DIFFERENTIAL EQUATIONS WITH VARIABLE SEPARABLE
(XII, R. S. AGGARWAL)

EXERCISE 19A (Pg. No.: 929)

Very-Short-Answer Questions

Find the general solution of each of the following differential equations

1. $\frac{dy}{dx} = \left(1+x^2\right)\left(1+y^2\right)$

Sol. $\frac{dy}{dx} = \left(1+x^2\right)\left(1+y^2\right)$

On separating variables we get $\frac{dy}{\left(1+y^2\right)} = \left(1+x^2\right)dx$

Integrate on the both sides we get $\int \frac{dy}{\left(1+y^2\right)} = \int \left(1+x^2\right)dx$

$\Rightarrow \frac{1}{1}\tan^{-1}\frac{y}{1} = x + \frac{x^3}{3} + C \quad \Rightarrow \tan^{-1} y = x + \frac{x^3}{3} + C$

2. $x^4 \frac{dy}{dx} = -y^4$

Sol. $x^4 \frac{dy}{dx} = -y^4$

On separating variables we get $\frac{dy}{dx} = \frac{-y^4}{x^4}$

$\frac{dy}{y^4} = -\frac{1}{x^4}dx$

Integrate on the both sides we get $\int \frac{dy}{y^4} = \int -\frac{1}{x^4}dx$

$\int \frac{dy}{y^4} = -\int \frac{1}{x^4}dx$

$\int y^{-4}dy = -\int x^{-4}dx$

$y^{-4+1} = -x^{-4+1} + C$

$y^{-3} = -x^{-3} + C$

$y^{-3} + x^{-3} = C$

$\frac{1}{y^3} + \frac{1}{x^3} = C$

3. $\frac{dy}{dx} = 1 + x + u + xy$

Sol. $\frac{dy}{dx} = 1 + x + y + xy$

$$\frac{dy}{dx}=1(1+x)+y(1+x)$$

$$\frac{dy}{dx}=(1+x)(1+y)$$

On separating variables we get $\frac{dy}{(1+y)}=(1+x)dx$

Integrating on the both sides we get $\int\frac{dy}{(1+y)}=\int(1+x)dx$

$$\log|1+y|=x+\frac{x^2}{2}+C$$

4. $\frac{dy}{dx}=1-x+y-xy$

Sol. $\frac{dy}{dx}=1-x+y-xy$

$$\frac{dy}{dx}=1(1-x)+y(1-x)$$

$$\frac{dy}{dx}=(1-x)(1+y)$$

On separating variables we get $\frac{dy}{(1+y)}=(1-x)dx$

Integrating on the both sides we get $\int\frac{dy}{(1+y)}=\int(1-x)dx$

$$\log|1+y|=x-\frac{x^2}{2}+C$$

5. $(x+1)\frac{dy}{dx}=2x^3y$

Sol. $(x+1)\frac{dy}{dx}=2x^3y$

$$\frac{dy}{dx}=\frac{2x^3y}{(x+1)}$$

On separating variables we get $\frac{dy}{y}=\frac{2x^3}{(x+1)}dx$

Integrating on the both sides we get $\int\frac{dy}{y}=\int\frac{2x^3}{(x+1)}dx$

$$\int\frac{dy}{y}=2\int\frac{x^3}{(x+1)}dx$$

On dividing $x^3dy(x-1)$

$$\begin{array}{r|l} x-1 & x^3 \quad (\quad x^2+x+1 \\ & \underline{x^3-x^2} \\ & x^2 \\ & \underline{x^2-x} \\ & x-1 \\ & \underline{x-1} \\ & 0 \end{array}$$

$$\frac{x^3}{(x-1)}=x^2+x+1+\frac{1}{(x-1)}$$

$$\int\frac{dy}{y}=2\int\frac{x^3}{(x-1)}dx$$

$$\int\frac{dy}{y}=2\int\left[x^2+x+1+\frac{1}{(x-1)}\right]dx$$

$$\log|y|=2\left[\frac{x^3}{3}+\frac{x^2}{2}+x+\log|x-1|\right]+C$$

$$\log|y|=\frac{2x^3}{3}+\frac{2x^2}{2}+2x+2\log|x-1|+C$$

$$\log|y|=\frac{2x^3}{3}+x^2+2x+2\log|x-1|+C$$

6. $\frac{dy}{dx}=e^{x+y}$

Sol. $\frac{dy}{dx}=e^{x+y}$

$$\frac{dy}{dx}=e^x\ge e^y$$

On separating variables we get $\frac{dy}{e^y}=e^x dx$

Integrating on the both sides we get $\int\frac{dy}{e^y}=\int e^x dx$

$$\int e^{-y}dy=\int e^x dx$$

$$-e^{-y}=e^x+C$$

$$-e^{-y}-e^x=C$$

$$-\left(e^x+e^{-y}\right)=C$$

$$e^x+e^{-y}=\frac{C}{-1}$$

$$e^x+e^{-y}=C$$

7. $\left(e^x+e^{-x}\right)dy-\left(e^x-e^{-x}\right)dx=0$

Sol. $\left(e^x+e^{-x}\right)dy-\left(e^x-e^{-x}\right)dx=0$

$$\left(e^x+e^{-x}\right)dy=\left(e^x-e^{-x}\right)dx$$

$$\frac{dy}{dx}=\frac{\left(e^{x}-e^{-x}\right)}{\left(e^{x}+e^{-x}\right)}$$

On separating variables we get $dy=\frac{\left(e^{x}-e^{-x}\right)}{\left(e^{x}+e^{-x}\right)}dx$

Integrating on the both sides we get $\int dy=\int\frac{\left(e^{x}-e^{-x}\right)}{\left(e^{x}+e^{-x}\right)}dx$

Let $\left(e^{x}+e^{-x}\right)=t$

Diff. on the both sides will respect to t

$$e^{x}-e^{-x}=\frac{dt}{dx}$$

$$dx=\frac{dt}{\left(e^{x}-e^{-x}\right)}$$

$$\int dy=\int\frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}dx$$

$$\int dy=\int\frac{\left(e^{x}-e^{-x}\right)}{t}\times\frac{dt}{\left(e^{x}-e^{-x}\right)}$$

$$\int dy=\int\frac{1}{t}dt$$

$$y=\log\left|t\right|+C$$

$$y=\log\left|e^{x}+e^{-x}\right|+C$$

8. $\frac{dy}{dx}=e^{x-y}+x^{2}e^{-y}$

Sol. $\frac{dy}{dx}=e^{x-y}+x^{2}e^{-y}$

$$\frac{dy}{dx}=e^{x}\cdot e^{-y}+x^{2}e^{-y}$$

$$\frac{dy}{dx}=e^{-y}\left(e^{x}+x^{2}\right)$$

On separating variables we get $\frac{dy}{dx}=e^{-y}\left(e^{x}+x^{2}\right)$

$$\frac{dy}{e^{-y}}=\left(e^{x}+x^{2}\right)dx$$

$$e^{y}dy=\left(e^{x}+x^{2}\right)dx$$

Integrating on the both sides we get $\int e^{y}dy=\int\left(e^{x}+x^{2}\right)dx$

$$e^{y}=e^{x}+\frac{x^{3}}{3}+C$$

9. $e^{2x-3y}dx+e^{2y-3x}dy=0$

Sol. $e^{2x-3y}dx+e^{2y-3x}dy=0$

$$e^{2y-3x}dy = -e^{2x-3y}dx$$

$$\frac{dy}{dx} = \frac{-e^{2x-3y}}{e^{2y-3x}}$$

On separating variables we get $\frac{dy}{dx} = \frac{-e^{2x}e^{-3y}}{e^{2y}e^{-3x}}$

$$\frac{e^{2y}}{e^{-3y}}dy = \frac{-e^{2x}}{e^{-3x}}dx$$

$$e^{2y+3y}dy = -e^{2x+3x}dx$$

$$e^{5y}dy = -e^{5x}dx$$

Integrating on the both sides we get $\int e^{5y}dy = \int -e^{5x}dx$

$$\int e^{5y}dy = -\int e^{5x}dx$$

$$\frac{e^{5y}}{5} = -\frac{1}{5}e^{5x} + C$$

$$e^{5y} = -e^{5x} + C$$

$$e^{5y} + e^{5x} = C$$

$$e^{5x} + e^{5y} = C$$

10. $e^x \tan dx + \left(1-e^x\right)\sec^2 y dy = 0$

Sol. $e^x \tan y dx + \left(1-e^x\right)\sec^2 y dy = 0$

$$\left(1-e^x\right)\sec^2 y\, dy = -e^x \tan y\, dx$$

$$\frac{dy}{dx} = \frac{-e^x \tan y}{\left(1-e^x\right)\sec^2 y}$$

On separating variables we get $\frac{\sec^2 y}{\tan y}dy = -\frac{e^x}{\left(1-e^x\right)}dx$

Integrating on the both sides we get $\int \frac{\sec^2 y}{\tan y}dy = -\int \frac{e^x}{\left(1-e^x\right)}dx$

Let $\tan y = t$

Diff on the both sides w.r. to y

$$\sec^2 y = \frac{dt}{dy}$$

$$dy = \frac{dt}{\sec^2 y}$$

Let $\left(1-e^x\right) = m$

Diff on the both sides wr. To x

$$-e^x = \frac{dm}{dx}$$

$$dx = \frac{dm}{-e^x}$$

$$\int \frac{\sec^2 y}{t} \times \frac{dt}{\sec^2 y} = -\int \frac{e^x}{m} \times \frac{dm}{-e^x}$$

$$\int \frac{1}{t} dt = \int \frac{1}{m} dm$$

$$\log|t| = \log|m| + C$$

$$\log|\tan y| = \log|1-e^x| + \log|C|$$

$$\log|\tan y| = \log|(1-e^x)C|$$

$$\tan y = C(1-e^x)$$

11. $\sec^2 x \tan y dx + \sec^2 y \tan x \, dy = 0$ s

Sol. $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$

$$\sec^2 y \tan x \, dy = -\sec^2 x \tan y \, dx$$

$$\frac{dy}{dx} = \frac{-\sec^2 x \tan y}{\sec^2 y \tan x}$$

On separating on the both sides we get $\frac{dy}{dx} = \frac{-\sec^2 x \tan y}{\sec^2 y \tan x}$

$$\frac{\sec^2 y}{\tan y} dy = \frac{-\sec^2 x}{\tan x} dx$$

Integrating on the both sides $\int \frac{\sec^2 y}{\tan y} dy = \int \frac{-\sec^2 x}{\tan x} dx$

$$\int \frac{\sec^2 y}{\tan y} dy = -\int \frac{\sec^2 x}{\tan x} dx$$

Let $\tan y = U$

Diff on the both sides w.r. to y $\sec^2 y = \frac{du}{dy}$

$$dy = \frac{du}{\sec^2 y}$$

Let $\tan x = V$

Diff on the both sides w.r.t x

$$\sec^2 x = \frac{dV}{dx}$$

$$dx = \frac{dV}{\sec^2 x}$$

$$\int \frac{\sec^2 y}{U} \times \frac{du}{\sec^2 y} = -\int \frac{\sec^2 x}{V} \times \frac{du}{\sec^2 x}$$

$$\int \frac{1}{U} du = -\int \frac{1}{V} dv$$

$$\log|u| = -\log|v| + C$$

$$\log|\tan y| = -\log|\tan x| + C$$

$$\log|\tan y| + \log|\tan x| = C$$

$\log|\tan y \cdot \tan x| = \log|C|$

$\tan x \cdot \tan y = C$

12. $\cos x(1+\cos y)dx - \sin y(1+\sin x)dy = 0$

Sol. $\cos x(1+\cos y)dx - \sin y(1+\sin x)dy = 0$

$\cos x(1+\cos y)dx = \sin y(1+\sin x)dy$

$\sin y(1+\sin x)dy = \cos x(1+\cos y)dx$

$$\frac{dy}{dx} = \frac{\cos x(1+\cos y)}{\sin y(1+\sin x)}$$

On separating variables we get

$$\frac{dy}{dx} = \frac{\cos x(1+\cos y)}{\sin y(1+\sin x)}$$

$$\frac{\sin y}{(1+\cos y)}dy = \frac{\cos x}{(1+\sin x)}dx$$

Integrating on the both sides we get

$$\int \frac{\sin y}{(1+\cos y)}dy = \int \frac{\cos x}{(1+\sin x)}dx$$

Let $(1+\cos y) = u$

Diff on the both sides w.r.t y $0 + (-\sin y) = \frac{du}{dy}$

$$dy = \frac{du}{= \sin y}$$

Let $(1+\sin x) = V$

Diff on the both sides w.r. t x

$$0 + \cos x = \frac{dv}{dx}$$

$$dx = \frac{dv}{\cos x}$$

$$\int \frac{\sin y}{u} \times \frac{du}{\sin y} = \int \frac{\cos x}{v} \times \frac{dv}{\cos x}$$

$$-\int \frac{1}{u}du = \int \frac{1}{v}dv$$

$-\log|u| = \log|v| + C$

$-\log|u| - \log|v| = C$

$-(\log|u| + \log v) = C$

$\log|u| + \log|v| = C$

$\log|1+\cos y| + \log|1+\sin x| = C$

$\log|(1+\cos y)(1+\sin x)| = \log|C|$

$(1+\cos y)(1+\cos x) = C$

$(1+\sin x)(1+\cos y)=C$

For each of the following differential equations, find a particular solution satisfying the given condition

13. $\cos\left(\frac{dy}{dx}\right)=a,$ c where $a\in R$ and $y=2$ when $x=0$

Sol. $\cos\left(\frac{dy}{dx}\right)=a$

On separating variables we get $\frac{dy}{dx}=\cos^{-1}a$

$dy=\cos^{-1}a dx$

Integrating on the both sides we get

$\int dy=\int\cos^{-1}a dx$

$y=\cos^{-1}a\int dx$

$y=\cos^{-1}a x+C$ .... (i)

Putting when $x=0,$ then $y=2$

$2=\cos^{-1}a\times 0+C$

$C=2$

Now putting the value of $C=2$ equation (i)

$y=\cos^{-1}a\cdot x+C$

$y=\cos^{-1}a\cdot x+2$

$y^{-2}=\cos^{-1}a\cdot x$

$\frac{y-2}{x}=\cos^{-1}a$

$\cos\left(\frac{y-2}{x}\right)=a$

14. $\frac{dy}{dx}=-4xy^2$ it being given that $y=1$ when $x=0$

Sol. $\frac{dy}{dx}=-4xy^2$

On separating variables we get $\frac{dy}{dx}=-4xy^2$

$\frac{dy}{y^2}=-4x\,dx$

$y^{-2}dy=-4x\,dx$

Integrating on the both sides we get

$\int y^{-1}dy=\int -4x\,dx$

$\int y^{-2}dy=-4\int x\,dx$

$-y^{-1}=-4\times\frac{x^2}{2}+C$

$$-\frac{1}{y}=-2x^2+C \qquad \ldots \text{(i)}$$

Putting when $x=0$, they $y=1$

$$-\frac{1}{1}=-2(0)^2+C$$

$$-1=C \quad \Rightarrow C=-1$$

Now putting the value of $C=-1$ in equation (i)

$$-\frac{1}{y}=-2x^2+(-1)$$

$$-\frac{1}{y}=-\left(2x^2+1\right)$$

$$\frac{1}{y}=\left(2x^2+1\right)$$

$$\left(2x^2+1\right)y=1$$

$$y=\frac{1}{\left(2x^2+1\right)}$$

15. $x\,dy=\left(2x^2+1\right)dx\left(x\neq 0\right)$, given that $y=1$ when $x=1$

Sol. $x\,dy=\left(2x^2+1\right)dx$

On separating variables we get $x\,dy=\left(2x^2+1\right)dx$

$$\frac{dy}{dx}=\frac{\left(2x^2+1\right)}{x}$$

$$dy=\left(\frac{2x^2+1}{x}\right)dx$$

Integrating on the both sides we get $\int dy=\int\left(\frac{2x^2+1}{x}\right)dx$

$$\int dy=\int\left(\frac{2x}{x}+\frac{1}{x}\right)dx$$

$$\int dy=2\int x\,dx+\int\frac{1}{x}dx$$

$$y=2\frac{x}{2}+\log|x|+C$$

$$y=x^2+\log|x|+C \qquad \ldots \text{(i)}$$

Putting when $x=1$ then $y=1$

$$1=1^2+\log|1|+C$$

$$1=1+0+C$$

$$1=1+0+C$$

$$1-1=C$$

$$C=0$$

Now putting the values of $C=0$ in equation (i)

$y = x^2 + \log|x| + 0$

$y = x^2 + \log|x|$

16. $\frac{dy}{dx} = y \tan x$, it being given that $y = 1$ when $x = 0$

Sol. $\frac{dy}{dx} = y \tan x$

On separating variables we get $\frac{dy}{dx} = y \tan x$

$\frac{dy}{y} = \tan x \, dx$

Integrating on the both sides we get

$\int \frac{dy}{y} = \int \tan x \, dx$

$\log|y| = \log|\sec x| + C$

$y = \sec x + C$ ... (i)

Putting when $x = 0$, then $y = 1$

$1 = \sec 0 + C$

$1 = 1 + C$

$1 - 1 = C$

$\therefore C = 0$

EXERCISE 19B (Pg.No.: 931)

Very-Short-Answer Questions

Find the general solution of each of the following differential equations

1. $\frac{dy}{dx}=\frac{x-1}{y+2}$

Sol. Given differential equations $\frac{dy}{dx}=\frac{x-1}{y+2}$

$\Rightarrow (y+2)dy=(x-1)dx$ [separating the variables]

$\Rightarrow \int(y+2)dy=\int(x-1)dx$

$\Rightarrow \frac{y^2}{2}+2y=\frac{x^2}{2}-x+c_1$

$\Rightarrow y^2+4y \Rightarrow y^2+4y=x^2-2x+2c_1$

$\Rightarrow y^2+4y-x^2+2x=2c_1$

$\Rightarrow y^2+4y-x^2+2x=c$ {Here $c=24$ this is the required solution of given differential equation

2. $\frac{dy}{dx}=\frac{x}{(x^2+1)}$

Sol. Given differential equation is $\frac{dy}{dx}=\frac{x}{x^2+1}$

$\Rightarrow dy=\frac{xdx}{x^2+1}$ {separating the variables

$\Rightarrow \int dy=\int\frac{xdx}{x^2+1} \Rightarrow y=\frac{1}{2}\log(x^2+1)+c$

This is the required solution

3. $\frac{dy}{dx}=(1+x)(1+y^2)$

Sol. Given, $\frac{dy}{dx}=(1+x)(1+y^2)$

$\Rightarrow dy=(1+x)(1+y^2)dx$

$\Rightarrow \frac{dy}{1+y^2}=(1+x)dx$ [On separating the variables]

$\Rightarrow \int\frac{1}{1+y^2}dy=\int(1+x)dx$ [Integrating both sides] $\therefore \tan^{-1}(y)=x+\frac{x^2}{2}+c$

4. $(1+x^2)\frac{dy}{dx}=xy$

Sol. Given, $(x^2+1)\frac{dy}{dx}=xy \Rightarrow (x^2+1)dy=xy\,dx \Rightarrow \frac{dy}{y}=\frac{x}{x^2+1}dx$ [On separating the variables]

$\Rightarrow \int\frac{1}{y}dy=\int\frac{x}{x^2+1}dx$ [Integrating both sides]

$\Rightarrow \log y=\frac{1}{2}\log\left|x^2+1\right|+\log c$

$\Rightarrow \log\left|y^2\right|=\log\{(x^2+1).c\}$

$\therefore\ y^2=(x^2+1)+c$

5. $\dfrac{dy}{dx}+y=1(y\neq 1)$

Sol. Given, $\dfrac{dy}{dx}+y=1 \Rightarrow \dfrac{dy}{dx}=1-y \quad \Rightarrow \dfrac{dy}{1-y}=dx$ [On separating the variables]

$\Rightarrow \int \dfrac{1}{1-y}dy=\int dx$ [Integrating both sides]

$\Rightarrow \dfrac{\log|1-y|}{-1}=x+c$

$\Rightarrow -\log|1-y|=x+c$

$\therefore c=x+\log|1-y|$

6. $\dfrac{dy}{dx}+\sqrt{\dfrac{1-y^2}{1-x^2}}=0$

Sol. $\dfrac{dy}{dx}+\sqrt{\dfrac{1-y^2}{1-x^2}}=0$

$\Rightarrow \dfrac{1}{\sqrt{1-y^2}}dy+\dfrac{1}{\sqrt{1-x^2}}dx=0$ [on separating the variables]

$\Rightarrow \int \dfrac{dy}{\sqrt{1-y^2}}+\int \dfrac{dx}{\sqrt{1-x^2}}=C$ [integrating both sides]

$\Rightarrow \sin^{-1} y+\sin^{-1} x=C$

Hence $\sin^{-1} y+\sin^{-1} x=C$ is the required solution

7. $x\dfrac{dy}{dx}+y=y^2$

Sol. $x\dfrac{dy}{dx}+y=y^2$

Sol. Given, $x\dfrac{dy}{dx}+y=y^2$

$\Rightarrow x\dfrac{dy}{dx}=y^2-y$

$\Rightarrow x\,dy=(y^2-y)dx$

$\Rightarrow \int \dfrac{dy}{y(y-1)}=\int \dfrac{dx}{x}$ [Integrating both sides]

$\Rightarrow -\log y+\log|y-1|=\log x+\log c$

$\Rightarrow \log|y-1|=\log(xyc) \quad \therefore\ y-1=xyc$

8. $x^2(y+1)dx+y^2(x-1)dy=0$

Sol. Given differential equation is $x^2(y+1)dx+y^2(x-1)dy=0$

$\Rightarrow y^2(x-1)dy=-x^2(y+1)dx$

$$\Rightarrow \frac{y^2+dy}{y+1} = -\frac{x^2}{x-1}dx$$

$$\Rightarrow \int \frac{y^2 dy}{y+1} = -\int \frac{x^2 dx}{x-1}$$

$$\Rightarrow \int \frac{y^2-1+1}{y+1}dy = -\int \frac{x^2-1+1}{x-1}dx$$

$$\Rightarrow \int \frac{(y-1)(y+1)+1}{y+1}dy = -\int \frac{(x-1)(x+1)+1}{x-1}dx$$

$$\Rightarrow \int \left\{y-1+\frac{1}{y+1}\right\}dy = -\int \left\{x+1+\frac{1}{x-1}\right\}dx$$

$$\Rightarrow \frac{y^2}{2} - y + \log y + 1 = -\frac{x^2}{2} - x - \log|x-1| + c_1$$

$$\Rightarrow y^2 - 2y + 2\log|y+1| = -x^2 - 2x - 2\log|x-1| + 2c_1$$

$$\Rightarrow x^2 + y^2 + 2x + 2y + 2\log|(y+1)(x-1)| = c$$

Here $2c_1 = c$

This is the required solution of given differential equation

9. $y(1-x^2)\frac{dy}{dx} = x(1+y^2)$

Sol. Given, $y(1-x^2)dy = x(1+y^2)dx \quad \Rightarrow \frac{y}{1+y^2}dy = \frac{x}{1-x^2}dx$ [On separating the variables]

$$\Rightarrow \int \frac{y}{1+y^2}dy = \int \frac{x}{1-x^2}dx \qquad \text{[Integrating both sides]}$$

Let $1+y^2 = t$ & $1-x^2 = z$, $2y = \frac{dt}{dy}$ & $-2x = \frac{dz}{dx}$ $\Rightarrow y\,dy = \frac{dt}{2}$ & $x\,dx = -\frac{dz}{2}$

$$\Rightarrow \frac{1}{2}\int \frac{1}{t}dt = -\frac{1}{2}\int \frac{1}{z}\,dz \quad \Rightarrow \log|t| + \log|z| = \log c \quad \Rightarrow (t\,z) = c \quad \therefore\ (1+y^2)(1-x^2) = c$$

10. $y\log y dx - xdy = 0$

Sol. Given differential equation is $y\cdot\log dx - x\cdot dy = 0$

$$\Rightarrow xdy = y\log y dx$$

$$\Rightarrow \frac{dy}{y\log y} = \frac{dx}{x}$$

$$\Rightarrow \int \frac{dy}{y\log y} = \int \frac{dx}{x}$$

$$\Rightarrow \log(\log y) = \log|x| + \log c_1$$

$$\Rightarrow \log(\log y) = \log c_1|x|$$

$$\Rightarrow (\log y) = c_1|x| \Rightarrow \log y = \pm c_1 x$$

$$\Rightarrow \log y = cx$$

{here $c = \pm c_1$,

This is the required solution of given differential equation

11. $x\left(x^2-x^2y^2\right)dy+y\left(y^2+x^2y^2\right)dx=0$

Sol. Given differential equation is $x\left(x^2-x^2y^2\right)dy+y\left(y^2+x^2y^2\right)dx=0$

$\Rightarrow x^3\left(1-y^2\right)dy+y^3\left(1+x^2\right)dx=0$

$\Rightarrow x^3\left(1-y^2\right)dy=-y^3\left(1+x^2\right)dx$

$\Rightarrow \frac{\left(1-y^2\right)}{y^3}dy=-\frac{\left(1+x^2\right)}{x^3}dx$

$\Rightarrow \left(\frac{1}{y^3}-\frac{1}{y}\right)dy=-\left(\frac{1}{x^3}+\frac{1}{x}\right)dx$

$\Rightarrow \int\left\{\frac{1}{y^3}-\frac{1}{y}\right\}dy=-\int\left(\frac{1}{x^3}+\frac{1}{x}\right)dx$

$\Rightarrow -\frac{1}{2y^2}-\log|y|=\frac{1}{2x^2}-\log|x|+c$

$\Rightarrow -\frac{1}{2x^2}-\frac{1}{2y^2}+\log|x|-\log|y|=c$

$\Rightarrow -\frac{1}{2x^2}-\frac{1}{2y^2}+\log\left|\frac{x}{y}\right|=c$

This is the required solution of given differential equation

12. $\left(1-x^2\right)dy+xy\left(1-y\right)dx=0$

Sol. Given differential equation is $\left(1-x^2\right)dy+xy\left(1-x\right)dx=0$

$\Rightarrow \left(1-x^2\right)dy=-xy\left(1-y\right)dx$

$\Rightarrow \frac{dy}{y(1-y)}=\frac{-xdx}{1-x^2}$

$\Rightarrow \int\frac{dy}{y(1-y)}=\frac{1}{2}\int\frac{-2xdx}{1-x^2}$

$\Rightarrow \int\left\{\frac{1}{y}+\frac{1}{y-1}\right\}dy=\frac{1}{2}\int\frac{-2xdx}{1-x^2}$

$\Rightarrow \log|t|=\log|1-y|=\frac{1}{2}\log\left|1-x^2\right|+\log c_1$

$\Rightarrow \log\left|\frac{y}{1-y}\right|=\log\left(\sqrt{1-x^2}\right)c_1$

$\Rightarrow \left|\frac{y}{1-y}\right|=c_1\sqrt{1-x^2}\quad \Rightarrow \frac{y}{1-y}=\pm c_1\sqrt{1-x^2}\quad \Rightarrow y=c\left(1-y\right)\sqrt{1-x^2}$

{ where $c=\pm c_1$

This is the required solution of given diff equation

13. $\left(1-x^2\right)\left(1-y\right)dx=xy\left(1+y\right)dy$

Sol. Given, $\left(1-x^2\right)\left(1-y\right)dx=xy\left(1+y\right)dy$

$\Rightarrow \frac{1-x^2}{x}dx = \frac{y(1+y)}{1-y}dx$ [On separating the variable]

$\Rightarrow \frac{1-x^2}{x}dx = \frac{y(1+y)}{1-y}dy$ [On intergrading both sides]

$\Rightarrow \int\frac{1}{x}dx - \int x\,dx = \int\frac{y^2}{1-y}dy + \int\frac{y}{1-y}\,dy \quad \Rightarrow \log|x| - \frac{x^2}{2} = -\int\frac{1-y^2-1}{1-y}dy + \int\frac{y}{1-y}dy$

$\Rightarrow \log|x| - \frac{x^2}{2} = -\int\frac{(1-y)(1+y)}{1-y}dy + \int\frac{1}{1-y}\,dy + \int\frac{y}{1-y}dy$

$\Rightarrow \log|x| - \frac{x^2}{2} = -\int(1+y)dy - \log|1-y| - \int\frac{(1-y)-1}{1-y}\,dy$

$\Rightarrow \log|x| - \frac{x^2}{2} = -y - \frac{y^2}{2} - \log|1-y| - \int dy + \int\frac{1}{1-y}\,dy$

$\Rightarrow \log|x| - \frac{x^2}{2} = -y - \frac{y^2}{2} - \log|1-y| - y - \log|1-y|$

$\Rightarrow \log|x| - \frac{x^2}{2} = -2y - \frac{y^2}{2} - 2\log|1-y| \quad \Rightarrow \log|x| - \frac{x^2}{2} = -2y - \frac{y^2}{2} - \log|1-y|^2$

$\Rightarrow \log|x| + \log|1-y|^2 = -2y - \frac{y^2}{2} + \frac{x^2}{2} \quad \therefore \log\left(x(1-y)^2\right) = \frac{x^2}{2} - 2y - \frac{y^2}{2} + c$

14. $(y+xy)dx + (x-xy^2)dy = 0$

Sol. Given, $(y+xy)dx + (x-xy^2)dy = 0 \quad \Rightarrow (y+xy)dx = -(x-xy^2)dy$

$\Rightarrow y(1+x)dx = x(y^2-1)dy \quad \Rightarrow \frac{1+x}{x}dx = \frac{y^2-1}{y}\,dy$ [On separating the variable]

$\Rightarrow \int\frac{1+x}{x}dx = \int\frac{y^2-1}{y}dy \quad \Rightarrow \int\frac{1}{x}dx + \int dx = \int y\,dy - \int\frac{1}{y}\,dy$

$\Rightarrow \log|x| + x = \frac{y^2}{2} - \log|y| + c \quad \therefore \log|xy| + x - \frac{y^2}{2} = c$

15. $(x^2 - yx^2)dy + (y^2 + xy^2)dx = 0$

Sol. Given, $(x^2 - y\,x^2)dy + (y^2 + x\,y^2)dx = 0 \quad \Rightarrow x^2(1-y)dy = -y^2(1+x)dx$

$\Rightarrow \frac{1-y}{y^2}dy = -\frac{1+x}{x^2}dx$ [On separating the variable]

$\Rightarrow \int\frac{1-y}{y^2}dy = -\int\frac{1+x}{x^2}dx$ [Integrating both sides]

$\Rightarrow \int\frac{1}{y^2}dy - \int\frac{y}{y^2}dx = -\int\frac{1}{x^2}dx - \int\frac{x}{x^2}dx \quad \Rightarrow -\frac{1}{y} - \log|y| = \frac{1}{x} - \log|x|$

$\Rightarrow \log|x| - \log|y| = \frac{1}{x} + \frac{1}{y} + c \quad \therefore \log\left|\frac{x}{y}\right| = \frac{1}{x} + \frac{1}{y} + c$

16. $(x^2y - x^2)dx + (xy^2 - y^2)dy = 0$

Sol. Given, $(x^2y - x^2)dx + (xy^2 - y^2)dy = 0 \quad \Rightarrow (x^2y - x^2)dx = -(xy^2 - y^2)dy$

$$\Rightarrow x^2(y-1)dx = -y^2(x-1)dy$$

$$\Rightarrow \int \frac{x^2}{x-1}dx = \int \frac{y^2}{1-y}dy$$

$$\Rightarrow \int \frac{(x^2-1)+1}{x-1}dx = -\int \frac{(y^2-1)+1}{y-1}dy$$

$$\Rightarrow \int \frac{(x-1)(x+1)}{x-1}dx + \int \frac{1}{x-1}dx = -\left[\int \frac{(y-1)(y+1)}{y-1}dy + \int \frac{1}{y-1}dy\right]$$

$$\Rightarrow \int (x+1)dx + \log|x-1| = -\left[\int (y+1)dy + \log|y-1|\right] + c$$

$$\Rightarrow \frac{x^2}{2} + x + \log|x-1| = -\left[\frac{y^2}{2} + y + \log|y-1|\right] + c$$

$$\Rightarrow \frac{x^2}{2} + \frac{y^2}{2} + x + y + \log|x-1| + \log|y-1| = c$$

$$\therefore \frac{1}{2}(x^2+y^2) + (x+y) + \log|(x-1)(y-1)| = c$$

17. $x\sqrt{1+y^2}dx + y\sqrt{1+x^2}dy = 0$

Sol. Given, $x\sqrt{1+y^2}dx + y\sqrt{1+x^2}dy = 0$

$$\Rightarrow x\sqrt{1+y^2}\,dx = -y\sqrt{1+x^2}\,dy$$

$$\Rightarrow \frac{x}{\sqrt{1+x^2}}dx = -\frac{y}{\sqrt{1+y^2}}dy \qquad \text{[On separating both sides]}$$

$$\Rightarrow \int \frac{x}{\sqrt{1+x^2}}dx = -\int \frac{y}{\sqrt{1+y^2}}dy \qquad \text{[Integrating both sides]}$$

Let $1+x^2 = t$ & $1+y^2 = z$

$$\Rightarrow 2x = \frac{dt}{dx} \ \& \ 2y = \frac{dz}{dy}$$

$$\Rightarrow xdx = \frac{dt}{2} \ \& \ ydy = -\frac{dz}{2}$$

Now, $\frac{1}{2}\int \frac{1}{\sqrt{t}}dt = -\frac{1}{2}\int \frac{1}{\sqrt{z}}dz$

$$\Rightarrow \int t^{-\frac{1}{2}}dt = -\int z^{-\frac{1}{2}}dz$$

$$\Rightarrow \frac{t^{1/2}}{1/2} = -\frac{z^{1/2}}{1/2} + c$$

$$\Rightarrow \sqrt{1+x^2} + \sqrt{1+y^2} = c$$

18. $\frac{dy}{dx} = e^{x+y} + x^2e^y$

Sol. We have $\frac{dy}{dx} = e^{x+y} + x^2 \cdot e^y$

$$= e^x \cdot e^y + x^2 \cdot e^y \quad = (e^x + x^2) \cdot e^y$$

$\Rightarrow e^{-y}dy = \left(e^x + x^2\right)dx$ [separating the variables]

$\Rightarrow \int e^{-y}dy = \int\left(e^x + x^2\right)dx$

$\Rightarrow -e^{-y} = e^x + \frac{x^3}{3} + C_1$

$\Rightarrow e^x + e^{-y} + \frac{x^3}{3} = C,$ where $C = -C_1$

Hence $e^x + e^{-y} + \frac{x^3}{3} = C$ is the required solution

19. $\frac{dy}{dx} = \frac{3e^{2x} + 3e^{4x}}{e^x + e^{-x}}$

Sol. Given differential equation is $\frac{dy}{dx} = \frac{3e^{2x} + 3e^{4x}}{e^x + e^{-x}}$

$\Rightarrow dy = \frac{3e^{2x} + 3e^{4x}}{e^x + e^{-x}}dx$

$\Rightarrow dy = \left\{\frac{3e^{2x} + 3e^{4x}}{e^x + e^{-x}}\right\}\cdot\frac{e^x}{e^x}dx$

$\Rightarrow dy = \frac{3e^{3x} + 3e^{5x}}{e^{2x} + 1}dx$

$\Rightarrow dy = \frac{3e^{3x}\left\{1 + e^{2x}\right\}}{e^{2x} + 1}dx$

$\Rightarrow dy = 3e^{3x}dx \Rightarrow \int dy = 3\int e^{3x}dx$

$\Rightarrow y = 2x \times \frac{1}{9}\cdot e^{3x} + c$

$\Rightarrow y = e^{3x} + c$

Is the required solution of given differential equation

20. $3e^x \tan y dx + \left(1 - e^x\right)\sec^2 y dy = 0$

Sol. Given equation is $3e^x \tan y dx + \left(1 - e^x\right)\sec^2 y dy = 0$

$\Rightarrow \left(1 - e^x\right)\sec^2 y dy = -3e^x \tan y dx$

$\Rightarrow \frac{\sec^2 y dy}{\tan y} = \frac{-3e^x dx}{1 - e^x}$

$\Rightarrow \int\frac{\sec^2 y dy}{\tan y} = 3\int\frac{-e^x dx}{1 - e^x}$

$\Rightarrow \log\left|\tan y\right| = 3\log\left|1 - e^x\right| + \log c_1$

$\Rightarrow \log\left|\tan y\right| = \log c_1\left|1 - e^x\right|^3$

$\Rightarrow \left|\tan y\right| = c_1\left|1 - e^x\right|^3$

$\Rightarrow \tan y = \pm c_1\left(1 - e^x\right)^3$

$\Rightarrow \tan y = c\left(1 - e^x\right)$

Where $c = \pm c_1$

This is the required solution of given differention equation

21. $e^y\left(1+x^2\right)dy - \frac{x}{y}dx = 0$

Sol. Given differential equation is $e^y\left(1+x^2\right)dy - \frac{x}{y}dx = 0$

$$\Rightarrow e^y\left(1+x^2\right)dy = \frac{x}{y}dx \quad \Rightarrow ye^y dy = \frac{xdx}{1+x^2}$$

$$\Rightarrow \int y \cdot e^y dy = \int \frac{xdx}{1+x^2}$$

$$\Rightarrow y\int e^y dy - \int\left\{\frac{dy}{dx}\int e^y dy\right\}dy = \frac{1}{2}\int \frac{2xdx}{1+x^2}$$

$$\Rightarrow y \cdot e^y - \int e^y dy = \frac{1}{2}\log\left(1+x^2\right)+c$$

$$\Rightarrow ye^y - e^y = \frac{1}{2}\log\left(1+x^2\right)+c$$

This is the required solution of given differential equation

22. $\frac{dy}{dx} = e^{x+y} + e^{x-y}$

Sol. Given differential equation is $\frac{dy}{dx} = e^{x+y} + e^{x-y}$

$$\Rightarrow \frac{dy}{dx} = e^x \cdot e^y + \frac{e^x}{e^y} \quad \Rightarrow \frac{dy}{dx} = e^x\left\{\frac{e^{2y}+1}{e^y}\right\}$$

$$\Rightarrow \frac{e^y}{e^2 y+1}dy = e^x dx \quad \Rightarrow \int \frac{e^y dy}{\left(e^y\right)^2+1} = \int e^x dx \quad \Rightarrow \tan^{-1}\left(e^y\right) = e^y + c$$

This is the required solution of given differential equation

23. $\left(e^y+1\right)\cos xdx + e^y \sin xdy = 0$

Sol. Given differential equation is $\left(e^y+1\right)\cos xdx + e^y \sin xdy = 0$

$$\Rightarrow e^y \sin xdy = -\left(e^y+1\right)\cos xdx$$

$$\Rightarrow \frac{e^y dy}{e^y+1} = -\frac{\cos xdx}{\sin x} \quad \Rightarrow \int \frac{e^y dy}{e^y+1} = -\int \cot xdx$$

$$\Rightarrow \log\left|e^y+1\right| = -\log\left|\sin x\right| + \log c \quad \Rightarrow \log\left|e^y+1\right| + \log\left|\sin x\right| = \log c_1$$

$$\Rightarrow \log\left|\left(e^y+1\right)\sin x\right| = \log c_1 \quad \Rightarrow \left|\left(e^y+1\right)\sin x\right| = c_1$$

$$\Rightarrow \left(e^y+1\right)\sin x = \pm c_1 \quad \Rightarrow \left(e^y+1\right)\sin x = c \quad \{\text{where } \pm c_1 - c\}$$

This is the required solution of given differential equation

24. $\frac{dy}{dx} + \frac{xy+y}{xy+x} = 0$

Sol. Given, $\frac{dy}{dx} + \frac{xy+y}{xy+x} = 0 \quad \Rightarrow \frac{dy}{dx} = -\frac{y(x+1)}{x(y+1)} \quad \Rightarrow \frac{y+1}{y}dy = -\left(\frac{x+1}{x}\right)dx$ [On separating the variables]

$\Rightarrow \int \frac{y+1}{y}dy = -\int \frac{x+1}{x}dx$ [Integrating both sides]

$\Rightarrow \int dy + \int \frac{1}{y}dy = -\int dx - \int \frac{1}{x}dx \Rightarrow y + \log y = -x - \log x + c \quad \therefore\ x + y + \log(xy) = c$

25. $\sqrt{1-x^4}\,dy = xdx$

Sol. Given differential equation is $\sqrt{1-x^4}\,dy = xdx$

$\Rightarrow dy = \frac{x\,dx}{\sqrt{1-x^4}}$

$\Rightarrow \int dy = \frac{1}{2}\int \frac{2x\,dx}{\sqrt{1-(x^2)^2}} \Rightarrow \int dy = \frac{1}{2}\int \frac{dz}{\sqrt{1-z^2}}$ { Let $z = x^2$

$\Rightarrow y = \frac{1}{2}\sin^{-1} z + c_1 \Rightarrow y = \frac{1}{2}\sin^{-1}(2x) + c_1$

26. $\operatorname{cosec} x \log y \frac{dy}{dx} + x^2 y = 0$

Sol. Given differential equation is $\operatorname{cosec} cx \cdot \log y \frac{dy}{dx} + x^2 y = 0$

$\Rightarrow \operatorname{cosec} x \cdot \log y \frac{dy}{dx} = -x^2 y \Rightarrow \int \frac{\log y}{y}dy = \int(-x^2 \sin x)dx$

$\Rightarrow \int zdx = -\left[x^2 \int \sin xdx - \int\left\{\frac{dx^2}{dx}\int \sin xdx\right\}dx\right]$

$\left\{\text{Let } z = \log y \therefore dz = \frac{dy}{y}\right\}$

$\Rightarrow \frac{z^2}{2} = -\left[-x^2 \cos x + 2\int x \cos xdx\right] + c$

$\Rightarrow \frac{z^2}{2} = x^2 \cos x - 2\left[x\int \cos dx - \int\left\{\frac{dx}{dx}\int \cos xdx\right\}dx\right] + c$

$\Rightarrow \frac{z^2}{2} = x^2 \cos x - 2\left[x \sin x - \int \sin xdx\right] + c \Rightarrow \frac{z^2}{2} = x^2 \cos x - 2x \sin x - 2\cos x + c$

$\Rightarrow \frac{1}{2}(\log y)^2 = (x^2 - 2)\cos x - 2x \sin x + c \Rightarrow \frac{1}{2}(\log y)^2 + (2 - x^2)\cos x + 2x \sin x = c$

This is the required solution of given differential equation

27. $ydx + (1+x^2)\tan^{-1} xdy = 0$

Sol. Given differential equation is $ydx + (1+x^2)\tan^{-1} xdy = 0$

$\Rightarrow (1+x^2)\tan^{-1} xdy = -ydx \Rightarrow \frac{dy}{y} = \frac{-dx}{(1+x^2)\tan^{-1} x}$

$\Rightarrow \int \frac{dy}{dx} = -\int \frac{dx}{(1+x^2)\tan^{-1} x} \Rightarrow \log|y| = -\log\left|\tan^{-1} x\right| + \log c_1$

$\Rightarrow \log\left|y \tan^{-1} x\right| = \log c_1 \Rightarrow \left|y \tan^{-1} x\right| = c_1 \Rightarrow y \tan^{-1} x = \pm c_1 \Rightarrow y \tan^{-1} x = c$

{where $c = \pm c_1$ }

This is the required solution of given differential equation

28. $\frac{1}{x} \cdot \frac{dy}{dx} = \tan^{-1} x$

Sol. Given differential equation $\frac{1}{x} \cdot \frac{dy}{dx} = \tan^{-1} x$

$$\Rightarrow dy = x \tan^{-1} x dx \Rightarrow \int dy = \int \left(\tan^{-1} x\right) \cdot x dx$$

$$\Rightarrow \int dy = \tan^{-1} x \int x dx - \int \left\{ \frac{d}{dx} \left(\tan^{-1} x\right) \int x dx \right\} dx$$

$$\Rightarrow \int dy = \frac{x^2}{2} \tan^{-1} x - \int \frac{1}{1+x^2} \cdot \frac{x^2}{2} dx$$

$$\Rightarrow \int dy = \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \frac{x^2}{1+x^2} dx$$

$$\Rightarrow \int dy = \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \frac{1+x^2-1}{1+x^2} dx$$

$$\Rightarrow \int dy = \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \left\{ 1 - \frac{1}{1+x^2} \right\} dx$$

$$\Rightarrow y = \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} x + \frac{1}{2} \tan^{-1} x + c$$

$$\Rightarrow y = \frac{1}{2}\left(x^2+1\right) \tan^{-1} x - \frac{1}{2} x + c$$

This is the required solution of given differential equation

29. $e^x \sqrt{1-y^2} dx + \frac{y}{x} dy = 0$

Sol. Given differential equation is

$$e^x \sqrt{1-y^2} dx + \frac{y}{x} dy = 0$$

$$\Rightarrow x^2 dy = -e^x \sqrt{1-y^2} dx$$

$$\Rightarrow \frac{y dx}{\sqrt{1-y^2}} = -x \cdot e^x dx$$

Integrating both sides we have $\int \frac{y dx}{\sqrt{1-y^2}} = -\int x \cdot e^x dx$

$$\Rightarrow -\frac{1}{2} \int \frac{-2y dy}{\sqrt{1-y^2}} = -\int x \cdot e^x dx$$

$$\Rightarrow -\frac{1}{2} \int \frac{dz}{\sqrt{z}} = -\left[x \int e^x dx - \int \left\{ \frac{dx}{dy} \int e^x dx \right\} dx \right] + C$$

$\{$Let $Z = 1 - y^2$ $\}$

$$\Rightarrow -\frac{1}{2} \cdot 2\sqrt{Z} = -x \cdot e^x + e^x + C$$

$$\Rightarrow xe^x - e^x - \sqrt{1-y^2} = C$$

$$\Rightarrow e^x \left(x-1\right) - \sqrt{1-y^2} = C$$

This is the required general solution of given differential equation

30. $\frac{dy}{dx}=\frac{1-\cos x}{1+\cos x}$

Sol. Given, $\frac{dy}{dx}=\frac{1-\cos x}{1+\cos x}$ $\Rightarrow dy=\frac{1-\cos x}{1+\cos x}dx$ $\Rightarrow dy=\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}dx$

$\Rightarrow dy=\tan^2\frac{x}{2}dx$ [On integrating both sides]

$\Rightarrow \int dy=\int \tan^2\frac{x}{2}dx$ [Integrating both sides]

$\Rightarrow y=\int\left(\sec^2\frac{x}{2}-1\right)dx$ $\Rightarrow y=\frac{\tan\frac{x}{2}}{\frac{1}{2}}-x+c$ $\therefore y=2\tan\frac{x}{2}-x+c$

31. $(\cos x)\frac{dy}{dx}+\cos 2x=\cos 3x$

Sol. Given differential equation is $\cos x\frac{dy}{dx}+\cos 2x=\cos 3x$

$\Rightarrow \cos x\frac{dy}{dx}=\cos 3x-\cos 2x$

$\Rightarrow dy=\frac{\cos 3x-\cos 2x}{\cos 2x}dx$

$\Rightarrow dy=\frac{4\cos^3 x-3\cos x-2\cos^2 x+1}{\cos x}dx$

$\Rightarrow dy=\left\{4\cos^2 x-3-2\cos x+\sec x\right\}dx$

$\Rightarrow dy=\left\{\frac{4(1+\cos 2x)}{2}-3-2\cos x+\sec x\right\}dx$

$\Rightarrow dy=\left\{2+2\cos x-3-2\cos x+\sec x\right\}dx$

Integrating both sides we have $\int dy=\int\left\{2\cos 2x-2\cos x+\sec x-1\right\}dx$

$\Rightarrow y=\sin 2x-2\sin x-x+\log|\sec x+\tan x|+C$

This is the required grated solution of given differential equation

32. $\frac{dy}{dx}+\frac{(1+\cos 2y)}{(1-\cos 2y)}=0$

Sol. Given, $\frac{dy}{dx}+\frac{1+\cos 2y}{1-\cos 2x}=0$ $\Rightarrow \frac{dy}{dx}=-\frac{1+\cos 2y}{1-\cos 2x}$ $\Rightarrow \frac{1}{1+\cos 2y}dy=-\frac{1}{1-\cos 2x}dx$

$\Rightarrow \frac{1}{2\cos^2 y}dy=-\frac{1}{2\sin^2 x}dx$ [On separating the variables]

$\Rightarrow \frac{1}{2}\sec^2 y\,dy=-\frac{1}{2}\text{cosec}^2 x\,dx$ [Integrating both sides]

$\Rightarrow \frac{1}{2}\int\sec^2 y\,dy=-\frac{1}{2}\int\text{cosec}^2 x\,dx$ $\therefore$ $\tan y=\cot x+c$

33. $\frac{dy}{dx}+\frac{\cos x\sin y}{\cos y}=0$

Sol. Given differential equation is $\frac{dy}{dx}+\frac{\cos x\sin y}{\cos y}=0$

$\Rightarrow \frac{dy}{dx}=-\frac{\cos x-\sin y}{\cos y}$

$\Rightarrow \frac{\cos y}{\sin y}dy=-\cos dx$

$\Rightarrow \cot y\cdot dy=-\cos xdx$

Integrating both sides we have $\int\cot y\cdot dy=-\int\cos xdx$

$\Rightarrow \log|\sin y|=-\sin x+C$

$\Rightarrow \log|\sin y|+\sin x=C$

This is the required solution of given differential equation

34. $\cos x(1+\cos y)dx-\sin y(1+\sin x)dy=0$

Sol. We have $\cos x(1+\cos y)dx-\sin y(1+\sin x)dy=0$ ... (i)

$\Rightarrow \frac{\cos x}{(1+\sin x)}dx-\frac{\sin y}{(1+\cos y)}dy=0$

$\Rightarrow \int\frac{\cos x}{(1+\sin x)}dx-\int\frac{\sin y}{(1+\cos y)}dy=\log C,$ where C is a constant

$\Rightarrow \log|1+\sin x|+\log|1+\cos y|=\log C$

$\Rightarrow \log|(1+\sin x)(1+\cos y)|=\log C$

$\Rightarrow (1+\sin x)(1+\cos y)=C$

Hence $(1+\sin x)(1+\cos y)=C$ is the required solution

35. $\sin^3 xdx-\sin ydy=0$

Sol. Given differential equation is $\sin^3 xdx-\sin ydy=0$

$\Rightarrow \sin ydy=\sin^3 xdx$

Integrating both sides we have $\int\sin ydy=\int\sin^3 x\,dx$

$\Rightarrow \int\sin ydy=\frac{1}{4}\int(3\sin x-\sin 3x)dx$

$\Rightarrow -\cos y=-\frac{3}{4}\cos x+\frac{1}{12}\cos 3x+C_1$

$\Rightarrow -12\cos y=-9\cos x+\cos 3x+124$

$\Rightarrow \cos 3x-9\cos 3x-12\cos y+C=0$

Where $12c=c$ this is the required solution of given differential equation

36. $\frac{dy}{dx}+\sin(x+y)=\sin(x-y)$

Sol. Given differential equation is $\frac{dy}{dx}+\sin(x+y)=\sin(x-y)$

$\Rightarrow \frac{dy}{dx}+\sin(x+y)-\sin(x-y)=0$

$\Rightarrow \frac{dy}{dx}+2\cos x\cdot\sin x=0$

$\Rightarrow \frac{dy}{dx}=-2\cos x\cdot\sin y$

$\Rightarrow \sec y dy=-2\cos x dx$

Integrating both sides w.r.t x we have

$\int \sec y dy=-2\int \cos x dx$

$\Rightarrow \log|\sec y+\tan y|=-2\sin x+C$

$\Rightarrow \log|\sec y+\tan y|+2\sin x=C$

This is the required general solution at given differential equation

37. $\frac{1}{x}\cos^2 y dy+\frac{1}{y}\cos^2 x dx=0$

Sol. Given differential equation is $\frac{1}{x}\cdot\cos^2 y dy+\frac{1}{y}\cos^2 x dx=0$

$\Rightarrow \frac{1}{x}\cdot\cos^2 y\, dy=-\frac{1}{y}\cos^2 x dx$

$\Rightarrow y\cos^2 y dy=-x\cos^2 x$

$\Rightarrow y\frac{1+\cos 2y}{2}dy=-x\frac{1+\cos 2x}{2}dx$

$\Rightarrow \left(\frac{y}{2}+\frac{y}{2}\cos 2y\right)dy=\left(\frac{-x}{2}-\frac{x\cos 2x}{2x}\right)dx$

$\Rightarrow y\,dy+y\cos 2y\,dy=-xdx-x\cdot\cos 2xdx$

Integrating both sides we have $\int ydy+\int y\cos 2y\,dy=-\int x\,dx-\int x\cos 2xdx$

$\Rightarrow \frac{y^2}{2}+y\int\cos 2y\,dy-\int\left\{\frac{dy}{dx}\int\cos 2y\,dy\right\}dy=-\frac{x^2}{2}-\left[x\int\cos 2x-\int\left\{\frac{dx}{dx}\int\cos 2x\right\}dx\right]+C_1$

$\Rightarrow \frac{y^2}{2}+\frac{1}{2}y\sin 2y-\frac{1}{2}\int\sin 2y\,dy=-\frac{x^2}{2}-\frac{1}{2}x\sin 2x+\frac{1}{2}\int\sin 2x\,dx$

$\Rightarrow \frac{y^2}{2}+\frac{1}{2}\cdot y\cdot\sin 2y+\frac{1}{4}\cos 2y=-\frac{x^2}{2}-\frac{1}{2}x\sin 2x-\frac{1}{4}\cos 2x+C_1$

$\Rightarrow \frac{x^2}{2}+\frac{y^2}{2}+\frac{1}{2}x\sin 2x+\frac{1}{2}y\sin 2y+\frac{1}{4}\cos 2y+\frac{1}{4}\cos 2x=C_1$

$\Rightarrow 2(x^2+y^2)+2\{x\sin 2x+y\sin 2y\}+\{\cos 2x+\cos 2y\}=C$

{where $C=4C_1$

This is the required general solution of given differential equation

38. $\frac{dy}{dx}=\sin^3 x\cos^2 x+xe^x$

Sol. Given differential equation is $\frac{dy}{dx}=\sin^3 x\cdot\cos^2 x+x\cdot e^x$

$\Rightarrow dy = \sin^3 x \cos^2 xdx + xe^x dx$

$\Rightarrow \int dy = \int \sin^3 x \cdot \cos^2 xdx + \int xe^x dx$

$\Rightarrow \int dy = \int \sin^2 x \cdot \cos^2 x \cdot \sin dx + \int x \cdot e^x dx$

$\Rightarrow \int dy = \int \left(1 - \cos^2 x\right) \cos^2 x \sin xdx + \int xe^x dx$

Let $\cos x = Z \Rightarrow \sin xdx = -dz$

$\Rightarrow \int dy = -\int \left(1 - z^2\right) \cdot z^2 dz + x\int e^x dx - \int \left\{\frac{dx}{dx} \int e^x dx\right\} dx$

$\Rightarrow y = -\int z^2 dz + \int z^4 dz + x \cdot e^x - e^x + c$

$\Rightarrow Y = -\frac{z^3}{3} + \frac{z^5}{5} + e^x (x-1) + c$

$\Rightarrow y = -\frac{1}{3}\cos^3 + \frac{1}{5}\cos^5 x + e^x (x-1) + c_1$

This is the Required general solution of given differential equation

39. Find the particular solution of the differential equation $\frac{dy}{dx} = 1 + x + y + xy$, given that $y = 0$ when $x = 1$

Sol. Given differential equation is $\frac{dy}{dx} = 1 + x + y + xy$

$\Rightarrow \frac{dy}{dx} = (1+x) + y(1+x)$

$\Rightarrow \frac{dy}{dx} = (1+x)(1+y)$

$\Rightarrow \frac{dy}{1+y} = (1+x)dx$

$\Rightarrow \int \frac{dy}{1+y} = \int (1+x)dx \cdot$ {integrating both side}

$\Rightarrow \log|1+y| = x + \frac{x^2}{2} + c$ ...... (i)

Putting $x = 1$ and $y = 0$ in (i) we have $\log|1+0| = 1 + \frac{1}{2} + c$

$\Rightarrow 0 = \frac{3}{2} + c \quad \Rightarrow c = -\frac{3}{2}$

Putting $c = -\frac{3}{2}$ in (i) we have $\log|1+y| = x + \frac{x^2}{2} - \frac{3}{2}$

This is the required particular solution of given differential equation

40. Find the particular solution of the differential equation $x\left(1+y^2\right)dx - y\left(1+x^2\right)dy = 0,$ given that $y = 1$ when $x = 0$

Sol. Given differential equation is $x\left(1+y^2\right)dx - y\left(1+x^2\right)dy = 0$

$\Rightarrow x\left(1+y^2\right)dx = y\left(1+x^2\right)dy$

$\Rightarrow \frac{ydy}{1+y^2} = \frac{xdx}{1+x^2}$

Integrating both sides we have $\int \frac{ydy}{1+y^2} = \int \frac{xdx}{1+x^2}$

$\Rightarrow \int \frac{2ydy}{1+y^2} = \int \frac{2xdx}{1+x^2}$

$\Rightarrow \log|1+y^2| = \log|1+x^2| + \log c$

$\Rightarrow \log|1+y^2| = \log c|1+x^2|$

$\Rightarrow |1+y^2| = c_1|1+x^2|$

$\Rightarrow (1+y^2) = \pm(1+x^2)$

$\Rightarrow (1+y^2) = c(1+x^2)$ { Let $c = \pm c_1$ .... (i)

Putting $x=0$ and $y=1$ in (i) we have $2=c$

Putting $c=2$ (i) we have $(1+y^2) = 2(1+x^2)$

$\Rightarrow y^2 = 2+2x^2-1 \Rightarrow y^2 = 2x^2+1$

This is the particular solution of given differential equation

41. Find the particular solution of the differential equation $\log\left(\frac{dy}{dx}\right) = 3x+4y$, given that $y=0$ where $x=0$

Sol. Given differential equation is $\log\left(\frac{dy}{dx}\right) = 3x+4y$

$\Rightarrow \frac{dy}{dx} = e^{3x+4y} \Rightarrow \frac{dy}{dx} = e^{3x} \cdot e^{4y}$

$\Rightarrow e^{-4y}dy = e^{3x} \cdot dx$

Integrating both sides we have $\int e^{-4y} \cdot dy = \int e^{3x} \cdot dx$

$\Rightarrow -\frac{1}{4} \cdot e^{-4y} = \frac{1}{3}e^{3x} + c$ ...... (i)

Putting $x=0$ and $y=0$ we have $-\frac{1}{4} \cdot e^0 = \frac{1}{3}e^0 + c$

$c = -\frac{1}{4} - \frac{1}{3} \Rightarrow c = \frac{-7}{12}$

Putting $c = -\frac{7}{12}$ in (i) we have

$-\frac{1}{4} \cdot e^{-4y} = \frac{1}{3} \cdot e^{3x} - \frac{7}{12}$

$\Rightarrow -3.e^{-4y} = 4 \cdot e^{3x} - 7$

$\Rightarrow 4e^{3x} + 3e^{-4y} = 7$

This is the required particular solution of given differential equation

42. Solve the differential equation $(x^2 - yx^2)dy + (y^2 + x^2y^2)dx = 0$, given that $y=1$ when $x=1$

Sol. Given differential equation is $x^2(1-y)dy + y^2(1+x^2)dx = 0$

$$\Rightarrow x^2(1-y)dy = -y^2(1+x^2)dx$$

$$\Rightarrow \frac{(1-y)dy}{y^2} = \frac{-(1+x^2)}{x^2}dx$$

Integrating both sides we have $\int \frac{(1-y)dy}{y^2} = -\int \frac{1+x^2}{x^2}dx$

$$\Rightarrow \int \left\{\frac{1}{y^2} - \frac{1}{y}\right\}dy = -\int \left\{\frac{1}{x^2} + 1\right\}dx$$

$$\Rightarrow -\frac{1}{y} - \log|y| = \frac{1}{x} - x + c \quad \text{.... (i)}$$

Putting $x = 1$ and $y = 1$ we get $-1 - \log 1 = 1 - 1 + c$

$c = -1$

Putting $c = -1$ in (i) we have $-\frac{1}{y} - \log|y| = \frac{1}{x} - x - 1$

$\Rightarrow x - \frac{1}{x} - \frac{1}{y} - \log|y| + 1 = 0$ is the particular solution of given differential equation

43. Find the particular solution of the differential equation $e^x\sqrt{1-y^2}\,dx + \frac{y}{x}dy = 0$ given that $y = 1$ when $x = 0$

Sol. Given differential equation is $e^x\sqrt{1-y^2}\,dx + \frac{y}{x}dy = 0$

$$\Rightarrow e^x\sqrt{1-y^2}\,dx = -\frac{y}{x}dy$$

$$\Rightarrow x.e^x dx = -\frac{ydy}{\sqrt{1-y^2}}$$

$$\Rightarrow \int x \cdot e^x dx = \int \frac{-ydy}{\sqrt{1-y^2}}$$

$$\Rightarrow x\int e^x dx - \int \left\{\frac{dx}{dx}\int e^x dx\right\}dx = \frac{1}{2}\int \frac{-2ydy}{\sqrt{1-y^2}}$$

$$\Rightarrow x \cdot e^x - e^x = \frac{1}{2}2\sqrt{1-y^2} + c$$

$$\Rightarrow e^x(x-1) = \sqrt{1-y^2} + c$$

Putting $x = 0$ and $y = 1$ we have = c

Putting $c = 1$, in (i) we have $e^x(x-1) = \sqrt{1-y^2} = 1$

This is the required particular solution of given differential equation

44. Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{x(2\log x + 1)}{(\sin y + y\cos y)}$, given that $y = \frac{\pi}{2}$ when $x = 1$

Sol. Given differential equation is $\frac{dy}{dx} = \frac{x(2\log x + 1)}{\sin y + y\cos y}$

$$\Rightarrow (\sin y + y\cos y)dy = x(2\log x + 1)dx$$

Integrating both sides we have

$$\int(\sin y + y\cos y)dy = \int x(2\log x + 1)dx$$

$$\Rightarrow \int \sin y\,dy + \int y\cos y dy = 2\int x\log x dx + \int x dx$$

$$\Rightarrow -\cos y + y\int \cos y dy - \int\left\{\frac{dy}{dy}\int \cos y dy\right\}dy$$

$$= 2\left[\log x\int x dx - \int\left\{\frac{d\log x}{dx}\int x dx\right\}dx\right] + \frac{x^2}{2} + c$$

$$\Rightarrow -\cos y + y\sin y - \int \sin y\,dy = 2\left[\frac{x^2}{2}\log x - \int \frac{1}{x}\frac{x^2}{2}dx\right] + \frac{x^2}{2} + c$$

$$\Rightarrow -\cos y + y\sin y + \cos y = x^2 \cdot \log x - \int x dx + \frac{x^2}{2} + c$$

$$\Rightarrow y\sin y = x^2\log x - \frac{x^2}{2} + \frac{x^2}{2} + c \qquad \ldots\ldots \text{(i)}$$

Putting $x = 1$ and $y = \frac{\pi}{2}$ we have $\frac{\pi}{2}\cdot\sin\frac{\pi}{2} = x^2\log y + c$

$$\Rightarrow \frac{\pi}{2} = +c$$

$$\Rightarrow c = \frac{\pi}{2}$$

Putting $c = \frac{\pi}{2} - \frac{1}{2}$ in (i) we have $y\sin y = x^2\cdot\log x + \frac{\pi}{2}$

This is the required solution of given differential equation

45. Solve the differential equation $\frac{dy}{dx} = y\sin 2x$, given that $y(0) = 1$

Sol. Given differential equation is $\frac{dy}{dx} = y\cdot\sin 2x$

$$\Rightarrow \frac{dy}{y} = \sin x \cdot dx$$

Integrating both sides we have $\int\frac{dy}{y} = \int \sin 2x\,dx$

$$\Rightarrow \log|y| = -\frac{1}{2}\cos 2x + c \qquad \ldots\ldots \text{(i)}$$

Putting $x = 0$ and $y - 1$ we have $0 = -\frac{1}{2} + c$

$$\Rightarrow c = \frac{1}{2}$$

Putting $= -\frac{1}{2}$ in (i) we have $\log|y| = -\frac{1}{2}\cos 2x + \frac{1}{2}$

$$\Rightarrow 2\log|y| = 1 - \cos 2x \quad \Rightarrow 2\log|y| = 2\sin^2 x$$

$$\Rightarrow \log|y| = \sin^2 c \quad \Rightarrow |y| = e^{\sin^2 x}$$

This is the required solution of given differential equation

46. Solve the differential equation $(x+1)\frac{dy}{dx}=2xy$, given that $y(2)=3$

Sol. Given differential equation is $(1+x)\cdot\frac{dy}{dx}=2xy$

$\Rightarrow \frac{dy}{y}=\frac{2x}{x+1}dx$

Integrating both sides we have $\int\frac{dy}{y}=2\int\frac{xdx}{x+1}$

$\Rightarrow \int\frac{dy}{y}=2\int\frac{x+1-1}{x+1}dx$

$\Rightarrow \int\frac{dy}{y}=2\int\left\{1-\frac{1}{1+x}\right\}dx$

$\Rightarrow \log|y|=2x-2\log|x+1|+c_1$

$\Rightarrow \log|y|+\log(x+1)^2=c_1+2x$

$\Rightarrow \log\left|y(x+1)^2\right|=C_1+2x$

$\Rightarrow e^{C_1+2x}=\left|y(x+1)^2\right|$

$\Rightarrow \pm e^{C_1}\cdot e^{2x}=y(x+1)^2$

$\Rightarrow C\cdot e^{2x}=y(x+1)^2$ ..... (i)

Putting $x=2$ and $y=3$ we get $C\cdot e^4=3(2+1)^2$

$\Rightarrow C=\frac{27}{e^4}$

Putting $C=\frac{27}{e^4}$ in (i) we have $\frac{27}{e^4}\cdot e^{2x}=y(x+1)^2$

$\Rightarrow y(x+1)^2=27e^{(2x-4)}$

This is the required solution of given differential equation

47. Solve $\frac{dy}{dx}=x(2\log x+1)$, given that $y=0$ when $x=2$

Sol. Given differential equation is $\frac{dy}{dx}=x(2\log x+1)$

$\Rightarrow dy=x(2\log x+1)dx$

Integrating both sides we have $\int dy=2\int x\log x\,dx+6\int xdx$

$y=2\left[\log x\int xdx-\int\left\{\frac{d}{dx}(\log x)\int x\,dx\right\}dx\right]+\int xdx+C$

$\Rightarrow 2\cdot\frac{x^2}{2}\log x-2\int\frac{x^2}{2}\cdot\frac{1}{x}\cdot dx+\int xdx+C$

$\Rightarrow y=x^2\cdot\log x-\int xdx+\int xdx+C$

$\Rightarrow y=x^2\log x+C$ .... (i)

Putting $x=2$ and $y=0$ we have $0=4\log 2+C$

$\Rightarrow C = -4\log 2$

Putting $C = -4\log 2$ in (i) we have $y = x^2 \cdot \log x - 4\log 2$

$\Rightarrow y = x^2 \cdot \log x - 4\log 2$

This is the required solution of given differential equation

48. Solve $\left(x^3 + x^2 + x + 1\right)\frac{dy}{dx} = 2x^2 + x$, given that $y = 1$ when $x = 0$

Sol. Given differential equation is $\left(x^3 + x^2 + x + 1\right)\frac{dy}{dx} = 2x^2 + x$

$\Rightarrow \frac{2x^2 + x}{x^3 + x^2 + x + 1}dx = dy$

Integrating both sides we get

$\int \frac{\left(2x^2 + x\right)dx}{x^2(x+1) + (x+1)} = \int dy$

$\Rightarrow \int \frac{2x^2 + x}{(x+1)\left(x^2 + 1\right)} = \int dy \quad \text{.... (i)}$

Let $\frac{2x^2 + x}{(x+1)\left(x^2+1\right)} = \frac{A}{x+1} + \frac{Bx}{x^2+1} + \frac{C}{x^2+1}$

$\Rightarrow 2x^2 + x = A\left(x^2+1\right) + Bx(x+1) + C(x+1)$

Putting $x = -1$ we have $2 - 1 = 2A + 0 + 0$

$\Rightarrow C = -A = -\frac{1}{2}$

Putting $x = -2$, we have $8 - 2 = 5A + 2B - C$

$\Rightarrow 6 = +\frac{5}{2} + 2B + \frac{1}{2}$

$\Rightarrow 6 - \frac{5}{2} - \frac{1}{2} = 2B$

$\Rightarrow B = \frac{3}{2}$

Now $\frac{2x^2 + x}{(x+1)\left(x^2+1\right)} = \frac{1}{2(x+1)} + \frac{3x}{2\left(x^2+1\right)} - \frac{1}{2\left(x^2+1\right)}$

Now from (i) $\int dy = \int \frac{\left(2x^2 + x\right)}{(x+1)\left(x^2+1\right)}dx$

$\int dy = \frac{1}{2}\int \frac{dx}{x+1} + \frac{3}{2}\int \frac{xdx}{x^2+1} - \frac{1}{2}\int \frac{dx}{x^2+1}$

$\Rightarrow y = \frac{1}{2}\log|x+1| + \frac{3}{4}\log\left(x^2+1\right) - \frac{1}{2}\tan^{-1}x + C \quad \text{.... (ii)}$

Putting $x = 0$ and $y = 1$ we have $1 = \frac{1}{2}\log 1 + \frac{3}{2}\log 2 - \frac{1}{2}\tan^{-1}0 + C$

$\Rightarrow 1 = 0 + 0 - 0 + C$

$\Rightarrow C = 1$

Putting $C=1$ in (ii) we have

$y=\frac{1}{2}\log|x+1|+\frac{3}{4}\log\left(x^2+1\right)-\frac{1}{2}\tan^{-1}x+1$

This is the required solution of given differential equation

49. Solve $\frac{dy}{dx}=y\tan x$, given that $y=1$ and $x=0$

Sol. Given differential equation is $\frac{dy}{dx}=y\tan x$

$\Rightarrow \frac{dy}{y}=\tan x dx$

Integrating both sides we get $\int\frac{dy}{y}=\int\tan x\,dx$

$\Rightarrow \log|y|=\log|\sec x|+\log C_1$

$\Rightarrow \log|y|=\log C_1|\sec x|$

$\Rightarrow |y|=C_1|\sec x|$

$\Rightarrow y=\pm C_1\sec x$

$\Rightarrow y=\pm C\cdot\sec x \quad \pm C_1=e$ let

$\Rightarrow y\cos x=C$

Putting $x=0$ and $y=1$ we have

$1\times\cos 0=e$

$\Rightarrow c=1$

Putting $C=1$ in equation (ii) we have $y\cos x=1$

This is the required solution of given differential equation

50. Solve $\frac{dy}{dx}=y^2\tan 2x$, given that $y=2$ when $x=0$

Sol. Given differential equation is $\frac{dy}{dx}=y^2\tan 2x$

$\Rightarrow y^2dy=\tan 2x\cdot dx$

Integrating both sides we have $\int y^{-2}dy=\int\tan 2x\,dx$

$\Rightarrow -\frac{1}{y}=\frac{1}{2}\log|\sec 2x|+C$

$\Rightarrow -\frac{1}{y}=-\frac{1}{2}|\cos 2x|$

$\Rightarrow -\frac{1}{y}=-\frac{1}{2}\log|\cos 2x|+C \qquad \text{.... (i)}$

Putting $x=0$ and $y=2$ we have $-\frac{1}{2}=-\frac{1}{2}\log|\cos|+C$

$\Rightarrow -\frac{1}{2}=C$

Putting $C=-\frac{1}{2}$ in (i) we get $-\frac{1}{y}=-\frac{1}{2}\log|\cos 2x|-\frac{1}{2}$

$\Rightarrow 2=y\{\log|\cos 2x|+1\}$

This is the required solution of given differential equation

51. Solve $\frac{dy}{dx}=y\cot 2x$, given that $y=2$ when $x=\frac{\pi}{4}$

Sol. Given differential equation is $\frac{dy}{dx}\cdot\cot 2x$

$\Rightarrow \frac{dy}{y}=\cot 2x\cdot dx$

Integrating both sides we have

$\Rightarrow \int\frac{dy}{y}=\int\cot 2x\cdot dx$

$\Rightarrow \log|y|=\frac{1}{2}\log|\sin x|+\log C_1$

$\Rightarrow \log|y|=\log\left(C_1\sin^2 2x\right)$

$\Rightarrow |y|=C_1\sin^2 2x$

$\Rightarrow y=\pm C_1\sin^2 2x$

$\Rightarrow y=C\sin^2 2x$ $\{$Let $\pm C_1=C$ ….. (i)

Putting $x=\frac{\pi}{4}$ and $y=2$ in (i) we have $2=C\sin^2\frac{\pi}{2}$

$\Rightarrow C=2$ putting $C=2$ in (i) we have $y=2\sin^2 2x$

This is the required particular solution of given differential equation

52. Solve $\left(1+x^2\right)\sec^2 ydy+2x\tan ydx=0$ given that $y=\frac{\pi}{4}$ when $x=1$

Sol. Given, $\left(1+x^2\right)\sec^2 y\ dy+2x\ \tan y\ dx=0 \Rightarrow \left(1+x^2\right)\sec^2 y\ dy=-2x\ \tan\ y\ dx$

$\Rightarrow \frac{\sec^2 y}{\tan y}dy=-\frac{2x}{1+x^2}dx \quad \Rightarrow \int\frac{\sec^2 y}{\tan y}dy=-\int\frac{2x}{1+x^2}dx$

Let $\tan y=t$ & $1+x^2=z \Rightarrow \sec^2 y\ dy=dt$ & $2x\ dx=dz$

$\Rightarrow \int\frac{1}{t}dt=-\int\frac{1}{z}dz \Rightarrow \log|t|+\log|z|=\log c \Rightarrow t.z=c$

$\Rightarrow \tan y.\left(1+x^2\right)=c$ …(1)

Given that $y=\frac{\pi}{4}$ when $x=1$, $\tan\left(\frac{\pi}{4}\right)(1+1)=c \Rightarrow 1\,.\,2=c \quad \therefore\ c=2$

Putting the value of c in equation (1), we get, $\left(1+x^2\right)\tan y=2$.

53. Find the equation of the curve passing through the point $\left(0,\frac{\pi}{4}\right)$ whose differential equation is $\sin x\cos ydx+\cos x\sin ydy=0$

Sol. Given differential equation is $\sin x\cdot\cos ydx+\cos x\cdot\sin ydy=0$

$\Rightarrow \cos x\cdot\sin y\cdot dy=-\sin x\cdot\cos ydx$

$\Rightarrow \dfrac{\sin y}{\cos y} dy = -\dfrac{\sin x}{\cos x} dx \Rightarrow \tan y\, dy = -\tan x\, dx$

Integrating both sides we get $\int \tan y\, dy = -\int \tan x\, dx$

$\log y|\sec y| = -\log|\sec x| + \log C_1$

$\Rightarrow \log|\sec y| + \log|\sec x| = \log C_1$

$\Rightarrow \log|\sec x \cdot \sec y| = \log C_1$

$\Rightarrow |\sec x \cdot \sec y| = C_1$

$\Rightarrow \sec x \cdot \sec y = \pm C_1$

$\Rightarrow \sec x \geq \sec y = C \quad \{\text{Let } \pm C_1 = e$

$\because$ the curve passing through $\left(0, \dfrac{\pi}{4}\right)$

$\therefore \sec 0 \cdot \sec \dfrac{\pi}{4} = C$

$\Rightarrow \sqrt{2} = C$

Hence required solution is $\sec x \cdot \sec y = \sqrt{2}$

54. Find the equation of a curve which passes through the origin and whose differential equation is $\dfrac{dy}{dx} = e^x \sin x$

Sol. Given differential equation is $\dfrac{dy}{dx} = e^x \cdot \sin x$

$\Rightarrow dy = e^x \cdot \sin x \cdot dx$

Integrating both sides w.r.t x we have

$\Rightarrow \int dy = \int e^x \cdot \sin x\, dx$

$\Rightarrow y = I_1 + C \quad \{\text{Let } I_1 = \int e^x \sin x dx$

Now $I_1 = \int e^x \sin x\, dx$

$\Rightarrow I_1 = \sin x \int e^x dx \int \left\{ \dfrac{d}{dx}(\sin x) \int e^x dx \right\} dx$

$\Rightarrow I_1 = e^x . \sin x - \int \cos x \cdot e^x dx$

$\Rightarrow I_1 = e^x \sin x - \left[\cos x \int e^x dx - \int \left\{ \dfrac{d}{dx}(\cos x) \int e^x dx \right\} dx \right]$

$\Rightarrow I_1 = e^x \cdot \sin x - \left[e^x \cos x + \int \sin x \cdot e^x dx \right]$

$\Rightarrow e^x \cdot \sin x - e^x \cdot \cos x - I_1$

$\Rightarrow 2I_1 = e^x (\sin x - \cos x)$

$\Rightarrow I_1 = \dfrac{1}{2} \cdot e^x (\sin x - \cos x) \quad \ldots\ldots (ii)$

From (i) and (ii) we have

$y = \dfrac{1}{2} e^x (\sin x - \cos x) + C \quad \ldots.. (iii)$

Since it passes through origin

$\therefore 0=\frac{1}{2}e^{0}(\sin 0-\cos 0)+C$

$\Rightarrow 0=\frac{1}{2}(0-1)+C \quad \Rightarrow C=\frac{1}{2}$

Putting $C=\frac{1}{2}$ in equation (iii) we have

$y=\frac{1}{2}\cdot e^{x}(\sin x-\cos x)+\frac{1}{2}$

$\Rightarrow 2y=e^{x}(\sin x-\cos x)+1$

This is the required solution of given differential equation

55. A curve passes through the point $(0,-2)$ and at any point (x,y) of the curve, the product of the slope of its tangent and y-coordinate of the point is equal to the x-coordinate of the point. Find the equation of the curve

Sol. We know that slope of tangent at (x,y) is $m=\frac{dy}{dx}$

According to question $y\cdot\frac{dy}{dx}=x$

$\Rightarrow ydy=xdx$

Integrating both sides we have $\int ydy=\int x\,dx$

$\Rightarrow \frac{y^2}{2}=\frac{x^2}{2}+C_1$

$\Rightarrow y^2=x^2+2C_1$

$\Rightarrow y^2=x^2+C \quad \{\text{let } 2C_1=C$ .... (i)

Since the curve passes through $(0,-2)$

$\therefore (-2)^2=0+C$

$\Rightarrow C=4$

Putting $C=4$ in (i) we have $y^2=x^2+4$

This is the required equation of curve

56. A curve passes through the point $(-2,1)$ and at any point (x,y) of the curve, the slope of the tangent is twice the slope of the line segment joining the point of contact to the point $(-4,-3)$. Find the equation of the curve

Sol. We know that slope of tangent of (x,y) is $m=\frac{dy}{dx}$

Equation $\frac{dy}{dx}=2\left(\frac{y+3}{x+4}\right)$

$\Rightarrow \frac{dy}{y+3}=\frac{2dx}{x+4}$

Integrating both sides we have $\int\frac{dy}{y+3}=2\int\frac{dx}{x+4}$

$\Rightarrow \log|y+3|=2\log|x+4|+\log C_1$

$\Rightarrow \log|y+3| = \log C_1 (x+4)^2$

$\Rightarrow |y+3| = C_1 (x+4)^2$

$\Rightarrow y+3 = \pm C_1 (x+4)^2$

$\Rightarrow y+3 = C(x+4)^2$ { let $C = \pm C_1$

Since the curve through $(-2,1)$

$\therefore 1+3 = C(-2+4)^2$

$\Rightarrow 4 = 4C$

$\Rightarrow C = 1$

Putting $C = 1$ in equation (i) we have

$y+3 = (x+4)^2$

$\Rightarrow y = x^2 + 8x + 16 - 3$

$\Rightarrow y = x^2 + 8x + 15$

This is the required equation of curve

57. In a bank principal increases at the rate of r% per annum. Find the value of r Rs. 100 double itself in 10 years. (Given $\log_e 2 = 0.9631$)

Sol. Let P be the principal at any time t_1

$$\therefore \frac{dp}{dt} = \frac{pr}{100}$$

$$\Rightarrow \frac{dp}{p} = \frac{r}{100} dt$$

Integrating both sides we have $\int \frac{dp}{p} = \int \frac{rdt}{100}$

$$\Rightarrow \log(p) = \frac{rt}{100} + C \quad \text{...... (i)}$$

At $t = 0$, we have $p = p_0$

$\therefore \log P_0 = C$

$$\therefore \log P = \frac{rt}{100} + \log p_0$$

$$\Rightarrow \log p - \log p_0 = \frac{rt}{100}$$

$$\Rightarrow \log\left(\frac{p}{p_0}\right) = \frac{rt}{100} \quad \text{..... (ii)}$$

Putting $p_0 = 100, p = 2\ p_0 = 200$

And $t = 10$, we have $\log 2 = \frac{r}{10}$

$\Rightarrow r = 10 \log 2 = 6.931$

58. In a bank principal increases at the rate of 5% per annum An amount of Rs. 1000 is deposited in the bank. How much will it worth after 10 years? (Given $e^{0.5} = 1.648$)

Sol. Let P be the principal at any instant t

$\therefore \frac{dp}{dt} = \frac{5p}{50}$

$\Rightarrow \frac{dp}{p} = \frac{5dt}{100}$

$\Rightarrow \frac{dp}{p} = \frac{dt}{20}$

Integrating both sides we have $\int \frac{dp}{p} = \frac{1}{20}\int dt$

$\Rightarrow \log|p| = \frac{t}{20} + \log C$

At $t = 0$ we have $p = 1000$

$\therefore \log 1000 = \log C$

$\therefore \log p = \frac{t}{20} + \log 1000$

Putting $t = 10$, we have $\log p = \frac{10}{20} + \log 1000$

$\Rightarrow \log \frac{p}{1000} = \frac{1}{2}$

$\Rightarrow \frac{p}{1000} = e^{0.5}$

$\Rightarrow p = 1648$

Hence $p = 1648$ after 10 years

59. The volume of a spherical balloon being inflated changes at a constant rate if initially its radius is 3 units and after 3 seconds it is 6 units. Find the radius of the balloon after t seconds

Sol. The volume of a spherical balloon of radius r is given by $V = \frac{4}{3}\pi r^3$

Now, $\frac{dV}{dt} = -k$, where $k > 0$ [note that V is decreasing]

$\Rightarrow \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = -k \Rightarrow \left(4\pi r^2\right)\frac{dr}{dt} = -k$

$\Rightarrow \int\left(4\pi r^2\right)dr = \int(-k)dt$

$\Rightarrow \frac{4}{3}\pi r^3 = -kt + C$ ..... (i), where C is an arbitary constant

Putting $t = 0$ and $r = 3$ in (i), we get $C = 36\pi$

$\therefore \frac{4}{3}\pi r^3 = -\alpha t + 36\pi$ ...... (iii)

It is being given that when $t = 3$, then $r = 6$

Putting $t = 3$ and $r = 6$ in (ii), we get $k = -84\pi$

Putting $k = -84\pi$ in (ii), we get $r^3 = (63t + 27) \Rightarrow r = (63t + 27)^{1/3}$

60. In a culture the bacteria count is 100000. The number is increased by 10% in 2 hours in how many hours will the count reach 200000, if the rate of growth of bacteria is proportional to the number present ?

Sol. Let at any time t, the bacteria count be N. then

$$\frac{dN}{dt} \propto N \Rightarrow \frac{dN}{dt} = kN \Rightarrow \int \frac{1}{N} dN = \int kdt \Rightarrow \log N = kt + \log(C)$$

At $t = 0$, we have $N = 100000$

$\therefore \log C = \log 100000$

$\Rightarrow \lg oN = kt + \log 100000$

At $t = 2$, we have N = 110000

Putting these values in (i), we get $k = \frac{1}{2}\log\frac{11}{10}$ .... (i)

$\therefore \log N = \frac{1}{2} t \log\left(\frac{11}{10}\right) + \log 100000$ ..... (ii)

When $N = 200000$, let $t = T$, then $\log 20000 = \frac{T}{2}\log\left(\frac{11}{16}\right) + \log 100000 \Rightarrow T = \frac{2\log 2}{\log\left(\frac{11}{10}\right)}$