Class-XII

Session - 2022-23

Subject - Mathematics (041)

Sample Question Paper - 23

With Solution

BLUE PRINT

Ch. No.	Chapter Name	Per Unit Marks	Section-A (1 Mark)		Section-B (2 Marks)	Section-C (3 Marks)	Section-D (5 Marks)	Section-E (4 Marks)	Total Marks
			MCQ	A/R	VSA	SA	LA	Case-Study	
1	Relations and Functions	8					Q.32		5
2	Inverse Trigonometry Functions			Q.20	Q.21				3
3	Matrices	10	Q.1,6						2
4	Determinants		Q.3,8,11				Q.33		8
5	Continuity and Differentiability	35	Q.4,15		Q.22				4
6	Applications of Derivatives				Q.23			Q.36,38	10
7	Integrals		Q.2,7			Q.26,28,30			11
8	Applications of Integrals						Q.35		5
9	Differential Equations		Q.10,18			Q.27			5
10	Vector Algebra	14	Q.9,13,17		Q.24				5
11	Three Dimensional Geometry		Q.5	Q.19	Q.25		Q.34		9
12	Linear Programming	5	Q.14,16			Q.31			5
13	Probability	8	Q.12			Q.29		Q.37	8
	Total Marks (Total Questions)		18(18)	2(2)	10(5)	18(6)	20(5)	12(3)	80(38)

Note : The number given inside the bracket denotes question number, asked in the sample paper, while the number given outside the bracket are the number of questions from that particular chapter.

Time : 3 Hours **Max. Marks : 80**

General Instructions

1. *This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.*
2. *Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.*
3. *Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.*
4. *Section C has 6 Short Answer (SA)-type questions of 3 marks each.*
5. *Section D has 4 Long Answer (LA)-type questions of 5 marks each.*
6. *Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.*

SECTION-A (Multiple Choice Questions)

Each question carries 1 mark.

1. If $\begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$, then value of $a + b - c + 2d$ is:

(a) 8 (b) 10 (c) 4 (d) –8

2. $\int \frac{e^x(1+x)}{\cos^2(e^x x)} dx$ equals

(a) $-\cot(ex^x) + C$ (b) $\tan(xe^x) + C$ (c) $\tan(e^x) + C$ (d) $\cot(e^x) + C$

3. If area of triangle is 4 sq. units with vertices (–2, 0), (0, 4) and (0, k), then k is equal to

(a) 0, –8 (b) 8 (c) – 8 (d) 0, 8

4. If $f(x) = \begin{cases} \frac{\sin 5x}{x^2+2x}, & x \neq 0 \\ k + \frac{1}{2}, & x = 0 \end{cases}$ is continuous at x = 0, then the value of k is

(a) 1 (b) – 2 (c) 2 (d) $\frac{1}{2}$

5. The angle between two lines $\frac{x+1}{2} = \frac{y+3}{2} = \frac{z-4}{-1}$ and $\frac{x-4}{1} = \frac{y+4}{2} = \frac{z+1}{2}$ is:

(a) $\cos^{-1}\left(\frac{1}{9}\right)$ (b) $\cos^{-1}\left(\frac{4}{9}\right)$ (c) $\cos^{-1}\left(\frac{2}{9}\right)$ (d) $\cos^{-1}\left(\frac{3}{9}\right)$

6. If $[1 \ x \ 1] \begin{bmatrix} 1 & 3 & 2 \\ 0 & 5 & 1 \\ 0 & 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ 1 \\ -2 \end{bmatrix} = 0$, then x is

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) 1 (d) – 1

7. The value of $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right) dx$ is

(a) 1 (b) 0 (c) – 1 (d) $\frac{\pi}{4}$

8. If A_{ij} denotes the cofactor of the element a_{ij} of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, then value of

$a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$ is

(a) 0 (b) 5 (c) 10 (d) -5

9. If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + 3\hat{j} + \hat{k}$, $\vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\alpha\vec{a} + \beta\vec{b} + \gamma\vec{c} = -3(\hat{i} - \hat{k})$, then the ordered triplet (α, β, γ) is

(a) $(2, -1, -1)$ (b) $(-2, 1, 1)$ (c) $(-2, -1, 1)$ (d) $(2, 1, -1)$

10. The integrating factor of the differential equation $x\frac{dy}{dx} - y = 2x^2$ is

(a) e^{-x} (b) e^{-y} (c) $\frac{1}{x}$ (d) x

11. If the system of equations $x + \lambda y + 2 = 0$, $\lambda x + y - 2 = 0$, $\lambda x + \lambda y + 3 = 0$ is consistent, then

(a) $\lambda = \pm 1$ (b) $\lambda = \pm 2$ (c) $\lambda = 1, -2$ (d) $\lambda = -1, 2$

12. If A and B be two events such that P(A) = 0.6, P(B) = 0.2 and P(A/B) = 0.5, then $P(A'/B')$ is equal to

(a) $\frac{1}{10}$ (b) $\frac{3}{10}$ (c) $\frac{3}{8}$ (d) $\frac{6}{7}$

13. If $\lambda(3\hat{i} + 2\hat{j} - 6\hat{k})$ is a unit vector, then the values of λ are

(a) $\pm\frac{1}{7}$ (b) ± 7 (c) $\pm\sqrt{43}$ (d) $\pm\frac{1}{\sqrt{43}}$

14. A vertex of bounded region of inequalities $x \geq 0$, $x + 2y \geq 0$ and $2x + y \leq 4$ is

(a) (1, 1) (b) (0, 1) (c) (3, 0) (d) (0, 1)

15. If $x = a \sec\theta$, $y = b \tan\theta$, then $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{6}$ is :

(a) $\frac{-3\sqrt{3}b}{a^2}$ (b) $\frac{-2\sqrt{3}b}{a}$ (c) $\frac{-3\sqrt{3}b}{a}$ (d) $\frac{-b}{3\sqrt{3}a^2}$

16. The constraints $-x_1 + x_2 \leq 1$, $-x_1 + 3x_2 \leq 9$, $x_1, x_2 \geq 0$ define on

(a) Bounded feasible space (b) Unbounded feasible space

(c) Both bounded and unbounded feasible space (d) None of these

17. If $|\overline{a}| = 3, |\overline{b}| = 4$, then a value of λ for which $\overline{a} + \lambda\overline{b}$ is perpendicular to $\overline{a} - \lambda\overline{b}$ is :

(a) $\frac{9}{16}$ (b) $\frac{3}{4}$ (c) $\frac{3}{2}$ (d) $\frac{4}{3}$

18. The order and degree of the differential equation $\left(1 + 3\frac{dy}{dx}\right)^{2/3} = 4\frac{d^3y}{dx^3}$ are

(a) $(1, \frac{2}{3})$ (b) (3, 1) (c) (3, 3) (d) (1, 2)

(ASSERTION-REASON BASED QUESTIONS)

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) ***Both A and R are true and R is the correct explanation of A.***

(b) ***Both A and R are true but R is not the correct explanation of A.***

(c) ***A is true but R is false.***

(d) ***A is false but R is true.***

19. **Assertion:** The pair of lines given by $\vec{r} = \hat{i} - \hat{j} + \lambda(2\hat{i} + \hat{k})$ and $\vec{r} = 2\hat{i} - \hat{k} + \mu(\hat{i} + \hat{j} - \hat{k})$ intersect.
Reason: Two lines intersect each other, if they are not parallel and shortest distance = 0.

20. **Assertion:** The domain of the function $\sec^{-1} x$ is the set of all real numbers.
Reason: For the function $\sec^{-1}x$, x can take all real values except in the interval (–1, 1).

SECTION-B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. Find the value of $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) + \cos^{-1}\left(-\frac{1}{2}\right) - \tan^{-1}\left(-\sqrt{3}\right) + \cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$.

22. Find the value of k which makes $f(x) = \begin{cases} \sin(1/x), & x \neq 0 \\ k, & x = 0 \end{cases}$ continuous at x = 0.

OR

If $x = a\sec^3\theta$ and $y = a\tan^3\theta$, find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{3}$.

23. Find the maximum value of $f(x) = \sin x + \cos x$.

OR

Show that the function given by f (x) = 3x + 17 is strictly increasing on R.

24. Find the unit vector in the direction of the sum of the vectors $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} + 3\hat{k}$.

25. If a line makes angles 90°, 60° and θ with x, y and z-axis respectively, where θ is acute, then find θ.

SECTION-C

This section comprises of short answer type questions (SA) of 3 marks each.

26. Evaluate: $\int(3\text{cosec}^2 x - 5x + \sin x)\,dx$.

OR

Evaluate: $\int_2^3 \frac{x}{x^2+1}dx$

27. Find the particular solution of the differential equation $\frac{dy}{dx} = 1 + x + y + xy$, given that y = 0 when x = 1.

OR

Solve $x^2\frac{dy}{dx} - xy = 1 + \cos\left(\frac{y}{2x}\right)$, $x \neq 0$ and $x = 1, y = \frac{\pi}{2}$.

28. $\int_0^{\pi/2} \frac{\cos^2 x}{\cos^2 x + 4\sin^2 x}dx$

29. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both diamonds. Find the probability of the lost card being a diamond?

30. Find the value of $\int \frac{1}{\sqrt{(x-1)(x-2)}}dx$.

OR

Prove : $\int_0^{\pi/4} 2\tan^3 x\,dx = 1 - \log 2$

31. Find graphically, minimum and maximum values of Z = x + 2y subject to $x + 2y \geq 100,\ 2x - y \leq 0,\ 2x + y \leq 200;\ x, y \geq 0$.

SECTION-D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Let $A = R - \{3\}$ and $B = R - \{1\}$. Let $f: A \to B$ defined as $f(x) = \frac{x-2}{x-3}$, $\forall x \in A$. Then show that f is bijective.

33. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$. Find A^{-1}. Using A^{-1}. Solve the following system of linear equations $2x - 3y + 5z = 11$, $3x + 2y - 4z = -5$, $x + y - 2z = -3$

34. Find the shortest distance between lines $\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$ and $\vec{r} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$

OR

Find the image of the point (1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$.

35. Find the area bounded by the curve $y = x^3$, the x-axis and the ordinates $x = -2$ and $x = 1$.

OR

Find the area of the region bounded by $x^2 = 4y$, $y = 2$, $y = 4$ and the y-axis in the first quadrant.

SECTION-E

This section comprises of 3 case study/passage - based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.

36. **Case - Study 1:** Read the following passage and answer the questions given below.
A teacher discussed the shape of window with certain information to get the maximum light and air through it.

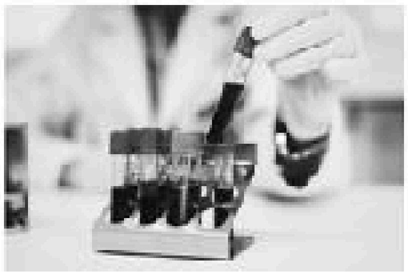
In the figure, a window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m.
If x be the width of window and r be the radius of semicircular opening, then students were asked the following questions.

(i) What is the relations between width *x* and radius *r*?
(ii) Find the area (*A*) of window in terms of radius *r* only.
(iii) Find the dimensions of window to admit the maximum light and air.

OR

Find the maximum area (*A*) of the window? $\left(\text{Use } \pi = \frac{22}{7}\right)$

37. **Case - Study 2:** Read the following passage and answer the questions given below.
Suppose that the reliability of a COVID-19 test is specified as follows:
Of people having COVID-19, 95% of the test detect the disease but 5% go undetected. Of people free of COVID-19, 90% of the test are judged COVID-19 –ve but 10% are diagnosed as showing COVID-19 + ve. From a large population of which only 10% have COVID-19 one person is selected at random, given the COVID-19 test, and the pathologist reports him/her as COVID-19 +ve.

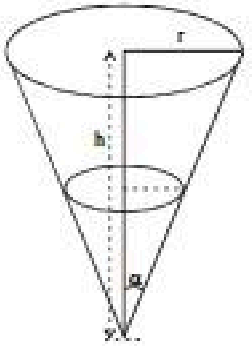


(i) Find the probability of report is positive when person having COVID-19
(ii) Find the probability of report is positive when person not having COVID-19
(iii) The probability that the person actually has COVID-19

OR

Find the probability of report is positive.

38. Case - Study 3: Read the following passage and answer the questions given below.
A group of class XII students had to analyse the water in a water tank has the shape of an inverted right circular cone with its axis vertical and vertex lowermost. Its semi-vertical angle is $\tan^{-1}(0.5)$. Water is poured into it at a constant rate of 5 cubic metre per hour. The figure of the water tank is given below. [Use $\pi = \frac{22}{7}$]



(i) Find the rate at which the level of water is rising at instant when the depth of water in the tank is 4 m.
(ii) Find the relation between volume (V), surface area (S) and radius (r)

SOLUTIONS

SAMPLE PAPER-1

1. **(a)** $2a + b = 4$ ..... (i)
$a - 2b = -3$ ..... (ii)
$5c - d = 11$ ..... (iii)
$4c + 3d = 24$ ..... (iv)

Solving equations (i), (ii), (iii) and (iv), we get
$a = 1,$
$b = 2,$
$c = 3,$
$d = 4$
$\therefore \quad a + b - c + 2d = 8$

2. **(b)** $\int \frac{e^x(1+x)}{\cos^2(e^x x)}dx$

Let $xe^x = t$

$\Rightarrow \quad \left(xe^x + e^x\right) = \frac{dt}{dx} \Rightarrow dx = \frac{dt}{e^x(x+1)}$

$\therefore \quad \int \frac{e^x(1+x)}{\cos^2(e^x x)}dx = \int \frac{e^x(1+x)}{\cos^2 t} \times \frac{dt}{e^x(1+x)}$

$= \int \frac{1}{\cos^2 t}dt = \int \sec^2 t\, dt$

$= \tan t + C = \tan\left(xe^x\right) + C$

3. **(d)** Given $\frac{1}{2}\begin{vmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{vmatrix} = 4 \Rightarrow |-2(4-k) + 1(0-0)| = 8$

$\Rightarrow -2(4-k) + 1(0-0) = \pm 8 \Rightarrow (-8 + 2k) = \pm 8$
Taking positive sign,
$2k - 8 = 8 \Rightarrow 2k = 16 \Rightarrow k = 8$
Taking negative sign,
$2k - 8 = -8$
$\Rightarrow 2k = 0 \Rightarrow k = 0 \therefore k = 0, 8$

4. **(c)** LHL $= \lim_{h\to 0} f(0-h) = \lim_{h\to 0}\frac{\sin 5(0-h)}{(0-h)^2 + 2(0-h)}$

$= -\lim_{h\to 0}\frac{\frac{\sin 5h}{5h}}{\frac{1}{5}(h-2)} = \frac{5}{2}$

RHL $= \lim_{x\to 0^+}\frac{\sin 5x}{x^2 + 2x} = \lim_{x\to 0^+}\frac{\sin 5x}{5x}\cdot \lim_{x\to 0^+}\frac{1}{(x+2)} = \frac{5}{2}$

$f(0) = k + \frac{1}{2}$

Since, it is continuous at $x = 0 \therefore$ LHL = RHL = f(0)

$\Rightarrow \frac{5}{2} = k + \frac{1}{2} \Rightarrow k = 2$

5. **(b)** **Note:** The angle θ between the two lines

$\frac{x - x_1}{a_1} = \frac{y - y_1}{a_2} = \frac{z - z_1}{a_3}$

and $\frac{x - x_2}{b_1} = \frac{y - y_2}{b_2} = \frac{z - z_2}{b_3}$ is given by:

$\cos\theta = \frac{a_1b_1 + a_2b_2 + a_3b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2}\sqrt{b_1^2 + b_2^2 + b_3^2}}$

Now in the given equation: $a_1 = 2, a_2 = 2, a_3 = -1$
$b_1 = 1, b_2 = 2, b_3 = 2$

$\therefore \cos\theta = \frac{2\times 2 + 2\times 2 + (-2)\times 1}{\sqrt{4+4+1}\sqrt{4+4+1}} \Rightarrow \theta = \cos^{-1}\left(\frac{4}{9}\right).$

6. **(b)** We have $\begin{bmatrix} 1 & x & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 2 \\ 0 & 5 & 1 \\ 0 & 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ 1 \\ -2 \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} 1 & 5x+6 & x+4 \end{bmatrix}\begin{bmatrix} x \\ 1 \\ -2 \end{bmatrix} = 0$

$\Rightarrow x + 5x + 6 - 2x - 8 = 0 \Rightarrow 4x - 2 = 0 \Rightarrow x = \frac{1}{2}$

7. **(b)** $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx = \int_0^1 \tan^{-1}\left[\frac{x + (x-1)}{1 - x(x-1)}\right]dx$

$I = \int_0^1 \left[\tan^{-1} x + \tan^{-1}(x-1)\right]dx$ ... (i)

let $I = \int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx$

$= \int_0^1 [\tan^{-1} x + \tan^{-1}(x-1)]\, dx$

$= \int_0^1 [\tan^{-1}(1-x) - \tan^{-1}(1-x-1)]\, dx$

$= \int_0^1 [-\tan^{-1}(x-1) - \tan^{-1} x]\, dx,$

$I = -\int_0^1 [\tan^{-1} x + \tan^{-1}(x-1)]\, dx$ ...(ii)

Adding (i) & (ii) $2I = 0$ or $I = 0$

8. **(a)** Given determinant is $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$

We have $M_{31} = \begin{vmatrix} -3 & 5 \\ 0 & 4 \end{vmatrix} = -12 - 0 = -12$

$\Rightarrow A_{31} = M_{31} = -12$

$M_{32} = \begin{vmatrix} 2 & 5 \\ 6 & 4 \end{vmatrix} = 8 - 30 = -22 \Rightarrow A_{32} = -M_{32} = 22$

$M_{33} = \begin{vmatrix} 2 & -3 \\ 6 & 0 \end{vmatrix} = 0 + 18 = 18 \Rightarrow A_{33} = M_{33} = 18$

$\therefore a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$
$= (2)(-12) + (-3)(22) + (5)(18)$
$= -24 - 66 + 90 = -90 + 90 = 0$

9. **(a)** Equating the components in
$\alpha(\hat{i} + 2\hat{j} + 3\hat{k}) + \beta(2\hat{i} + 3\hat{j} + \hat{k}) + \gamma(3\hat{i} + \hat{j} + 2\hat{k})$
$= -3(\hat{i} - \hat{k})$, we have

$\alpha + 2\beta + 3\gamma = -3$ ....(i) $\quad 2\alpha + 3\beta + \gamma = 0$ ....(ii)

$3\alpha + \beta + 2\gamma = 3$ ....(iii)

Solving the equations (i), (ii), & (iii), we get
$\alpha = 2, \beta = -1, \gamma = -1$.

10. **(c)** $x\dfrac{dy}{dx} - y = 2x^2$ or $\dfrac{dy}{dx} - \dfrac{y}{x} = 2x$

I.F. $= e^{\int \frac{-1}{x} dx} = e^{-\log x} = e^{\log \frac{1}{x}} = \dfrac{1}{x}$

11. **(a)** The system of equations will be consistent if

$\Delta = \begin{vmatrix} 1 & \lambda & 2 \\ \lambda & 1 & -2 \\ \lambda & \lambda & 3 \end{vmatrix} = 0$

To evaluate Δ we use $R_1 \to R_1 + R_2$ followed by $C_2 \to C_2 - C_1$ to obtain

$\Delta = \begin{vmatrix} \lambda+1 & \lambda+1 & 0 \\ \lambda & 1 & -2 \\ \lambda & \lambda & 3 \end{vmatrix} = \begin{vmatrix} \lambda+1 & 0 & 0 \\ \lambda & 1-\lambda & -2 \\ \lambda & 0 & 3 \end{vmatrix}$

$= 3(\lambda + 1)(1 - \lambda) = 3(1 - \lambda^2)$

For the system to be consistent, we must have
$1 - \lambda^2 = 0$ or $\lambda = \pm 1$.

12. **(c)** Given $P(A/B) = 0.5 \Rightarrow \dfrac{P(A \cap B)}{P(B)} = 0.5$

$\Rightarrow P(A \cap B) = (0.5) \times P(B) = 0.5 \times 0.2 = 0.1$

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.6 + 0.2 - 0.1 = 0.7$

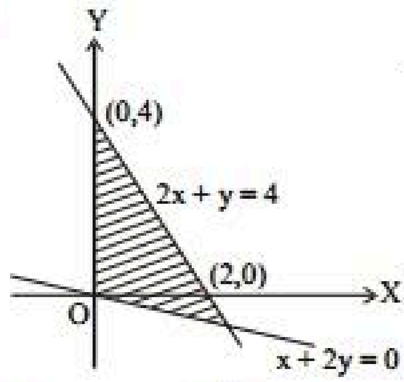
Hence $P(A'/B') = \dfrac{P(A' \cap B')}{P(B')} = \dfrac{P\left((A \cup B)'\right)}{1 - P(B)}$

$= \dfrac{1 - P(A \cup B)}{1 - P(B)} = \dfrac{1 - 0.7}{1 - 0.2} = \dfrac{3}{8}$

13. **(a)** Since $\lambda(3\hat{i} + 2\hat{j} - 6\hat{k})$ is a unit vector therefore
$\left|\lambda(3\hat{i} + 2\hat{j} - 6\hat{k})\right| = 1 \Rightarrow |\lambda|\left|(3\hat{i} + 2\hat{j} - 6\hat{k})\right| = 1$

$\Rightarrow |\lambda|\sqrt{9 + 4 + 36} = 1 \Rightarrow |\lambda|\sqrt{49} = 1 \Rightarrow \lambda = \pm\dfrac{1}{7}$

14. **(d)**



15. **(a)** $\because x = a\sec\theta \Rightarrow \dfrac{dx}{d\theta} = a\tan\theta\sec\theta$

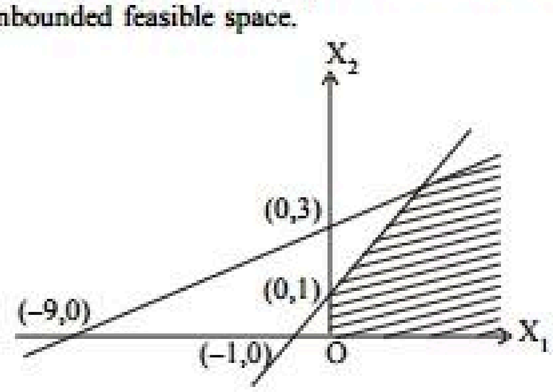
and $y = b\tan\theta \Rightarrow \dfrac{dy}{d\theta} = b\sec^2\theta$

$\therefore \dfrac{dy}{dx} = \dfrac{dy}{d\theta}\Big/\dfrac{dx}{d\theta} = \dfrac{b}{a}\text{cosec }\theta$

$\Rightarrow \dfrac{d^2y}{dx^2} = \dfrac{-b}{a}\text{cosec}\,\theta \cdot \cot\theta \cdot \dfrac{d\theta}{dx} = \dfrac{-b}{a^2}\cot^3\theta$

$\therefore \left.\dfrac{d^2y}{dx^2}\right]_{\theta = \frac{\pi}{6}} = \dfrac{-3\sqrt{3}b}{a^2}$

16. **(b)** It is clear from the graph, the constaints define the unbounded feasible space.



17. **(b)** If $\bar{a} + \lambda\bar{b}$ is perpendicular to $\bar{a} - \lambda\bar{b}$, then
$(\bar{a} + \lambda\bar{b}).(\bar{a} - \lambda\bar{b}) = |\bar{a} + \lambda\bar{b}||\bar{a} - \lambda\bar{b}|.\cos 90°$

$\Rightarrow (\bar{a} + \lambda\bar{b}).(\bar{a} - \lambda\bar{b}) = 0$

$\Rightarrow \bar{a}.\bar{a} - \lambda.\bar{a}.\bar{b} + \lambda.\bar{b}.\bar{a} - \lambda^2.\bar{b}.\bar{b} = 0$

$\Rightarrow a^2 - \lambda^2 b^2 = 0 \Rightarrow \lambda^2 = \dfrac{a^2}{b^2} \Rightarrow \lambda^2 = \dfrac{3^2}{4^2}$

$\Rightarrow \lambda = \dfrac{3}{4}$.

18. **(c)** $\left(1+3\frac{dy}{dx}\right)^2=\left(\frac{4d^3y}{dx^3}\right)^3$

$\Rightarrow \left(1+3\frac{dy}{dx}\right)^2=64\left(\frac{d^3y}{dx^3}\right)^3$

Order = 3, degree 3

19. **(a)** Here, $\vec{a}_1=\hat{i}-\hat{j}, \vec{b}_1=2\hat{i}+\hat{k}$

$\vec{a}_2=2\hat{i}-\hat{k}, \vec{b}_2=\hat{i}+\hat{j}-\hat{k}$

$\because \quad \vec{b}_1 \neq \lambda \vec{b}_2$, for any scalar λ.

$\therefore$ Given lines are not parallel.

$\vec{a}_2-\vec{a}_1=(2\hat{i}-\hat{k})-(\hat{i}-j)=\hat{i}+j-\hat{k}$

$$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 0 & 1\\ 1 & 1 & -1\end{vmatrix}$$

$=\hat{i}(0-1)-\hat{j}(-2-1)+\hat{k}(2-0)$

$=-\hat{i}+3\hat{j}+2\hat{k}$

$|\vec{b}_1\times\vec{b}_2|=\sqrt{(-1)^2+(3)^2+(2)^2}=\sqrt{1+9+4}=\sqrt{14}$

$$SD=\left|\frac{(\vec{a}_2-\vec{a}_1).(\vec{b}_2-\vec{b}_1)}{|\vec{b}_1\times\vec{b}_2|}\right|$$

$$=\left|\frac{(\hat{i}+j-\hat{k}).(-\hat{i}+3j+2\hat{k})}{\sqrt{14}}\right|=\left|\frac{-1+3-2}{\sqrt{14}}\right|=0$$

Hence, two lines intersect each other.

Two lines intersect each other, if they are not parallel and shortest distance = 0.

20. **(d)** The domain of the function $\sec^{-1}x$ is R – (–1, 1).

21. The given expression is

$=-\frac{\pi}{4}+\frac{2\pi}{3}+\frac{\pi}{3}+\frac{2\pi}{3}$ [1 Mark]

$=\frac{5\pi}{3}-\frac{\pi}{4}=\frac{17\pi}{12}$ [1 Mark]

22. $\lim_{x\to 0}\sin\left(\frac{1}{x}\right)=$ any real ≤ 1 or ≥ -1 which is finite but is not definite [1 Mark]

$\therefore$ Limit does not exist. Hence the given function is not continuous for any value of k. [1 Mark]

OR

$\frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{3a\tan^2\theta\sec^2\theta}{3a\sec^3\theta\tan\theta}=\frac{\tan\theta}{\sec\theta}=\sin\theta$ [1 Mark]

Hence, $\left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{3}}=\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$ [1 Mark]

23. $f(x)=\sin x+\cos x$

$=\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x\right)$

$=\sqrt{2}\sin\left(\frac{\pi}{4}+x\right)$ [1 Mark]

$\because \quad -1\leq\sin\left(\frac{\pi}{4}+4\right)\leq 1\Rightarrow-\sqrt{2}\leq\sqrt{2}\sin\left(\frac{\pi}{4}+4\right)\leq\sqrt{2}$

$\therefore$ Maximum $f(x)=\sqrt{2}$ [1 Mark]

OR

$f(x)=3x+17 \quad \therefore f'(x)=3>0\ \forall\ x\in R$

$\Rightarrow$ f is strictly increasing on R. [2 Marks]

24. Let $\vec{c}$ denote the sum of $\vec{a}$ and $\vec{b}$. We have

$\vec{c}=\left(2\hat{i}-\hat{j}+2\hat{k}\right)+\left(-\hat{i}+\hat{j}+3\hat{k}\right)=\hat{i}+5\hat{k}$

[½ Mark]

Now, $|\vec{c}|=\sqrt{1^2+5^2}=\sqrt{26}$ [½ Mark]

Thus, the required unit vector is

$\hat{c}=\frac{\vec{c}}{|\vec{c}|}=\frac{1}{\sqrt{26}}\left(\hat{i}+5\hat{k}\right)=\frac{1}{\sqrt{26}}\hat{i}+\frac{5}{\sqrt{26}}\hat{k}$ [1 Mark]

25. $\cos^2(90°)+\cos^2(60°)+\cos^2\theta=1$

$\Rightarrow 0^2+\left(\frac{1}{2}\right)^2+\cos^2\theta=1$ [1 Mark]

$\Rightarrow \cos^2\theta=1-\frac{1}{4}=\frac{3}{4}$

$\Rightarrow \cos\theta=\frac{\sqrt{3}}{2}$ (θ is acute.)

$\therefore \theta=30°$ [1 Mark]

26. Let $I=\int(3\operatorname{cosec}^2x-5x+\sin x)dx$

$=3\int\operatorname{cosec}^2x\,dx-5\int x\,dx+\int\sin x\,dx$ [1 Mark]

$=3(-\cot x)-5\frac{x^2}{2}-\cos x+C$ [1 Mark]

$=-3\cot x-\frac{5x^2}{2}-\cos x+C$ [1 Mark]

OR

Let $I=\int_2^3\frac{x}{x^2+1}dx=\frac{1}{2}\int_2^3\frac{2x}{x^2+1}dx$. [1 Mark]

$\therefore \quad I=\frac{1}{2}[\log(x^2+1)]_2^3=\frac{1}{2}[\log 10-\log 5]$ [1 Mark]

$\left[\because\int\frac{f'(x)}{f(x)}dx=\log|x|\right]$

$=\frac{1}{2}\log\frac{10}{5}=\frac{1}{2}\log 2.$ [1 Mark]

27. Consider

$\frac{dy}{dx}=1+x+y+xy=1+x+y(1+x)$

$=(1+x)(1+y)$ [½ Mark]

$\Rightarrow \frac{dy}{1+y}=(1+x)dx\Rightarrow\int\frac{dy}{1+y}=\int(1+x)dx$

$\Rightarrow \log(1+y) = x + \frac{x^2}{2} + C$ [½ Mark]

Putting $y = 0$ and $x = 1$, we get

$\log 1 = 1 + \frac{1}{2} + C \Rightarrow C = \frac{-3}{2}$ [1 Mark]

$\therefore$ Particular solution is

$\log(1+y) = x + \frac{x^2}{2} - \frac{3}{2}$ [1 Mark]

OR

Given equation can be written as

$x^2 \frac{dy}{dx} - xy = 2\cos^2\left(\frac{y}{2x}\right), x \neq 0$ [½ Mark]

$\Rightarrow \frac{x^2 \frac{dy}{dx} - xy}{2\cos^2\left(\frac{y}{2x}\right)} = 1 \Rightarrow \frac{\sec^2\left(\frac{y}{2x}\right)}{2}\left[x^2 \frac{dy}{dx} - xy\right] = 1$

[½ Mark]

Dividing both sides by x^3, we get

$$\frac{\sec^2\left(\frac{y}{2x}\right)}{2}\left[\frac{x\frac{dy}{dx} - y}{x^2}\right] = \frac{1}{x^3}$$

$\Rightarrow \frac{d}{dx}\left[\tan\left(\frac{y}{2x}\right)\right] = \frac{1}{x^3}$ [1 Mark]

Integrating both sides, we get :

$\tan\left(\frac{y}{2x}\right) = \frac{-1}{2x^2} + k$ [½Mark]

Substituting $x = 1, y = \frac{\pi}{2}$, we get $k = \frac{3}{2}$,

therefore, $\tan\left(\frac{y}{2x}\right) = -\frac{1}{2x^2} + \frac{3}{2}$ is the required solution.

[½ Mark]

28. $\int_0^{\pi/2} \frac{\cos^2 x}{\cos^2 x + 4(1 - \cos^2 x)} dx$

$= \int_0^{\pi/2} \frac{\cos^2 x \, dx}{4 - 3\cos^2 x}$ [½ Mark]

$= -\frac{1}{3}\int_0^{\pi/2} dx + \frac{4}{3}\int_0^{\pi/2} \frac{dx}{4 - 3\cos^2 x}$

$= -\frac{1}{3}\left(\frac{\pi}{2}\right) + \frac{4}{3}\int_0^{\pi/2} \frac{\sec^2 x}{4\sec^2 x - 3} dx$ [1 Mark]

$= -\frac{\pi}{6} + \frac{4}{3}\int_0^{\pi/2} \frac{\sec^2 x}{4(1 + \tan^2 x) - 3} dx$

Put $\tan x = t$, so that $\sec^2 x \, dx = dt$ when $x = 0$, $t = 0$, and when $x = \frac{\pi}{2}, t = \infty$ [½ Mark]

$I = -\frac{\pi}{6} + \frac{4}{3}\int_0^{\infty} \frac{dt}{4(1+t^2) - 3} = -\frac{\pi}{6} + \frac{4}{3}\cdot\frac{1}{4}\int_0^{\infty} \frac{dt}{t^2 + \frac{1}{4}}$

$= -\frac{\pi}{6} + \frac{1}{3}\cdot 2\left[\tan^{-1}\frac{t}{1/2}\right]_0^{\infty}$

$= \frac{-\pi}{6} + \frac{\pi}{3} = \frac{\pi}{6}$ [1 Mark]

29. E_1 = Event that lost card is diamond,
E_2 = Event that lost card is not diamond.
There are 13 diamond cards, out of a pack or 52 cards

$P(E_1) = \frac{{}^{13}C_1}{{}^{52}C_1} = \frac{13}{52} = \frac{1}{4}$

There are 39 cards which are not diamond.

$P(E_2) = \frac{39}{52} = \frac{3}{4}$ [1 Mark]

(i) When one diamond card is lost , 12 diamond cards are left and in total 51 cards are left. Out of 12 cards 2 may be drawn in ${}^{12}C_2$ way.

$\therefore$ Probability of getting 2 diamond cards when one diamond card is lost

$P(A/E_1) = \frac{{}^{12}C_2}{{}^{51}C_2} = \frac{12 \times 11}{51 \times 50}$ [½ Mark]

Where A denotes the lost card

When diamond card is not lost, there are 13 diamond cards. The probability of drawing 2 diamond cards

$= \frac{{}^{13}C_2}{{}^{51}C_2} = \frac{13 \times 12}{51 \times 50}$ [½ Mark]

Probability that the lost card is diamond

$= P(E_1/A)$

$= \frac{P(E_1)\,P(A/E_1)}{(P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2))}$

$= \frac{\frac{1}{4} \times \frac{12 \times 11}{51 \times 50}}{\frac{1}{4} \times \frac{12 \times 11}{51 \times 50} + \frac{3}{4} \times \frac{13}{51} \times \frac{12}{50}} = \frac{11}{50}$ [1 Mark]

30. Let $I = \int \frac{1}{\sqrt{(x-1)(x-2)}} dx$

$= \int \frac{1}{\sqrt{x^2 - 3x + 2}} dx$ [½ Mark]

$= \int \frac{1}{\sqrt{\left(x - \frac{3}{2}\right)^2 + 2 - \frac{9}{4}}} dx$ [1 Mark]

$= \int \frac{1}{\sqrt{\left(x - \frac{3}{2}\right)^2 - \frac{1}{4}}} dx = \int \frac{1}{\sqrt{\left(x - \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2}} dx$

[½ Mark]

$= \log\left|\left(x - \frac{3}{2}\right) + \sqrt{\left(x - \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2}\right| + C$

$\left(\because \int \frac{dx}{\sqrt{x^2-a^2}} = \log|x+\sqrt{x^2-a^2}|\right)$ [½ Mark]

$= \log\left|\left(x-\frac{3}{2}\right)+\sqrt{(x-1)(x-2)}\right|+C$ [½ Mark]

OR

$\text{L.H.S.} = 2\int_0^{\pi/4} \tan^2 x \tan x \, dx$

$= 2\int_0^{\pi/4} (\sec^2 x - 1)\tan x \, dx$ [½ Mark]

$(\because \sec^2 A = 1 + \tan^2 A)$

$= 2\int_0^{\pi/4} \sec^2 x \tan x \, dx - 2\int_0^{\pi/4} \tan x \, dx$

$= 2I_1 - 2I_2$ (say) ...(i) [½ Mark]

Here, $I_1 = \int_0^{\pi/4} \tan x \sec^2 x \, dx$

Put $\tan x = t \Rightarrow \frac{d}{dx}\tan x = \frac{dt}{dx}$

$\Rightarrow \sec^2 x \, dx = dt$

when $x = 0$, $t = \tan 0 = 0$ and

when $x = \frac{\pi}{4}$, $t = \tan\frac{\pi}{4} = 1$ [½ Mark]

$\therefore \quad I_1 = \int_0^1 t \, dt = \left[\frac{t^2}{2}\right]_0^1 = \frac{1}{2} - 0 = \frac{1}{2}$ [½ Mark]

And, $I_2 = \int_0^{\pi/4} \tan x \, dx = -[\log \cos x]_0^{\pi/4}$

$(\because \int \tan x \, dx = -\log \cos x)$ [½ Mark]

$= -\left[\log \cos\frac{\pi}{4} - \log \cos 0\right]$

$= -\left[\log\frac{1}{\sqrt{2}} - \log 1\right] = -\log\frac{1}{\sqrt{2}}$ $(\because \log 1 = 0)$

$\Rightarrow \quad I_2 = -\log 2^{-1/2} = \frac{1}{2}\log 2$ $(\because \log a^b = b \log a)$

Putting the values of I_1 & I_2 in (i), we get

$I = 2I_1 - 2I_2 = 2\left(\frac{1}{2}\right) - 2.\frac{1}{2}\log 2 \Rightarrow I = 1 - \log 2 = \text{R.H.S.}$

(Hence proved). [½ Mark]

31. Consider $x + 2y \geq 100$

Let $x + 2y = 100 \Rightarrow \frac{x}{100} + \frac{y}{50} = 1$

Now $x + 2y \geq 100$ represents which does not include (0,0) as it does not made it true.

Again consider $2x - y \leq 0$

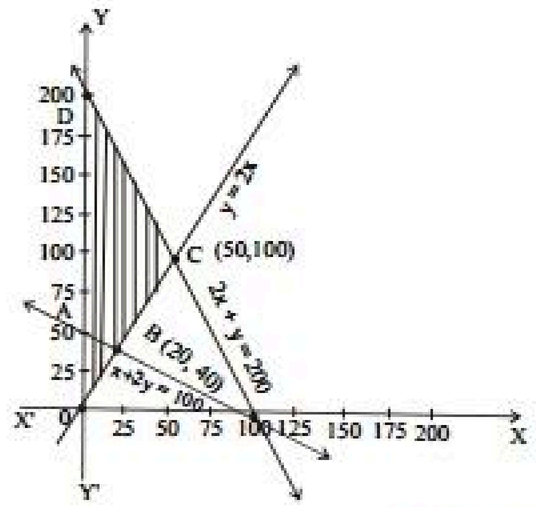
Let $2x - y = 0$ or $y = 2x$

x	0	25	50	100
y	0	50	100	200

Now let the test point be (10,0)

$2 \times 10 - 0 \leq 0$ which is false.

$\therefore$ the required half does not contain (10,0). [1 Mark]



[1 Mark]

Again consider $2x + y \leq 200$

Let $2x + y = 200 \Rightarrow \frac{x}{100} + \frac{y}{200} = 1$

Now (0, 0) satisfies $2x + y \leq 200$

$\therefore$ the required half plane contains (0, 0).

Now triple shaded region is ABCDA which is the required feasible region.

At A (0, 50), $Z = x + 2y = 0 + 2 \times 50 = 100$

At B (20, 40), $Z = 20 + 2 \times 40 = 100$

At C (50, 100), $Z = 50 + 2 \times 100 = 250$

At D (0, 200), $Z = 0 + 2 \times 200 = 400$

Thus maximum Z = 400 at x = 0, y = 200 and minimum Z = 100 at x = 0, y = 50 or x = 20, y = 40

[1 Mark]

32. **One-one/Many-one :** Let $x_1, x_2 \in R - \{3\}$ are the elements such that

$f(x_1) = f(x_2)$: then $f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$ [½ Mark]

$\Rightarrow (x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$

$\Rightarrow x_1x_2 - 2x_2 - 3x_1 + 6 = x_2x_1 - 2x_1 - 3x_2 + 6$

$\Rightarrow x_2 = x_1, \therefore f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

$\Rightarrow$ f is one-one function [1½ Marks]

Onto/Into : Let $y \in R - \{1\}$ (co-domain)

Then one element $x \in R - \{3\}$ in domain is such that

$f(x) = y \Rightarrow \frac{x-2}{x-3} = y \Rightarrow x - 2 = xy - 3y$

$\Rightarrow \quad x = \left(\frac{3y-2}{y-1}\right)$ [1½ Marks]

$\therefore$ The pre-image of each element of co-domain $R - \{1\}$ exists in domain $R - \{3\}$.

$\Rightarrow$ f is onto [1½ Marks]

33. Now, $|A| = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix} = -1 \neq 0$ [1 Mark]

$\therefore A^{-1}$ non singular hence the given equations have a unique solution.

$A_{11} = 0 \quad A_{21} = -1 \quad A_{31} = 2$
$A_{12} = 2 \quad A_{22} = -9 \quad A_{32} = 23$
$A_{13} = 1 \quad A_{23} = -5 \quad A_{33} = 13$ [1 Mark]

$$A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{-1}\begin{bmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{bmatrix}^{1}$$

$$= \frac{1}{-1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$ [1 Mark]

We have $AX = B$

Where, $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

and $B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

$X = A^{-1}B$ ...(i) [1 Mark]

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\Rightarrow x = 1, y = 2$ and $z = 3$. [1 Mark]

34. The shortest distance $= \left|\frac{(\bar{a}_1 - \bar{a}_2)\cdot(\bar{b}_1 \times \bar{b}_2)}{|\bar{b}_1 \times \bar{b}_2|}\right|$

$\bar{a}_1 - \bar{a}_2 = (6\hat{i} + 2\hat{j} + 2\hat{k}) - (-4\hat{i} - \hat{k})$

$= 10\hat{i} + 2\hat{j} + 3\hat{k}$ [1 Mark]

$$|\bar{b}_1 \times \bar{b}_2| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix} = 8\hat{i} + 8\hat{j} + 4\hat{k}$$ [1½ Marks]

$|\bar{b}_1 \times \bar{b}_2| = \sqrt{8^2 + 8^2 + 4^2} = 12$ [1 Mark]

$$\therefore \quad \text{S.D.} = \frac{(\bar{a}_1 - \bar{a}_2)\cdot(\bar{b}_1 + \bar{b}_2)}{|\bar{b}_1 + \bar{b}_2|} = 9$$ [1½ Marks]

OR

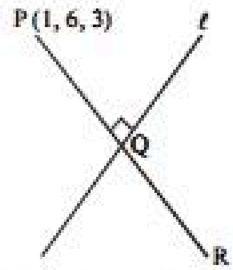
R (1, 0, 7)

Hint: For image of P (1, 6, 3) in L draw a line $PR \perp \ell$ then R is its image of Q is mid point of PR and $PR \perp \ell$. Let λ, μ, ν be the d.r's of PR. $PR \perp \ell$.

$\Rightarrow \lambda \times 1 + \mu \times 2 + \nu \times 3 = 0$ ...(i)

$\Rightarrow \lambda + 2\mu + 3\nu = 0$ ...(ii)

and equ. of PR is $\frac{x-1}{\lambda} = \frac{y-6}{\mu} = \frac{z-3}{\nu}$ [1½ Marks]



[½ Mark]

Any point on it is $(\lambda k + 1, \mu k + 6, \nu k + 3)$ let it be θ. As θ lies on l, so. [½ Mark]

$$\frac{xk-1}{1} = \frac{\mu k + 6 - 1}{2} = \frac{\nu k + 3 - 2}{2} \Rightarrow \frac{\lambda k + 1}{1} = \frac{\mu k + 5}{3}$$

$$R(x', y', z') = \frac{1(\lambda k + 1) + 2(\mu k + 5) + 3(\nu k + 1)}{1 \times 1 + 2 \times 2 + 3 \times 3}$$

$$= \frac{14 + (\lambda + 2\mu + 3\nu)k}{14} = 1$$

$\Rightarrow \quad \lambda k = 0, \mu k = -3, \nu k = 2$ [½ Mark]

$\Rightarrow Q(0 + 1, -3 + 6, 2 + 3) = (1, 3, 5)$

As Q is the mid point of PR, so [1 Mark]

$$\frac{1 + x'}{2} = 1, \frac{6 + y'}{2} = 3, \frac{3 + z'}{2} = 5$$

$\Rightarrow x' = 1, y' = 0, z' = 7$

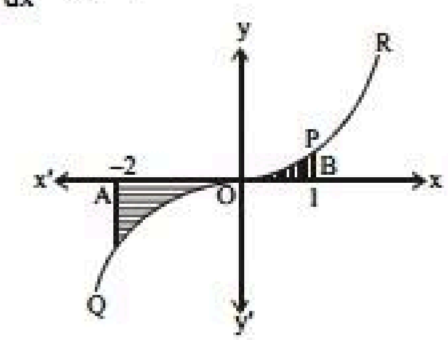
$\Rightarrow R(1, 0, 7)$. Which is the image of P. [1 Mark]

35. The curves $y = x^3$

Differentiating $\frac{dy}{dx} = 3x^2 (+\text{ve})$

$\therefore$ curve is an increasing curve

$\frac{dy}{dx} = 0, x = 0$



[2 Marks]

$\therefore$ x-axis is the tangent at $x = 0$,

$(-x)^3 = -x^3$

$\therefore$ $f(-x) = -f(x)$

curve is symmetrical in opposite quadrants,

Area bounded by the curve $y = x^3$, the x-axis,

$x = -2, x = 1$

= Area of the region AQOBPOA

= Area of the region AQOA + Area of the region BPOB

$$= \left|\int_{-2}^{0} y\,dx\right| + \int_{0}^{1} y\,dx = \left(\int_{-2}^{0} x^3\,dx\right) + \int_{0}^{1} x^3\,dx$$

[2 Marks]

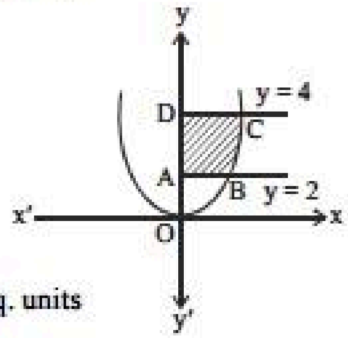
$$= \frac{16}{4} + \frac{1}{4} = \frac{17}{4}$$ [1 Mark]

OR

Required area = area ABCD

$= \int_2^4 x\,dy = \int_2^4 2\sqrt{y}\,dy$

$= 2\int_2^4 \sqrt{y}dy$ [1 Mark]



$= 2\left[\frac{y^{3/2}}{\frac{3}{2}}\right]_2^4$ [2 Marks]

$= \left(\frac{32-8\sqrt{2}}{3}\right)$ sq. units [2 Marks]

36. (i) $\because$ Perimeter = 10

$2x + \pi r + 2r = 10$

$2x + (\pi + 2)r = 10$ [1 Mark]

(ii) A = sum of areas of rectangle and semicircle

$= 2rx + \frac{1}{2}\pi r^2 = r[10-(\pi+2)r] + \frac{1}{2}\pi r^2$

$= 10r - \left(\frac{1}{2}\pi + 2\right)r^2$ [1 Mark]

(iii) $\frac{dA}{dr} = 10 - (\pi + 4)r$

For critical point

$\frac{dA}{dt} = 0 \Rightarrow 10 - (\pi+4)r = 0 \Rightarrow r = \frac{10}{(\pi+4)}$ [1 Mark]

$\frac{d^2A}{dt^2} = -(\pi+4) \Rightarrow \left(\frac{d^2A}{dt^2}\right)_{(r)} = -(\pi+4) < 0$

$\Rightarrow r = \frac{10}{\pi+4}$ is point of maxima

$\because 2x + (\pi+2)r = 10$

$\Rightarrow x = \frac{10}{\pi+4}$

$\therefore$ Length of rectangle = 2r

$= \frac{20}{\pi+4}$ and width $= \frac{10}{\pi+4}$

$\therefore$ Required dimension is

$\frac{20}{\pi+4}, \frac{10}{\pi+4}$ [2 Marks]

OR

$\because$ A is maximum for

$r = \frac{10}{\pi+4} = \frac{10}{\frac{22}{7}+4} = \frac{7}{5}$ [1 Mark]

$\therefore A = 10r - \left(\frac{1}{2}\pi + 4\right)r^2$

$= 10 \times \frac{7}{5} - \left(\frac{1}{2} \times \frac{22}{7} + 4\right) \times \left(\frac{7}{5}\right)^2$

$= 14 - 10.92 = 3.08\ m^2$ [1 Mark]

37. (i) Probability of report is positive when person having COVID-19 $= \frac{95}{100}$ [1 Mark]

(ii) Probability of report is positive when person not having COVID-19 $= \frac{10}{100}$ [1 Mark]

(iii) Probability that person actually has COVID-19

$= \frac{\frac{10}{100} \times \frac{95}{100}}{\frac{10}{100} \times \frac{95}{100} + \frac{90}{100} \times \frac{10}{100}}$ [1 Mark]

$= \frac{95}{1000} \times \frac{1000}{185} = \frac{19}{37}$ [1 Mark]

OR

Probability of report positive

$= \frac{10}{100} \times \frac{95}{100} + \frac{90}{100} \times \frac{10}{100}$ [1 Mark]

$= \frac{10}{100}\left[\frac{185}{100}\right] = 0.185$ [1 Mark]

38. (i) $\tan(\alpha) = \frac{r}{h} \Rightarrow \alpha = \tan^{-1}\left(\frac{r}{h}\right)$

It is given that $\alpha = \tan^{-1}(0.5)$

$\therefore \tan^{-1}\left(\frac{r}{h}\right) = \tan^{-1}(0.5)$

$\Rightarrow \frac{r}{h} = 0.5 = \frac{1}{2} \Rightarrow h = 2r$ [1 Mark]

$\because V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{h}{2}\right).h = \frac{1}{12}\pi h^3$

$\frac{dV}{dt} = \frac{\pi}{4}h^2\frac{dh}{dt}$

$\Rightarrow \frac{dh}{dt} = \frac{4}{\pi h^2} \cdot \frac{dV}{dt}$ $\left[\because h = 4m, \frac{dV}{dt} = 5\ m^3/hr\right]$

$\Rightarrow \frac{dh}{dt} = \frac{4}{\pi(4)^2} \times 5$

$\Rightarrow \frac{dh}{dt} = \frac{35}{88} m/hr$ [1 Mark]

(ii) $\because S = \pi rl + \pi r^2$

$\Rightarrow l = \frac{S - \pi r^2}{\pi r}$ ...(i) [1 Mark]

$\because V = \frac{1}{3}\pi r^2 h \Rightarrow V^2 = \frac{1}{9}\pi^2 r^4 h^2$

$\Rightarrow V^2 = \frac{1}{9}\pi^2 r^4 (l^2 - r^2) [\because l^2 = r^2 + h^2]$

$\Rightarrow V^2 = \frac{1}{9}\pi^2 r^4 \left[\left(\frac{S-\pi r^2}{\pi r}\right)^2 - r^2\right]$ [Using eqn (i)]

$\Rightarrow V^2 = \frac{1}{9}Sr^2(S - 2\pi r^2)$ [1 Mark]