ALLIGATIONS AND MIXTURES

4

INTRODUCTION

In the previous chapter, we have seen in details, situations of averages and weighted averages. To recollect, whenever one faces situations where one needs to find the average of a few groups of readings, the simple average formula no longer works. In such situations, we move to dealing with the situation using the approach of weighted averages.

However, before we move there, I am sure you are wondering why I am talking about weighted averages in the introduction of the chapter on Alligations. Well, as you are about to see, Alligations is nothing but a smarter and visual way of solving weighted average questions (and dealing with weighted average situations), when two groups are involved. We will be eventually moving to discussing those situations, but to start off, let us first recollect what weighted averages was.

To recollect, the weighted average is used when a number of smaller groups are mixed together to form one larger group.

If the average of the measured quantity was

A_1	for group	1	containing	n_1 elements
A_2	for group	2	containing	n_2 elements
A_3	for group	3	containing	n_3 elements
A_k	for group	k	containing	n_k elements

$Aw = (n_1A_1 + n_2A_2 + n_3A_3 + \ldots\ldots + n_kA_k)/(n_1 + n_2 + n_3 \ldots + n_k)$

That is, the weighted average

$$= \frac{\text{Sum total of all groups}}{\text{Total number of elements in all groups together}}$$

In this formula, we have also seen in the theory of the averages chapter, that we do not specifically require to use the values of n_1, n_2, etc. But we actually just need to use the ratios of the values of n_1, n_2 etc. Thus, if you purchase 12kg of rice @ ₹40/kg and 18kgs of rice @ ₹70/kg, we can calculate the average price, assume that you have purchased only 2kg @ ₹40/kg and 3kg @ ₹70/kg and the required answer would be obtained by using: $\frac{2\times40+3\times70}{5}$ = ₹58/kg, rather than doing: $\frac{12\times40+18\times70}{5}$ = ₹58/kg.

Let us now move to understand how the same situation can also be handled using Alligations (since in this situation, we are dealing with weighted average of two groups).

THEORY

In the case of the situation where just two groups are being mixed, we can write the weighted average formula as:

$$A_w = (n_1A_1 + n_2A_2)/(n_1 + n_2)$$

Rewriting this equation we get: $(n_1 + n_2)\,A_w = n_1A_1 + n_2A_2$

$$n_1(A_w - A_1) = n_2\,(A_2 - A_w)$$

or $n_1/n_2 = (A_2 - A_w)/(A_w - A_1)$ → the alligation equation.

This equation can be read in language and logical terms as: The difference between A_2 and A_w is proportional to n_1 (the weight on A_1) and the difference between A_w and A_1 is proportional to n_2 (the weight on A_2). In other words, there is a cross proportionality between the difference of the averages and the weights on the individual averages.

Alligation Situation: Consider the Following Situations:

Two groups of elements are mixed together to form a third group containing the elements of both the groups.

If the average of the first group is A_1 and the number of elements is n_1 and the average of the second group is A_2 and the number of elements is n_2, then to find the average of the new group formed, we can use either the weighted average equation or the alligation equation.

As a convenient convention, we take $A_1 < A_2$. Then, by the principal of averages, we get $A_1 < Aw < A_2$.

Illustration 1

Weighted Average is Unknown:

Two varieties of rice at ₹10 per kg and ₹12 per kg are mixed together in the ratio 1 : 2. Find the average price of the resulting mixture.

Solution using Alligation: $1/2 = (12 - Aw)/(Aw - 10) \rightarrow Aw - 10 = 24 - 2Aw$

$\Rightarrow 3Aw = 34 \Rightarrow Aw = ₹11.33/\text{kg}.$

Solution using weighted averages:

$$\frac{1\times10+2\times12}{3} = ₹11.33/\text{kg}.$$

Illustration 2

Ratio of the Weights is Unknown:

On combining two groups of students having 30 and 40 marks respectively in an exam, the resultant group has an average score of 34. Find the ratio of the number of students in the first group to the number of students in the second group.

Solution using alligations $n_1/n_2 = (40 - 34)/(34 - 30) = 6/4 = 3/2$

Note: In this situation, though you can still solve this using the weighted averages approach, as you can yourself imagine, it would take quite a bit of writing and time to get to the same answer.

Thus, the solution would start with: $\frac{n_1 \times 30 + n_2 \times 40}{n_1 + n_2} = 34$.

After a few transformations, you would be able to identify the ratio of n_1/n_2.

Illustration 3

Average of One of the Smaller Groups is Unknown:

In a class containing girls and boys in the ratio 2:3, during a test, the girls scored 40 marks on average, while the overall average of the class was 52 marks. How much did the boys score on average?

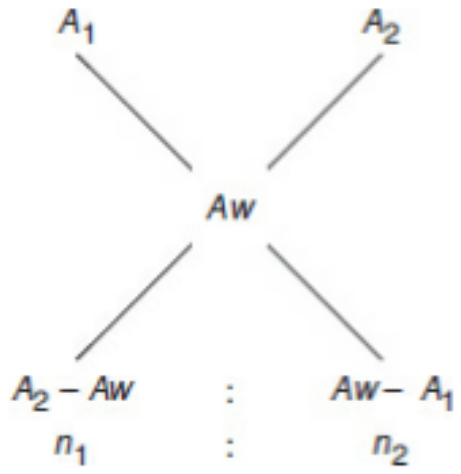
Solution using alligations: $n_1/n_2 = 2/3 = (x - 52)/(52 - 40)$ → On solving, we get $x = 60$.

Solution using weighted averages: $\frac{2 \times 40 + 3 \times x}{5} = 52$ → $x = 60$.

As you can see for yourself, each of the alligation situations illustrated above can be solved using the weighted average formula. So, why does one need to equip oneself with the alligations thought? Well the answer to that will emerge out of the following discussion – where we are discussing the graphical methods to solve each of the above situations using a graphical (or visual) approach to viewing the alligation equation that we have derived above. There are essentially two methods to think visually about the components of the alligation equation. These are: (i) the cross method and (ii) The straight line method (or the number line method).

Graphical Representation of Alligation

The formula illustrated above can be represented by the following cross diagram:



Note: The cross method yields nothing but the alligation equation. Hence, the cross method is nothing but a graphical representation of the alligation equation.

As we have seen, there are five variables embedded inside the alligation equation. These being:

the three averages → A_1, A_2 and Aw

and the two weights → n_1 and n_2

Based on the problem situation, one of the following cases may occur with respect to the knowns and the unknown in the problem.

Case	*Known*	*Unknown*
1	A_1, A_2, n_1, n_2	Aw
2	A_1, A_2, Aw	$n_1 : n_2$
3	A_1, Aw, n_1, n_2	A_2 (**Note:** In this situation, you could also know A_2 and need to find A_1.)

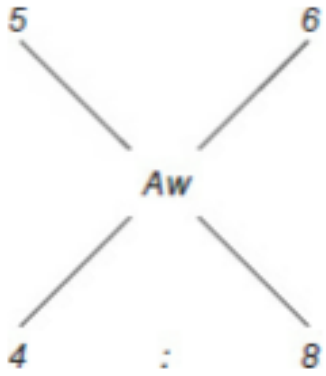
Now, let us try to evaluate the effectiveness of the cross method for each of the three cases illustrated above:

Case 1: A_1, A_2, n_1 and n_2 are known, Aw is in known.

Illustration 4

Four kg of rice at ₹5 per kg is mixed with 8kg of rice at ₹6 per kg. Find the average price of the mixture.

Solution



$$= (6 - Aw) : (Aw - 5)$$

$$\Rightarrow (6 - Aw)/(Aw - 5) = 4/8 \rightarrow 12 - 2\,Aw = Aw - 5$$

$$3\,Aw = 17$$

$$\therefore \quad Aw = 5.66\ ₹/kg$$

Task for students: Solve through the alligation formula approach and through the weighted average approach to get the solution. Notice, the amount of time required in doing the same.

> ***Note:*** The cross method becomes quite cumbersome in this case, as this method results in the formula being written. Hence, there seems to be no logic in using the cross method in this case.

Case 2: A_1, A_2, Aw are known; may be one of n_1 and n_2 is unknown.

To find: $n_1 : n_2$ and n_2 if n_1 is known OR n_1 if n_2 is known.

Let us illustrate through an example:

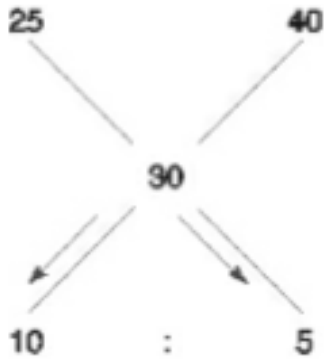
Illustration 5

On mixing two classes of students having average marks 25 and 40 respectively, the overall average obtained is 30 marks. Find

(a) The ratio of students in the classes

(b) The number of students in the first class if the second class had 30 students.

Solution



(a) Hence, solution is 2 : 1.

(b) If the ratio is 2 : 1 and the second class has 30 students, then the first class has 60 students.

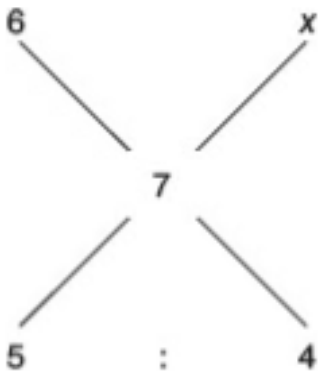
Note: The cross method becomes pretty effective in this situation when all the three averages are known and the ratio is to be found out.

Case 3: A_1, Aw, n_1 and n_2 are known; A_2 is unknown.

Illustration 6

Five kg of rice at ₹6 per kg is mixed with four kg of rice to get a mixture costing ₹7 per kg. Find the price of the costlier rice.

Solution Using the cross method:



$$= (x - 7) : 1$$

$$\therefore \quad (x - 7)/1 = 5/4 \rightarrow 4x - 28 = 5$$

$$\therefore \quad x = ₹8.25$$

Task for student: Solve through the alligation formula approach and through the weighted average approach to get the solution. Notice the amount of time required in doing the same.

> ***Note:*** The cross method becomes quite cumbersome in this case since this method results in the formula being written. Hence, there seems to be no logic in using the cross method in this case.

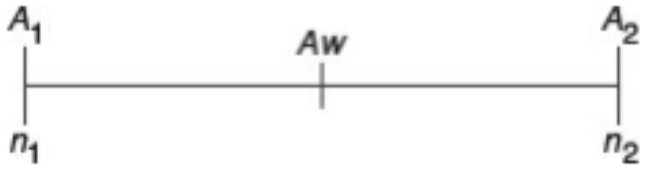
However, if we look at the straight line method or the number line method, we will see that the question and its' solution becomes visually solvable with little or no writing:

Straight Line Method (or the number line method)

As we have seen, the cross method becomes quite cumbersome in **Case 2 and**

3. We will now proceed to modify the cross method so that the question can be solved graphically in all the three cases.

Consider the following diagram, which results from closing the cross like a pair of scissors. Then the positions of A_1, A_2, Aw, n_1 and n_2 are as shown.



Visualise this as a fragment of the number line with points A_1, Aw and A_2 in that order from left to right.

Then,

(a) n_2 is responsible for the distance between A_1 and Aw or n_2 corresponds to $Aw - A_1$

(b) n_1 is responsible for the distance between Aw and A_2 or n1 corresponds to $A_2 - Aw$

(c) $(n_1 + n_2)$ is responsible for the distance between A_1 and A_2 or $(n_1 + n_2)$ corresponds to $A_2 - A_1$

The processes for the three cases illustrated above can then be illustrated below:

Case 1: A_1, A_2, n_1 and n_2 are known; Aw is unknown.

Illustration 7

Four kg of rice at ₹5 per kg is mixed with eight kg of rice at ₹6 per kg. Find the average price of the mixture.

Solution

is the same as

Then, by unitary method:

$n_1 + n_2$ corresponds to $A_2 - A_1$

→ 1 + 2 corresponds to 6 − 5

That is, 3 corresponds to 1

∴ n_2 will correspond to $\frac{(A_2 - A_1) \times n_2}{(n_1 + n_2)}$

In this case, (1/3) × 2 = 0.66.

Hence, the required answer is 5.66.

Note: In this case, the problem associated with the cross method is overcome and the solution becomes graphical.

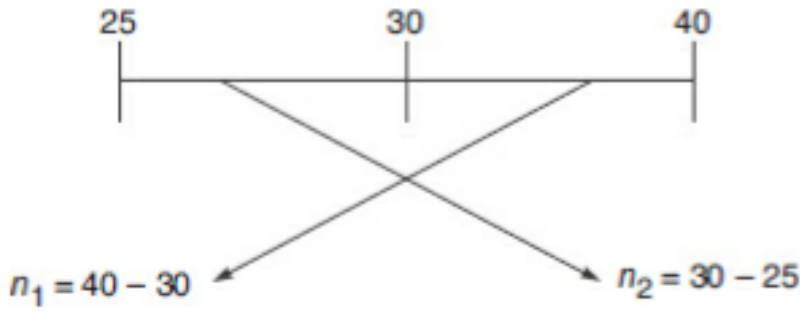
Illustration 8

On mixing two classes of students having average marks 25 and 40 respectively, the overall average obtained is 30 marks. Find

(a) the ratio in which the classes were mixed

(b) the number of students in the first class if the second class had 30 students

Solution



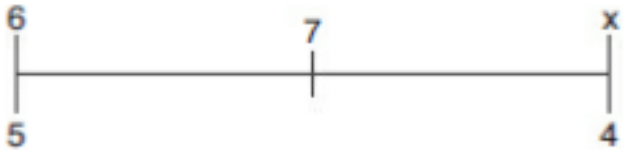
Hence, ratio is 2 : 1, and the second class has 60 students (if the first class has 30).

Case 3: A_1, Aw, n_1 and n_2 are known; A_2 is unknown.

Illustration 9

Five kg of rice at ₹6 per kg is mixed with four kg of rice to get a mixture costing ₹7 per kg. Find the price of the costlier rice.

Using straight line method:



4 corresponds to 7 – 6 and 5 corresponds to x – 7.

The thought process should go like:

$$4 \rightarrow 1$$

$$\therefore \quad 5 \rightarrow 1.25$$

Hence, $x - 7 = 1.25$

and $x = 8.25$

Some typical situations where alligations can be used

Given below are typical alligation situations, which students should be able to recognise. This will help them improve upon the time required in solving questions. Although in this chapter, we have illustrated problems based on alligation

at level 1 only. Alligation is used in more complex problems where the weighted average is an intermediate step in the solution process.

The following situations should help the student identify alligation problems better as well as spot the way A_1, A_2, n_1 and n_2 and Aw are mentioned in a problem.

In each of the following problems, the following magnitudes represent these variables:

$A_1 = 20$, $A_2 = 30$, $n_1 = 40$, $n_2 = 60$

Each of these problems will yield an answer of 26 as the value of Aw.

1. A man buys 40kg of rice at ₹20/kg and 60kg of rice at ₹30/kg. Find his average price. (26/kg)
2. Pradeep mixes two mixtures of milk and water. He mixes 40 litres of the first containing 20% water and 60 litres of the second containing 30% water. Find the percentage of water in the final mixture. (26%)
3. Two classes are combined to form a larger class. The first class having 40 students scored an average of 20 marks on a test while the second having 60 students scored an average of 30 marks on the same test. What was the average score of the combined class on the test? (26 marks)
4. A trader earns a profit of 20% on 40% of his goods sold, while he earns a profit of 30% on 60% of his goods sold. Find his percentage profit on the whole. (26%)
5. A car travels at 20 km/h for 40 minutes and at 30 km/h for 60 minutes. Find the average speed of the car for the journey. (26 km/hr)
6. Forty percent of the revenues of a school came from the junior classes while 60% of the revenues of the school came from the senior classes. If the school raises its fees by 20% for the junior classes and by 30% for the

senior classes, find the percentage increase in the revenues of the school. (26%)

7. A man sells a chair and a table at a profit of 20% and 30% respectively. If the ratio of the cost price of the table and the chair is 2:3, then what is the percentage profit for the man in the overall transaction? (26%)

Some keys to spot A_1, A_2 and A_w and differentiate these from n_1 and n_2

1. Normally, there are 3 averages mentioned in the problem, while there are only 2 quantities. This is not foolproof though, since at times the question might confuse the student by giving 3 values for quantities representing n_1, n_2 and $n_1 + n_2$ respectively.
2. A_1, A_2 and A_w are always rate units, while n_1 and n_2 are quantity units.
3. The denominator of the average unit corresponds to the quantity unit (i.e. unit for n1 and n_2).
4. All percentage values represent the average values.

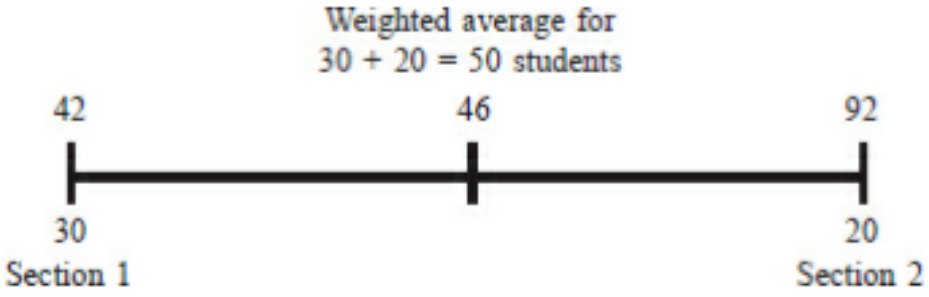
Can Alligations be Applied to Three Groups:

By definition, alligations is only applicable to two groups. However, in order to deal with the three group situation, we can use alligation twice – first to merge two groups into one larger group; then merging this larger group with the third group to get the answer for the entire problem. This process can be understood by considering the following situation;

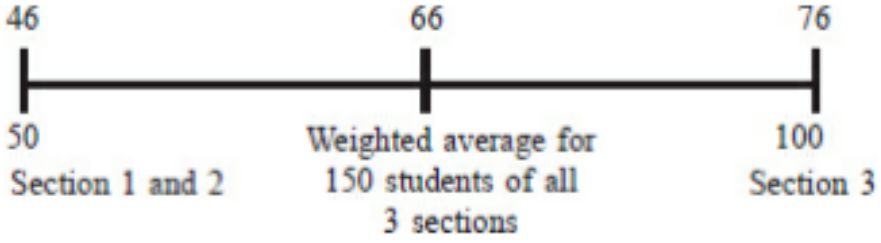
In a test, 30 students of Section 1, scored an average of 42 marks, while 20 students of Section 2 scored an average of 52 marks. Also, 100 students of Section 3, scored an average of score of 76 marks. What is the average score of all three sections taken together?

In this situation, the first step can be to find the average score of Sections 1 and 2. Using alligation, you should be able to work out that the average score of a total of 50 students of sections 1 and 2 would be 46 marks (average of 42 and 52 in the ratio 3:2). Now, we have reached a situation in the problem where we have this combined group of 50 students with an average of 46 and we have 100 students of section 3, with an average score of 76 marks. Applying alligation again, on these two averages (46 and 76) in the ratio of 50:100, i.e. 1:2 (for the number of students), we would get the final average of 66.

This would appear as follows: First alligation between Sections 1 and 2:



Second alligation between the combined sections 1 and 2, with section 3:



Application of Alligations to Questions Based on Mixtures:

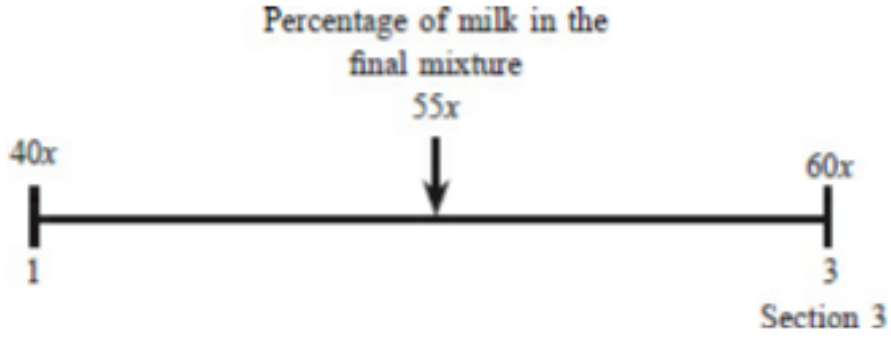
Questions based on mixtures is one of the favorite question types of the CAT and all other aptitude exams. Alligation also provides us a view into how to solve questions based on mixtures. Let us now look at how this can be used through a couple of examples:

Note: A detailed discussion on mixtures problems can be found in the chapter of Ratio, Proportions and Variations. In this discussion, we are dealing with only viewing how these questions can be solved using alligations. For the other methods, that one has I would encourage you to also refer to that discussion in the Ratio Proportion and Variations chapter of this book.

Illustration 10

Vessel A contains 10 litres of milk and water in the ratio 2:3, while vessel B contains 30 litres of milk and water in the ratio 3:2. Find the ratio of milk and water if the contents of vessel A and B are mixed together.

Solution: 2:3 milk water can be read to mean 40% milk (since milk is 2/5th of the mixture). Likewise, 3:2 milk water mixture can be read to mean 60% milk (since milk is 3/5th of the mixture). Also, since you are mixing 10 litres of the first to 30 litres of the second, the ratio of mixing is 1:3. The following alligation solution would yield you the percentage of milk:



Thus, we get 55% milk and 45% water in the final mixture, which essentially converts to a ratio of 11:9 between milk and water.

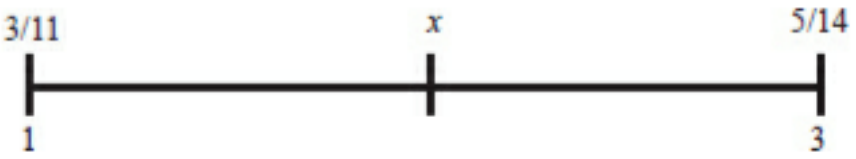
However, all such questions cannot be converted into percentage values and solved – especially if the ratios are not conducive to conversion into percentages. Consider the following:

Illustration 11

Vessel A contains 10 litres of milk and water in the ratio 3:8, while vessel B contains 30 litres of milk and water in the ratio 5:9. Find the ratio of milk and water if the contents of vessel A and B are mixed together.

Solution: As you can see in this case, it is not feasible to work with milk percentages. Yet we can still solve this problem using alligation as follows- We first check, which vessel has a higher proportion of milk to understand which fraction we want to keep for A_2, in our Alligation figure (Remember, we always use $A_1 < A_2$ as a convention) The first vessel has 3/11 milk, while the second vessel has 5/14 milk.

Since, 3/11 is smaller than 5/14, our alligation figure would be:



The solution of this, to solve for x would be:

$$\frac{\frac{5}{14}-x}{x-\frac{3}{11}}=\frac{1}{3} \rightarrow \frac{(5-14x)\times 3}{14}=\frac{11x-3}{11} \rightarrow 165 - 462x$$

$$= 154x - 42 \rightarrow 616x = 207$$

$$\therefore \quad x = \frac{207}{616}$$

Hence, the final mixture has 207/616 milk, which means that the milk-water ratio in the final mixture is 207:409 and this would be the required answer.

A typical problem

A typical problem related to the topic of alligation goes as follows:

Four litres of wine are drawn from a cask containing 40 litres of wine. It is replaced by water. The process is repeated three times.

(a) What is the final quantity of wine left in the cask?

(b) What is the ratio of wine to water finally?

If we try to chart out the process, we get: Out of 40 litres of wine, 4 are drawn out.

This leaves 36 litres wine and 4 litres water. (Ratio of 9 : 1)

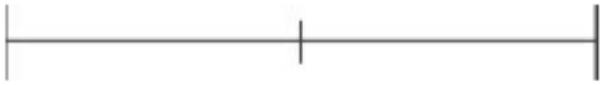
Now, when 4 litres are drawn out of this mixture, we will get 3.6 litres of wine and 0.4 litres of water (as the ratio is 9 : 1). Thus, at the end of the second step we get: 32.4 litres of wine and 7.6 litres of water. Further, the process is repeated, drawing out 3.24 litres wine and 0.76 litres water leaving 29.16 litres of wine and 10.84 litres of water.

This gives the final values and the ratio required.

A closer look at the process will yield that we can get the amount of wine left by:

$$40 \times 36/40 \times 36/40 \times 36/40 = 40 \times (36/40)3$$

$$\Rightarrow 40 \times (1 - 4/40)3$$



This yields the formula:

Wine left : Capacity × (1 − fraction of wine withdrawn)n for *n* operations.

Thus, you could have multiplied:

$40 \times (0.9)3$ to get the answer

That is, reduce 40 by 10% successively thrice to get the required answer.

Thus, the thought process could be:

40 – 10% → 36 – 10% → 32.4 – 10% → 29.16

LEVEL OF DIFFICULTY (I)

1. If 5kg of salt costing ₹5/kg and 3kg of salt costing ₹4/kg are mixed, find the average cost of the mixture per kilogram.

   (a) ₹4.5

   (b) ₹4.625

   (c) ₹4.75

   (d) ₹4.125

2. Two types of oils having the rates of ₹4/kg and ₹5/kg respectively are mixed in order to produce a mixture having the rate of ₹4.60/kg. What should be the amount of the second type of oil, if the amount of the first type of oil in the mixture is 40kg?

   (a) 75kg

   (b) 50kg

   (c) 60kg

   (d) 40kg

3. How many kilograms of sugar worth ₹3.60 per kg should be mixed with 8kg of sugar worth ₹4.20 per kg, such that by selling the mixture at ₹4.40 per kg, there may be a gain of 10%?

   (a) 6kg

   (b) 3kg

   (c) 2kg

   (d) 4kg

4. A mixture of 125 gallons of wine and water contains 20% water. How much water must be added to the mixture in order to increase the percentage of water to 25% of the new mixture?

   (a) 10 gals

   (b) 8.5 gals

   (c) 8 gals

   (d) 8.33 gals

5. Ravi lends ₹3600 on simple interest to Harsh for a period of five years. He lends a part of the amount at 4% interest and the rest at 6% and receives ₹960 as the amount of interest. How much money did he lend on 4% interest rate?

   (a) ₹2800

   (b) ₹2100

   (c) ₹2400

   (d) ₹1200

6. Four hundred students took a mock exam in Delhi. Sixty percent of the boys and 80% of the girls cleared the cut off in the examination. If the total percentage of students qualifying is 65%, how many girls appeared in the examination?

   (a) 100

   (b) 120

(c) 150

(d) 300

7. A man purchased a cow and a calf for ₹1300. He sold the calf at a profit of 20% and the cow at a profit of 25%. In this way, his total profit was $23\frac{1}{13}\%$. Find the cost price of the cow.

   (a) ₹1100

   (b) ₹600

   (c) ₹500

   (d) ₹800

8. The average salary per head of all employees of a company is ₹600. The average salary of 120 officers is ₹4000. If the average salary per head of the rest of the employees is ₹560, find the total number of workers in the company.

   (a) 10200

   (b) 10320

   (c) 10500

   (d) 10680

9. A dishonest milkman purchased milk at ₹10 per litre and mixed 5 litres of water in it. By selling the mixture at the rate of ₹10 per litre he earns a profit of 25%. The quantity of the amount of the mixture that he had was

   (a) 15 litres

   (b) 20 litres

   (c) 25 litres

(d) 30 litres

10. A cistern contains 50 litres of water. Five litres of water is taken out of it and replaced by wine. The process is repeated again. Find the proportion of wine and water in the resulting mixture.

    (a) 1 : 4

    (b) 41 : 50

    (c) 19 : 81

    (d) 81 : 19

11. A container has a capacity of 20 gallons and is full of spirit. Four gallons of spirit is drawn out and the container is again filled with water. This process is repeated five times. Find out how much spirit is left in the resulting mixture finally.

    (a) $6\frac{257}{525}$ gallons

    (b) $6\frac{346}{625}$ gallons

    (c) 6.5 gallons

    (d) 6.25 gallons

12. A vessel is full of refined oil. One fourth of the refined oil is taken out and the vessel is filled with mustard oil. If the process is repeated four times and 10 litres of refined oil is finally left in the vessel, what is the capacity of the vessel?

    (a) 33 litres

    (b) $\frac{2460}{81}$ litres

(c) 6.5 gallons

(d) 6.25 gallons

12. A vessel is full of refined oil. One fourth of the refined oil is taken out and the vessel is filled with mustard oil. If the process is repeated four times and 10 litres of refined oil is finally left in the vessel, what is the capacity of the vessel?

(a) 33 litres

(b) $\frac{2460}{81}$ litres

(c) $\frac{2560}{81}$ litres

(d) 30 litres

13. In what ratio should two qualities of coffee powder having the rates of ₹47 per kg and ₹32 per kg be mixed in order to get a mixture that would have a rate of ₹37 per kg?

(a) 1 : 2

(b) 2 : 1

(c) 1 : 3

(d) 3 : 1

14. A thief steals four gallons of liquid soap kept in a train compartment's bathroom from a container that is full of liquid soap. He then fills it with water to avoid detection. Unable to resist the temptation he steals four gallons of the mixture again, and fills it with water. When the liquid soap is checked at a station, it is found that the ratio of the liquid soap now left in the container to that of the water in it is 36 : 13. What was the initial

amount of the liquid soap in the container if it is known that the liquid soap is neither used nor augmented by anybody else during the entire period?

(a) 7 gallons

(b) 14 gallons

(c) 21 gallons

(d) 28 gallons

15. In what ratio should water be mixed with soda costing ₹12 per litre so as to make a profit of 25% by selling the diluted liquid at ₹13.75 per litre?

(a) 10 : 1

(b) 11 : 1

(c) 1 : 11

(d) 12 : 1

16. A sum of ₹36.90 is made up of 90 coins that are either 20 paise coins or 50 paise coins. Find out how many 20 paise coins are there in the total amount.

(a) 47

(b) 43

(c) 27

(d) 63

17. A dishonest grocer professes to sell pure butter at cost price, but he mixes it with adulterated fat and thereby gains 25%. Find the percentage of

adulterated fat in the mixture assuming that adulterated fat is freely available.

(a) 20%

(b) 25%

(c) 33.33%

(d) 40%

18. A mixture of 70 litres of alcohol and water contains 10% of water. How much water must be added to the above mixture to make the water 12.5% of the resulting mixture?

(a) 1 litre

(b) 1.5 litre

(c) 2 litres

(d) 2.5 litres

19. A mixture of 20 litres of brandy and water contains 10% water. How much water should be added to it to increase the percentage of water to 25%?

(a) 2 litres

(b) 3 litres

(c) 2.5 litres

(d) 4 litres

20. A merchant purchased two qualities of pulses at the rate of ₹200 per quintal and ₹260 per quintal. In 52 quintals of the second quality, how much pulse of the first quality should be mixed so that by selling the resulting mixture at ₹300 per quintal, he gains a profit of 25%?

    (a) 100 quintals

    (b) 104 quintals

    (c) 26 quintals

    (d) None of these

21. A man buys milk at ₹8.5 per litre and dilutes it with water. He sells the mixture at the same rate and thus gains 11.11%. Find the quantity of water mixed by him in every litre of milk.

    (a) 0.111 litres

    (b) 0.909 litres

    (c) 0.1 litre

    (d) 0.125 litres

22. There are two mixtures of honey and water, the quantity of honey in them being 25% and 75% of the mixture. If two gallons of the first are mixed with three gallons of the second, what will be the ratio of honey to water in the new mixture?

    (a) 11 : 2

    (b) 11 : 9

    (c) 9 : 11

    (d) 2 : 11

23. There are two kinds of alloys of tin and copper. The first alloy contains tin and copper such that 93.33% of it is tin. In the second alloy, there is 86.66% tin. What weight of the first alloy should be mixed with some weight of the second alloy so as to make a 50kg mass containing 90% of tin?

    (a) 15kg

    (b) 30kg

    (c) 20kg

    (d) 25kg

24. Two containers of equal capacity are full of a mixture of oil and water. In the first, the ratio of oil to water is 4 : 7 and in the second it is 7 : 11. Now both the mixtures are mixed in a bigger container. What is the resulting ratio of oil to water?

    (a) 149 : 247

    (b) 247 : 149

    (c) 143 : 241

    (d) 241 : 143

25. Two vessels contain spirit and water mixed respectively in the ratio of 1 : 3 and 3 : 5. Find the ratio in which these are to be mixed to get a new mixture in which the ratio of spirit to water is 1 : 2.

    (a) 2 : 1

    (b) 3 : 1

    (c) 1 : 2

(d) 1 : 3

26. The price of a pen and a pencil is ₹35. The pen was sold at a 20% profit and the pencil at a 10% loss. If in the transaction a man gains ₹4, how much is cost price of the pen?

(a) ₹10

(b) ₹25

(c) ₹20

(d) None of these

27. A person purchased a cupboard and a cot for ₹18,000. He sold the cupboard at a profit of 20% and the cot at a profit of 30%. If his total profit was 25.833%, find the cost price of the cupboard.

(a) ₹10,500

(b) ₹12,000

(c) ₹7500

(d) ₹10,000

28. A vessel is full of a mixture of kerosene and petrol in which there is 18% kerosene. Eight litres are drawn off and then the vessel is filled with petrol. If the kerosene is now 15%, how much does the vessel hold?

(a) 40 litres

(b) 32 litres

(c) 36 litres

(d) 48 litres

29. Two solutions of 90% and 97% purity are mixed resulting in 21 litres of mixture of 94% purity. How much is the quantity of the first solution in the resulting mixture?

    (a) 15 litres

    (b) 12 litres

    (c) 9 litres

    (d) 6 litres

30. In the Singapore zoo, there are deer and there are ducks. If the heads are counted, there are 180, while the legs are 448. What will be the number of deer in the zoo?

    (a) 136

    (b) 68

    (c) 44

    (d) 22

31. A bonus of ₹9,85,000 was divided among 300 workers of a factory. Each male worker gets 5000 rupees and each female worker gets 2500 rupees. Find the number of male workers in the factory.

    (a) 253

    (b) 47

    (c) 94

    (d) 206

32. What will be the ratio of petrol and kerosene in the final solution formed by mixing petrol and kerosene that are present in three vessels of equal capacity in the ratios 4 : 1, 5 : 2 and 6 : 1 respectively?

(a) 166 : 22

(b) 83 : 22

(c) 83 : 44

(d) None of these

33. A mixture worth ₹3.25 a kg is formed by mixing two types of flour, one costing ₹3.10 per kg while the other ₹3.60 per kg. In what proportion, must they have been mixed?

(a) 3 : 7

(b) 7 : 10

(c) 10 : 3

(d) 7 : 3

34. A 20 percent gain is made by selling the mixture of two types of ghee at ₹480 per kg. If the type costing 610 per kg was mixed with 126kg of the other, how many kilograms of the former was mixed?

(a) 138kg

(b) 34.5kg

(c) 69kg

(d) Cannot be determined

35. In what proportion must water be mixed with milk so as to gain 20% by selling the mixture at the cost price of the milk? (Assume that water is freely available)

(a) 1 : 4

(b) 1 : 5

(c) 1 : 6

(d) 1 : 12

36. A bartender stole champagne from a bottle that contained 50% of spirit and he replaced what he had stolen with champagne having 20% spirit. The bottle then contained only 25% spirit. How much of the bottle did he steal?

(a) 80%

(b) 83.33%

(c) 85.71%

(d) 88.88%

37. A bag contains a total of 105 coins of ₹1, 50 p and 25 p denominations. Find the total number of coins of ₹ 1 if there are a total of 50.5 rupees in the bag and it is known that the number of 25 paise coins are 133.33% more than the number of 1 rupee coins.

(a) 56

(b) 25

(c) 24

(d) None of these

38. A man possessing ₹6800, lent a part of it at 10% simple interest and the remaining at 7.5% simple interest. His total income after $3\frac{1}{2}$ years was ₹1904. Find the sum lent at 10% rates.

(a) ₹1260

(b) ₹1700

(c) ₹1360

(d) None of these

39. If a man decides to travel 80 kilometres in 8 hours partly by foot and partly on a bicycle, his speed on foot being 8 km/h and that on bicycle being 16 km/h, what distance would he travel on foot?

(a) 20 km

(b) 30 km

(c) 48 km

(d) 60 km

40. Two vessels contain a mixture of spirit and water. In the first vessel, the ratio of spirit to water is 8 : 3 and in the second vessel the ratio is 5 : 1. A 35 litre cask is filled from these vessels so as to contain a mixture of spirit and water in the ratio of 4 : 1. How many litres are taken from the first vessel?

(a) 11 litres

(b) 22 litres

(c) 16.5 litres

(d) 17.5 litres

ANSWER KEY

Level of Difficulty (l)

1. (b)
2. (c)
3. (d)
4. (d)
5. (d)
6. (a)
7. (d)
8. (b)
9. (c)
10. (c)
11. (b)
12. (c)
13. (a)
14. (d)
15. (c)
16. (c)
17. (a)
18. (c)
19. (d)
20. (c)
21. (a)
22. (b)
23. (d)
24. (a)

25. (c)
26. (b)
27. (c)
28. (d)
29. (c)
30. (c)
31. (c)
32. (b)
33. (d)
34. (d)
35. (b)
36. (b)
37. (c)
38. (c)
39. (c)
40. (a)

Solutions and Shortcuts

Level of Difficulty (I)

1. Solving the following alligation figure:

   The answer would be 4.625/kg.

2. Mixing ₹4/kg and ₹5/kg to get ₹4.6 per kg, we get that the ratio of mixing is 2:3. If the first oil is 40kg, the second would be 60kg.

3. Since by selling at ₹4.40, we want a profit of 10%, it means that the aver-

age cost required is ₹4 per kg. Mixing sugar worth ₹3.6/kg and ₹4.2/kg to get ₹4/kg means a mixture ratio of 1:2. Thus, to 8kg of the second variety we need to add 4kg of the first variety to get the required cost price.

4. In 125 gallons, we have 25 gallons water and 100 gallons wine. To increase the percentage of water to 25%, we need to reduce the percentage of wine to 75%. This means that 100 gallons of wine = 75% of the new mixture. Thus, the total mixture = 133.33 gallons. Thus, we need to mix 133.33 – 125 = 8.33 gallons of water in order to make the water equivalent to 25% of the mixture.

5. Since, Ravi earns ₹960 in five years, it means that he earns an interest of 960/5 = ₹192 per year. On an investment of 3600, an annual interest of 192 represents an average interest rate of 5.33%.

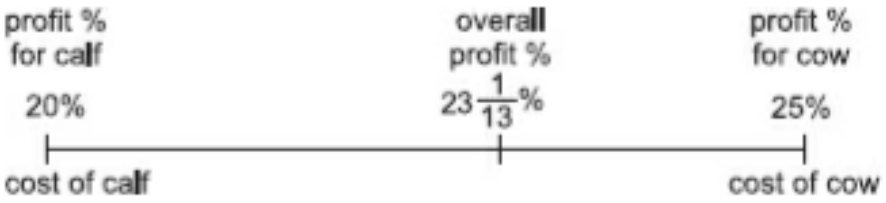
   Then using the alligation figure below:

   We get the ratio of investments as 1:2. Hence, he lent 1 × 3600/3 = 1200 at 4% per annum.

6. The ratio of boys and girls appearing for the exam can be seen to be 3:1, using the following alligation figure.

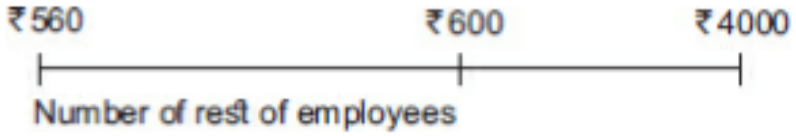
   This means that out of 400 students, there must have been 100 girls who appeared in the exam.

7. The ratio of the cost of the cow and the calf would be 40:25 or 8:5 as can be seen from the following alligation figure:



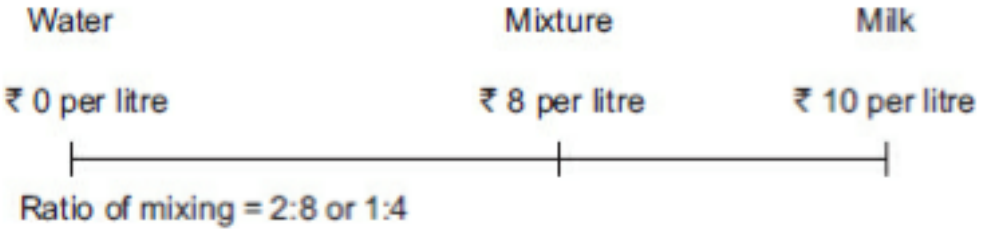
Thus, the cost of the cow would be ₹800.

8.



From the figure, it is clear that the ratio of the number of officers to the number of other employees would be 40:3400. Since there are 120 officers, there would be 3400 × 3 = 10200 workers in the company. Thus, the total number of employees would be 10200 + 120 = 10320.

9. The cost price of the mixture would have been ₹8 per litre for him to get a profit of 25% by selling at ₹10 per litre. The ratio of mixing would have been 1:4 (water is to milk) as can be seen in the figure:



Since we are putting in 5 litres of water, the amount of milk must be 20 litres. The amount of mixture then would become 25 litres.

10. Amount of water left = 50 × 9/10 × 9/10 = 40.5 litres. Hence, wine = 9.5 litres. Ratio of wine and water = 19:81. Option (c) is correct.

11. The amount of spirit left = 20 × 4/5 × 4/5 × 4/5 × 4/5 × 4/5 = 4096/625 = 6 (346/625).

12. Let the quantity of refined oil initially be Q. Then we have $Q \times 3/4 \times 3/4 \times 3/4 \times 3/4 = 10 \rightarrow Q = 2560/81$ litres.

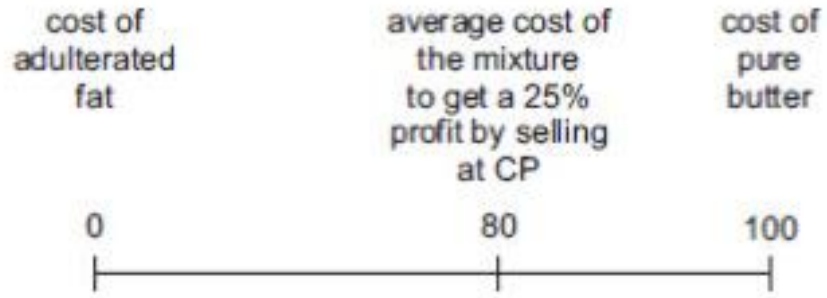
13. The ratio would be 1:2 as seen from the figure:

14. It can be seen from the ratio 36:13 that the proportion of liquid soap to water is 36/49 after two mixings. This means that 6/7th of the liquid soap must have been allowed to remain in the container and hence 1/7th of the conatiner's original liquid soap would have been drawn out by the thief. Since he takes out 4 gallons every time, there must have been 28 gallons in the container (as 4 should be 1/7th of 28).

15. In order to sell at a 25% profit by selling at 13.75, the cost price should be 13.75/1.25 = 11. Also since water is freely available, we can say that the ratio of water and soda must be 1:11.

16. The average value of a coin is 41 paise and there are only 20 paise and 50 paise coins in the sum. Hence, the ratio of the number of 20 paise coins to 50 paise coins would be 9:21 = 3:7. Since, there are a total of 90 coins, the number of 20 paise coins would be 3 × 90/10 = 27 coins.

17.

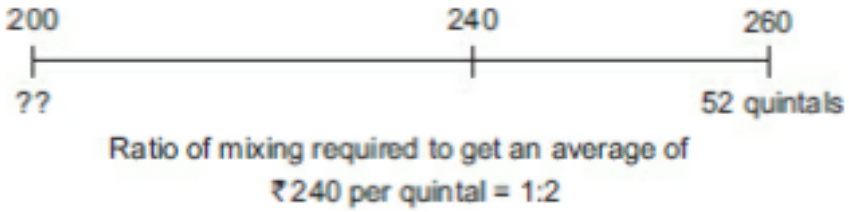


The ratio of mixing required would be 1:4 which means that the percentage of adulterated fat would be 20%.

18. Solve using options. Initially, there are 7 litres of water in 70 litres of the mixture. By mixing 2 litres of water, we will have 9 litres of water in 72 litres of the mixture – which is exactly 12.5%.

19. Again solve this question using options. Initially there are 2 litres of water in 20 litres of the mixture. To take the water to 25% of the mixture, we would need to add 4 litres of water as that would give us 6 litres of water in 24 litres of the mixture.

20. By selling at 300, if we need to get a profit of 25% it means that the cost price would be 300/1.25 = 240.



Thus, in 52 quintals of the second we need to mix 26 quintals of the first.

21. The requisite 11.11% profit can be obtained by mixing 0.111 litres of water in 1 litre of milk. In such a case, the total milk quantity would be 1.111 litres and the price would be for 1 litre only. The profit would be 0.111/1 = 11.11%.

22. The percentage of honey in the new mixture would be:

    $(2 \times 25 + 3 \times 75)/5 = 275/5 = 55\%$. The ratio of honey to water in the new mixture would be 55:45 = 11:9.

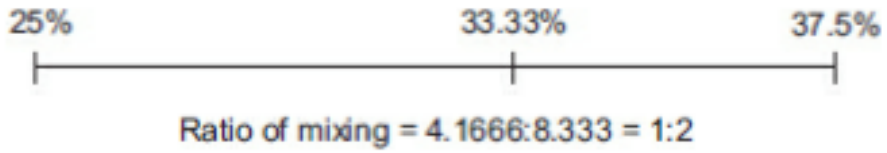
23. In order to mix two tin alloys containing 86.66% tin and 93.33% tin to get 90% tin, the ratio of mixing should be 1:1. Thus, each variety should be 25kg each.

24. Assume the capacity of the two containers is 198 litres each. When we mix 198 litres of the first and 198 litres of the second the amount of oil would be

    $198 \times 4/11 + 198 \times 7/18 = 72 + 77 = 149$ litres.

    Consequently, the amount of water would be 396 – 149 = 247 litres. Option (a) is correct.

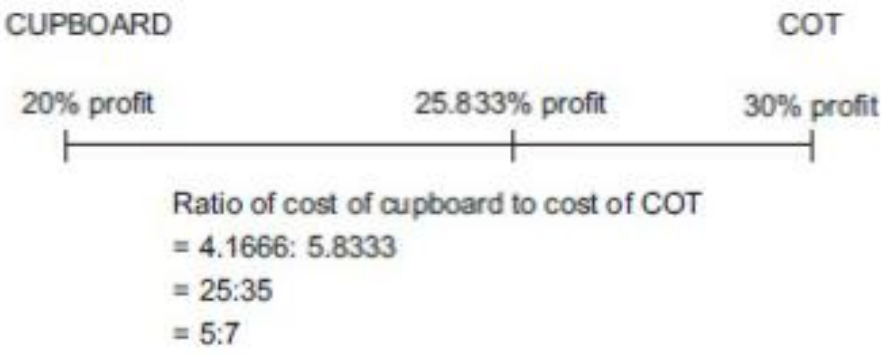
25. The first vessel contains 25% spirit while the second vessel contains 37.5 % spirit. To get a 1:2 ratio we need 33.33% spirit in the mixture. The ratio, of mixture can be seen using the following alligation figure:



Ratio of mixing = 4.1666:8.333 = 1:2

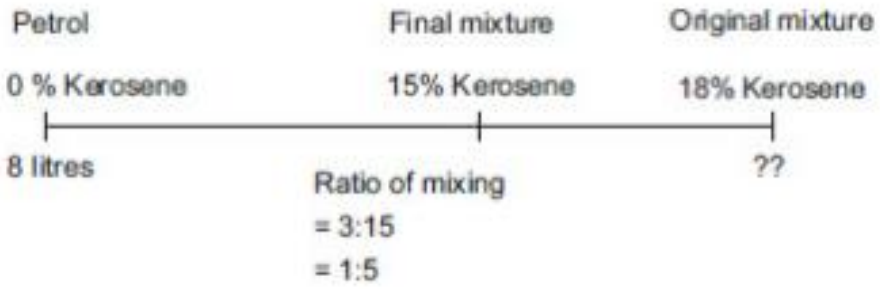
26. Solve using options as that would be the best way to tackle this question. Option (b) fits the situation perfectly as if we take the price of the pen as ₹25, the cost of the pencil would be ₹10. The profit in selling the pen would be ₹5 while the loss in selling the pencil would be ₹1. The total profit would be ₹4 as stipulated by the problem.

27. The following alligation visualisation would help us solve the problem:



Cost of cupboard = 5 × 18000/12 = 7500.

28. The following visualisation would help:



From the figure, we can see that the original mixture would be 40 litres and the petrol being mixed is 8 litres. Thus, the vessel capacity is 48 litres.

29. 90% and 97% mixed to form 94% means that the mixing ratio is 3:4. The first solution would be 3 × 21/7 = 9 litres.

30. If all the animals were ducks, we would have 180 heads and 360 legs. If we reduce the number of ducks by 1 to 179 and increase the number of deer by 1 to 1, we would get an incremental 2 legs.

Since, the number of legs we need to increment is 88 (448 – 360 = 88), we need to have 44 deer and 136 ducks.

31. Solve using options. Option (a) does not fit because 253 × 5000 + 47 × 2500 is not equal to 985000.

    Similarly option (b) does not fit because 253 × 2500 + 47 × 5000 is not equal to 985000.

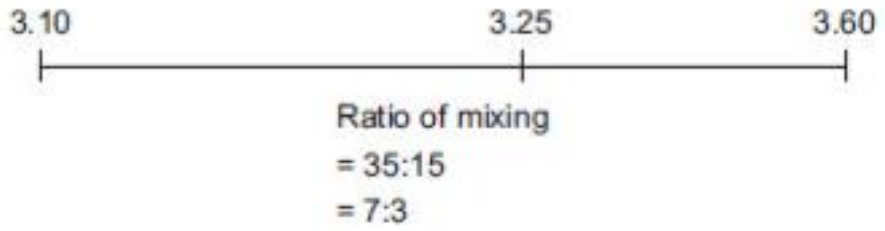
    Option (c) fits as 94 × 5000 + 206 × 2500 = 985000.

32. In order to solve this, we need to assume a value for the amounts in the vessels. If we assume 35 litres as the quantities in all the three vessels we will get:

    28 litres + 25 liters + 30 litres = 83 litres of petrol and 22 liters of kerosene in 105 litres of the mixture.

    The required ratio is 83:22.

33. The required ratio would be 7:3 as seen in the following figure.



34. We cannot determine the answer to this question as we do not know the price per kg of the other type of ghee. Hence, we cannot find the ratio of mixing which would be required in order to move further in this question.

35. To gain 20% by selling at cost price, milk should comprise 83.33% of the total mixture. The ratio of mixing that achieves this is 1:5.

36. Twenty percent spirit is mixed with 50% spirit to get 25% spirit. The ratio of mixing would be 5:1. This means he stole 5/6th of the bottle or 83.33% of the bottle.

37. $O + F + T = 105$

$O + 0.5F + 0.25T = 50.5$

$T = 2.333O$

Solving we get: 24 coins of one rupee each.

38. Annual interest income = 1904 /3.5 = 544. Interest of ₹544 on a lending of ₹6800 implies an 8% average rate of interest. This 8% is generated by mixing two loans @ 7.5% and 10% respectively. The ratio in which the two loans should be allocated would be 4:1. The amount lent at 10% would be 1 × 6800/5 = 1360.

39. Solve using options. If he travels 48 km on foot he would take 6 hours on foot. Also, in this case he would travel 32 km on bicycle @ 16kmph – which would take him 2 hours. Thus, a total of 8 hours. Option (c) satisfies the conditions of the question.

40. Solving through options is the best way to tackle this question. Option (a) fits the conditions of the problem as if there are 11 litres in the first vessel, there would be 8 litres of spirit. Also it means that we would be taking 24 litres from the second vessel, out of which there would be 20 litres of spirit. Thus, total spirit would be 28 out of 35 litres giving us 7 litres of water.

REVIEW CAT SCAN

Measure your progress through the CAT Scans (block review tests) and see whether you have improved after doing these chapters. Questions to ask yourself: Have I improved my accuracy? Have I improved my speed? You might want to go back to the chapters of the block, if you see more need for improvement.

REVIEW CAT Scan 1

1. Rakshit bought 19 erasers for ₹10. He paid 20 paise more for each white eraser than for each brown eraser. What could be the price of a white eraser and how many white erasers could he have bought?

   (a) 60 paise, 8

   (b) 60 paise, 12

   (c) 50 paise, 8

   (d) 50 paise, 10

2. After paying all your bills, you find that you have ₹7.20 in your pocket. You have equal number of 50 paise and 10 paise coins; but no other coins nor any other currency notes. How many coins do you have?

   (a) 8

   (b) 24

   (c) 27

   (d) 30

3. Suresh Kumar went to the market with ₹100. If he buys three pens and six pencils he uses up all his money. On the other hand, if he buys three pencils and six pens he would fall short by 20%. If he wants to buy equal number of pens and pencils, how many pencils can he buy?

   (a) 4

   (b) 5

   (c) 6

   (d) 7

4. For the above question, what is the amount of money he would save if he were to buy three pens and three pencils?

   (a) ₹50

   (b) ₹25

   (c) ₹75

   (d) ₹40

5. Abdul goes to the market to buy bananas. If he can bargain and reduce the price per dozen by ₹2, he can buy three dozen bananas instead of two dozen with the money he has. How much money does he have?
   (a) ₹6

   (b) ₹12

   (c) ₹18

   (d) ₹24

6. Two oranges, three bananas and four apples cost ₹15. Three oranges, two bananas and one apple cost ₹10. I bought three oranges, three bananas and three apples. How much did I pay?

   (a) ₹10

   (b) ₹8

   (c) ₹15

   (d) Cannot be determined

7. John bought five mangoes and ten oranges together for ₹40. Subsequently, he returned one mango and got two oranges in exchange. The price of an orange would be

   (a) ₹1

   (b) ₹2

   (c) ₹3

   (d) ₹4

REVIEW CAT Scan 2

1. Two towns *A* and *B* are 100 km apart. *A* school is to be built for 100 students of Town *B* and 30 students of Town *A*. The expenditure on transport is ₹1.20 per km per person. If the total expenditure on transport by all 130 students is to be as small as possible, then the school should be built at

   (a) 33 km from Town *A*

(b) 33 km from Town *B*

(c) Town *A*

(d) Town *B*

2. A person who has a certain amount with him goes to the market. He can buy 50 oranges or 40 mangoes. He retains 10% of the amount for taxi fare and buys 20 mangoes and of the balance, he purchases oranges. Number of oranges he can purchase is

   (a) 36

   (b) 40

   (c) 15

   (d) 20

3. Seventy two hens costs ₹_96.7_. Then what does each hen cost, where numbers at "_" are not visible or are written in illegible hand?

   (a) ₹3.43

   (b) ₹5.31

   (c) ₹5.51

   (d) ₹6.22

Directions for Questions 4 to 7: There are 60 students in a class. These students are divided into three groups *A*, *B* and *C* of 15, 20 and 25 students each. The groups *A* and *C* are combined to form group *D*.

4. What is the average weight of the students in group *D*?

   (a) More than the average weight of *A*

   (b) More than the average weight of *C*

   (c) Less than the average weight of *C*

   (d) Cannot be determined

5. If one student from Group *A* is shifted to group *B*, which of the following will be true?

   (a) The average weight of both groups increases

   (b) The average weight of both groups decreases

   (c) The average weight of the class remains the same

   (d) Cannot be determined

6. If all the students of the class have the same weight then which of the following is false?

   (a) The average weight of all the four groups is the same.

   (b) The total weight of *A* and *C* is twice the total weight of *B*.

   (c) The average weight of *D* is greater than the average weight of *A*.

   (d) The average weight of all the groups remains the same even if a number of students are shifted from one group to another.

7. The average marks of a student in ten papers are 80. If the highest and the lowest score are not considered, the average is 81. If his highest score is 92, find the lowest.

   (a) 55

(b) 60

(c) 62

(d) Cannot be determined

REVIEW CAT Scan 3

1. A shipping clerk has five boxes of different but unknown weights each weighing less than 100kg. The clerk weighs the boxes in pairs. The weights obtained are 110, 112, 113, 114, 115, 116, 117,118, 120 and 121kg. What is the weight of the heaviest box?

   (a) 60kg

   (b) 62kg

   (c) 64kg

   (d) Cannot be determined

2. The total expenses of a boarding house are partly fixed and partly varying linearly with the number of boarders. The average expense per boarder is ₹700 when there are 25 boarders and ₹600 when there are 50 boarders. What is the average expense per boarder when there are 100 boarders?

   (a) 550

   (b) 580

   (c) 540

   (d) 570

3. A yearly payment to a servant is ₹90 plus one turban. The servant leaves the job after 9 months and receives ₹65 and a turban, then find the price of the turban.

(a) ₹10

(b) ₹15

(c) ₹7.50

(d) Cannot be determined

4. A leather factory produces two kinds of bags, standard and deluxe. The profit margin is ₹20 on a standard bag and ₹30 on a deluxe bag. Every bag must be processed on machine *A* and on machine *B*. The processing times per bag on the two machines are as follows:

	Time required (hours/bag)	
	Machine A	Machine B
Standard bag	4	6
Deluxe bag	5	10

The total time available on machine *A* is 700 hours and on machine *B* is 1250 hours. Among the following production plans, which one meets the machine availability constraints and maximises the profit?

(a) Standard 75 bags, deluxe 80 bags

(b) Standard 100 bags, deluxe 60 bags

(c) Standard 50 bags, deluxe 100 bags

(d) Standard 60 bags, deluxe 90 bags

5. Three Math classes: *X*, *Y*, and *Z*, take an algebra test.

The average score of class *X* is 83.

The average score of class Y is 76.

The average score of class Z is 85.

What is the average score of classes X, Y, Z ?

(a) 81.5

(b) 80.5

(c) 83

(d) Cannot be determined

6. Prabhat ordered four Arrow shirts and some additional Park Avenue shirts. The price of one Arrow shirt was twice that of one Park Avenue shirt. When the order was executed, it was found that the number of the two brands had been interchanged. This increased the bill by 40%. The ratio of the number of Arrow shirts to the number of Park Avenue shirts in the original order was

(a) 1:3

(b) 1:4

(c) 1:2

(d) 1:5

7. Three groups of companies: Tata, Birla and Reliance announced the average of the annual profit for all years since their establishment.

The average profit of Tata is ₹75,000 lakh

The average profit of Birla is ₹64,000 lakh

The average profit of Reliance is ₹73,000 lakh

The average profit of all results of Tata and Birla together is ₹70,000 lakh.

The average profit of all results of Birla and Reliance together is ₹69,000 lakh.

Approximately, what is the average profit for all the three group of companies?

(a) ₹70,800 lakh

(b) ₹71,086 lakh

(c) ₹70,666 lakh

(d) Cannot be determined

ANSWER KEY

REVIEW CAT Scan 1

1. (b)
2. (b)
3. (a)
4. (b)
5. (b)
6. (c)
7. (b)

REVIEW CAT Scan 2

1. (d)
2. (d)
3. (c)
4. (d)
5. (c)

6. (c)
7. (b)

REVIEW CAT Scan 3

1. (b)
2. (a)
3. (a)
4. (a)
5. (d)
6. (a)
7. (b)

BLOCK III
ARITHMETIC AND WORD-BASED PROBLEMS

BACK TO SCHOOL...

As you are already aware, this block consists of the following chapters:

Percentages,

Profit and Loss,

Interest,

Ratio, Proportion and Variation,

Time and Work,

Time, Speed and Distance

To put it very simply, the reason for these seemingly diverse chapters to be under one block of chapters is: **Linear Equations**

Yes, the solving of linear equations is the common thread that binds all the chapters in this block.

But before we start going through what a linear equation is, let us first understand the concept of a variable and its need in the context of solving mathematical expressions.

Let us start off with a small exercise first:

Think of a number.

Add 2 to it.

Double the number to get a new number.

Add half of this new number to itself.

Divide the number by 3.

Take away the original number from it.

The number you now have is..... 2!!

How do I know this result?

The answer is pretty simple. Take a look. I am assuming that you had taken the initial number as 5 to show you what has happened in this entire process.

Instruction	*You*	*Me*
Think of a number	5	X
Add 2 to it	5 + 2 = 7	$X + 2$
Double the number to get a new number	7 ¥ 2 = 14	$2X + 4$
Add half of this new number to itself	14 + 7 = 21	$3X + 6$
Divide the result by 3	21/3 = 7	$X + 2$
Take away the original number from it	7 – 5 = 2	$X + 2 – X = 2$
The number you now have is….. 2 and is independent of the value again	The number you now have is….. 2!	The number you now have is….. 2!

The above is a perfect illustration of what a variable is and how it operates.

In this entire process, it does not matter to me as to what number you have assumed. All I set up is a kind of a parallel world wherein the number in your mind is represented by the variable X in my mind.

By ensuring that the final value does not have an X in it, I have ensured that the answer is independent of the value you would have assumed. Thus, even if someone had assumed 7 as the original value, his values would go as: 7, 9, 18, 27, 9, 2.

What you need to understand is that in Mathematics, whenever we have to solve for the value of an unknown, we represent that unknown by using some letter (like x, y, a, etc.) These letters are then called as the variable representations of the unknown quantity.

Thus, for instance, if you come across a situation where a question says: The temperature of a city increases by 1°C on Tuesday from its value on Monday, you assume that if Monday's temperature was t, then Tuesday's temperature will be $t + 1$.

Thus, although you do not have the actual value in your mind, you can still move ahead in the question by assuming a variable to represent the value of the unknowns. All problems in Mathematics ultimately take you to a point which will give the value of the unknown—which then becomes the answer to the question.

The opposite of a variable is a constant. Thus, if it is said in the same problem that the temperature on Wednesday is 34° C, then 34 becomes a constant value in the context of the problem.

As a trainer and mentor to lacs of students over the past 25 odd years, one of the things that has interested me a lot has been to identify exactly what goes wrong in the thinking of a student when he/she is not able to solve a question. This has particularly intrigued me, since in the answer to this question lies the path to changing the skills of solving quants for my readers and my students. In order to explain my point of view, I would first discuss what it takes to solve a question in a word-based chapter (like the ones in block II and III).

Any quants question elicits a reaction from the solver. If you are solving a question in QA and you can come up with the correct reactions (and not make any errors in the calculations), you should largely be able to solve any question you come across. This then that if you are solving a question and the question requires a particular reaction at a particular time/step during the solution, and you are not able to come up with that particular reaction – then you would not be able

to get an answer to the question. I refer to these points in any question that elicit a reaction from you as **'Triggers'. Improving your ability in QA, then means improving the quality and precision of the reactions that your mind has – at various trigger points during the solving of the questions.**

Language triggers: The beauty of QA and Mathematics, and the ease of these subjects, emerges from the fact that reactions and processes of solving questions are standardised – throughout the subject. And since they are standardised, each process and/or reaction has to be learnt only once. Unfortunately, a vast majority of students never see the subject for the essential simplicity it has – but rather look at the subject with a great deal of suspicion and fear.

If instead, you were able to bring yourself around to trying to discover in a systematic fashion what these standard triggers are and what their reactions are, skilling yourself up in solving QA problems could become child's play.

A closer look at problem-solving situations would show you that there are two kinds of triggers that you face in QA questions:

Language triggers: As the name itself suggests, a language trigger is essentially a trigger in the question, that is completely communicated in language terms to the solver. Consider the following question from CAT 2017, which would clarify for you, what a language trigger is:

Of the total number of shirts produced, in a factory, 15% of the shirts are defective. (*Language trigger 1: Your thought: Assume 100 shirts produced, 15 are defective).* Of the remaining shirts, 20% are sold in the domestic market. (*Language trigger 2: Your thought: I was left with 85 shirts – and 20% of those are defective. Thus, 20% of 85 = 17 are defective. That leaves me with 100-15-17 = 68 shirts).* The remaining 8840 shirts are exported. (*Language trigger 3: Your thought: Out of 100, 68 shirts are exported. Given: 8840 shirts are exported. Means that the 68 in the context of the assumption of 100 shirts produced, is actually 8840. If 68 is 8840,*

it means that 1 is 130.) Find the number of shirts produced. (*Question statement: Your thought: 1 is 130, hence 100 is 100 × 130 = 13000.)*

As you could see in the above question, what is required to successfully solve the question, is the reaction with each language trigger the question was throwing at you. If you can react to each of the language triggers, you would get to the answer of the question automatically.

Situation triggers: However, many a times, while solving a question in Mathematics/Quants, just reacting to language triggers is not sufficient. In such questions, during the process of solving, one reaches a specific point/s in the question, where the quantitative situation you are faced with tells you what to do (i.e. the language of the question would not tell you what to do next). In such situations, you need your mind to be equipped to think correctly from that situation and come up with the correct thoughts that would take you to the answer.

Thus, in summary, there are only two reasons that you are not able to solve a question in QA would be:

1. You are not able to react to a language trigger in the question;
2. You are not able to react to a situation trigger in the question.

There is simply no third reason, for not being able to solve a question.

Hence, the focus of your preparation as you start to enter the remaining part of this book (and not just this block) has to be to focus on (i) identifying and programming your reactions for the language triggers in the chapter;

(ii) identifying and programming your reactions for the situation triggers in the chapter.

Once, the programs are done – allow the beauty of repetitions and practice to carry you to the skill level of someone who is well-equipped with Mathematical and quantitative thinking.

Thus, your progress while studying any chapter in this book should be:

Learning objective 1: Get control of the language of the chapter;

Learning objective 2: Get control of the quantitative situations that questions in the chapter throw up regularly;

Learning objective 3: Get control of the typical mathematical equations that the chapter throws up – learn to write them and solve them;

Learning objective 4: Get control of the calculations.

While what we have talked about in the above discussion applies to all chapters of this book, an interesting observation arises when you think of the equations that emerge in the chapters of Block III (and indeed of block II too). A majority of the times, when you are solving questions from chapters of these blocks, you would be dealing with linear equations. Hence, when we are talking about Learning objective 3 above, the focus for these blocks of chapters has to be on linear equations.

Depending upon the number of variables in a problem, a linear equation might have one variable, two variables or even three or more variables. The only thing you should know is that in order to get the value of a variable, the number of equations needed is always equal to the number of variables. In other words, if you have more variables in a system of equations than the number of equations, you cannot solve for the individual values of the variables.

The basic mathematical principle goes like this:

For a system of equations to be solvable, the number of equations should be equal to the number of variables in the equations.

Thus for instance, if you have two variables, you need two equations to get the values of the two variables, while if you have three variables, you will need three equations.

This situation is best exemplified by the situation where you might have the following equation: $x + y = 7$. If it is known that both x and y are natural numbers, it yields a set of possibilities for the values of x and y as follows: (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1). One of these possibilities has to be the answer.

In fact, it might be a good idea to think of all linear equation situations in this fashion. Hence, before you go ahead to read about the next equation, you should set up this set of possibilities based on the first equation.

Consider the following situation where a question yields a set of possibilities:

Four enemies A, B, C and D gather together for a picnic in a park with their wives. A's wife consumes 5 times as many glasses of juice as A. B's wife consumes 4 times as many glasses of juice as B. C's wife consumes 3 times as many glasses of juice as C and D's wife consumes 2 times as many glasses of juice as D. In total, the wives of the four enemies consume a total of 44 glasses of juice. If A consumes at least 5 glasses of juice while each of the other men have at least one glass, find the least number of drinks that could have been consumed by the 4 enemies together.

(1) 9

(2) 12

(3) 11

(4) 10

In the question above, we have 8 variables—A, B, C, D and a, b, c, d — the number of glasses consumed by the four men and the number of glasses consumed by the four wives.

Also, the question gives us five informations which can be summarised into five equations as follows.

$a = 5A$

$b = 4B$

$c = 3C$

$d = 2D$

and $a + b + c + d = 44$

Also, $A > 5$.

Under this condition, you do not have enough information to get all values and hence you will get a set of possibilities.

Since the minimal value of A is 5, *it* can take the values 25, 30, 35 and 40 when A takes the values 5, 6, 7 and 8, respectively. Based on these, and on the realisation that b has to be a multiple of 4, c a multiple of 3 and d a multiple of 2, the following possibilities emerge:

At $A = 5$

a	(multiple of 5)	*25*	*25*	*25*	*25*	*25*
b	(multiple of 4)	12	8	8	4	4
c	(multiple of 3)	3	9	3	3	9
d	(multiple of 2)	4	2	8	12	6
	$a + b + c + d$	44	44	44	44	44

a		*A = 6, a = 30*	*A = 7, a = 35*	*A = 8, a = 40*
b	(multiple of 4)	4	4	No solution
c	(multiple of 3)	6	3	
d	(multiple of 2)	4	2	
	$a + b + c + d$	44	44	

In this case, the answer will be 10, since in the case of $a = 35$, $b = 4$, $c = 3$ and $d = 2$, the values for A, B, C and D will be respectively 7,1,1 and 1. This solution is the least number of drinks consumed by the four enemies together as in all the other

possibilities the number of glasses is greater than 10.

Such utilisations of linear equations are very common in CAT and top-level aptitude examinations.

Before you begin studying the individual chapters of this block then, I would suggest that you first take yourself through the following CAT Scan tests, that would help you identify where you stand with respect to the up-coming chapters. The block of chapters would close with review CAT Scans that would help to give you an indication of how much you have learnt and indeed whether your skills have gone up while studying these chapters.

CAT Scan 1

1. Three runners *A*, *B* and *C* run a race, with runner *A* finishing 24 metres ahead of runner *B* and 36 metres ahead of runner *C*, while runner *B* finishes 16 metres ahead of runner *C*. Each runner travels the entire distance at a constant speed. What was the length of the race?

   (a) 72 metres

   (b) 96 metres

   (c) 120 metres

   (d) 144 metres

2. A dealer buys dry fruits at ₹100, ₹80 and ₹60 per kilogram. He mixes them in the ratio 4:5:6 by weight, and sells at a profit of 50%. At what price per kilogram does he sell the dry fruit?

   (a) ₹116

   (b) ₹106

   (c) ₹115

(d) None of these

3. There are two containers: the first contains 500 ml of alcohol, while the second contains 500 ml of water. Five cups of alcohol from the first container is taken out and is mixed well in the second container. Then, five cups of this mixture is taken out from the second container and put back into the first container. Let X and Y denote the proportion of alcohol in the first and the proportion of water in the second container. Then what is the relationship between X and Y? (Assume the size of the cups to be identical)

   (a) $X > Y$

   (b) $X < Y$

   (c) $X = Y$

   (d) Cannot be determined

4. Akhilesh took five papers in an examination, where each paper was of 200 marks. His marks in these papers were in the proportion of 7: 8: 9 : 10 : 11. In all papers together, the candidate obtained 60% of the total marks. Then, the number of papers in which he got more than 50% marks is

   (a) 1

   (b) 3

   (c) 4

   (d) 5

5. *A* and *B* walk up an escalator (moving stairway). The escalator moves at a constant speed, *A* takes six steps for every four of *B*'s steps. *A* gets to the top of the escalator after having taken 50 steps, while *B* (because his slower pace lets the escalator do a little more of the work) takes only 40 steps to reach the top. If the escalator were turned off, how many steps would they have to take to walk up?

   (a) 80
   (b) 100
   (c) 120
   (d) 160

6. Fifty per cent of the employees of a certain company are men, and 80% of the men earn more than ₹2.5 lacs per year. If 60% of the company's employees earn more than ₹2.5 lacs per year, then what fraction of the women employed by the company earn more than ₹2.5 lacs per year?

   (a) 2/5
   (b) 1/4
   (c) 1/3
   (d) 3/4

7. A piece of string is 80 centimetres long. It is cut into three pieces. The longest piece is three times as long as the middle-sized and the shortest piece is 46 centimetres shorter than the longest piece. Find the length of the shortest piece (in cm).

   (a) 14
   (b) 10

(c) 8

(d) 18

CAT Scan 2

1. Three members of a family A, B, and C, work together to get all household chores done. The time it takes them to do the work together is six hours less than A would have taken working alone, one hour less than B would have taken alone, and half the time C would have taken working alone. How long did it take them to do these chores working together?

   (a) 20 minutes

   (b) 30 minutes

   (c) 40 minutes

   (d) 50 minutes

2. Fresh grapes contain 90% water by weight while dried grapes contain 20% water by weight. What is the weight of dry grapes available from 20kg of fresh grapes?

   (a) 2 kg

   (b) 2.4 kg

   (c) 2.5 kg

   (d) None of these

3. At the end of the year 2008, a shepherd bought twelve dozen goats. Henceforth, every year he added p% of the goats at the beginning of the year and sold q% of the goats at the end of the year where $p > 0$ and $q > 0$. If the

shepherd had twelve dozen goats at the end of the year 2012, (after making the sales for that year), which of the following is true?

(a) $p = q$

(b) $p < q$

(c) $p > q$

(d) $p = q/2$

Directions for Questions 4 and 5: Answer the questions based on the following information.

An Indian company purchases components *X* and *Y* from UK and Germany, respectively. *X* and *Y* form 40% and 30% of the total production cost. Current gain is 25%. Due to change in the international exchange rate scenario, the cost of the German mark increased by 50% and that of UK pound increased by 25%. Due to tough competitive market conditions, the selling price cannot be increased beyond 10%.

4. What is the maximum current gain possible?

   (a) 10%

   (b) 12.5%

   (c) 0%

   (d) 7.5%

5. If the UK pound becomes cheap by 15% over its original cost and the cost of German mark increased by 20%, what will be the gain if the selling price is not altered?

   (a) 10%

(b) 20%

(c) 25%

(d) 7.5%

6. A college has raised 80% of the amount it needs for a new building by receiving an average donation of ₹800 from the people already solicited. The people already solicited represent 50% of the people, the college will ask for donations. If the college is to raise exactly the amount needed for the new building, what should be the average donation from the remaining people to be solicited?

(a) 300

(b) 200

(c) 400

(d) 500

7. A student gets an aggregate of 60% marks in five subjects in the ratio 10: 9: 8: 7: 6. If the passing marks are 45% of the maximum marks and each subject has the same maximum marks, in how many subjects did he pass the examination?

(a) 2

(b) 3

(c) 4

(d) 5

1. After allowing a discount of 12.5%, a trader still makes a gain of 40%. At what per cent above the cost price does he mark on his goods?

   (a) 45%

   (b) 60%

   (c) 25%

   (d) None of these

2. The owner of an art shop conducts his business in the following manner. Every once in a while he raises his prices by X%, then a while later he reduces all the new prices by X%. After one such up-down cycle, the price of a painting decreased by ₹441. After a second up-down cycle, the painting was sold for ₹1944.81. What was the original price of the painting (in ₹)?

   (a) 2756.25

   (b) 2256.25

   (c) 2500

   (d) 2000

3. Manas, Mirza, Shorty and Jaipal bought a motorbike for $60,000. Manas paid 50% of the amounts paid by the other three boys; Mirza paid one third of the sum of the amounts paid by the other boys; and Shorty paid one fourth of the sum of the amounts paid by the other boys. How much did Jaipal have to pay?

(a) $15000

(b) $13000

(c) $17000

(d) None of these

4. A train *X* departs from station A at 11.00 a.m. for station B, which is 180 km away. Another train *Y* departs from station B at 11.00 a.m. for station A. Train *X* travels at an average speed of 70 km/hr and does not stop anywhere until it arrives at station B. Train *Y* travels at an average speed of 50 km/hr, but has to stop for 10 minutes at station C, which is 60 km away from station B enroute to station A. Ignoring the lengths of the trains, what is the distance, to the nearest km, from station A to the point where the trains cross each other?

   (a) 110

   (b) 112

   (c) 116

   (d) None of these

5. In a survey of political preferences, 81% of those asked were in favour of at least one of the three budgetary proposals *A*, *B* and *C*. Fifty percent of those asked favoured proposal *A*, 30% favoured proposal *B* and 20% favoured proposal *C*. If 5% of those asked favoured all the three proposals, what percentage of those asked favoured more than one of the three proposals?

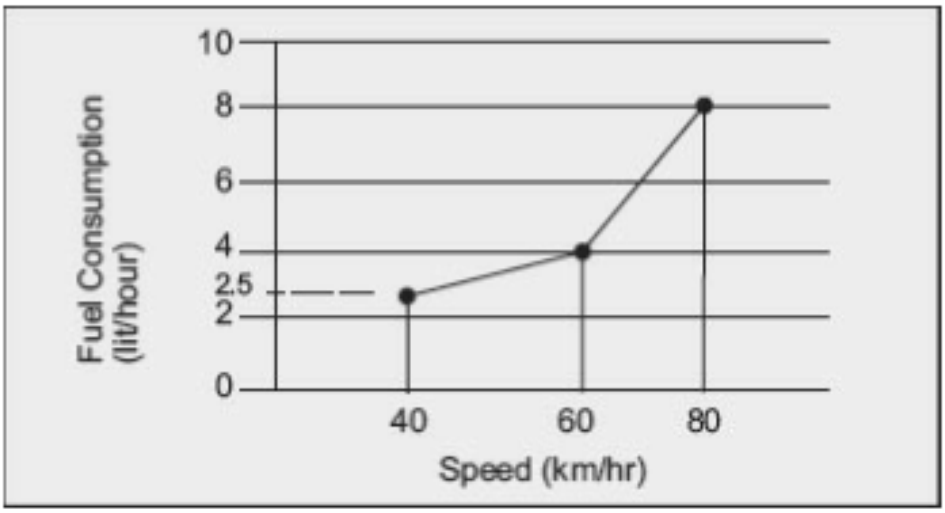
(a) 10%

(b) 12%

(c) 9%

(d) 14%

Directions for Questions 6 and 7: The petrol consumption rate of a new model car 'Palto' depends on its speed and may be described by the graph below:



6. Manasa makes the 240 km trip from Mumbai to Pune at a steady speed of 60 km per hour. What is the amount of petrol consumed for the journey?

   (a) 12.5 litres

   (b) 16 litres

   (c) 15 litres

   (d) 19.75 litres

7. Manasa would like to minimise the fuel consumption for the trip by driving at the appropriate speed. How should she change the speed?

   (a) Increase the speed

   (b) Decrease the speed

(c) Maintain the speed at 60 km/hr

(d) Cannot be determined

CAT Scan 4

Directions for Questions 1 and 2: Answer the questions based on the following information:

There are five machines—*A*, *B*, *C*, *D,* and *E* situated on a straight line at distances of 10 m, 20 m, 30 m, 40 m and 50 m, respectively from the origin of the line. A robot is stationed at the origin of the line. The robot serves the machines with raw material whenever a machine becomes idle. All the raw materials are located at the origin. The robot is in an idle state at the origin at the beginning of a day. As soon as one or more machines become idle, they send messages to the robot-station and the robot starts and serves all the machines from which it received messages. If a message is received at the station while the robot is away from it, the robot takes notice of the message only when it returns to the station. While moving, it serves the machines in the sequence in which they are encountered, and then returns to the origin. If any messages are pending at the station when it returns, it repeats the process again. Otherwise, it remains idle at the origin till the next message(s) is (are) received.

1. Suppose on a certain day, machines A and D have sent the first two messages to the origin at the beginning of the first second, C has sent a message at the beginning of the 7th second, B at the beginning of the 8th second and E at the beginning of the 10th second. How much distance has the robot traveled since the beginning of the day, when it notices the message of E? Assume that the speed of movement of the robot is 10m/s.

(a) 140 m

(b) 80 m

(c) 340 m

(d) 360 m

2. Suppose there is a second station with raw material for the robot at the other extreme of the line which is 60 m from the origin, i.e. 10 m from E. After finishing the services in a trip, the robot returns to the nearest station. If both stations are equidistant, it chooses the origin as the station to return to. Assuming that both stations receive the messages sent by the machines and that all the other data remains the same, what would be the answer to the above question?

   (a) 120

   (b) 160

   (c) 140

   (d) 170

3. One bacteria splits into eight bacteria of the next generation. But due to environment, only 50% of a generation survives. If the eighth generation number is 8192 million, what is the number in the first generation?

   (a) 1 million

   (b) 2 million

   (c) 4 million

   (d) 8 million

4. I bought 10 pens, 14 pencils and 4 erasers. Ravi bought 12 pens, 8 erasers and 28 pencils for an amount which was half more what I had paid. What percent of the total amount paid by me was paid for the pens?

   (a) 37.5%

   (b) 62.5%

   (c) 50%

   (d) None of these

ANSWER KEY

CAT Scan 1

1. (b)
2. (a)
3. (c)
4. (c)
5. (b)
6. (a)
7. (c)

CAT Scan 2

1. (c)
2. (c)
3. (c)
4. (a)
5. (c)
6. (b)
7. (d)

CAT Scan 3

1. (b)
2. (a)
3. (b)
4. (a)
5. (d)
6. (b)
7. (b)

CAT Scan 4

1. (a)
2. (a)
3. (b)
4. (b)