Probability and statistics

* Two events A and B are said to be mutually exclusive if $P(A \cap B) = 0$ (i.e. A & B can not occur simultaneously)

* Two events A and B are said to be independent if $P(A \cap B) = P(A).P(B)$ (i.e. occurance of one event is not affected by the other.

* Two events A & B are said to be equally likely if $P(A) = P(B)$

* For any two event A & B

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad \{\text{Additive theorem}\}$$

either A or <u>B</u>

Question A Number is selected at random from first two hundred (200) number. what is the chance that it is divisible by either 4 or 6.

<u>Ans</u> $S = \{1, 2, 3 \ldots 200\}$

A : divisible by 4 $\quad \frac{200}{4} = 50$

B : divisible by 6 $\quad \frac{200}{6} = 33$

L.CM of 4 & 6 $= 12$

$\Rightarrow \frac{200}{12} = 16$

So $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{50}{200} + \frac{33}{200} - \frac{16}{200}$$

$$= \frac{67}{200}.$$

Conditional Probability:-

The conditional probability of an event B, given A is denoted by $P(B/A)$ and is given by

$$P(B/A) = \frac{P(B \cap A)}{P(A)} \qquad P(A) \neq 0$$

Question Two dice are rolled once. If the number on first dice is greater than the number on the second dice, then the probability that the sum is 7

Sol^n First condition

(2, 1) — 1

(3,1) (3, 2) — 2

(4,1) (4,2) (4, 3) — 3

(5,1) (5, 2) (5,3) (5,4) — 4

(6,1) (6,2) (6,3) (6,4) (6,5) — 5

15

second condition sum is 7

so prob. $= \frac{3}{15} = \frac{1}{5}$ Ans

Question Two dice are rolled once the prob. the the number on first dice is greater than the number on the second dice, Given that sum is 7.

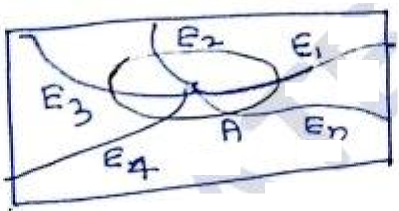
Soln Sum is 7

(6,1) (5,2), (4,3) Prob. $= \frac{3}{6} = \frac{1}{2}$.

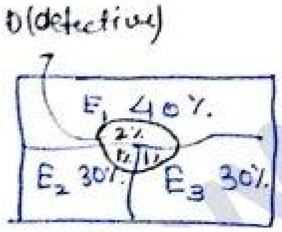
(1,6) (2,5) (3,4)

Bayes theorem:— if $E_1, E_2, \dots E_n$ are n mutually event in the sample space S and A is any arbitary event in S than

$$P\left(\frac{E_i}{A}\right) = \frac{P(E_i)\, P\left(\frac{A}{E_i}\right)}{\sum_{i=1}^{n} P(E_i) P\left(\frac{A}{E_i}\right)}$$



eg



$$P\left(\frac{E_1}{D}\right) = \frac{P(E).\ P\left(\frac{A}{E_1}\right)}{P(E_1).P\left(\frac{A}{E_1}\right) + P(E_2).P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

Prob. of defective product from unit 1

$$= \frac{(0.40)(0.02)}{(0.40)(0.02) + (0.30)(0.01) + (0.30)(0.01)}$$

$$P\left(\frac{E_1}{D}\right) = \frac{4}{7}$$

Q.1

(3,6) (4,6) (5,6) (6,6) (4,5)

(6,3), (6,4), (6,5), (5,4), (5,5)

$$P(\text{sum} > 8) = P(\text{sum } 9) + P(\text{sum } 10) + P(\text{sum } 11) + P(\text{sum } 12)$$

- sum 9: (3,6) (4,5) (5,4), (6,3)
- sum 10: (4,6) (5,5) (6,4)
- sum 11: (5,6) (6,5)
- sum 12: (6,6)

$$= \frac{4}{36} + \frac{3}{36} + \frac{2}{36} + \frac{1}{36} = \frac{10}{36}$$

tricks

$$\underbrace{2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7}_{\left(\frac{\text{sum}-1}{36}\right)} \qquad \underbrace{8 \quad 9 \quad 10 \quad 11 \quad 12}_{\left(\frac{13-\text{sum}}{36}\right)}$$

Q.2

$$P = \frac{0.30}{0.50 + 0.30 + 0.20} = \frac{0.3}{1} = 3/10$$

Q.3

$$50 \times 3C \rightarrow 150$$
$$30 \times 2C \rightarrow 60$$
$$20 \times 1C \rightarrow 20$$
$$\overline{\quad 230 \quad}$$

$$P = \frac{60}{230} = 6/23$$

Q.3

10^{96} Multiples

$10^{96} \times 1 = 10^{96}$

$10^{96} \times 2 =$

$\vdots$

$10^{96} \times 100 = 10^{98}$

$\vdots$

$10^{96} \times 1000 = 10^{99}$ total = 1000 muliple

$$\frac{10^{99}}{10^{96} \times n} = \frac{10^3}{n}$$

$n = 1, 2, 4, 5, 8, 10, 20, 25$
$40, 50, 100, 125, 200$
$250, 500, 1000 \rightarrow 16$

Diviser

$10^1 \longrightarrow 1, 2, 5, 10 \longrightarrow 4$

$10^2 \rightarrow 1, 2, 5, 10, 20, 25, 50, 100 \rightarrow 9$

$10^3 \rightarrow 1, 2, 5, 8, 10, 20, 25, 40, 50, 100, 125, 200, 250, 500, 1000 \rightarrow 16$

$10^n \longrightarrow (n+1)^2$

$10^{99} \longrightarrow (99+1)^2 = 100^2$

So Prob. $= \dfrac{\text{total No. of devisor which are multiple of } 10^{96}}{\text{total devisor of } 10^{99}}$

$= \dfrac{16}{10000} = \dfrac{1}{625}$

Q.4 chanse of one day

M	T	W	T	F	S	S
$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$

chanse of occur all on one day $\underbrace{\left(\frac{1}{7}\right)^7}_{\text{one day}} \times \underbrace{(7)}_{\text{total 7 days}}$

$= \dfrac{7}{7^7} = \dfrac{1}{7^6}$

Q.5

①	②	③	
H	T	T	– 1
H	T	H	– 2
H	H	T	– 2
H	H	H	– 3

Total = 4

exactly 2 = 2

So $P = \frac{2}{4} = \frac{1}{2}$

Q.6

Box 1	3	6	9	12	15
Box 2	6	11	16	21	26

$$P = \frac{5+5+3+3+3}{5 \times 5} = \frac{19}{25}.$$

Product is even = even①.odd② or odd① even② or even① even②

$$P = \frac{{}^2C_1 \cdot {}^2C_1 + {}^3C_1 \cdot {}^3C_1 + {}^2C_1 \cdot {}^2C_1}{{}^5C_1 \cdot {}^5C_1} = \frac{19}{25}$$

Q.7

①	②
4 – R	3 – b
3 – G	4 – G

$$\frac{{}^4C_1 \cdot}{{}^7C_1 \cdot} \times \frac{{}^3C_1}{{}^7C_1}$$

$$\frac{{}^4C_1 \cdot {}^3C_1}{{}^7C_1 \, {}^7C_1} = \frac{12}{49} \Rightarrow \frac{4 \times 3}{7 \times 7} = \frac{12}{49}$$

Q.8

4 - R

6 - B

$$\cancel{\frac{{}^4C_1 \cdot {}^6C_1 \cdot {}^5C_1}{{}^{10}C_3}} = \cancel{\frac{4 \times 6 \times 5}{10 \times 9 \times 8}}$$

$$= \frac{{}^4C_1 \cdot {}^6C_2}{{}^{10}C_3} = \frac{1}{2}$$

Q.9 2nd Method

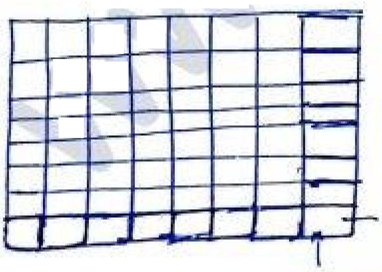
R B B   B R B   B B R

$$\frac{{}^4C_1}{{}^{10}C_1} \cdot \frac{{}^6C_1}{{}^9C_1} \cdot \frac{{}^5C_1}{{}^8C_1} + \frac{{}^6C_1 \cdot {}^4C_1 \cdot {}^5C_1}{{}^{10}C_1 \cdot {}^9C_1 \cdot {}^8C_1} + \frac{{}^6C_1 \cdot {}^5C_1 \cdot {}^4C_1}{{}^{10}C_1 \cdot {}^9C_1 \cdot {}^8C_1}$$

$$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}$$

Q.10 Chess Board

Total $= {}^{64}C_2$



7 in a row $\Rightarrow 7 \times 8$

7 in a column $\Rightarrow 7 \times 8$

$$P = \frac{(7 \times 8) + (7 \times 8)}{{}^{64}C_2}$$

$$P = \frac{2 \times 7 \times 8 \times 2}{64 \times 63} = \frac{1}{18}$$

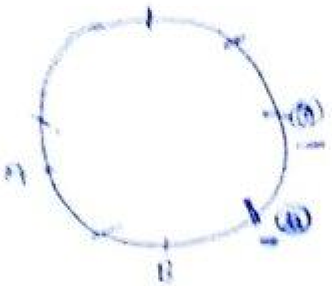
Q.10 PROBABILITY = 11

BB

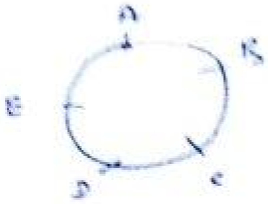
II

$$P = \frac{\frac{9!}{2!} \times 2}{\frac{11!}{2!\,2!}} = \frac{9! \times 2 \times 2}{11 \times 10 \times 9!} = \frac{2}{55}$$

Q.11



$$\frac{(}{(n-1)!}$$



$n \to \text{②},\ (n-2)$

$1 + (n-2)$

$\underbrace{1 + (n-2)}_{(n-1)} \Rightarrow$

When they sit togethe $= (n-2)! \times 2!$

A B ~ 2 possible way.
B A ~

Prob. that they sit toghether $= \dfrac{2!(n-2)!}{(n-1)!} = \dfrac{2}{(n-1)}$

So when they do not sit together $= 1 - \dfrac{2}{(n-1)}$

$$= \frac{n-3}{n-1} \quad \text{An}$$

Q12

6 +ve
8 -ve

$$\frac{{}^6C_2 \cdot {}^8C_2 + {}^6C_4 + {}^8C_4}{{}^{14}C_4} = \frac{505}{1001}$$

Q.13

$4 \to$ 4: (1,3), (2,2), (3,1)

8: (2,6), (3,5), (4,4), (5,3), (6,2) — 5

12: (6,6)

$6 \to$ 6: (3,4)... — (3,3), (4,2), (1,5), (5,1) — 5

12: (6,6)

$= \cancel{\frac{15}{36} = \frac{5}{12}}$

$\Rightarrow \frac{9}{36} + \frac{6}{36} - \frac{1}{36}$

$\Rightarrow 7/18$

Q.14 $8 \to (2,6), (6,2)$ $\quad 9 \to (3,6)\ (6,3)$

$(3,5), (5,3)$ $\quad (4,5)\ (5,4)$

$(4,4)$

either 8 or 9 $\Rightarrow \frac{9}{36} = \frac{1}{4}$ so Neither 8 nor 9 $= 1 - \frac{1}{4} = 3/4$

Q.13 $P(\text{sum } 4) + P(\text{sum } 6) + P(\text{sum } 8) + P(\text{sum } 12)$

$\Rightarrow \frac{3}{36} + \frac{5}{36} + \frac{5}{36} + \frac{1}{36} = \frac{7}{18}$

Q.15 sum 7 $(3,4)\ (4,3)$

$(2,5)\ (5,2) = \frac{1}{2}$

$(1,6)\ (6,1)$

Q.16 $P(P_1) = 0.3$ $\quad P(P_1/P_2) = 0.6$

$P(P_2) = 0.2$ $\quad \frac{P(P_1 \cap P_2)}{P(P_2)} = 0.6$

$P(P_1 \cap P_2) = 0.6 \times 0.2 = 0.12$

Q.17 $\boxed{w - 2}$ $\boxed{r - 3}$ $\boxed{b - 4}$

$$= \frac{{}^2C_1}{{}^9C_1} \cdot \frac{{}^1C_1}{{}^8C_1} \cdot \frac{{}^3C_1}{{}^7C_1} \cdot \frac{{}^2C_1}{{}^6C_1} \cdot \frac{{}^1C_1}{{}^5C_1} \cdot \frac{{}^4C_1}{{}^4C_1} \cdot \frac{{}^3C_1}{{}^3C_2} \cdot \frac{{}^2C_1}{{}^2C_1} \cdot \frac{{}^1C_1}{{}^1C_1}$$

$$= \frac{1}{1260}$$

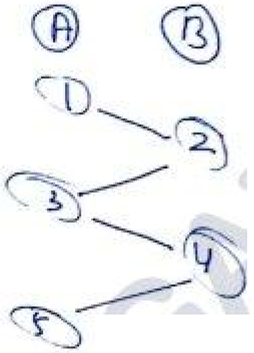
Q. 18

$$\begin{array}{ccccccccc} ① & & ①&②&③ & & ①&②&③&④&⑤ \\ H & & T&T&H & & T&T&T&T&H \\ \frac{1}{2} & & \frac{1}{2}&\frac{1}{2}&\frac{1}{2} & & \frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2} \end{array} \quad \cdots$$

$$\frac{1}{2}+\left(\frac{1}{2}\right)^3+\left(\frac{1}{2}\right)^5 \cdots = \frac{1/2}{1-1/4} = \frac{2}{3}$$

Q.19

$\frac{1}{6}$ +



$$\begin{array}{cccccccccc} ① & & ①&②&③ & & ①&②&③&④&⑤ \\ A & & A'&B'&A & & A'&B'&A'&B'&A \end{array} \quad \cdots$$

$$\frac{1}{6} + \frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6} + \frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6} \cdots$$

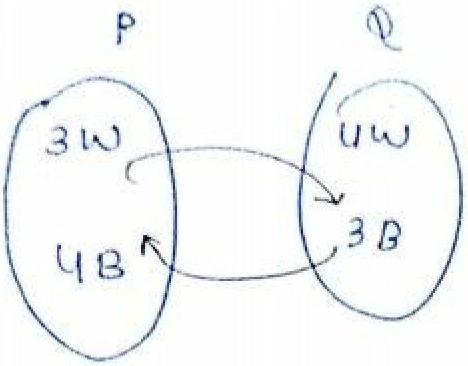
$$\frac{1}{6}\left\{1+\frac{25}{36}+\left(\frac{25}{36}\right)^2 \cdots\right)$$

$$\frac{1}{6}\left\{\frac{1}{1-25/36}\right) = \frac{6}{11}$$

Q.20 $\underbrace{(1,4)\ (2,3)\ (3,2)\ (4,1)}\ (1,6)\ (2,5)\ (3,4)\ (4,3)\ (5,2)\ (6,1)$

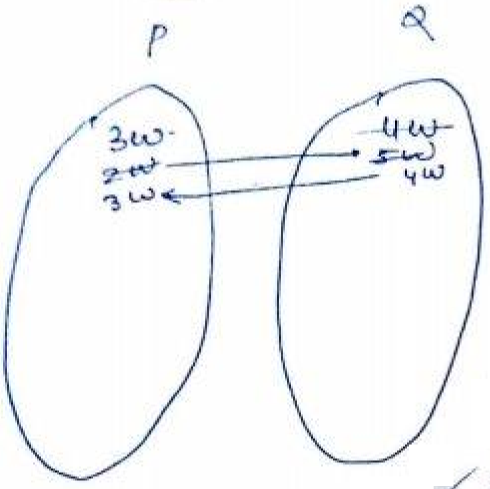
$$= \frac{4}{10} = \frac{2}{5}$$

Q.21



www or wBw or Bww or BBW

WW WB BW BB



WWW

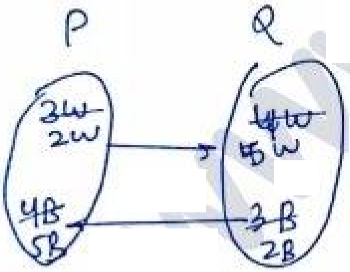
$$\frac{3}{7}\cdot\frac{5}{8}\cdot\frac{3}{7}$$

BWW

$$\frac{4}{7}\cdot\frac{4}{8}\cdot\frac{4}{7}$$

BBW

$$\frac{4}{7}\times\frac{4}{8}\times\frac{3}{7}$$



WBW

$$\frac{3}{7}\cdot\frac{3}{8}\cdot\frac{2}{7}$$

$$\Rightarrow \frac{3}{7}\times\frac{5}{8}\times\frac{3}{5}+\frac{3}{7}\times\frac{3}{8}\times\frac{2}{7}+\frac{4}{7}\times\frac{4}{8}\times\frac{4}{7}+\frac{4}{7}\ \frac{4}{8}\times\frac{3}{7}$$

$$\Rightarrow \frac{45+18+64+48}{392}=\frac{25}{56}$$

Q.22

4 diff $\xrightarrow{D}$ $\frac{{}^4C_1}{{}^6C_1}$ $\boxed{3D,\ 2I}$ $\Rightarrow \frac{{}^2C_1}{{}^5C_1} \Rightarrow \frac{4}{6}\times\frac{2}{5}=\frac{8}{30}$

2 Identicle $\xrightarrow{I}$ $\frac{{}^2C_1}{{}^6C_1}$ 4D 1I $\rightarrow \frac{{}^1C_1}{{}^5C_1} \rightarrow \frac{2}{6}\cdot\frac{1}{5}=\frac{2}{30}$

total $=\frac{10}{30}=\frac{1}{3}$

23

Given

+ve 0.01 $\xrightarrow{\text{Incorrect}}$ 0.12 = 0.01 × 0.12

+

−ve 0.99 $\xrightarrow{\text{incorrect}}$ 0.15 = 0.99 × 0.15

$$(0.01 \times 0.12) + (0.99 \times 0.15) = 0.1497$$

Q.24

knows	Guess
1	1/4
2/3	1/3

$$P\left(\frac{\text{knows}}{\text{Correct}}\right) = \frac{\frac{2}{3}\cdot 1}{\frac{2}{3}\cdot 1 + \frac{1}{3}\cdot\frac{1}{4}} = \frac{8}{9}$$

Q.25

70% ⟶ 80% / 20% (loss / not loss)

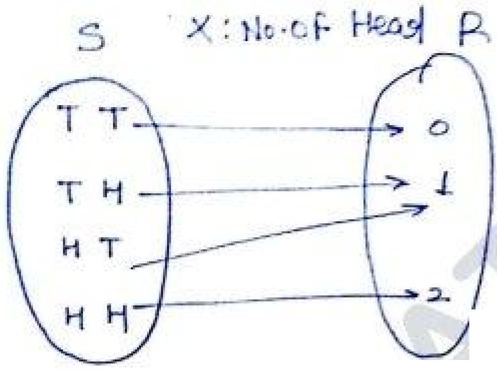
30% ⟶ 10% / 90% (loss / Not loss)

$$P\left(\frac{\text{Non Rain}}{\text{gain}}\right) = \frac{(0.30)(0.90)}{(0.70)(0.20) + (0.30)(0.90)} = \frac{27}{41}$$

Random Variable :-

Random variable is a real valued function define on a sample space of random experiment

Eg :- Tossing of Two coins



X	0	1	2
p(X)	1/4	2/4	1/4

Random variable

- Discrete R.V.
- Continuous R.V.

Discrete R.V.

* it takes Countable Values
* The probability are given by prob. mass function

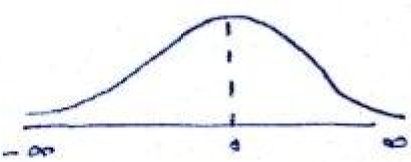
$$\sum p(x) = 1$$

p(x)	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$

Continuous R.V.

* It takes Value in an interval
* The probabilities given by prob. density function

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$



Mean or Mathematical Expectation :-

The mean or mathematical expectation of a random variable x is given by

$$E(x) = \mu = \begin{cases} \sum x \cdot p(x), & \text{if } X \text{ is Descrete} \\ \int_{-\infty}^{\infty} x \cdot f(x)\, dx, & \text{if } X \text{ is continuous} \end{cases}$$

* $E(x) = c$: where c constant

* $E(cx) = c\,E(x)$

* if x & Y are two R.Vs then $\boxed{E(x+Y) = E(x) + E(Y)}$

* if X & P are independent R.Vs then $E(XY) = E(x) \cdot E(Y)$.

Variance :- The variance of a r.v. X is given by

$$Var = \sigma^2 = E\left((x-\mu)^2\right) = \begin{cases} E(x-\mu)^2 \cdot p(x) & \text{if } X\text{- Discreate} \\ \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx & \text{if } X\text{- Continuous.} \end{cases}$$

$$\boxed{Var\, X = E(x^2) - (E(x))^2}$$

* $\text{Var}(c) = 0$ where c is const.

* $\text{Var}(cX) = c^2 \text{Var}X$

* $\text{Var}(-X) = (-1)^2 \text{Var}X = \text{Var}X$

* standard deviation $= \sigma = \sqrt{\text{Var}X}$

* If X & Y are independent R.V.s then

$$\text{Var}(X+Y) = \text{Var}X + \text{Var}Y$$

Q.27

$$f(x) = e^{-x}, \quad 0 < x < \infty$$

$$P(X>1) = \int_1^\infty e^{-x}dx = -e^{-x}\Big|_1^\infty$$

$$= -e^{-\infty} - (-e^{-1})$$

$$= 1/e = 0.368.$$

Q.26

face	1	2	3	4	5	6
Prob	k	2k	3k	4k	5k	6k

$$\Sigma P(x) = 1 \Rightarrow (k + 2k \cdots + 6k) = 1 \Rightarrow k = \frac{1}{21}$$

Prob of geting odd face $= \frac{1}{21} + \frac{3}{21} + \frac{5}{21} = \frac{3}{7}$

Q.29

X	1	2	3
$P(X)$	$\frac{2+5P}{5}$	$\frac{1+3P}{5}$	$\frac{1.5+2P}{5}$

$$\Sigma P(X) = 1$$

$$2+5P+1+3P+1.5+2P = 5$$

$$10P = 0.5$$

$$P = 0.05$$

$$E(X) = \Sigma X P(X)$$

$$= \frac{1(2+0.25)}{5} + \frac{2(1+0.15)}{5} + \frac{3(1.5+0.1)}{5}$$

$$= \frac{2.25}{5} + \frac{2.30}{5} + \frac{4.5}{5}$$

$$E(X) = 0.45 + 0.46 + 0.96 = 1.87$$

Q.28

$$f(x) = \begin{cases} \lambda(x-1)(2-x) & 1 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$\int_1^2 \lambda(x-1)(2-x)\, dx = 1$$

$$\lambda\left\{\frac{3x^2}{2} - \frac{x^3}{3} - 2x\right\}\Big|_1^2 = 1 \Rightarrow \lambda\left(\frac{-4+5}{6}\right) = 1$$

$$\boxed{\lambda = 6}$$

Q. 30

x	0	1	2
$p(x)$	$1/6$	$2/3$	$1/6$

Mean $E(x) = \mu = 0 \times \frac{1}{6} + \frac{2}{3} + \frac{2}{6} = 1$

$\text{Var}\,x = E(x^2) - (E(x))^2$

$E(x^2) = \sum x^2 p(x)$

$E(x^2) = \frac{2}{3} + \frac{4}{6} = \frac{4}{3}$

$\text{Var}\,x = \frac{4}{3} - 1 = \frac{1}{3}$

Q. 31

Binomal Distribution

The probability mass function of Binomial distribution is given by

$$P(x) = {}^{n}C_{x}\, p^{x}\, q^{n-x} \qquad x = 0, 1, 2 \ldots$$

mean = np

Variance = npq

Hint
Repeated exp.
only two outcoms

where

n : no of trials

x : the random variable

p : prob. of success of the R.V.

q : prob of failure of the R.V.

$$p + q = 1$$

Q.31 $n = 4$ X : Heads $p = P(H) = \frac{1}{2}$, $q = \frac{1}{2}$

H T or H T

3 1 4 0

$P(X=3) + P(X=4)$

$${}^{4}C_{3}\left(\tfrac{1}{2}\right)^{3}\left(\tfrac{1}{2}\right)^{1} + {}^{4}C_{4}\left(\tfrac{1}{2}\right)^{4}\left(\tfrac{1}{2}\right)^{0}$$

$$= \frac{4}{16} + \frac{1}{16} = \frac{5}{16}$$

Q.32 10^{th} toss

$\downarrow$

$H \left(\frac{1}{2}\right)$

In 9 trial we have to get exactly 3 Heads

$n = 9$

x = Heads $p = \frac{1}{2}$ $q = \frac{1}{2}$

Prob of exactly 3 Heads

$= {}^9C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)$

→ to Get Head in 10th

$= 0.082$

Q.33 $n = 5$ X: Face $P(\text{Face 3}) = \frac{2}{6} = \frac{1}{3}$

$q = \frac{2}{3}$

$P(\text{Sum 12}) = P(\text{Fourtimes we get Face 3})$

$= {}^5C_4 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^1$

Q.34 $n = 11$ X: Forward $p = P(F) = 0.4$, $q = 0.6$

	F	B
one back	5	6
one forward	6	5

$P(X = 5) + P(X = 6)$

${}^nC_5\, p^5 q^{n-5} + {}^nC_6\, p^6 q^{n-6}$

${}^{11}C_5 (0.4)^5 (0.6)^6 + {}^{11}C_6 (0.4)^6 (0.6)^5$

$= 462 (0.4)^5 (0.6)^5 (0.4 + 0.6)$

$= 462 \left(\frac{6}{25}\right)^5$

Q.35 $n = 900$

$p = 0.1, \quad q = 0.9$

mean $= np = 900 \times 0.1 = 90$

s.d. $= \sqrt{Var} = \sqrt{npq} = \sqrt{900 \times 0.1 \times 0.9} = 9$

Poisson Distribution :-

The probability mass function of Poisson distribution is given by

$$P(x) = \frac{e^{-\lambda} \lambda^{x}}{x!}, \quad x = 0, 1, 2 \ldots$$

mean $= \lambda$

Variance $= \lambda$

Note Poission distribution is the limiting case of Binomial distribution.

i.e. if n is large and P is very small then $\lambda = np$

Q.36 mean $= \lambda = 5.2$ $\qquad P(x) = \dfrac{e^{-5.2}(5.2)^{x}}{x!}$

$$P(0) = \frac{e^{-5.2}(5.2)^{0}}{0!}$$

$$P(x<2) = e^{-5.2}\left\{1 + \frac{(5.2)}{1}\right\} = 0.034$$

Q.37 $\lambda = 3$ for one year

for two year $\lambda = 6$

$x \le 2 \quad x = 0, 1, 2.$

$$P(x \le 2) = P(0) + P(1) + P(2)$$

$$= \frac{e^{-6}(6)^0}{0!} + \frac{e^{-6}(6)}{1!} + \frac{e^{-6}(6)^2}{2!}$$

$$P(x \le 2) = e^{-6}(25) = 0.0619 \simeq 0.062$$

Q.38 $n = 100 \;\; \lambda = np \qquad P(x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$

avg defe $= 1 = \lambda$

for $\quad P(x \le 3) = e^{-1}\left\{\frac{1^0}{0!} + \frac{1^1}{1!} + \frac{1^2}{2!} + \frac{1^3}{3!}\right)$

$$= e^{-1}\left\{2 + \frac{1}{2} + \cancel{\frac{1}{6}}\right\} = e^{-1}\left(\cancel{\frac{8}{3}}\,\frac{5}{2}\right)$$

$$\frac{12+3+1}{6}$$

So for $x \ge 3 \quad = 1 - e^{-1}\cancel{\frac{8}{3}}\,\frac{5}{2}$

$$P(x \ge 3) = 1 - P(X < 3)$$

$$= 1 - e^{-1}\frac{5}{2}$$

Q.39 $X = 2, 4, 6, 8, 10$, $X^2 = 4, 16, 36, 64, 100$

$E(X) = \text{mean} = \frac{30}{5} = 6$ $E(X^2) = 44$

$Var(x) = 44 - 6 =$

Uniform distribution

If x follows uniform distribution in the interval $[\alpha, \beta]$ then the probability density function is $f(x) = \frac{1}{\beta - \alpha}$, $\alpha \le X \le \beta$

$$\text{Mean} = \frac{\alpha + \beta}{2}$$

$$\text{Variance} = \frac{(\beta - \alpha)^2}{12}$$

Q.39 $[2, 10]$ $\alpha = 2, \beta = 10$

$$Var. = \frac{(10-2)^2}{12} = \frac{64}{12} = \frac{16}{3}$$

Q. 40

$$f(x) = \frac{1}{5} e^{-x/5} \quad x \geq 0$$

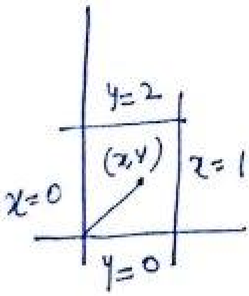
$$P(x) = \int_{-\infty}^{\infty} \frac{1}{5} e^{-x/5}\, dx$$

$$P(x>5) = \int_{5}^{\infty} \frac{1}{5} e^{-x/5} = \frac{1}{5}\left[-5 e^{-x/5}\right]_5^{\infty}$$

$$P(x>5) = -\left[e^{-\infty} - e^{-1}\right]$$

$$P(x>5) = 1/e$$

Question A point is randomly selected with uniform probability in xy plane within the rectangle bounded by $x=0$, $x=1$, $y=0$, $y=2$. if l is the length of the point then the expected value of l^2 is – a) $\frac{1}{3}$ b) $\frac{2}{3}$ c) $\frac{4}{5}$ d) $\frac{5}{3}$



$$l = \sqrt{x^2 + y^2}$$

$$l^2 = x^2 + y^2$$

$$E(l^2) = E(x^2+y^2) = E(x^2) + E(y^2)$$

$$= \int_0^1 x^2 f(x)\, dx + \int_0^2 y^2 f(y)\, dy$$

$$x \to [0,1] \Rightarrow f(x) = \frac{1}{1-0} = 1$$

$$y \to [0,2] \to f(y) = \frac{1}{2-0} = \frac{1}{2}$$

So $E(e^2) = \int_0^1 x^2\, dx + \int_0^2 \frac{y^2}{2}\, dy$

$$= \left[\frac{x^3}{3}\right]_0^1 + \left[\frac{y^3}{6}\right]_0^2$$

$$E(e^2) = \frac{1}{3} + \frac{8}{6} = \frac{5}{3}$$

Q.41

$$f(t) = \alpha e^{-\alpha t} \qquad 0 \le t \le \infty$$

$$100 < t < 200$$

$$P(t) = \int_{-\infty}^{\infty} f(t)\, dt = \int_{100}^{200} \alpha e^{-\alpha t}\, dt$$

$$P(t) = -\alpha\left[\frac{e^{-\alpha t}}{\alpha}\right]_{100}^{200}$$

$$= -\left[e^{-200\alpha} - e^{-100\alpha}\right]$$

$$= e^{-100\alpha} - e^{-200\alpha}$$

Exponential Distribution

The probability density function of exponential Distribution is given by

$$f(x) = \lambda e^{-\lambda x} \quad , x \geq 0$$

$$\text{mean} = \frac{1}{\lambda}$$

$$\text{variance} = \frac{1}{\lambda^2}$$

$e^{-st} f(x)$

Q. 46 $\quad f_x(x) = \frac{3}{2} e^{-3x} u(x) + a e^{4x} u(-x)$

$$\int_{-\infty}^{\infty} f_x(x)\, dx = 1$$

$$\int_{-\infty}^{\infty} \frac{3}{2} e^{-3x} u(x)\, dx + a \int_{-\infty}^{\infty} e^{4x} u(-x)\, dx$$

$u(x-a)$

$$\frac{3}{2} \left[\frac{e^{-0\times 3}}{3} \right] - a \left[\frac{(-1)}{-4} \right] = 1$$

$$\frac{1}{2} - \frac{a}{4} = 1 \Rightarrow \frac{a}{4} = 1 - \frac{1}{2} \Rightarrow a = 2$$

~~$P\{x<0\} = \int_{-\infty}^{0} \frac{3}{2}$~~ $\quad P(x<0) = \frac{1}{2}$

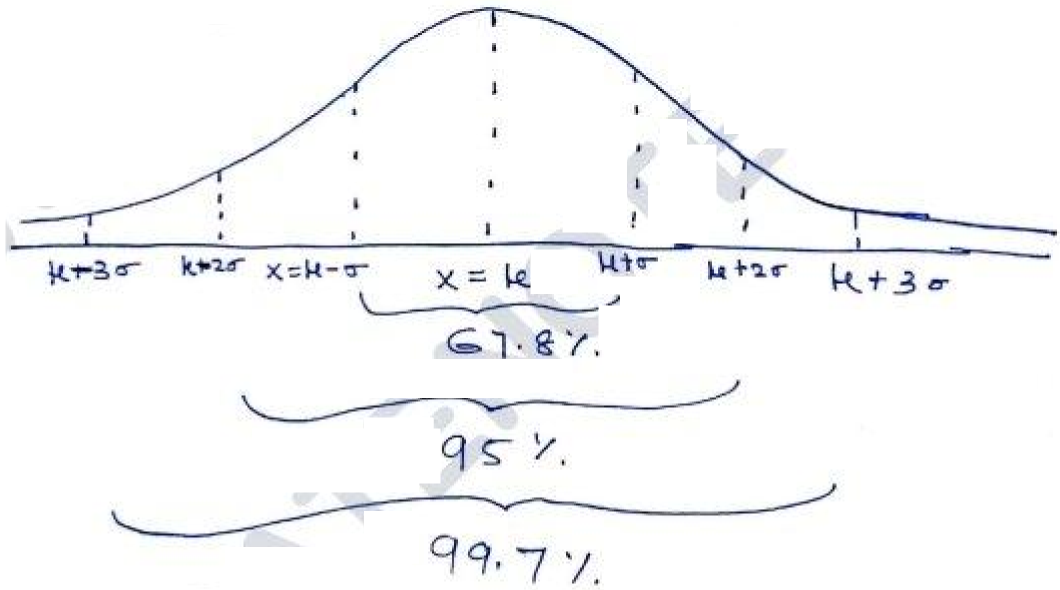
Normal distribution :-

The probability density function of normal distribution is given by

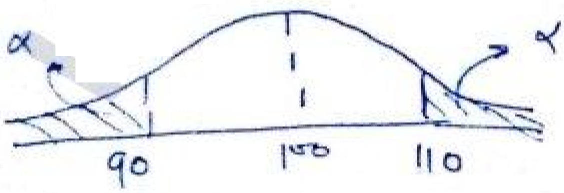
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \quad -\infty < x < \infty$$

mean $= \mu$

variance $= \sigma^2$



Q.42 $\quad \mu = 100 \quad P(\alpha \geq 110) = \alpha \quad P(90 < x < 110)$



$P(\alpha \geq 110) = \alpha$

$\therefore P(P \leq 90) = \alpha$ by Sym

it follow that

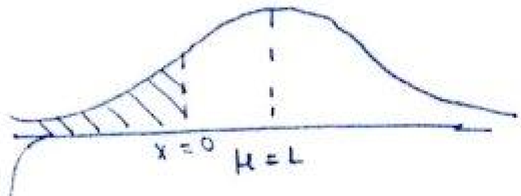
$P(90 \leq x \leq 110) = 1 - 2\alpha$

Q. 43

$\mu = 1$

$\sigma^2 = 4 \Rightarrow \sigma = 2$

$P\{X < 0\}$

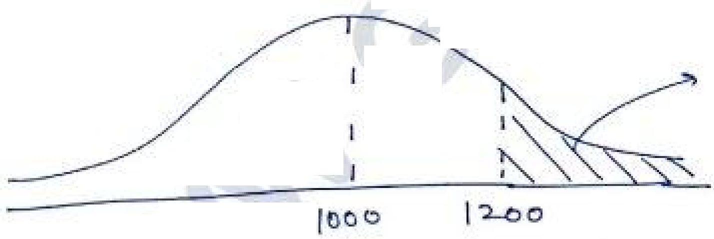


less than = 50 %

Q.44

$\mu = 1000$ mm

$\sigma^2 = 200 \Rightarrow \sigma = \sqrt{200}$



< 50 % Ans

$$Z = \frac{1200 - 1000}{\sqrt{200}} = 4.14$$

Q. 45

$P(X) = 0.40$, $P(X \cup Y^c) = 0.7$

$P(X \cap Y) = P(X)\,P(Y)$

$P(X \cup Y^c) = P(X) + P(Y^c) - P(X \cap Y^c)$

$= P(X) + 1 - P(Y) - P(X)\,(1 - P(Y))$

$= P(X) + 1 - P(Y)$

$=$

Q.46

$$f(x) = \frac{3}{2} e^{-3x} u(x) + a e^{4x} u(-x)$$

$$\int_{-\infty}^{\infty} f(x) = 1$$

$$\int_{-\infty}^{0} a e^{4x}\, dx + \int_{0}^{\infty} \frac{3}{2} e^{-3x}\, dx = 1$$

$$\frac{a}{4}\left[e^{4x}\right]_{-\infty}^{0} + \frac{3}{2}\left[\frac{-e^{-3x}}{3}\right]_{0}^{\infty} = 1$$

$$\frac{a}{4}\left[e^{0} - e^{-\infty}\right] - \frac{1}{2}\left[e^{-\infty} - e^{0}\right] = 1$$

$$\frac{a}{4} + \frac{1}{2} = 1 \Rightarrow \frac{a}{4} = \frac{1}{2} \Rightarrow a$$

$$a = 4/2 = 2$$

$$P(X<0) = \int_{-\infty}^{0} f(x)\, dx = \int_{-\infty}^{0} a e^{4x}\, dx$$

$$= \int_{-\infty}^{0} 2 e^{4x}\, dx$$

$$P(X<0) = \frac{2}{4}\left[e^{0} - e^{-\infty}\right] = \frac{1}{2}$$

Correlation & Regression :-

* The coefficient of correlation is given by

$$r = \frac{S_{xy}}{\sqrt{S_{xx}}\sqrt{S_{yy}}}$$ (Always -1 to 1)

where $S_{xy} = \sum_i (x_i - \bar{x})(y_i - \bar{y})$

$$S_{xx} = \sum_i (x_i - \bar{x})^2$$

$$S_{yy} = \sum_i (y_i - \bar{y})^2$$

* The regression line of y on x is given by.

$$y - \bar{y} = r\frac{\sigma_y}{\sigma_x}(x - \bar{x})$$

or

$$y - \bar{y} = b_{yx}(x - \bar{x}) \quad \text{where } b_{yx} = r\frac{\sigma_y}{\sigma_x}$$

* The regression line of x on y is given by

$$x - \bar{x} = r\frac{\sigma_x}{\sigma_y}(y - \bar{y})$$

or

$$x - \bar{x} = b_{xy}(y - \bar{y}) \quad \text{where } b_{xy} = r\frac{\sigma_x}{\sigma_y}$$

* The coefficient of correllation

$$\gamma = \sqrt{b_{yx} \cdot b_{xy}}$$

when two regrassion lines are given

$$\gamma = \sqrt{r \frac{\sigma_y}{\sigma_x} \cdot r \frac{\sigma_x}{\sigma_y}}$$

$$\gamma = \sqrt{\gamma^2} = \pm \gamma$$

Note: * If both the regrassion positive then the coefficient of correllation is also positive

* If both the regrassion coeff. are negative then the coeff. or Correllation is also negative

Ques Find the coeff. of Correllation to the following data

x	-1	0	3	2
y	1	2	3	2

$\bar{x} = 4/4 = 1$

$\bar{y} = 8/4 = 2$

$S_{xy} = (-1-1)(0-1)($

$$S_{xy} = (-1-1)(1-2) + (0-1)(2-2) + (3-1)(3-2) + (2-1)(2-2)$$

$$S_{xy} = 2 + 2 = 4$$

$$S_{xx} = (-1-1)^2 + (0-1)^2 + (3-1)^2 + (2-1)^2$$

$$S_{xx} = 4 + 1 + 4 + 1 = 10$$

$$S_{yy} = 1 + 0 + 1 + 0 = 2$$

$$r = \frac{S_{xy}}{\sqrt{S_{xx}}\sqrt{S_{yy}}} = \frac{4}{\sqrt{10}\sqrt{2}} = 2/\sqrt{5}$$

$$= 0.894$$

x	y	$X = x - \bar{x}$	$Y = y - \bar{y}$	XY	X^2	Y^2
-1	1	-2	-1	2	4	1
0	2	-1	0	0	1	0
3	3	2	1	2	4	1
2	2	1	0	0	1	0
				$S_{xy} = 4$	$S_{xx} = 10$	$S_{yy} = 2$

$\Sigma x = 4$ $\Sigma y = 8$

$\bar{x} = 1$ $\bar{y} = 2$

$$r = \frac{4}{\sqrt{10}\sqrt{2}} = 0.894$$

Q. If $y = x + 1$ & $x = 3y - 7$ are two regrassion lines then the coefficient of correllation

X wrong method
$$\left.\begin{aligned} y &= x+1 \\ x &= 3y-7 \end{aligned}\right\}$$

$r = \sqrt{3 \cdot 1} = \sqrt{3}$ Not Correct

because $\sqrt{3} > 1$

So

$y = x + 1 \Rightarrow x = y - 1$ $\quad b_{xy} = 1$

$x = 3y - 7 \Rightarrow y = \frac{x}{3} - \frac{7}{3}$ $\quad b_{yx} = \frac{1}{3}$

So $r = \sqrt{1 \cdot \frac{1}{3}} = \sqrt{\frac{1}{3}} = 0.577$