A = 180^\circ - 65^\circ - 30^\circ$

$= 180^\circ - 95^\circ$

$= 85^\circ$

Now, in ΔANC we have;

Ext. $\angle MNA = \angle NAC + 30^\circ$

$= \frac{1}{2}\angle A + 30^\circ$

$= \frac{85^\circ}{2} + 30^\circ$

$= \frac{85^\circ + 60^\circ}{2}$

$= \frac{145^\circ}{2}$

Therefore, $\angle MNA = \frac{145^\circ}{2}$

In ΔMAN. we have;

$\angle MAN = 180^\circ - \angle AMN - \angle MNA$

$= 180^\circ - 90^\circ - \angle MNA$ [since AM ⊥ BC, $\angle AMN = 90^\circ$]

$= 90^\circ - \frac{145^\circ}{2}$ [since $\angle MNA = \frac{145^\circ}{2}$]

$= \frac{180^\circ - 145^\circ}{2}$

$= \frac{35^\circ}{2}$

$= 17.5^\circ$

Thus, ∠MAN =

Question 26:

(i) False (ii) True (iii) False (iv) False (v) True (vi) True.