Short Answer Type Questions – II

[3 marks]

Que 1. Find the roots of the following quadratic equation by factorisation:

(i) $\sqrt{2x}^2 + 7x + 5\sqrt{2} = 0$ (ii) $2x^2 - x + \frac{1}{8} = 0$

Sol. (i) We have, $\sqrt{2x}^2 + 7x + 5\sqrt{2} = 0$

$\Rightarrow$ $\sqrt{2x}^2 + 5x + 2x + 5\sqrt{2} = 0$

$\Rightarrow$ $x(\sqrt{2x} + 5) + \sqrt{2}(\sqrt{2x} + 5) = 0$ $\Rightarrow$ $(\sqrt{2x} + 5)(x + \sqrt{2}) = 0$

$\therefore$ Either $\sqrt{2x} + 5 = 0$ or $x + \sqrt{2} = 0$

$\therefore$ $x = -\frac{5}{\sqrt{2}}$ or $x = -\sqrt{2}$

Hence, the roots are $-\frac{5}{\sqrt{2}}$ and $-\sqrt{2}$.

(ii) We have, $2x^2 - x + \frac{1}{8} = 0$

$\Rightarrow$ $\frac{16x^2 - 8x + 1}{8} = 0$ $\Rightarrow$ $16x^2 - 8x + 1 = 0$

$\Rightarrow$ $16x^2 - 4x - 4x + 1 = 0$ $\Rightarrow$ $4x(4x-1) - 1(4x-1) = 0$

$\Rightarrow$ $(4x-1)(4x-1) = 0$

So, either $4x - 1 = 0$ or $4x - 1 = 0$

$\therefore$ $x = \frac{1}{4}$ or $x = \frac{1}{4}$

Hence, the roots of given equation are $\frac{1}{4}$ and $\frac{1}{4}$.

Que 2. Find the roots of the following quadratic equation, if they exist, by the method of completing the square:

(i) $2x^2 + x - 4 = 0$ (ii) $4x^2 + 4\sqrt{3x} + 3 = 0$

Sol. (i) We have, $2x^2 + x - 4 = 0$

On dividing both sides by 2, we have

$x^2 + \frac{x}{2} - 2 = 0$

$\Rightarrow$ $x^2 + \frac{1}{2}x + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 - 2 = 0$ [b = $\frac{1}{2}$ (coefficient of x) = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$]

$\Rightarrow \quad \left(x + \frac{1}{4}\right)^2 - \frac{1}{16} - 2 = 0 \quad \Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{1}{16} + 2 = \frac{1+32}{16} = \frac{33}{16} > 0$

$\Rightarrow$ Roots exist.

$\therefore \quad x + \frac{1}{4} = \pm\sqrt{\frac{33}{16}} \Rightarrow \quad x + \frac{1}{4} = \pm\frac{\sqrt{33}}{4}$

$\Rightarrow \quad x + \frac{1}{4} = \frac{\sqrt{33}}{4}$ or $x + \frac{1}{4} = -\frac{\sqrt{33}}{4}$

$\therefore \quad x - \frac{1}{4} = +\frac{\sqrt{33}}{4}$ or $x = -\frac{1}{4} = -\frac{\sqrt{33}}{4}$

$\Rightarrow \quad x = \frac{\sqrt{33}-1}{4}$ or $x = \frac{-(\sqrt{33}+1)}{4}$

Hence, roots of given equation are $\frac{\sqrt{33}-1}{4}$ and $\frac{-(\sqrt{33}+1)}{4}$.

(ii) We have, $4x^2 + 4\sqrt{3x} + 3 = 0$

On dividing both sides by 4, we have

$x^2 + \sqrt{3x} + \frac{3}{4} = 0 \quad \Rightarrow x^2 + \sqrt{3x} + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 + \frac{3}{4} = 0$

$\Rightarrow \quad \left(x + \frac{\sqrt{3}}{2}\right)^2 - \frac{3}{4} + \frac{3}{4} = 0 \quad \Rightarrow \left(x + \frac{\sqrt{3}}{2}\right)^2 = 0 \quad \ldots (i)$

$\Rightarrow$ Roots exist. $\quad \therefore (i) \quad \Rightarrow x = -\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}$.

Hence, roots of given equation are $-\frac{\sqrt{3}}{2}$ and $-\frac{\sqrt{3}}{2}$.

Que 3. Find the roots of the following quadratic equation by applying the quadratic formula.

(i) $2x^2 - 7x + 3 = 0$ **(ii) $4x^2 + 4\sqrt{3x} + 3 = 0$**

Sol. (i) We have, $2x^2 - 7x + 3 = 0$

Here, a = 2, b = - 7 and c = 3

Therefore, $D = b^2 - 4ac$

$\Rightarrow \quad D = (-7)^2 - 4 \times 2 \times 3 = 49 - 24 = 25$

$\therefore$ D > 0, $\therefore$ roots exist.

Thus, $\quad x = \frac{-b \pm\sqrt{D}}{2a} = \frac{-(-7) \pm \sqrt{25}}{2 \times 2} = \frac{7 \pm 5}{4}$

$= \frac{7+5}{4}$ or $\frac{7-5}{4}$

$= 3 \quad \text{or} \quad \frac{1}{2}$

So, the roots of given equation are 3 and $\frac{1}{2}$.

(ii) We have, $4x^2 + 4\sqrt{3x} + 3 = 0$

Here, a = 4, b = 4 $\sqrt{3}$ and c = 3

Therefore, D = $b^2 - 4ac = (4\sqrt{3})^2 - 4 \times 4 \times 3 = 48 - 48 = 0$

∴ D = 0, roots exist and are equal.

Thus, $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-4\sqrt{3} \pm 0}{2 \times 4} = \frac{-\sqrt{3}}{2}$

Hence, the roots of given equation are $\frac{-\sqrt{3}}{2}$ and $\frac{-\sqrt{3}}{2}$.

Que 4. Using quadratic formula solve the following quadratic equation:
$p^2x^2 + (p^2 - q^2)\,x - q^2 = 0$

Sol. We have, $p^2x^2 + (p^2 - q^2)\,x - q^2 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we have

$a = p^2$, $b = p^2 - q^2$ and $c = -q^2$

∴ $D = b^2 - 4ac = (p^2 - q^2)^2 - 4 \times p^2 \times (-q^2)$

$= (p^2 - q^2)^2 + 4p^2q^2 = (p^2 + q^2)^2 > 0$

So, the given equation has real roots given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{(p^2 - q^2) + (p^2 + q^2)}{2p^2} = \frac{2q^2}{2p^2} = \frac{q^2}{p^2}$$

and $$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-(p^2 - q^2) - (p^2 + q^2)}{2p^2} = \frac{-2q^2}{2p^2} = -1$$

Hence, roots are $\frac{q^2}{p^2}$ and – 1.

Que 5. Find the roots of the following equation:
$\frac{1}{x+3} - \frac{1}{x-6} = \frac{9}{20};\ x \neq -3, 6$

Sol. Given, $\frac{1}{x+3} - \frac{1}{x-6} = \frac{9}{20};\ x \neq -3, 6$

$\Rightarrow \quad \frac{(x-6)-(x+3)}{(x+3)(x-6)} = \frac{9}{20} \quad \Rightarrow \quad \frac{-9}{(x+3)(x-6)} = \frac{9}{20}$

$\Rightarrow \quad (x+3)(x-6) = -20 \quad \text{or} \quad x^2 - 3x + 2 = 0$

$\Rightarrow \quad x^2 - 2x - x + 2 = 0 \quad \Rightarrow \quad x(x-2) - 1(x-2) = 0$

$\Rightarrow \quad (x-1)(x-2) = 0 \quad \Rightarrow \quad x = 1 \quad \text{or} \quad x = 2$

Both x = 1 and x = 2 are satisfying the given equation. Hence, x = 1, 2 are the solution of
the equation.

Que 6. Find the nature of the roots of the following quadratic equation. If the real roots exist, find them:

(i) $3x^2 - 4\sqrt{3x} + 4 = 0$ **(ii) $2x^2 - 6x + 3 = 0$**

Sol. (i) We have, (i) $3x^2 - 4\sqrt{3x} + 4 = 0$

Here, a = 3, b = - 4 $\sqrt{3x}$ and c = 4

Therefore, D = $b^2 - 4ac = (-4\sqrt{3x})^2 - 4 \times 3 \times 4 = 48 - 48 = 0$

Hence, the given quadratic equation has real and equal roots.

Thus, $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-4\sqrt{3} \pm \sqrt{0}}{2 \times 3} = \frac{2\sqrt{3}}{3}$.

Hence, equal roots of given equation are $\frac{2\sqrt{3}}{3}$, $\frac{2\sqrt{3}}{3}$.

(ii) We have, $2x^2 - 6x + 3 = 0$

Here a = 2, b = - 6, c = 3

Therefore, D = $b^2 - 4ac$

$= (-6)^2 - 4 \times 2 \times 3 = 36 - 24 = 12 > 0$

Hence, given quadratic equation has real and distinct roots.

Thus, $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-6) \pm \sqrt{12}}{2 \times 2} = \frac{6 \pm 2\sqrt{3}}{4} = \frac{3 \pm \sqrt{3}}{2}$

Hence, roots of given equation are $\frac{3+\sqrt{3}}{2}$ and $\frac{3-\sqrt{3}}{2}$.

Que 7. Find the value of k for each of the following quadratic equation, so that they have two equal roots.

(i) $2x^2 + kx + 3 = 0$ **(ii) $kx(x - 2) + 6 = 0$**

Sol. (i) We have, $2x^2 + kx + 3 = 0$

Here, a = 2, b = k, c = 3

∴ D = $b^2 - 4ac = k^2 - 4 \times 2 \times 3 = k^2 - 24$

For equal roots

D = 0 *i.e.*, $k^2 - 24 = 0$

⇒ $k^2 = 24$

⇒ $k = \pm\sqrt{24}$ ⇒ $k = \pm 2\sqrt{6}$

(ii) We have, $kx(x - 2) + 6 = 0$ ⇒ $kx^2 - 2kx + 6 = 0$

Here, a = k, b = - 2k, c = 6

For equal roots, we have

D = 0

i.e., $b^2 - 4ac = 0$ ⇒ $(-2k)^2 - 4 \times k \times 6 = 0$

⇒ $4k^2 - 24k = 0$ ⇒ $4k(k - 6) = 0$

Either $4k = 0$ or $k - 6 = 0$ ⇒ $k = 0$ or $k = 6$

But $k \neq 0$ (because if k = 0 then given equation will not be a quadratic equation).

So, k =6.

Que 8. If the roots of the equation (a – b) x^2 + (b – c) x + (c – a) = 0 are equal, prove that 2a = b + c.

Sol. Since the equation $(a - b)x^2 + (b - c)x + (c - a) = 0$ has equal roots, therefore discriminant

$D = (b - c)^2 - 4(a - b)(c - a) = 0$

⇒ $b^2 + c^2 - 2bc - 4(ac - a^2 - bc + ab) = 0$ ⇒ $b^2 + c^2 - 2bc - 4ac + 4a^2 + 4bc - 4ab = 0$

⇒ $4a^2 + b^2 + c^2 - 4ab + 2bc - 4ac = 0$

⇒ $(2a)^2 + (-b)^2 + (-c)^2 + 2(2a)(-b) + 2(-b)(-c) + 2(-c)2a = 0$

⇒ $(2a - b - c)^2 = 0$ ⇒ $2a - b - c = 0$ ⇒ $2a = b + c$. Hence proved

Que 9. If the equation (1 + m²) x^2 + 2mcx + c² – a² = 0 has equal root, show that c² = a² (1 + m²).

Sol. The given equation is $(1 + m^2)x^2 + (2mc)x + (c^2 - a^2) = 0$

Here, $A = 1 + m^2$, $B = 2mc$ and $C = c^2 - a^2$

Since the given equation has equal roots, therefore D = 0 ⇒ $B^2 - 4AC = 0$.

⇒ $(2mc)^2 - 4(1 + m^2)(c^2 - a^2) = 0$

⇒ $4m^2c^2 - 4(c^2 - a^2 + m^2c^2 - m^2a^2) = 0$

⇒ $m^2c^2 - c^2 + a^2 - m^2c^2 + m^2a^2 = 0$ [Dividing throughout by 4]

⇒ $-c^2 + a^2(1 + m^2) = 0$ ⇒ $c^2 = a^2(1 + m^2)$ Hence proved

Que 10. If sin θ and cos θ are roots of the equation ax^2 + bx + c = 0, prove that a² – b² + 2ac = 0.

Sol. Sum of the roots = $\frac{-B}{A}$ ⇒ $\sin\theta + \cos\theta = \frac{-b}{a}$ … (i)

Product of the roots = $\frac{C}{A}$ ⇒ $\sin\theta . \cos\theta = \frac{c}{a}$ … (ii)

Now, we have, $\sin^2\theta + \cos^2\theta = 1$

⇒ $(\text{Sin}\,\theta + \cos\theta)^2 - 2\sin\theta\cos\theta = 1$ ⇒ $\left(\frac{-b}{a}\right)^2 - 2.\frac{c}{a} = 1$

⇒ $\frac{b^2}{a^2} - \frac{2c}{a} = 1$ or $b^2 - 2ac = a^2$

$\Rightarrow \qquad a^2 - b^2 + 2ac = 0$

Que 11. Determine the condition for one root of the quadratic equation $ax^2 + bx + c = 0$ to be thrice the other.

Sol. Let the roots of the given equation be α and 3α.

Then sum of the roots = $\alpha + 3\alpha = 4\alpha = \frac{-b}{a}$ … (i)

Product of the roots = $(\alpha)(3\alpha) = 3\alpha^2 = \frac{c}{a}$ … (ii)

From (i), $\alpha = \frac{-b}{4a}$

$(ii) \Rightarrow 3\left(\frac{-b}{4a}\right)^2 = \frac{c}{a} \quad \Rightarrow \quad \frac{3b^2}{16a^2} = \frac{c}{a}$

$\Rightarrow$ $3b^2 = 16ac$, which is the required condition.

Que 12. Solve for x: $2\left(\frac{2x-1}{x+3}\right) - 3\left(\frac{x+3}{2x-1}\right) = 5;\ x \neq -3, \frac{1}{2}$

Sol. $2\left(\frac{2x-1}{x+3}\right) - 3\left(\frac{x+3}{2x-1}\right) = 5 \quad \Rightarrow \quad \left(\frac{4x-2}{x+3}\right) - \left(\frac{3x+9}{2x-1}\right) = 5$

$(4x - 2)(2x - 1) - (3x + 9)(x + 3) = 5(x + 3)(2x - 1)$

$(8x^2 - 4x - 4x + 2) - (3x^2 + 9x + 9x + 27) = 5(2x^2 - x + 6x - 3)$

$8x^2 - 8x + 2 - 3x^2 - 18x - 27 = 10x^2 + 25x - 15$

$5x^2 - 26x - 25 = 10x^2 + 25x - 15$

$5x^2 + 51x + 10 = 0$

$5x^2 + 50x + x + 10 = 0$

$5x(x + 10) + 1(x + 10) = 0$

$(5x + 1)(x + 10) = 0$

$5x + 1 = 0$ or $x + 10 = 0$

$x = \frac{-1}{5}$ or $x = -10$

Que 13. Solve the equation $\frac{4}{x} - 3 = \frac{5}{2x+3};\ x \neq 0, -\frac{3}{2}$, for x,

Sol. $\frac{4}{x} - 3 = \frac{5}{2x+3} \quad \Rightarrow \quad \frac{4-3x}{x} = \frac{5}{2x+3}$

$(4 - 3x)(2x + 3) = 5x \quad \Rightarrow \quad 8x - 6x^2 + 12 - 9x = 5x$

$6x^2 + 6x - 12 = 0 \quad \Rightarrow \quad x^2 + x - 2 = 0$

$x^2 + 2x - x - 2 = 0 \quad \Rightarrow \quad x(x+2) - 1(x+2) = 0$

$(x - 1)(x + 2) = 0$

$\Rightarrow \quad x - 1 = 0$ or $x + 2 = 0$

$x = 1$ or $x = -2$

Que 14. Solve for x: $\frac{16}{x} - 1 = \frac{15}{x+1}$ **;** $x \neq 0, -1$

Sol. $\frac{16}{x} - 1 = \frac{15}{x+1} \quad \Rightarrow \quad \frac{16-x}{x} = \frac{15}{x+1}$

$(16 - x)(x + 1) = 15x \quad \Rightarrow \quad 16x - x^2 + 16 - x = 15x$

$x^2 + 15x - 15x - 16 = 0 \quad \Rightarrow \quad x^2 = 16$

$x = \pm 4$

Que 15. Solve for x. $x^2 + 5x - (a^2 + a - 6) = 0$

Sol. $x^2 + 5x - (a^2 + a - 6) = 0$

$x^2 + 5x - (a^2 + 3a - 2a - 6) = 0$

$x^2 + 5x - [a(a + 3) - 2(a + 3)] = 0$

$x^2 + 5x - (a - 2)(a + 3) = 0$

$\therefore \quad x^2 + (a + 3)x - (a - 2)x - (a - 2)(a + 3) = 0$

$x[x + (a + 3)] - (a - 2)[x + (a + 3)] = 0$

$[\{x + (a + 3)\}\{x - (a - 2)\}] = 0$

$\therefore \quad x = -(a + 3) \; x = (a - 2)$

$\Rightarrow \quad x = -(a + 3), (a - 2)$

Alternative method

$x^2 + 5x - (a^2 + a - 6) = 0$

$\therefore \quad x = \frac{-5 \pm \sqrt{5^2 - 4 \times 1 \times [-(a^2 + a - 6)]}}{2 \times 1}$

$= \frac{-5 \pm \sqrt{25 - 4a^2 + 4a - 24}}{2} = \frac{-5 \pm \sqrt{4a^2 + 4a + 1}}{2}$

$= \frac{-5 \pm \sqrt{(2a)^2 + 2.(2a).1 + 1^2}}{2} = \frac{-5 \pm \sqrt{(2a+1)^2}}{2}$

$= \frac{-5 \pm 2a+1}{2} = \frac{-5+2a+1}{2}, \frac{-5-2a-1}{2}$

$= \frac{2a-4}{2}$, $-\frac{2a-6}{2}$ = (a – 2), - (a + 3)

Que 16. Solve for x: $\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0, x \neq 3, -3/2$

Sol. $\frac{2x(2x+3)+(x-3)+(3x+9)}{(x-3)(2x+3)} = 0$

$\Rightarrow 2x(2x + 3) + (x - 3) + (3x + 9) = 0$

$\Rightarrow 4x^2 + 10x + 6 = 0 \quad \Rightarrow \quad 2x^2 + 5x + 3 = 0$

$\Rightarrow (x + 1)(2x + 3) = 0 \quad \Rightarrow \quad x = -1, x = -\frac{3}{2}$

But $x \neq -\frac{3}{2}$ $\quad \therefore \quad x = -1$

Que 17. Solve for x: $\frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} = \frac{2}{3}, x \neq 1,2,3.$

Sol. $\frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} = \frac{2}{3} \Rightarrow \quad \frac{(x-3)+(x-1)}{(x-1)(x-2)(x-3)} = \frac{2}{3}$

$\Rightarrow 3(x - 3 + x - 1) = 2(x - 1)(x - 2)(x - 3) \quad \Rightarrow \quad 3(2x - 4) = 2(x - 1)(x - 2)(x - 3)$

$\Rightarrow 3 \times 2(x - 2) = 2(x - 1)(x - 2)(x - 3) \quad \Rightarrow \quad 3 = (x - 1)(x - 3)$ *i.e.,* $x^2 - 4x = 0$

$\Rightarrow x(x - 4) = 0 \quad \therefore \quad x = 0, x = 4$