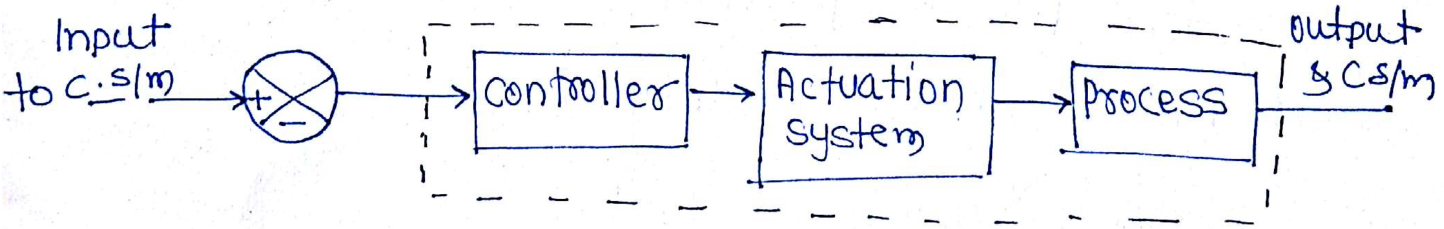
Control system

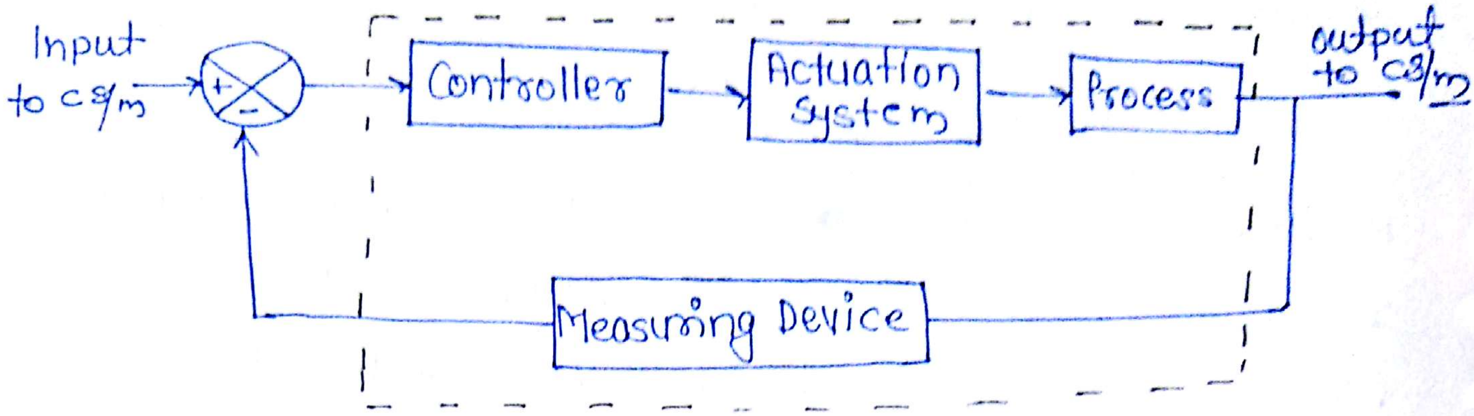
A Group of component connected together to perform a desired task is called control system.
Every Control System by default include the regulatory mechanism.
Control system are devided in two types
① open loop Control ~~Control~~ System.
② closed loop Control System

① open loop Control System:- In this case controller doen't get any information for process Variable, as there is no feedback.

Note:- Idealy control system output should be equal to input applied to the control system.



② closed loop Control System:- In this case Controller Gets the information about the process variable as feedback is present.



Note :⇒ Idealy controller should make error value zero in less time.

⇒ In open loop system as well as closed loop system, we should train controller by predefined algorithm.

⇒ To ~~Gener~~ write the algorithm we should have mathematical model of a physical system.

Mathematic Modeling :-

The process of developing mathematical expression for a physical process or system called mathematic modeling :-

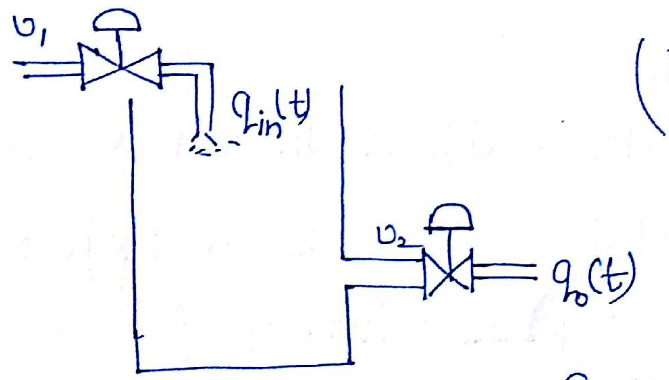
Steps to find mathematic model.

① Apply conservation of energy or force or flow rate or voltage etc. based on the nature of the physical system.

② Re-arrange the differential equation (which is a result of 1st step), and find the order of the system.

③ Apply Laplace transform on both sides to get the transfer function of the system. or apply differential equation concepts to find require variable.

Consider a hydraulic system, where the water level increases with the time as shown below.

i) 

$\begin{pmatrix}\text{input flow}\\ \text{rate}\end{pmatrix} - \begin{pmatrix}\text{output flow}\\ \text{rate}\end{pmatrix} =$ Accumulation of water in the tank

$$q_{in}(t) - q_o(t) = A\frac{dh(t)}{dt}$$

$$q_{in}(t) - \frac{h(t)}{K} = A\frac{dh(t)}{dt}$$

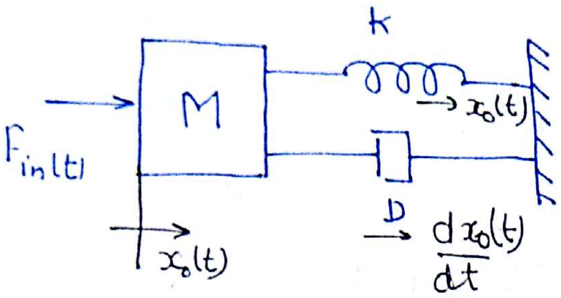
$$\frac{dh(t)}{dt} + \frac{h(t)}{AK} = \frac{1}{A} q_{in}(t)$$

take laplace both side

$$sH(s) - \cancel{H(0^-)}^{0} + \frac{H(s)}{AK} = \frac{1}{A} Q(s)$$

Ex (II) Mass- Damper - spring_System:-

Consider mass damper spring system.



$x_o(t) \longrightarrow 1^{st}$ state $\Rightarrow P_1(t)$

$\frac{dx_o(t)}{dt} \longrightarrow 2^{nd}$ state $\Rightarrow P_2(t)$

2 order Syste so 2stage

$$F_{in}(t) = M.\frac{d^2x_o(t)}{dt} + D.\frac{dx_o(t)}{dt} + K\,x_o(t)$$

state space representation →

Representing the mathematical model of a physical system in matrix form is a major purpose of state space representation.

Every physical system can be mathemattical model, every mathematical model can be represented in matix form.

Convert mathematical model of mass damper spring system to state space representation.

$$\frac{d^2x_o(t)}{dt^2} + \frac{D}{M}\frac{dx_o(t)}{dt} + \frac{k}{m}x_o(t) = \frac{1}{M}F_{in}(t)$$

$$x_o(t) = P_1(t) \qquad \dot{P}_1(t) = P_2(t)$$

$$\frac{dx_o(t)}{dt} = P_2(t), \qquad \frac{d^2x_o(t)}{dt^2} = \dot{P}_2(t)$$

$$\Rightarrow \dot{P}_2(t) + \frac{D}{M} P_2(t) + \frac{K}{M} P_1(t) = \frac{1}{M} F_{in}(t)$$

$$\dot{P}_2(t) = -\frac{K}{M} P_1(t) - \frac{D}{M} P_2(t) + \frac{1}{M} F_{in}(t)$$

$$\dot{P}_1(t) = P_2(t)$$

$$\begin{bmatrix} \dot{P}_1(t) \\ \dot{P}_2(t) \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -K/M & -D/M \end{bmatrix}_{2\times 2}}_{\text{System matrix [A]}} \begin{bmatrix} P_1(t) \\ P_2(t) \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 1/M \end{bmatrix}_{2\times 1}}_{\text{input matrix [B]}} F_{in}(t)$$

$$x_o(t) = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{\text{Output Vector [C]}} \underbrace{\begin{bmatrix} P_1(t) \\ P_2(t) \end{bmatrix}_{2\times 1}}_{\text{State Vector}}$$

Controllability:-

If the states of the system are able to change from one value to another value in a finite time by a finite input we can say that system is controlable otherwise it is uncontrolable.

To check controllability consider controllability matrix (ϕ_c)

$$[\phi_c] = \begin{bmatrix} B & AB \end{bmatrix}_{2\times 2}$$ for 2nd order system

$$[\phi_c] = \begin{bmatrix} 0 & 1/M \\ 1/M & -D/M^2 \end{bmatrix}, \quad |\phi_c| = -\frac{1}{M^2}$$

$\Rightarrow$ if $|\phi_c| = 0$, The system is uncontrollable

$\Rightarrow$ if $|\phi_c| \neq 0$, The system is Controllable

Note:- Order of the Controllability matrix is equal to order of system matrix and equals to order of the system.

For 3rd order system.

$$[\phi_c] = \begin{bmatrix} B & AB & A^2B \end{bmatrix}_{3\times3}$$

Observability:-

If the system states are able to calculate, with the help of output at any instant of time then we can say the system is observable otherwise it's not.

2nd order Sys. $[\phi_o] = [C^T \quad AC^T]$; 3rd order sys $[\phi_o] = [C^T \quad AC^T \quad A^2C^T]$

if $|\phi_o| = 0$; then the system is not-obseravble

if $|\phi_o| \neq 0$; then the system is obseravble

⇒ ~~Consider~~ find the condition for controlability and observability in mass damper spring system

$$[\phi_c] = \begin{bmatrix} 0 & 1/M \\ 1/M & D/M^2 \end{bmatrix} \qquad |\phi_c| = -\frac{1}{M^2}$$

Que A physical system is mathematically order as a 3^{rd} order diff. equation ~~with~~ which is given below.

$$\frac{d^3 y(t)}{dt^3} + 4 \cdot \frac{d^2 y(t)}{dt^2} + 3 \cdot \frac{dy(t)}{dt} + 2\, y(t) = x(t)$$

where $x(t)$ is input to the system and $y(t)$ is output of system.

Find wheather the system is controllable or not.

Find the system is observable or not.

Sol^n

$$\frac{d^3 y(t)}{dt^3} + 4 \cdot \frac{d^2 y(t)}{dt^2} + 3 \cdot \frac{dy(t)}{dt} + 2 \cdot y(t) = x(t)$$

$y(t) \rightarrow P_1(t)$

$\frac{dy(t)}{dt} \rightarrow P_2(t) = \dot{P}_1(t)$

$\frac{d^2 y(t)}{dt^2} \rightarrow P_3(t) = \dot{P}_2(t) = \ddot{P}_1(t)$

$\frac{d^3 y(t)}{dt^3} \rightarrow \dot{P}_3(t)$

$$\dot{P}_3(t) + 4P_3(t) + 3P_2(t) + 2P_1(t) = x(t)$$

$$\dot{P}_3(t) = -2P_1(t) - 3P_2(t) - 4P_3(t) + x(t)$$

$$\begin{bmatrix} \dot{P}_1(t) \\ \dot{P}_2(t) \\ \dot{P}_3(t) \end{bmatrix}_{3\times1} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -3 & -4 \end{bmatrix}_{3\times3}}_{A} \begin{bmatrix} P_1(t) \\ P_2(t) \\ P_3(t) \end{bmatrix}_{3\times1} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}_{3\times1}}_{B} x(t)$$

$y(t) = \underline{P(t)}$

$$[y(t)] = \underbrace{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}_{1\times3}}_{C} \begin{bmatrix} P_1(t) \\ P_2(t) \\ P_3(t) \end{bmatrix}_{3\times1}$$

$$\begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -3 & -4 \end{vmatrix} \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -3 & -4 \end{vmatrix}$$

0 1 1

For $\underline{\text{controllability}}$

$3\times3 \quad 3\times1$

3×1

$$[\phi_c] = [B \quad AB \quad A^2B]_{3\times3}$$

$$[\phi_c] = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -4 \\ 1 & -4 & 13 \end{bmatrix}$$

$|\phi_c| = 1(-1) = -1$ the system is $\underline{\underline{\text{controllable}}}$

Obseravability

$$[\phi_o] = [C^T \quad AC^T \quad A^2C^T]$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -2 \\ 0 & -2 & 8 \end{bmatrix}$$

$$|\phi_o| = 1(-4) = -4$$ system is ~~obser~~ observable

Question:- A system is represented in state space model with $[A] = \begin{bmatrix} 1 & 2 \\ \alpha & 6 \end{bmatrix}$ & $[B] = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ the value of α for which system is uncontrollable.

$$[\phi_c] = [B \quad AB] = \begin{bmatrix} 1 & 3 \\ 1 & \alpha+6 \end{bmatrix}$$

for system should be uncontrollable

$$|\phi_c| = 0$$

$$1(\alpha+6) - (3) = 0$$

$$\boxed{\alpha = -3}$$

Question:- The state variable description of a third order system is given by

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & a_1 & 0 \\ 0 & 0 & a_2 \\ a_3 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

where y(t) is output of the system & u(t) is input of the system then the condition for Controllability of the system is.

$$[Q_c] = [B \quad AB \quad A^2B]$$

$$\begin{bmatrix} 0 & a_1 & 0 \\ 0 & 0 & a_2 \\ a_3 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & a_1 & 0 \\ 0 & 0 & a_2 \\ a_3 & 0 & 0 \end{bmatrix}$$

$0 \quad 0 \quad a_1 a_2$

$a_2 a_3$

$$[Q_c] = \begin{bmatrix} 0 & 0 \\ 0 & a_2 \\ 1 & 0 \end{bmatrix}$$