CBSE Test Paper 04

Chapter 12 Linear Programming

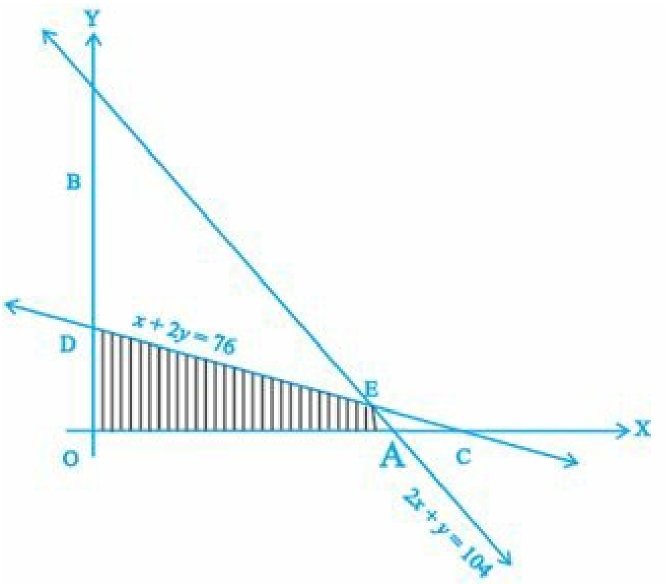
1. Minimize $Z = 5x + 10y$ subject to $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$, $x, y \geq 0$.

   a. Minimum Z = 310 at (60, 0)
   b. Minimum Z = 320 at (60, 0)
   c. Minimum Z = 330 at (60, 0)
   d. Minimum Z = 300 at (60, 0)

2. Corner points of the feasible region for an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). Let $F = 4x + 6y$ be the objective function. Maximum of F – Minimum of F =.

   a. 48
   b. 60
   c. 42
   d. 18

3. Determine the minimum value of $Z = 3x + 4y$ if the feasible region (shaded) for a LPP is shown in Figure above.



   a. 154
   b. 196
   c. 112

d. 132

4. Let R be the feasible region for a linear programming problem,and let Z = ax + by be the objective function. If R is bounded, then the objective function Z has both a maximum and a minimum value on R and

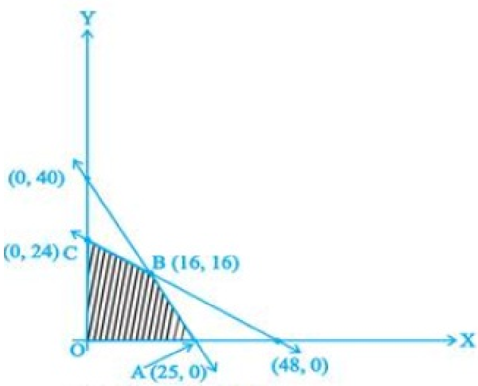
   a. each of these occurs at some points except corner points of R.
   b. each of these occurs at the centre of R.
   c. each of these occurs at the midpoints of the edges of R
   d. each of these occurs at a corner point (vertex) of R.

5. Minimize Z = 50x+60y , subject to constraints x +2 y ≤ 50 , x + y ≥ 30, x, y ≥ 0.

   a. 1800
   b. 1550
   c. 1700
   d. 1200

6. A problem which seeks to maximise or minimise a function is called an ________ problem.
7. The linear inequalities or restrictions on the variables of an LPP are called ________.
8. Any point in the feasible region that gives the optimal value of the objective function is called an ________ solution.

9. Determine the maximum value of Z = 4x + 3y if the feasible region for an LPP is shown in Figure.



10. Minimise Z = 3x + 5y subject to the constraints:

    $x + 2y \geqslant 10$

$x+y \geqslant 6$

$3x+y \geqslant 8$

$x, y \geqslant 0$

11. Maximise Z = 3x + 4y, subject to the constraints: $x+y \leqslant 1, x \geqslant 0, y \geqslant 0$.

12. Maximize Z = 100x + 170y subject to

    3x+2y$\leq$ 3600

    x+4y$\leq$1800

    $x \geq 0, y \geq 0$

13. Maximise and minimise Z = x + 2y

    subject to the constraints

    x + 2y $\geq$100

    2x - y $\leq$ 0

    2x + y $\leq$200

    x, y$\geq$ 0

    Solve the above LPP graphically.

14. A cooperative society of farmers has 50 hec of land to grow two crops A and B. The profit from crops A and B per hec are estimated at Rs 10500 and Rs 9000, respectively. To control weeds, a liquid herbicide has to be used for crops A and B at the rate of 20 L per hec and 10 L per hec, respectively. Further, not more than 800 L of herbicide should be used in order to protect fish and wildlife using a pond which collects drainage from this land. Keeping in mind that the protection of fish and other wildlife is more important than earning a profit. How much land should be allocated to each crop so as to maximise the total profit? Formulate the above as an LPP and solve it graphically. Do you agree with the message that the protection of wildlife is almost necessary to preserve the balance in the environment?

15. Maximise and Minimise Z = 3x– 4y. subject to

    $x-2y \leqslant 0$

    $-3x+y \leqslant 0$

    $x-y \leqslant 6$

    $x, y \geqslant 0$

16. A library has to accommodate two different types of books on a shelf. The books are 6 cm and 4 cm thick and weight 1 kg and $1\frac{1}{2}$ kg each, respectively. The 2 shelf is 96 cm long and almost can support a weight of 21 kg. How should the shelf be filled with the books of two types in order to include the greatest number of books? Make it as an LPP and solve it graphically.

17. Anil wants to invest at most Rs 12,000 in bonds A and B. According to the rules he has to invest at least Rs 2000 in bond A and at least Rs 4000 in bond B. If the rate of interest on bond A is 8% per annum and on bond B, it is 10% per annum, how should he invest the money for maximum interest.

18. A manufacturer has three machine I, II and III installed in his factory. Machines I and II are capable of being operated for at most 12 hours whereas machine III must be operated for at least 5 hours a day. She produces only two items M and N each requiring the use of all the three machines.
The number of hours required for producing 1 unit of each M and N on the three machines are given in the following table:

Items	**Number of hours required on machines**		
	I	II	III
M	1	2	1
N	2	1	1.25

She makes a profit of Rs. 600 and Rs. 400 on items M and N respectively. How many of each item should she produce so as to maximise her profit assuming that she can sell all the items that she produced? What will be the maximum profit?

Solution

1. d. Minimum Z = 300 at (60, 0)

Explanation:

Objective function is Z = 5x + 10 y ....(1).
The given constraints are : $x + 2y \le 120$, $x + y \ge 60$, $x - 2y \ge 0$, $x, y \ge 0$.

The corner points are obtained by drawing the lines x+2y =120, x+y = 60 and x-2y = 0. The points so obtained are (60,30),(120,0), (60,0) and (40,20)

Corner points	Z = 5x + 10y
D(60 ,30)	600
A(120,0)	600
B(60,0)	300.........................(Min.)
C(40,20)	400

Here , Z = 300 is minimum at (60, 0).

2. b. 60

Explanation:

Here the objective function is given by : F = 4x +6y .

Corner points	Z = 4x +6 y
(0, 2)	12...................(Min.)
(3,0)	12.....................(Min.)
(6,0)	24
(6 , 8)	72
(0 , 5)	30

Maximum of F – Minimum of F = 72 – 12 = 30 .

3. d. 132

Explanation:

Here , minimize Z = 3x + 4y ,

Corner points	**Z = 3x + 4y**
C(0 ,38)	132.......................(Min.)
B (52 ,0)	156
D(44 , 16)	196

The minimum value is 132

4. d. each of these occurs at a corner point (vertex) of R.

Explanation:

Let R be the feasible region for a linear programming problem,and let Z = ax + by be the objective function. If R is bounded, then the objective function Z has both a maximum and a minimum value on R and each of these occurs at a corner point (vertex) of R.

5. c. 1700

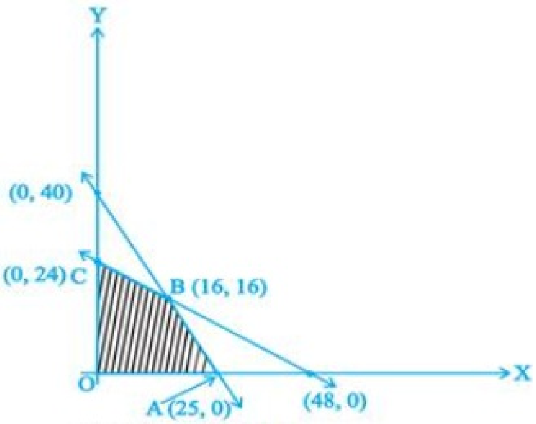
Explanation:

Here, Maximize Z = 50x+60y , subject to constraints x +2 y ≤ 50 , x + y ≥ 30, x, y ≥ 0.

Corner points	Z = 50x +60 y
P(50 ,0)	2500
Q(0 , 30)	1800
R(10, 20)	1700

Hence the minimum value is 1700

6. optimisation
7. constraints
8. optimal
9. We have to determine the maximum value of $Z = 4x + 3y$ if the feasible region for an LPP is shown in Fig.



The feasible region is bounded. Therefore, maximum of Z must occur at the corner point of the feasible region (Fig. 12.1).

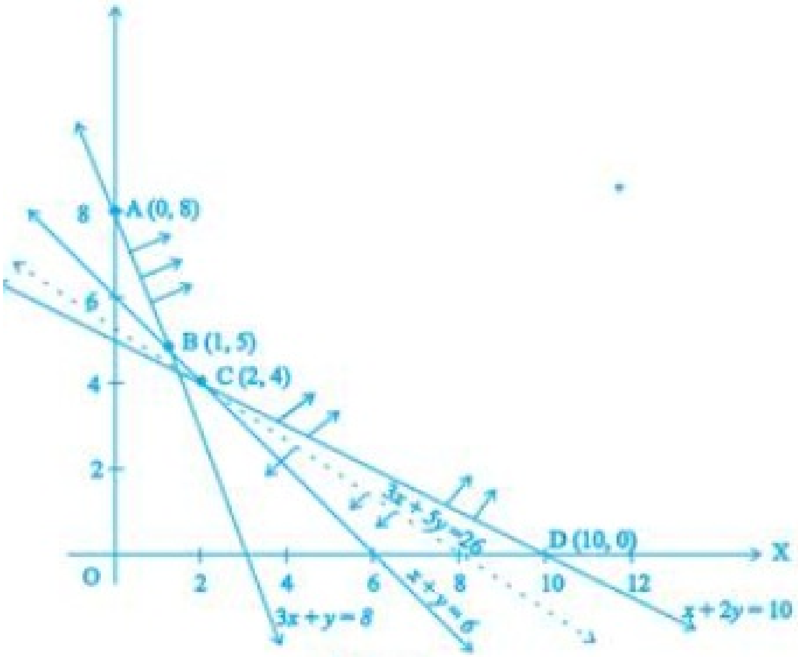
Corner Point	**Value of Z**
O, (0, 0)	4(0) + 3(0) = 0
A(25, 0)	4(25) + 3(0) = 100
B(16, 16)	4(16) + 3(16) = 112 (maximum)
C(0, 24)	4(0) + 3(24) = 72

Hence, the maximum value of Z is 112.

10. We first draw the graphs of $x + 2y = 10$, $x + y = 6$, $3x + y = 8$. The shaded region ABCD is the feasible region (R) determined by the above constraints. The feasible region is unbounded. Therefore, minimum of Z may or may not occur. If it occurs, it will be on the corner point.

Corner Point	**Value of Z**
A(0, 8)	40

B(1, 5)	28
C(2, 4)	26 (smallest)
D(10, 0)	30



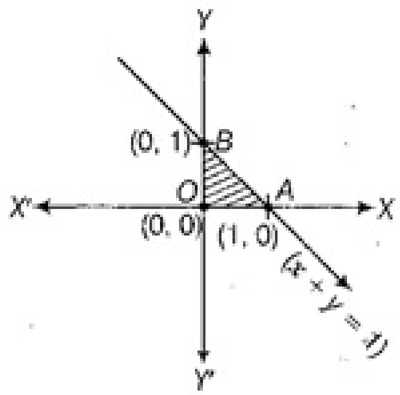
Let us draw the graph of 3x + 5y < 26 as shown in Fig by dotted line.

We see that the open half plane determined by 3x + 5y < 26 and R do not have a point in common. Thus, 26 is the minimum value of Z.

11. Maximise Z = 3x + 4y. Subject to the constraints

$x + y \leqslant 1, x \geqslant 0, y \geqslant 0$

The Shaded region shown in the figure as OAB is bounded and the coordinates of corner points O, A and B are (0, 0), (1, 0) and (0, 1), respectively.



Corner Points	Corresponding value of Z

(0, 0)	0
(1, 0)	3
(0, 1)	4 (Maximum)

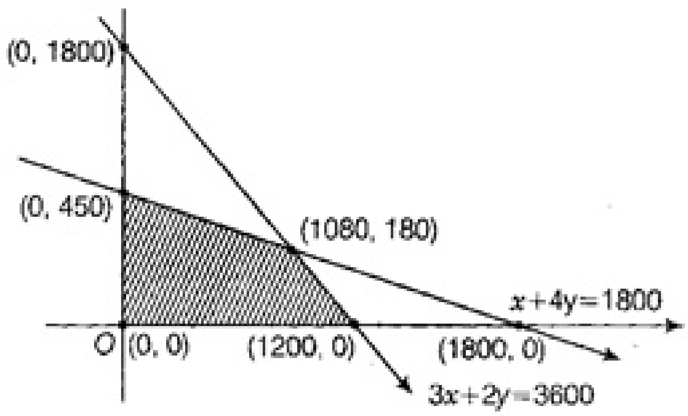
Hence, the maximum value of Z is 4 at (0, 1).

12. Maximise Z= 100x + 170y subject to

$$3x+2y \leqslant 3600, x+4y \leqslant 1800, x \geqslant 0, y \geqslant 0$$

From the shaded feasible region it is clear that the coordinates of corner points are (0,0),(1200,0),(1080,180) and (1080,180) and (0,450).

On solving x + 4y = 1800 and 3x + 2y = 3600 we get x = 1080 and y = 180.



Corner Points	Corresponding value of Z = 100x + 170y
(0, 0)	0
(1200, 0)	$1200 \times 100 = 12000$
(1080, 180)	$100 \times 1080 + 170 \times 180 = 138600\ (\max imum)$
(0, 450)	$0 + 170 \times 450 = 76500$

Hence, the maximum is 138600.

13. Our problem is to mimmise and maximise the given objective function given as Z = x + 2 y .....(i)
Subject to the given constraints,

x + 2y $\geq$100 ....(ii)
2x - y $\leq$ 0 ....(iii)
2x + y $\leq$200 ....(iv)
x$\geq$ 0, y$\geq$ 0 ....(v)
Table for line x + 2y = 100 is

x	0	100
y	50	0

So, the line x + 2y = 100 is passing through the points with coordinates (0, 50) and (100, 0).
On replacing the coordinates of the origin O (0, 0) in the inequality x + 2y $\geq$100, we get
$2 \times 0 + 0 \geq 100$
$\Rightarrow \quad 0 \geq 100$(which is False)
So, the half plane for the inequality of the line (ii) is away from the origin, which means that the point O(0,0) does not lie in the feasible region of the inequality of (ii)
Table for the line (iii) 2x - y = 0 is given as follows.

x	0	10
y	0	20

So, the line 2x - y = 0 is passing through the points (0, 0) and (10, 20).
On replacing the point (5, 0) in the inequality 2x - y$\leq$ 0, we get
$2 \times 5 - 0 \leq 0$
$\Rightarrow \quad 10 \leq 0$ (which is False)
So, the half plane for the inequality of (iii) is towards Y-axis.
Table of values for line 2x + y = 200 is given as follows.

x	0	100
y	200	0

So, the line 2x + y = 200 is passing through the points with coordinates (0, 200) and (100, 0).

On replacing O (0, 0) in the inequality 2 + y $\leq$ 200, we get

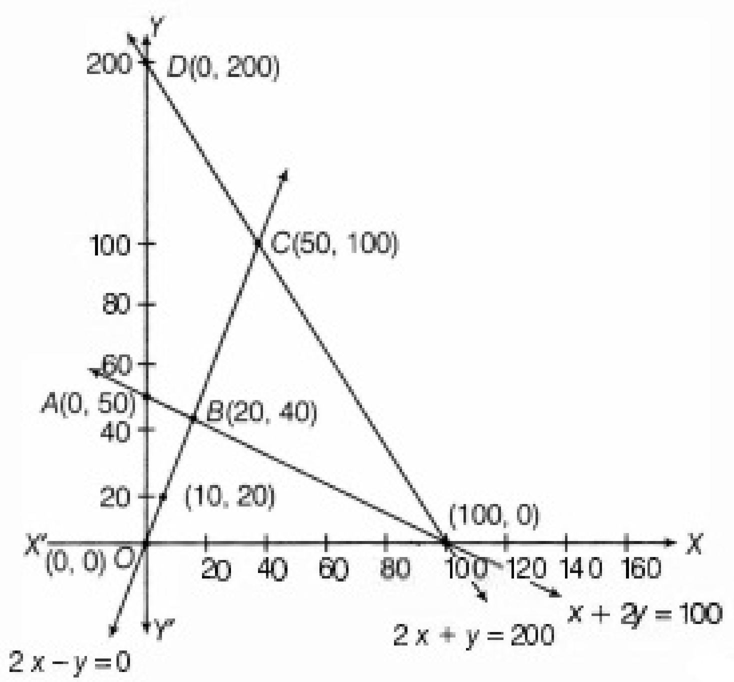
$2 \times 0 + 0 \leq 200$

$\Rightarrow \quad 0 \leq 200$ (which is true)

So, the half plane for the inequality of the line (iv) is towards the origin, which means that the point O (0,0) is a point in the feasible region.

Aslo, x, y$\geq$ 0

So, the region lies in the I quadrant only.



On solving equations 2x - y = 0 and x + 2y = 100, we get the point of intersection as B(20, 40).

Again, solving the equations 2x - y = 0 and 2x + y = 200, we get C(50, 100).

$\therefore$ Feasible region is ABCDA, which is a bounded feasible region.

The coordinates of the corner points of the feasible region are A(0, 50), B(20, 40), C(50, 100) and D(0, 200).

The values of Z at corner points are given below:

Corner points	**Z = x + 2y**
A(0, 50)	Z = 0 + 2$\times$50 = 100
B(20, 40)	Z = 20 + 2$\times$40 = 100
C(50, 100)	Z = 50 + 2$\times$100 = 250
D(0, 200)	Z = 0 + 2$\times$200 = 400

The maximum value of Z is 400 at D(0, 200) and the minimum value of Z is 100 at all the points on the line segment joining A(0, 50) and B(20, 40).

14. Let x hec for crop A and y hec for crop B be allocated.

	Crop A	**Crop B**	**Available**
land(hec)	x	y	50
Herbicide(L/hec)	20	10	800
Profit	10500	9000	

According to the question, we get the following LPP,
Maximise, $Z=10500x+9000y$
Subject to constraints
$x+y\leq 50$
$20x+10y\leq 800$
$2x+y\leq 80$
and $x,y\geq 0$
Let us consider the inequalities as equations, we get
$x+y=50$..........(i)
$2x+y=80$........(ii)
Table for line $x+y=50$ is

x	0	50
y	50	0

So, it passes through the points (0, 50) and (50, 0) On putting (0,0) in the inequality $x+y\leq 50$, we get
0 + 0 $\leq$ 50 = 0 $\leq$ 50 [which is true]
So, the half plane is towards the origin
Table for line $2x+y=80$ is

x	0	40
y	80	0

So, it passes though the points $(0, 80)$ and $(40, 0)$.

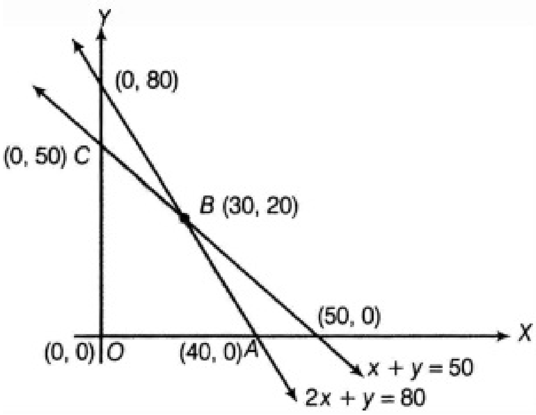
On putting (0,0) in the inequality 2x + y $\leq$ 80, we get

2(0) + 1(0) $\leq$ 80 $\Rightarrow$ 0$\leq$ 80 [which is true]

So, the half plane is towards the origin.

Also, x $\geq$0 and y $\geq$ 0, so the feasible region lies in the first quadrant.

Solving Eq. (i) and Eq. (ii), we get $x = 30$ and $y = 20$ So, the intersection point is $B(30, 20)$.



We observe that $OABCO$ is the feasible region which is bounded and extreme points are $O(0, 0), A(40, 0), B(30, 20)$ and $C(0, 50)$.

Corner Points	Value of Z =10500x+ 9000y
$O(0, 0)$	$Z = 0 + 0 = 0$
$A(40, 0)$	Z =$10500 \times 40 + 0 =$420000
$B(30, 20)$	Z= $10500 \times 30 + 9000 \times 20$=49500
$C(0, 50)$	Z = $0 + 9000 \times 50$= 450000

From table, maximum value of Z is 495000. Hence, 30 hectares land allocated for crop A and 20 hectares for crop B to maximize the profit.

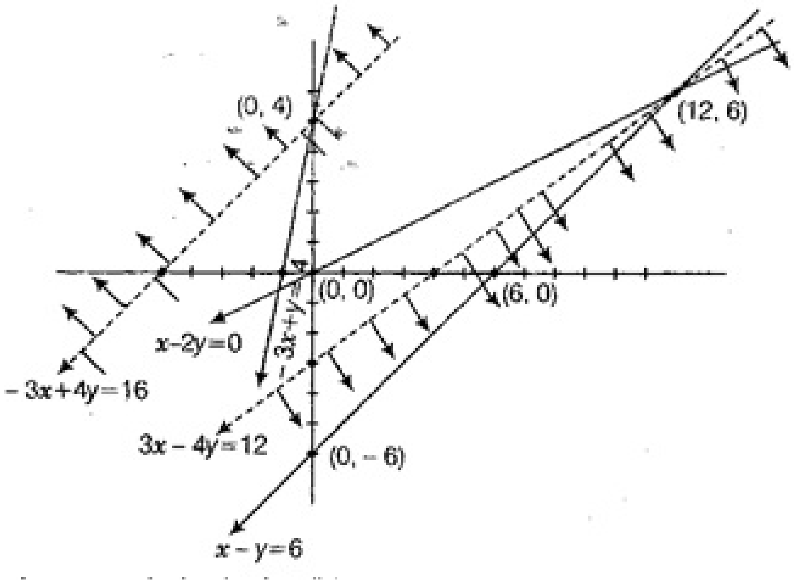
Yes, I agree with the message that the protection of wildlife is almost necessary to preserve the balance in environment.

15. Given LPP is

Maximise and minimise Z=3x-4y subject to

$x - 2y \leqslant 0, -3x + y \leqslant 4, x - y \leqslant 6, x, y \geqslant 0.$

[On solving x-y=6 and x-2y=0 we get x=12, y=6]



From the shown graph, for the feasible region, we see that it is unbounded and coordinates of corner points are (0,0),(12,6) and (0,4).

Corner	Corresponding value of Z = 3x - 4y
(0, 0)	0
(0, 4)	-16 (minimum)
(12, 6)	12 (maximum)

For given unbounded region the minimum value of Z may or may not be –16. So, for deciding this, we graph the inequality.

$3x - 4y \leq 16$

And check whether the resulting open half plane has common points with feasible region or not.

Thus, from the figures it shows it has common points with feasible region, So, it does not have any minimise value.

Also, similarly for maximum value, we graph the inequality 3x - 4y > 12

And see that resulting open half plane has no common points with the feasible region and hence maximum value of 12 exits for Z = 3x - 4y.

16. Let two types of books which should be accomodated in the two shelves be x and y, respectively. The required LPP is to find the maximum number of books to be accomodated in the shelves.

    Hence let Z be the objective function representing the total number of books those can be kept .Hence the equation of the objective function Z is given as Z = x + y, which is to be maximised, Subject to constraints

$6x+4y\leq 96$ (length constraint) (dividing throughout by 2 we get)

or $3x+2y\leq 48$

$x+\frac{3}{2}y\leq 21$ (weight constraint) (multiplying throughout by 2 we get)

or $2x+3y\leq 42$

and $x,y\geq 0$ (non negative constraints which will restrict the solution region to the first quadrant only)

On considering the inequalities as equations,

we get

3x + 2y = 48 ...(i)

2x + 3y = 42 ...(ii)

Table of values for line 3x + 2y = 48 is given below.

x	0	16
y	24	0

So, the line (i) passes through the points with coordinates (0, 24) and (16, 0).

On replacing the origin O (0, 0) in $3x+2y\leq 48$, we get

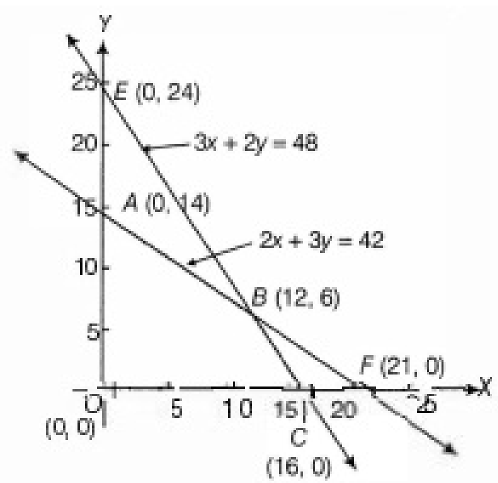
$0+0\leq 48\Rightarrow 0\leq 48$ (which is true)

So, the half plane for the inequality of the line (i) is towards the origin, which means that the point (0,0) which is the origin will be in the solution region of the inequality of the line (i) .

On solving Eqs. (i) and (ii), we get

x = 12 and y = 6

Thus, the point of intersection is B(12, 6)



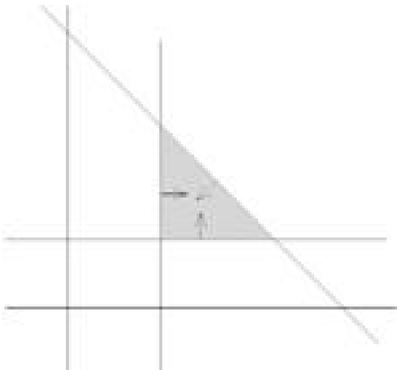
From the graph, OABCDO is the feasible region which is bounded. The coordinates of the corner points are O(0, 0), A(0, 14), B(12, 6) and C(16, 0).

The values of Z at corner points are as follows

Corner points	Value of Z = x + y
O(0,0)	Z = 0 + 0 = 0
A(0, 14)	Z = 0 + 14 = 14
B(12, 6)	Z = 12 + 6 = 18 (maximum)
C(16, 0)	Z = 16 + 0 = 16

From the table, we get the maximum value of Z as 18 at the point B(12, 6).

Hence, The maximum number of books is 18 and number of books of type I is 12 and the number of books of type II is 6, which could be accommodated in the shelves.

17. 

Let Anil invest x in bond A and Y in bond B,

$P = \frac{8x}{100} + \frac{10y}{100}$

$x + y \leqslant 12000$

x ≥ 2000

y ≥ 4000

x ≥ 0, y ≥ 0

Solving these equations, we get

x = 2000, y = 10,000

P = $\frac{8}{10}$ (2000) + $\frac{10}{100}$ (10,000) = 2600

18. Suppose x and y be the number of items M and N respectively.

Total profit on the production = $Rs. (600x + 400y)$

Maximize $Z = 600x + 400y$

Subject to constraints

$x + 2y \leq 12$ (Constraint on Machine I)

$2x + y \leq 12$ (Constraint on Machine II)

$x+\frac{5}{4}y \geq 5$ (Constraint on Machine III)

s.t. $x \geq 0, y \geq 0$

Reducing the linear constraints into the linear equations.

$x+2y=12$ ... (i)

$2x+y=12$ ... (ii)

$4x+5y=20$ ... (iii)

$x=0, y=0$ ... (iv)

Equations	Points of Intersection
(i) and (ii)	$\Rightarrow x=4, y=4$
	$\Rightarrow$ Point is (4, 4)
(i) and (iii)	$\Rightarrow x=\frac{-20}{3}$ and $y=\frac{-28}{3}$
	$\Rightarrow$ Point is $\left(\frac{-20}{3}, \frac{-28}{3}\right)$
(i) and (iv)	when x = 0 $\Rightarrow$ y = 6
	$\Rightarrow$ Point is (0, 6)
	when y = 0 $\Rightarrow$ x = 12
	$\Rightarrow$ Point is (12, 0)
(ii) and (iii)	x = 6 and $y=\frac{-4}{5}$
	$\Rightarrow$ Point is $\left(6, \frac{-4}{5}\right)$
(ii) and (iv)	when x = 0 $\Rightarrow$ y = 12
	$\Rightarrow$ Point is (0, 12)
	when y = 0, x = 6
	$\Rightarrow$ Point (6, 0)
(iii) and (iv)	when x = 0 $\Rightarrow$ y = 4
	$\Rightarrow$ Point is (0, 4)
	when y = 0 $\Rightarrow$ x = 5
	$\Rightarrow$ Point is (5, 0)

For $x+2y \leq 12$, let x = 0, y = 0

$\Rightarrow 0 \leq 12$ i.e., true

$\Rightarrow$ The shaded part will be toward origin.

For $2x + y \leq 12$, let x = 0, y = 0

$\Rightarrow 0 \leq 12$ i.e., true

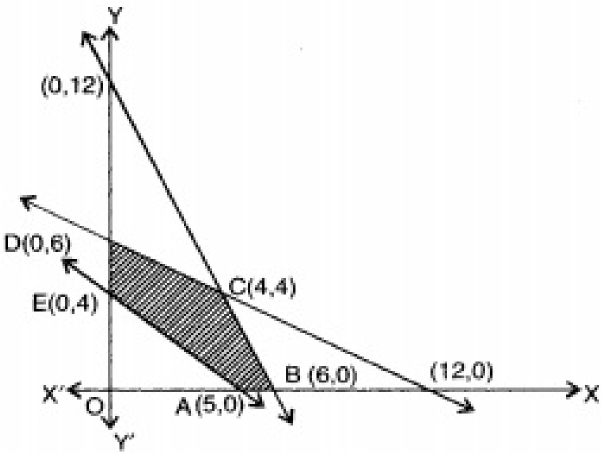
$\Rightarrow$ The shaded part will be toward origin

For $4x + 5y \geq 20$, let x = 0, y = 0

$\Rightarrow 0 \geq 20$ i.e., not true

$\Rightarrow$ The shaded part will be away from the origin.

Also, $x \geq 0, y \geq 0 \Rightarrow$ y > 0 The shaded part will exist in first quadrant.



From graph we observed that feasible region is $ABCDEA$, Le., bounded. Here corner points are $A(5,0), B(6,0) C(4,4) D(0,6), E(0,4)$

Corner points	Z = 600x + 400y
$A(5,0)$	3000
$B(6,0)$	3600
$C(4,4)$	4000 $\rightarrow$ Maximum
$D(0,6)$	2400
$E(0,4)$	1600

At the point (4, 4) the maximum value of Z exist i.e., Z = 4000.

The manufacturer has to produce 4 units of each item to get the maximum profit of Rs. 4000.