21. In a substance like rubber, 700 % strain can be produced. Such bodies are called elastomers.
22. When external force is applied to rubber, large deformation is produced in it. When deforming force is removed, it regains original state but not on original path. Here, energy spent in deforming the body is more than energy released by the body while regaining the shape. This phenomenon is known as elastic hysteresis. It is used in shockabsorbers.
23. Hooke's Law : For small deformations stress is directly proportional to strain.
24. For small deformations ratio of stress to strain is called modulus of elasticity. Young modulus (Y), Bulk modulus (B) and modulus of rigidity (η) are the moduli corresponding to longitudinal strain, volume strain and shearing strain respectively. Unit of modulus of elasticity is N m^{-2}.
25. When an axial force (tensile force or compressive force) is applied on a body, its length and lateral dimensions change. Ratio of fractional change in lateral dimension to fractional change in axial dimension is called Poisson's ratio. Its symbol is μ. It is dimensionless. Value of μ is less than 0.5.
26. When external force is applied, body achieves a new configuration due to deformation. And hence it gains potential energy. This potential energy is called elastic potential energy. It is given by

$$U = \frac{1}{2} \text{ stress} \times \text{strain} \times \text{volume}$$

EXERCISES

Choose the correct option from the given options :

1. A wire is stretched to double the length. Which of the following statements is false in this context ?

   (A) Its volume increases. (B) Its longitudinal strain is I.

   (C) Stress = Young's modulus (D) Stress = 2 Young's modulus.

2. Which is the dimensional formula for modulus of rigidity ?

   (A) $M^1L^1T^{-2}$ (B) $M^1L^{-1}T^{-2}$ (C) $M^1L^{-2}T^{-1}$ (D) $M^1L^{-2}T^{-2}$

3. When more then 20 kg mass is tied to the end of wire it breaks. What is maximum mass that can be tied to the end of a wire of same material with half the radius ?

   (A) 20 kg (B) 5 kg (C) 80 kg (D) 160 kg

4. Length of a metallic rod of mass *m*, and cross-sectional area A is L. If mass M is suspended at the lower end of this rod suspended vertically stress at the cross-section situated at $\frac{3l}{4}$ distance from its lower end is ....

   (A) M*g*/A (B) (M + *m*/4) *g*/A

   (C) (M + $\frac{3}{4}m$)g/A (D) M + *m*) *g*/A

5. Here, values of lengths and diameters of wires of same material are given. If same mass is suspended at the end which wire will have the maximum extension ?

(A) $l = 0.5$ m, $d = 0.05$ mm (B) $l = 1$m, $d = 1$mm

(C) $l = 2$m, $d = 2$mm (D) $l = 3$m, $d = 3$mm

6. When 100 N tensile force is applied to a rod of 10^{-6} m^2 cross sectional area, its length increases by 1 %. So, Young's modulus of material is ....

(A) 10^{12} Pa (B) 10^{11} Pa (C) 10^{10} Pa (D) 10^{2} Pa

7. A composite wire is made by joining ends of two wires of equal dimensions, one of copper and the other of steel. When a weight is attached to its end the ratio of increase in their lengths is .... $Y_{steel} = \frac{20}{7} Y_{copper}$

(A) 20:7 (B) 10:7 (C) 7:20 (D) 1:7

8. A rubber ball when taken to the bottom of a 100 m deep lake suffer decrease in volume by 1 %. Hence, the bulk modulus of rubber is .... ($g = 10$ m s^{-2})

(A) 10^6 Pa (B) 10^8 Pa (C) 10^7 Pa (D) 10^9 Pa

9. Young's modulus of a rigid body is ....

(A) 0 (B) 1 (C) $\propto$ (D) 0.5

10. Pressure on an object increases from 1.01×10^5 Pa to 1.165×10^5 Pa. Its volume decreases by 10% at constant temperature. Bulk modulus of material is ....

(A) 1.55×10^5 Pa (B) 51.2×10^5 Pa

(C) 102.4×10^5 Pa (D) 204.8×10^5 Pa

11. When 200 N force is applied on an object, its length increases by 1 mm. So potential energy stored in it due to this change is ....

(A) 0.2 J (B) 10 J (C) 20 J (D) 0.1 J

12. A wire is tied to a rigid support. Its length increases by l when force F acts at its free end. So work done is ....

(A) $\frac{1}{2}$ (B) Fl (C) 2Fl (D) $\frac{1}{2}$Fl

13. For perfect plastic body Young's modulus is ....

(A) l (B) zero (C) ∞ (D) 2

14. Dimensionally modulus of elasticity is equivalent to ....

(A) Force (B) Stress (C) Strain (D) none of these

15. Cross-sactional area of a wire of length L is A. Young's modulus of material is Y. If this wire acts as a spring what is the value of force constant ?

(A) $\frac{YA}{L}$ (B) $\frac{YA}{2L}$ (C) $\frac{2YA}{L}$ (D) $\frac{YL}{A}$

16. In a metal wire when 10 N tensile force is applied, its length becomes 5.001 m and when 20 N tensile force is applied, its length became 5.002 m so its original length is ....

(A) 5.001 m (B) 4.009 m (C) 5.0 m (D) 4.008 m

ANSWERS

1. (D)	**2.** (B)	**3.** (B)	**4.** (B)	**5.** (A)	**6.** (C)
7. (A)	**8.** (B)	**9.** (C)	**10.** (A)	**11.** (D)	**12.** (D)
13. (B)	**14.** (B)	**15.** (A)	**16.** (C)		

Answer the following questions in short :

1. Which forces are responsible for the formation of molecular crystals ?

2. Define perfect elastic body.

3. Give dimensional formula of strain.

4. Give the reason of restoring force produced in a body when an external force acts on it.

5. Define compressibility. Also give its dimensional formula.

6. Which is more elastic, rubber or steel ? Why ?

7. Give reason : Springs are made from steel and not from copper.

8. What happens to the energy spent in changing the dimensions of an elastic body ?

9. When a rod is stretched to increase its length by Δl, increase in its potential energy is U. What will be the change in its potential energy if it is compressed to decrease its length by Δl ?

10. For a wire breaking force is F. If the thickness of wire is doubled what will be the value of breaking force ?

Answer the following questions :

1. Write a short note on ionic crystals.

2. What is meant by strain ? Explain shearing strain with the help of an example.

3. Discuss the effect of force acting on a body making angle θ with a normal drawn to its surface.

4. Explain experimental method to determine Young's modulus.

5. Explain the difference between stress and pressure.

6. Define Poisson's ratio and show that its value is less then 0.5.

7. Derive an expression for elastic potential energy.

Solve the following problems :

1. A steel wire is hanged vertically. What should be its maximum length so that it does not break by its own weight. Density of steel = 7.8×10^3 kg m^{-3}, for steel breaking stress = 7.8×10^9 dyn/cm^2. (Ans : 1.02×10^4 m)

2. Figure shows a composite rod of cross-sectional area 10^{-4} m^2 made by joining three rods AB, BC and CD of different materials end to end. The composite rod is suspended vertically and an object of 10 kg is hung by it. L_{AB} = 0.1 m, L_{BC} = 0.2 m L_{CD} = 0.15 m. Calculate displacement of B, C and D. $Y_{AB} = 2.5 \times 10^{10}$ Pa, $Y_{BC} = 4 \times 10^{10}$ *Pa* $Y_{CD} = 1 \times 10^{10}$ *Pa*.

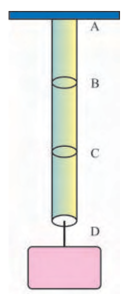


Figure 4.20

[**Ans. :** Displacement of B = 3.9×10^{-6} m, Displacement of C = 8.8×10^{-6} m, Displacement of D = 2.3×10^{-5} m]

3. A wire of length L and cross section area A is kept on a horizontal surface and one of its end is fixed at point 0. A ball of mass m is tied to its other end and the system is rotated with angular velocity ω. Show that increase in its length $\Delta l = \frac{m\omega^2 L^2}{AY}$. Y is Young's modulus.

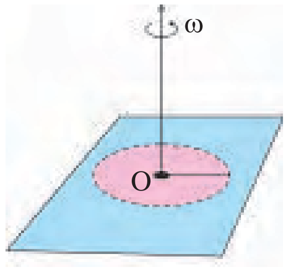


Figure 4.21

4. As shown in figure, masses of 2 kg and 4 kg are tied to two ends of a wire passed over a pulley. Cross-sectional area of wire is 2 cm^2. Calculate longitudinal strain produced in wire. $g = 10$ m s^{-2} Y = 2 × 10^{11} P*a*.

[**Ans :** 6.6 × 10^{-7}]

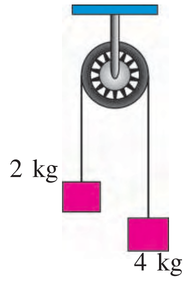


Figure 4.22

5. A wire of length 5 m and diameter 2 mm is hanging from a ceiling. A mass of 5 kg is suspended at its lower end. Calculate increase in its volume. Poisson's ratio of material = 0.2, Y = 2 × 10^{11} P*a*. $g = 10$ m s^{-2}. Also calculate change in potential energy of wire.

[**Ans. :** ΔV = 7.5 × 10^{-10} m^3, 10^{-2} J]

6. A steel wire of cross section 1 mm^2 is heated at 60ºC and tied between two ends firmly. Calculate change in tension when temperature becomes 30ºC co-efficient of linear expansion for steel is $\propto$ = 1.1 × 10^{-5} ºC^{-1}, Y = 2 × 10^{11} P_a. (change in length of wire due to change in temperature (Δt) is $\Delta l = \propto l \Delta t$) [**Ans. :** 66 N]

CHAPTER 5

FLUID MECHANICS

5.1 Introduction

A substance which can flow is known as fluid. As liquids and gases can flow, they are called fluids. Molten glass and tar can flow although slowly, they are also included in fluids.

Fluid mechanics comprises of fluid statics and fluid dynamics. In fluid statics the forces and pressures acting on a stationary fluid are studied while fluid dynamics includes motion of fluid and properties of its motion. Fluid dynamics is studied in two sections : Hydrodynamics and Aerodynamics.

We will discuss pressure of fluids and Pascal's Law in fluid statics. In fluid dynamics, characteristics of fluid flow, Bernoullie's theorem and its applications and viscosity will be studied. Finally, we will also discuss the surface tension of stationary liquids. Let us begin with fluid statics.

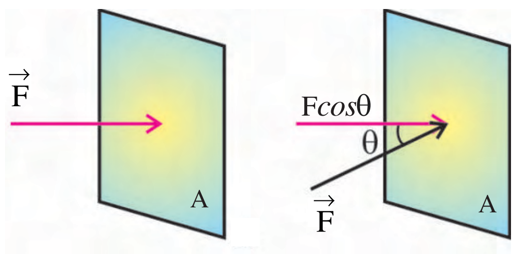
5.2 Pressure and Density

'Magnitude of force acting on a surface per unit area in a direction perpendicular to it, is called the pressure on the surface.'

If F is the magnitude of force acting perpendicular to area A, then the pressure acting on this surface is given by,

$$\text{Pressure (P)} = \frac{\text{Force(F)}}{\text{Area(A)}} \qquad (5.2.1)$$

If force is not perpendicular to the surface, then the component of the force perpendicular to the surface is taken into account for the pressure on the surface (See Figure 5.1).



Pressure

Figure 5.1

If the force ($\vec{F}$) makes an angle θ with the normal drawn to the surface, F*cos*θ is the force perpendicular to the surface. So, as per the definition, pressure is given by,

$$P = \frac{\text{Force(F)}}{\text{Area(A)}} \qquad (5.2.2)$$

Unit of pressure is Newton/(metre)^2, (N/m^2) which is also known as Pascal (P*a*), in memory of French scientist Blaise Pascal (1623–1662). Pressure is a scalar quantity.

Apart from pascal, other units of pressure are bar, atmosphere (atm) and torr.

$1 \;\; P_a = 1 \text{ N m}^{-2}$

$1 \;\; \text{bar} = 10^5 \; P_a$

and $1 \text{ atm} = 1.013 \times 10^5 \; P_a$

$1 \;\; \text{torr} = 133.28 \; P_a$

One atm pressure is the pressure equal to the pressure exterted by the atmosphere at sea level. It is also expressed in terms of height of mercury column, as cm–Hg or mm–Hg

1 atm = 76 cm Hg = 760 mm–Hg

Density : The ratio of mass to the volume of an object is known as density of the object. If the volume of a body of mass m is V, its density (ρ) is given by

$$\rho = \frac{m}{V} \qquad (5.2.3)$$

It is clear that unit of density is kg m^{-3}. Normally liquids are incompressible. (Percentage change in the volume of most of the liquids is of the order of 0.005 %) So, their densities are constant at a given temperature. Density of a gas depends on its pressure. In Table 5.1 densities of some fluids are given.

Table 5.1 : Densities of fluids at NTP (Only For Information)

Liquid	Density (kg m^{-3})	Gas	Density (kg m^{-3})
Water	1×10^3	Air	1.29
Sea water	1.03×10^3	Oxygen	1.43
Mercury	13.6×10^3	Hydrogen	9.0×10^{-2}
Ethyl Alcohol	0.806×10^3	Inter stellar space	10^{-18}–10^{-21}
Blood	1.06×10^3		

Water is taken as a standard substance. By comparing the density of a given body to that of water we get specific density. **'Specific density of an object is the ratio of density of an object to density of pure water at 277 K.** Thus,

Specific density

$$= \frac{\text{Density of an object}}{\text{Density of pure water at 277 K}}$$

Specific density is dimensionless. It is also known as relative density or specific gravity. Reciprocal of density is called specific volume.

If we take water having the same volume as that of the given object, the specific density can be obtained as,

$$\text{specific density} = \frac{\text{Mass of the object}}{\text{Mass of water of the same volume at 277 K}}$$

This equation is very useful in determining the specific density of a substance because, it is not necessary to know the density of given object to detemine the specific density.

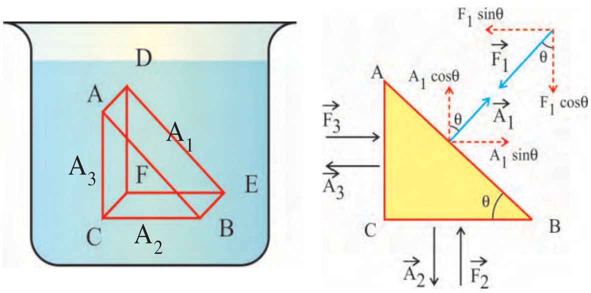
5.3 Pascal's Law and its Applications

Pascal's Law : 'Pressure in an incompressible fluid in equilibrium is the same everywhere, if the effect of gravity is neglected.'

This statement can easily be verified as follows :

Consider a small element in the interior of a liquid at rest. The liquid element is in the shape of a prism consisting of two right angled triangle surfaces.

Let the areas of surfaces ADEB, CFEB and ADFC be A_1, A_2 and A_3.



Verification of Pascal's Law

Figure 5.2

It is clear from Figure 5.2 that

$A_2 = A_1 \cos\theta$ and $A_3 = A_1 \sin\theta$

Also, since liquid element is in equilibrium,

$F_2 = F_1 cos\theta$ and $F_3 = F_1 sin\theta$

Now pressure on the surface ADEB is $P_1 = \frac{F_1}{A_1}$

Pressure on the surface CFEB is

$$P_2 = \frac{F_2}{A_2} = \frac{F_1 cos\theta}{A_1 cos\theta} = \frac{F_1}{A_1}$$

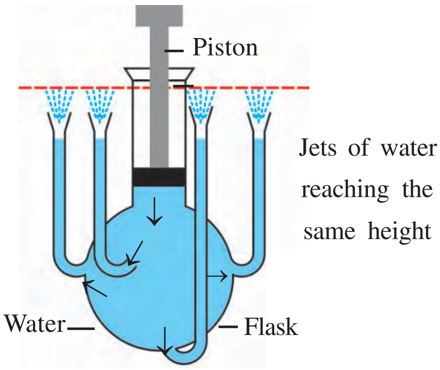
and pressure on the surface ADFC is

$$P_3 = \frac{F_3}{A_3} = \frac{F_1 cos\theta}{A_1 cos\theta} = \frac{F_1}{A_1}$$

So, $P_1 = P_2 = P_3$

Since θ is arbitrary this result holds for any surface. Thus Pascal's law is varified.

An obvious consequence of Pascal's law is that "A change in pressure applied to an enclosed is transmitted undiminished to every portion of the fluid and the walls of the container". It is perpendicular to the walls of the container. This statement is known as Pascal's law of transmission of fluid pressure.

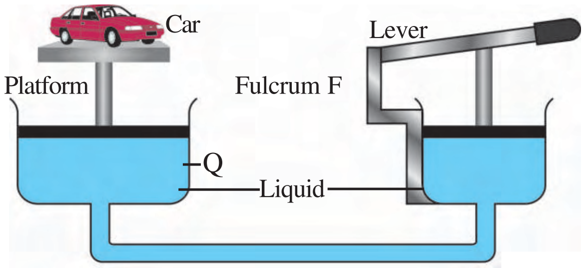


Transmission of pressure in the fluid

Figure 5.3

This can be demonstrated using a flask of glass, having small tubes jetting out from every part. (Figure 5.3) Fill some coloured water in it. Push the piston attached to it downwards. Water will rise in all tubes to the same height. This shows that change in pressure at any part of the enclosed liquid is transmitted equally in all directions.

Hydraulic Lift : Hydraulic lift works on Pascal's law. It is a device which consists of two cylinders of cross section A_1 and A_2, ($A_1 << A_2$) connected by a horizontal pipe. (Figure 5.4) These two cylinders are fitted with smooth, air tight pistons.



Hydraulic Jack

Figure 5.4

It contains liquid in it, as shown in the figure.

Suppose a force F_1 is applied on the pistion with cross sectional area A_1.

So, pressure produced due to it is,

$$P = \frac{F_1}{A_1}$$

This pressure exerted on liquid in a closed vessel is transmitted unchanged on to the piston with larger cross sectional area through liquid. Hence pressure on the second piston is

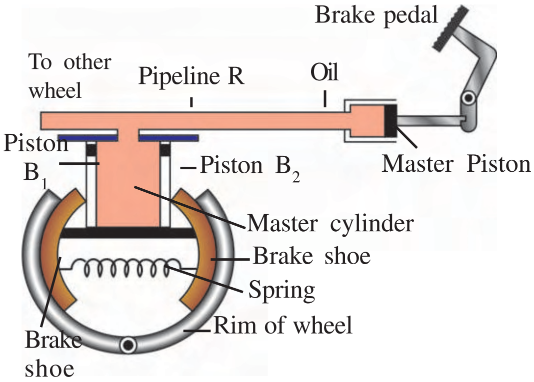
$$P = \frac{F_2}{A_2}$$

$$\therefore \frac{F_2}{A_2} = \frac{F_1}{A_1}$$

$$\therefore F_2 = F_1\left(\frac{A_2}{A_1}\right)$$

Here, $A_1 << A_2$, $\therefore F_1 << F_2$. Thus with less effort (F_1), heavy load can be lifted at the other end.

Hydraulic Brakes : Brake system used in automobiles are hydraulic brakes, which are based on Pascal's law. When driver applies a small force on the brake-pedal, the master piston moves in the master cylinder and the pressure caused by this is transmitted undiminished through the brake-oil, gets applied on the piston of larger area. Thus a greater force is applied on this piston, which pushes the brake-shoes to come in contact with the brake-liner. Thus, with a small force applied on the pedal, a greater retarding force is applied on the wheel.



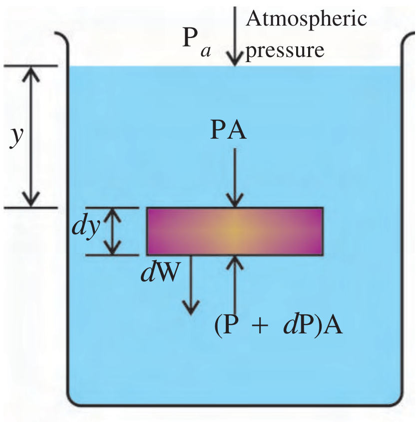
Hydraulic Brake

Figure 5.5

Door closers and shockabsorbers of automobiles work on Pascal's Law.

5.4 Pressure Due to Fluid Column

Suppose a liquid of density ρ is in static equilibrium in a container. Consider an imaginary cylindrical fluid element of height dy and cross-sectional area A at the depth y from the surface of liquid. As shown in Figure 5.6 the volume of this cylindrical element is Ady, its mass is $\rho \cdot A \cdot dy$ and its weight is $dw = \rho \cdot g \cdot A \cdot dy$.



Pressure due to liquid column

Figure 5.6

Suppose the pressure on upper and lower faces of the cylindrical element are P and P + dP respectively, as shown in Figure 5.6. Hence the force on upper surface in downward direction will be PA, while the force on lower face, in upward direction is (P + dp)A.

$$PA + dW = (P + dp)A$$

$$\therefore PA + \rho g A dy = PA + A dp$$

$$\therefore \rho g A dy = A dp$$

$$\therefore \frac{dp}{dy} = \rho g \qquad (5.4.1)$$

This equation shows that the rate of change in pressure with depth (or height) depends on physical quantity ρg, known as weight density (weight of a body per unit volume). Since most of the liquids are fairly incompressible, ρg is constant for small heights of liquid column. For fluids like air value of density ρ_1 depends on height from earth's suface, temperature etc. and hence value of weight density cannot be treated to be taken as constant for it.

As shown in Figure 5.6 container being open the pressure on upper free surface is equal to the atmospheric pressure. Thus for $y = 0$ P = P_a. Pressure P at depth $y = h$ can be determined by integrating equation 5.4.1,

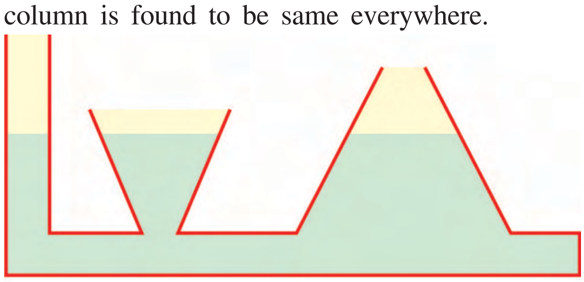
$$\int_{P_a}^{P} dp = \int_{0}^{h} \rho g dy$$

$$\therefore P - P_a = \rho g h$$

$$\therefore P = P_a + \rho g h \qquad (5.4.2)$$

Here, $P = P_a + \rho g h$ is known as absolute pressure, while the difference $P - P_a$ is known as the gauge pressure or hydrostatic pressure at that point.

The pressure at any point in a liquid neither depends on the shape of container in which it is filled nor on its area. This fact is known as **hydrostatic paradox.** (See Figure 5.7). When liquid is filled in containers of different shapes and sizes but interconnected, height of liquid column is found to be same everywhere.



Hydrostatic paradox

Figure 5.7

If two points are in the same horizontal level in the liquid equation 5.4.2 shows that pressure at these two points will be the same in stationary liquid.

5.5 Archimedes Principle : "When a body is partially or fully immersed in a liquid the buoyant force acting on it is equal to the weight of the liquid displaced by it and it acts in the upward direction at the centre of mass of the displaced liquid."

Thus if density of fluid is ρ_f and volume of the body immersed is V,

buoyant force is $F_b = \rho_f V g$

This is equal to decrease in weight of the body immersed.

Law of Floatation : When the weight W of a body is equal to the weight of the liquid displaced by the part of body immersed in it, the body floats on the surface of the liquid.

(i) If $W > F_b$, the body sinks in the liquid

(ii) If $W = F_b$, the body can remain in equilibrium at any depth in liquid.

(iii) If $W < F_b$, the body floats on the liquid surface, and remain partially immersed.

Illustration 1 : As shown in the figure 5.8 two cylindrical vessels A and B are interconnected. Vessel A contains water up to 2 m height and vessel B contains kerosene. Liquids are separated by movable, airtight disc C. If height of kerosene is to be maintained at 2 m, calculate the mass to be placed on the piston kept in vessel B. Also calculate the force acting on disc C due to this mass. Area of piston = 100 cm^2. Area of disc C = 10 cm^2. Density of water = 10^3 kg m^{-3}, specific density of kerosene = 0.8.

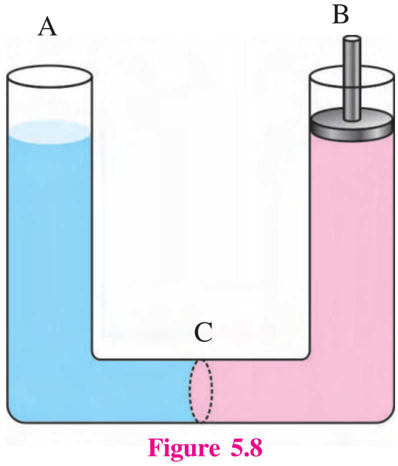


Figure 5.8

Solution : Area of piston A_1 = 100 cm^2 = 10^{-2} m^2

Area of disc A_2 = 10 cm^2 = 10^{-3} m^2

Density of water = 10^3 kg m^{-3} = ρ_ω

$$\text{Now, } \frac{\text{Density of kerosene}}{\text{Density of water}} = 0.8$$

$\therefore$ Density of kerosene ρ_k = 0.8 × density of water = 0.8 × 10^3 = 800 kg m^{-3}

If height of kerosene is maintained at 2 m,

Pressure of water column = $\frac{mg}{A_1}$ + Pressure of kerosene column

$$\therefore h\rho_w g = h\rho_k g + \frac{mg}{A_1}$$

$$\therefore 2 \times 10^3 = 2 \times 800 + \frac{m}{10^{-2}}$$

$$\therefore 2000 - 1600 = \frac{m}{10^{-2}}$$

$$\therefore 400 \times 10^{-2} = m$$

$$\therefore m = 4 \text{ kg}$$

Now pressure due to mass m is transmitted undiminished to disc C. So, pressure due to 4 kg mass

$$= \frac{\text{Force on disc C}}{\text{Area of disc C}}$$

$$\therefore \frac{mg}{A_1} = \frac{F_C}{A_2}$$

$$\therefore F_C = mg\frac{A_2}{A_1}$$

$$= \frac{4 \times 9.8 \times 10^{-3}}{10^{-2}}$$

$$= 3.92 \text{ N}$$

Illustration 2 : As shown in Figure 5.9. lower portion of the manometer tube contains fluid of density ρ_2 and the upper part contains fluid of density ρ_1 ($\rho_1 > \rho_2$). If pressures on the top of these two arms are P_1 and P_2, calculate pressure difference $(P_1 - P_2)$.

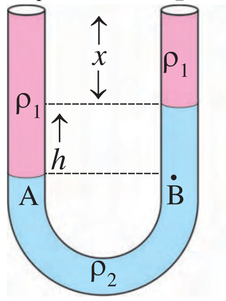


Figure 5.9

Solution : Consider points A and B, as shown in the diagram, at equal height from bottom.

For these points

$P_A = P_B$

$\therefore\ P_1 + (h + x)\rho_1 g = x\rho_1 g + h\rho_2 g + P_2$

$\therefore\ P_1 - P_2 = x\rho_1 g + h\rho_2 g - h\rho_1 g - x\rho_1 g$

$\therefore\ P_1 - P_2 = (\rho_2 - \rho_1)gh$

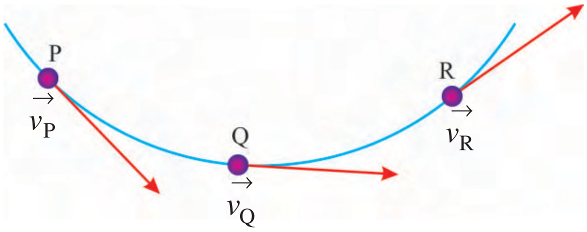
5.6 Fluid Dynamics

While studying the motion of a particle, we had to concentrate on the motion of the particle only and hence we didn't find it much difficult. But in the motion of a fluid when a very large number of particles are in motion, it becomes a formidable task to follow the motion of each of these particles. **J. L. Lagrange** developed a procedure in which he generalised the concepts of particle mechanics; but we shall not discuss it here. There is a treatment, developed by **Euler** which is more convenient for most purposes. In it we give up the attempt to specify the history of each fluid particle and instead specify the density, pressure and velocity of the fluid at each point in space at each instant of time. Of course, we can't afford to forget the particles of the fluid completely because, finally the motion of the fluid is attributed to the motion of its particles.

In the study of the motion of a fluid, we will consider ideal and simple situations. Let us first be familiar with some of the characteristics of the fluid flow.

Characteristics of Fluid Flow :

(1) Steady flow : If in a fluid flow, velocity of the fluid at each point remains constant with time, the flow is known as steady flow. This means that the velocity of any particle of the fluid remains the same while passing through a given point. To understand this, consider three representative points P, Q and R shown in the Figure 5.10. Let the velocities of each particle passing through these points be $\vec{v}_P$, $\vec{v}_Q$ and $\vec{v}_R$ respectively. These velocities remain constant with time. It is not necessary that the velocities of a particle at different points be the same, but velocity of the particles passing through the same point does not change with time, i.e. it is **not necessary** that $\vec{v}_P = \vec{v}_P = \vec{v}_R$, but $\vec{v}_P$, $\vec{v}_Q$ and $\vec{v}_R$ should remain constant with time. These conditions can be achieved at low flow speeds e.g. a gently flowing stream.



Characteristics of steady flow

Figure 5.10

(2) Unsteady flow : If in a fluid flow, velocity of the fluid at a given point keeps on changing with time, the flow is known as unsteady flow. For example, the motion of water during ebb and tide.

(3) Turbulent flow : In a fluid flow, if the velocity of the fluid changes erratically from point to point as well as from time to time, the flow is known as a turbulent flow. Waterfalls, breaking of the sea waves are the examples of turbulent flow.

(4) Irrotational flow : If the element of a fluid at each point has no net angular velocity about that point, the fluid flow is called irrotational.

If the flow is irrotational, a small paddle wheel placed in the flow (as shown in Figure 5.11) will move without rotating.

Motion of a small paddle-wheel

Figure 5.11

(5) Rotational flow : If the element of a fluid at each point has net angular velocity about that point, the fluid flow is called rotational. A paddle wheel placed in such a flow rotates while moving. The rotational motion is turbulent. Rotational flow includes vortex motion such as whirlpools, the air thrown out by exhaust fans etc.

(6) Incompressible flow : If the density of a fluid remains constant with time everywhere (in a given flow), the flow is said to be incompressible. Generally, liquids can usually be considered as flowing incompressibly. But even a highly compressible gas may sometimes undergo insignificant changes in density, its flow is then practically incompressible. For example, the flow of air relative to the wings of an aeroplane flying with velocity quite less than that of sound waves can be considered almost incompressible.

(7) Compressible flow : If in a fluid flow, the density changes with position and time, the flow is known as compressible flow.

(8) Non-viscous flow : The flow of a fluid having small co-efficient of viscosity is known as non-viscous flow. In other words, a flow of a readily flowing fluid is called non-viscous flow. The flow of water in normal conditions is an example of non-viscous flow.

(9) Viscous flow : The flow of a fluid which has large co-efficient of viscosity is called a viscous flow. Thus, a flow of a fluid which cannot flow readily is called a viscous flow. The flow of castor oil is an example of a viscous flow.

In the beginning, we shall consider steady, irrotational, incompressible and non-viscous flow only. But our assumptions are too ideal for the real situations. It is not possible to have such an ideal fluid.

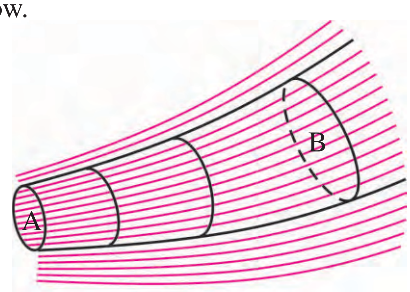
5.6.1 Streamlines, Tube of flow :

The path of motion of a fluid particle is called a **line of flow.** Normally, the direction and the magnitude of the velocity of a fluid particle keeps on changing on its path of motion and hence all the particles passing through a point in a flow may not move on the same path. But the situation in a steady flow is interesting.

In a steady flow, velocity of each particle arriving at a point remains constant with time. In Figure 5.10 let the velocity of a particle arriving at point P be $\vec{v}_P$. It does not change with time. Thus, each particle arriving at P has velocity $\vec{v}_P$ and at that point each particle proceeds in the same direction. When each particle passing through P goes to Q, its velocity $\vec{v}_Q$ (at point Q) also remains constant with time and it proceeds further to R, where its velocity $\vec{v}_R$ also remains constant. Thus, the path of motion of a particle passing through P is PQR. This path of motion does not change with time. Such a steady path of motion is called a **streamline.** The flow for which such streamlines can be defined is called streamline flow. In unsteady flow, flow lines can be defined but not the streamlines.

In a steady flow, streamlines can never intersect each other. If they do, at the point of intersection a particle may move in any direction out of two tangents drawn at that point, which is not possible.

Tube of flow : In principle, we can draw a streamline through every point in the fluid flow. As shown in Figure 5.12, if we imagine a bundle of streamlines passing through the boundary of any surface, this tubular region is called a tube of flow.



Flow tube

Figure 5.12

The wall of the tube of flow is made of streamlines. As streamlines can never intersect each other, a particle of a fluid cannot pass through the wall of a tube of flow. Hence, the tube behaves somewhat like a pipe of the same shape.

5.7 Equation of Continuity

Consider a tube of flow as shown in Figure 5.13. Let the velocity of fluid at cross section P, of area A_1, be v_1 and at cross section Q, of area A_2, be v_2.

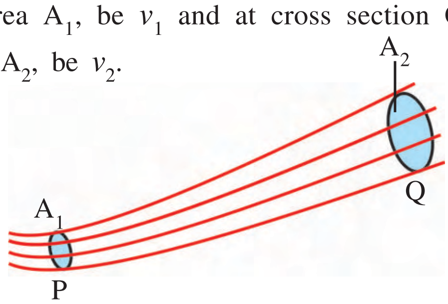


Figure 5.13

So, through the cross section P in unit time fluid can travel distance equal to v_1. Thus the volume of the fluid entering at P in unit time is A_1v_1. If the density of incompressible fluid is ρ, mass of fluid entering at P in unit time is $\rho A_1 v_1$.

Since, **mass of fluid passing through cross section per unit time is called mass flux,**

mass flux at P $= \rho A_1 v_1$ (5.7.1)

Similarly mass flux at Q $= \rho A_2 v_2$ (5.7.2)

Since the liquid cannot pass through the wall and the fluid can neither be destroyed nor be created, mass flux at P and Q should be equal. So, from 5.7.1 and 5.7.2.

$$\rho A_1 v_1 = \rho A_2 v_2$$

$$\therefore A_1 v_1 = A_2 v_2 \quad (5.7.3)$$

or for any cross-section of tube of flow

$$Av = \text{constant} \quad (5.7.4)$$

Equation 5.7.3 or 5.7.4 is known as the equation of continuity. The product of area of a cross-section and velocity of fluid at this cross section is called volume-flux. Equation 5.7.4 shows that speed of fluid is larger in narrower section of tube and vice versa. In a narrow part of the tube the density of streamlines is larger, fluid speed is more. So it can be concluded that closely packed stremlines indicate larger speed of fluid and vice versa.

5.8 Bernoulli's Equations and its Applications

Bernoulli's equation is a fundamental relation in fluid dynamics. This equation does not represent a new principle of fluid mechanics. It can be obtained using work-energy theorm.

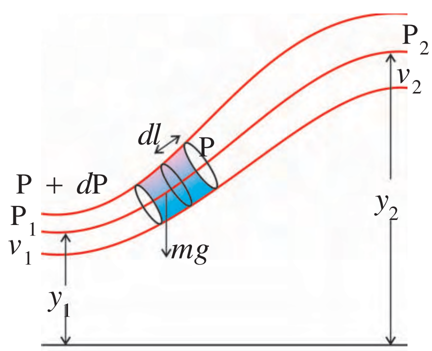


Figure 5.14

Let us consider a streamline flow which is steady, irrotational, incompressible and non-viscous, through flow-tube as shown in figure. Consider a small fluid-element, having area A and length dl. Central streamline passing through this fluid element passes through heights y_1 and y_2 from reference level. At height y_1, pressure and speed of fluid are P_1 and v_1 and at height y_2 they are P_2 and v_2 respectively. This fluid element of mass m, is acted upon by two forces (1) force due to pressure difference (Adp) and (2) gravitational force mg. Suppose this fluid element is displaced by distance dl.

So, here work done due to the first force is $Adl\,dp$ and work done against gravitational force is $-mgdy$ (change in potential energy) where dy is change in height. If initially its kinetic energy is $\frac{1}{2}mv^2$, change in kinetic energy during displacement dy, is $d(\frac{1}{2}mv^2) = mvdv$

As per work-energy theorem

$$mvdv = Adldp - mg\,dy \quad (5.8.1)$$

Since Adl is the volume of fluid element, equation 5.8.1 becomes

$$\frac{m}{Adl}vdv = -dp - \frac{m}{Adl}gdy \quad (5.8.2)$$

Here m/Adl is the density (ρ) of the fluid and since fluid is incompressible it is constant. So, equation 5.8.2 can be written as

$$\rho vdv = -dp - \rho gdy$$

$$\therefore \rho \int_{v_1}^{v_2} vdv = -\int_{P_2}^{P_2} dp - \rho g \int_{y_1}^{y_2} dy$$

$$\therefore \rho\left[\frac{v^2}{2}\right]_{v_1}^{v_2} = -[P]_{P_1}^{P_2} - \rho g[y]_{y_1}^{y_2}$$

$$\therefore \frac{1}{2}\rho(v_2^2 - v_1^2) = -(P_2 - P_1) - \rho g(y_2 - y_1)$$

$$\therefore P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2 \quad (5.8.3)$$

$$\therefore P + \rho g y + \frac{1}{2}\rho v^2 = \text{constant} \quad (5.8.4)$$

Equation 5.8.3 or 5.8.4 is known as Bernoulli's equation. It should be noted that all the terms of their equation are to be calculated on the same streamline. If the flow is irrotational it can be proved that the constant appearing in equation 5.8.4 is same for all streamlines.

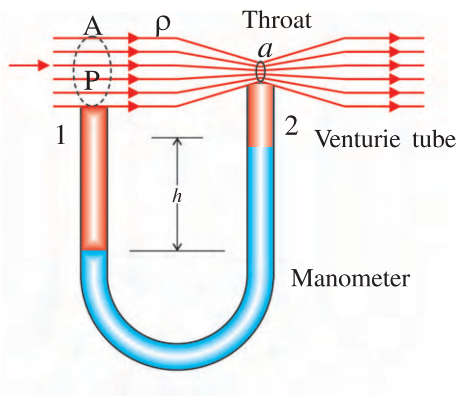
If equation 5.8.4 is divided by ρg we get

$$\frac{P}{\rho g} + \frac{v^2}{2g} + y = \text{constant} \qquad (5.8.5)$$

This is another form of Bernoulli's equation. The first term in this equation is known as pressure head, second term is known as velocity head and third terms as elevation head.

Applications of Bernoullis Equation

(1) Venturie meter : This apparatus is used to measure speed of fluid. The construction of venturie meter is shown in Figure 5.15. A manometer is connected with a venturie tube having specific design. The narrow part of the tube is called throat. Broader end of the apparatus has cross-sectional area 'A' and the cross-sectional area of throat is 'a'. Speeds near the broader end and near throat are v_1 and v_2. While, pressures are P_1 and P_2 respectively. Density of manometer – fluid is ρ_2 and that of fluid, where velocity is to be measured is ρ_1.



Venturie Meter

Figure 5.15

Using Bernoulli's equation at points '1' and '2'.

$$P_1 + \frac{1}{2}\rho_1 v_1^2 + \rho_1 g y_1 = P_2 + \frac{1}{2}\rho_1 g v_2^2 + \rho_1 g y_2$$

Points '1' and '2' are horizontal. $\therefore y_1 = y_2$.

$$\therefore P_1 + \frac{1}{2}\rho_1 v_1^2 = P_2 + \frac{1}{2}\rho_1 v_2^2$$

$$\therefore P_1 - P_2 = \frac{1}{2}\rho_1 (v_2^2 - v_1^2) \qquad (5.8.6)$$

Here, for manometer $P_1 - P_2 = (\rho_2 - \rho_1)gh$ (see illustration 2)

Inserting value of $P_1 - P_2$ in equation 5.8.6 we get

$$(\rho_2 - \rho_1)\, gh = \frac{1}{2}\rho_1 (v_2^2 - v_1^2) \qquad (5.8.7)$$

From equation of continuity $Av_1 = av_2$

$$\therefore v_2 = \frac{A v_1}{a}$$

Substituting this value of v_2 in equation 5.8.7 we get

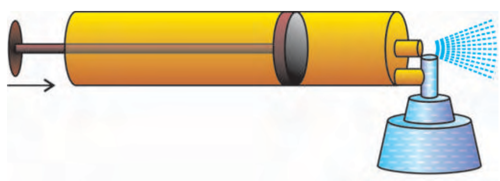
$$(\rho_2 - \rho_1)gh = \frac{1}{2}\rho_1 \left(\frac{A^2}{a^2}\, v_1^2 - v_1^2\right)$$

$$\therefore v_1^2 = \frac{2(\rho_2 - \rho_1)gh}{\rho_1} \cdot \frac{a^2}{A^2 - a^2}$$

$$\therefore v_1 = a\sqrt{\frac{2(\rho_2 - \rho_1)gh}{\rho_1 (A^2 - a^2)}} \qquad (5.8.8)$$

To find the volume flux or the rate of flow, $R = v_1 A$ or $v_2 a$ should be found.

Air flows through a venturie channel of a carburator in automobiles. At throat, pressure being low, the fluid is sucked in and proper mixture of air and fuel is made available for combustion.



Spray pump

Figure 5.16

The same principle is used in a spray pump, as shown in Figure 5.16. On pushing the piston in air comes out of the hole with high velocity. As a result of this, pressure near the hole is reduced and liquid is raised up in a capillary and it is sprayed along with air.

(2) The change in pressure with depth :

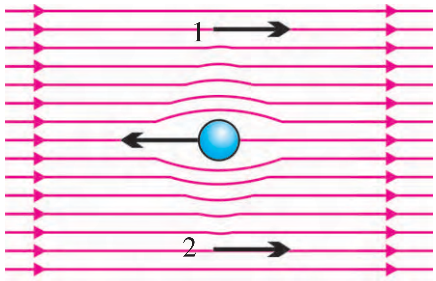
Earlier we have seen that $P - P_a = h\rho g$. This equation can be arrived at as a special case of Bernoulli's equation. If the fluid is stationary $v_1 = v_2 = 0$, and $P_2 = P_a$ (Pressure on free surface of liquid, see Figure 5.6). If the difference in height is taken to be $y_2 - y_1 = h$, from Bernoulli's equation we get $P_1 = P_a + \rho gh$.

(3) Dynamic Lift and Swing Bowling :

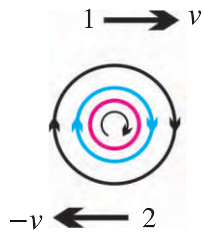
Figure 5.17(*a*) shows a ball moving in air. The streamlines are symmetric (w.r.t. the ball) around the ball (because the ball is symmetric). The velocity of air at points 1 and 2 is the same. According to the Bernoulli's equation the pressures at 1 and 2 would also be the same. Hence, the dynamic lift on the ball is zero.

Now, as shown in Figure 5.17(*b*) suppose the ball is having spin motion about an axis passing through its centre and perpendicular to the plane of the figure. As the surface of the ball is not quite smooth, it drags some air along with it. The streamlines produced due to such motion is shown in the figure.

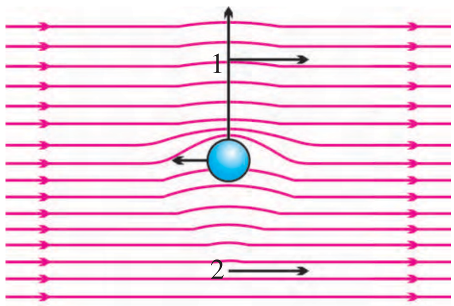
Figure 5.17(*c*) shows the streamline pattern of air when the ball performs linear as well as spin motions. The crowding of streamlines at point 1 indicates high velocity and low pressure, while the sparse streamlines at point 2 indicates low velocity and high pressure. So a ball thrown with such a spin will move up w.r.t. its trajectory. (Now, you might have understood why the bowlers play the michief with a ball to make its surface rough).



(*a*)



(*b*)

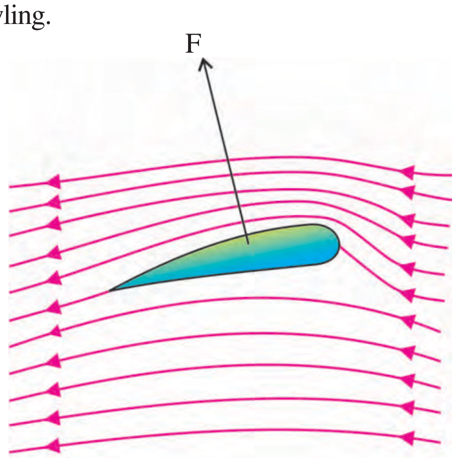


(*c*)

Swing Bowling

Figure 5.17

Now, if the ball is thrown making it spin about an axis lying in the plane of the figure and perpendicular to the direction of its motion, it may deviate towards the off stump or leg stump. This is the main reason of the swing of a ball in fast bowling.



Aerofoil

Figure 5.18

(4) Aerofoils : Figure 5.18 shows an aerofoil which is a solid piece shaped to provide an upward lift when it moves horizontally through air and hence it can float in air.

The shape of the wings of an aeroplane (shape of the cross-section perpendicular to the length of the wings) is like an aerofoil. As shown in the figure, the air has streamline flow about the wings. (Only when the angle between the wing and the direction of motion – called an angle of attack – is small, the streamline flow is possible). In Fig. 5.18, the streamlines are shown arround the wing. The crowded streamlines over the wings indicate high velocity and low pressure, while the sparse streamlines below the wings indicate low velocity and high pressure.

Due to this pressure difference an aeroplane experiences the upward thrust. Thus, the aeroplane in motion may float in air due to the dynamic lift.

Illustration 3 : The diameter of one end of a tube is 2 cm and that of another end is 3 cm. Velocity and pressure of water at narrow end are 2 ms^{-1} and 1.5×10^5 Nm^{-2} respectively. If the height difference between narrow and broad ends is 2.5 m, find the velocity and pressure of water at the broad end. (Density of water is 1×10^3 kg m^{-3}). The narrow end is higher.

Solution :

The narrow end of the flow tube

$d_1 = 2$ cm .

$\therefore\ r_1 = 1 \text{ cm} = 1 \times 10^{-2}$ m

$v_1 = 2\ ms^{-1}$

$P_1 = 1.5 \times 10^5\ Nm^{-2}$

The broad end of the flow tube

$d_2 = 3$ cm

$\therefore\ r_2 = 1.5 \text{ cm} = 1.5 \times 10^{-2}$ m

$v_2 = ?$

$P_2 = ?$

$A_1 v_1 = A_2 v_2$

$$\therefore\ v_2 = \frac{A_1}{A_2} \cdot v_1$$

$$= \frac{\pi r_1^2}{\pi r_2^2} \cdot v_1 = \frac{r_1^2}{r_2^2} \cdot v_1$$

$$= \frac{(1 \times 10^{-2})^2}{(1.5 \times 10^{-2})^2} \times 2$$

$= 0.89\ ms^{-1}$

According to Bernoulli's equation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_1^2 + \rho g y_2$$

$$\therefore\ P_2 = P_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) + \rho g(y_1 - y_2)$$

$$= (1.5 \times 10^5) + \frac{1}{2} \times 1 \times 10^3 \times [(2)^2 - (089)^2] + 1 \times 10^3 \times 9.8 \times 2.5$$

$P_2 = 1.76 \times 10^5\ Nm^{-2}$

Illustration 4 : Figure 5.19 shows a cylindrical vessel having cross-sectional area A_1 in which liquid of density ρ is filled. At the bottom of the container there is a small hole of cross-section A_2. Find the velocity of liquid coming out of the hole when the height of the liquid column is h from the hole. (Here, $A_1 >> A_2$)

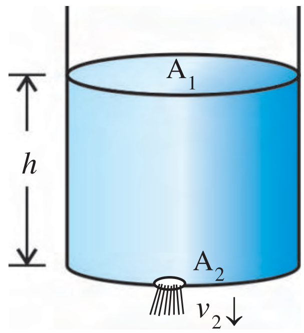


Figure 5.19

Solution : Suppose, the velocity of the liquid at cross-section A_1 and A_2 are v_1 and v_2 respectively. Both the cross-sections being open in the atmosphere, the pressure at the cross-sections is same as the atmospheric pressure P_a.

$$\therefore\ P_a + \frac{1}{2}\rho v_1^2 + \rho g h = P_a + \frac{1}{2}\rho v_2^2 \quad (1)$$

According to the equation of continuity,

$$A_1 v_1 = A_2 v_2$$

$$\therefore\ v_1 = \frac{A_2 v_2}{A_1} \qquad (2)$$

Substituting the value of v_1 from eqn. (2) in eqn. (1),

$$\frac{1}{2}\left(\frac{A_2}{A_1}\right)^2 v_2^{\,2} + gh = \frac{1}{2}v_2^{\,2}$$

$$\therefore\ v_2^{\,2} = \frac{2gh}{\left[1-\left(\frac{A_2}{A_1}\right)^2\right]} \cong 2gh\ \ (\because A_2 << A_1)$$

$$\therefore\ v_2 = \sqrt{2gh}$$

Note : The velocity of the liquid coming out of a hole at the depth h from the free surface of the liquid is the same as the final velocity of a particle falling freely from the same height. **This statement is called Torricelli's law.**

Illustration 5 : Water is filled in a container upto height H as shown in the Figure 5.20. A small hole is bored on the surface of a container at the depth h from the surface of water. What will be the distance of a point along the horizontal where the jet of the water strikes the ground ? For which value of h will this distance be maximum ? Also find this maximum distance.

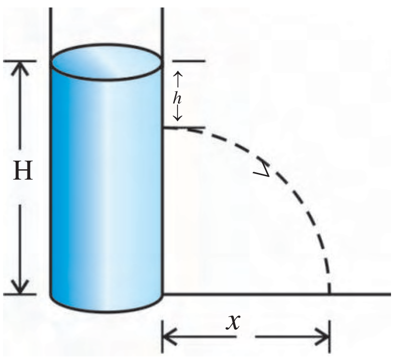


Figure 5.20

Solution : The velocity of water coming out of the hole in horizontal direction is

$$v = \sqrt{2gh} \qquad (1)$$

As the acceleration of water coming out of the hole is downward, it moves with constant velocity in horizontal direction while it moves with constant acceleration downward. (It is like projectile motion).

From the equations of motion, displacement in downward direction,

$$H - h = \frac{1}{2}gt^2 \qquad (2)$$

where t = time taken by the water to fall on the ground.

The distance travelled along horizontal,

$$x = vt \qquad (3)$$

Substituting the values of v and t from equations (1) and (2) in equation (3),

$$x = \sqrt{2gh}\left(\frac{2(H-h)}{g}\right)^{\frac{1}{2}}$$

$$= (4hH - 4h^2)^{\frac{1}{2}}$$

$$= [H^2 - (H-2h)^2]^{\frac{1}{2}} \qquad (4)$$

Equation (4) shows that x is maximum if $H = 2h$, i.e. $h = \frac{H}{2}$

$$\therefore\ h = \frac{H}{2}$$

Also, for $h = \frac{H}{2}$ equation (4) gives

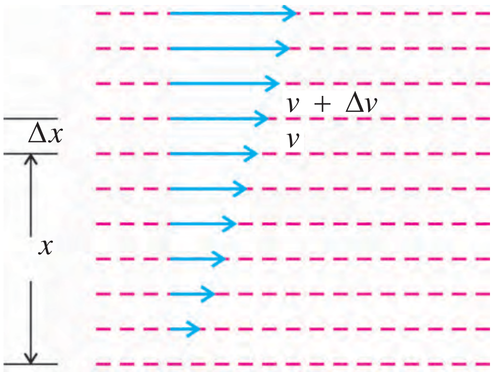
$$\therefore\ x = H$$

5.9 Viscosity

We know that the liquids like water and kerosene flow easily. While the liquids like honey, castor oil cannot flow easily. If we consider Bernoulli's equation for horizontal flow, i.e. $y_1 = y_2$, we have

$$P_1 + \frac{1}{2}\rho v_1^{\,2} = P_2 + \frac{1}{2}\rho v_2^{\,2}$$

This equation suggests that to maintain horizontal fluid flow with constant speed ($v_1 = v_2$), no pressure difference is required i.e. $P_1 = P_2$. But this is not found in reality. We need some pressure difference to maintain fluid flow with constant speed. This means that there must be some force opposing the motion of fluid. This force is due to viscosity. To understand this let us consider the steady flow of liquid on some horizontal stationary surface.



Laminar flow

Figure 5.21

The layer of liquid which is in contact with the surface, remains stuck to it due to adhesive force acting between the molecules of the liquid and molecules of the surface. The velocity of the layer at the top is the maximum.

In Figure 5.21, some of the layers are shown along with their velocity vectors. Thus in a steady flow different layers slide over each other without getting mixed up. This kind of flow is called **laminar flow.**

In a laminar flow any two consecutive layers of fluid have relative velocity between them. As a result, resistive force is produced tangentially at the surface of layers in contact. This internal force of friction is called **viscous force. The property of fluid due to which relative motion between two consecutive layers in opposed is known as viscosity of the fluid.** In order to maintain the relative velocity of the layers the minimum external force to be applied must balance the viscous force. In absence of such external force the relative motion between the layers decreases with time due to viscous force and fluid comes to rest. This is the reason why milk in cup comes to rest after sometimes after it has been stirred.

Velocity gradient : In a laminar flow the difference in velocity between two layers of liquid per unit perpendicular distance, in the direction perpendicular to the direction of flow is called velocity gradient.

As shown in the Figure 5.21 difference in velocity of two layers having separation Δx is Δv. So, $\frac{\Delta v}{\Delta x}$ is velocity gradient. If Δx is very small velocity gradient becomes

$$\lim_{\Delta x \to 0} \frac{\Delta v}{\Delta x} = \frac{dv}{dx}$$

In case of laminar flow velocity gradient is constant for all layers.

Its SI unit is s^{-1}.

Now let us come back to viscosity. Here viscosity is the force opposing the motion. According to Newton's experimental work at constant temperature viscous force is given by

$$F = \eta A \frac{dv}{dx} \qquad (5.9.1)$$

where F is the viscous force, A is contact-area between two layers and η is the constant of proportion known as co-efficient of viscosity. Value of η depends on type of fluid and its temperature.

If the value of η is larger, the viscous force is larger, and hence fluid can flow slowly and vice versa. Thus co-efficient of viscosity is the measure of viscosity of fluid. Also, value of η decreases with temperature of liquid but increase with increase in temperature of gases. From equation 5.9.1,

$$\eta = \frac{F}{A\frac{dv}{dx}}$$

If we take A = 1 unit and $\frac{dv}{dx}$ = 1 unit

$\eta = F$

Thus **"the viscous force acting per unit surface area of contact and per unit velocity gradient between two adjacent layers in a laminar flow, is known as the co-efficient of viscosity."**

CGS Unit of co-efficient of viscosity is dyne s cm^{-2}, and is called 'poise' in honour of Jean Lois Poiseuille, a French physician and physicist. Its SI unit is N s m^{-2} or Pa s. Its dimensional formula is $M^1L^{-1}T^{-1}$.

Value of co-efficient of viscosity for some fluids are given in Table 5.2.

Co-efficient of viscosity of some fluids

(For Information Only)

Table 5.2

Fluid	Temperature	Co-efficient of Viscosity (N s m^{-2})
Water	20°C	1×10^{-3}
	100°C	2.8×10^{-4}
Air	0°C	1.71×10^{-5}
	340°C	1.9×10^{-5}
Blood	38°C	1.5×10^{-3}
Seasem oil		4.0×10^{-2}
Engine oil	16°C	1.13×10^{-1}
	38°C	3.4×10^{-2}
Honey		2.0×10^{-1}
Water vapour	100°C	1.25×10^{-5}
Glycerine	20°C	8.30×10^{-1}
Aceton	25°C	3.6×10^{-4}

Illustration 6 : A disc of area 10^{-2} m^2 is placed over a layer of oil having thickness 2×10^{-3} m. If the co-efficient of viscosity of the oil is 1.55 N s m^{-2}, find the horizontal (tangential) force required to move the disc with the velocity of 3×10^{-2} ms^{-1}.

Solution :

$A = 10^{-2}\ m^2$

$\Delta v = 3 \times 10^{-2}\ ms^{-1}$

$\Delta x = 2 \times 10^{-3}\ m$

$\eta = 1.55\ N\ s\ m^{-2}$

Since,

$$F = \eta A \frac{\Delta v}{\Delta x}$$

$$= 1.55 \times 10^{-2} \times \frac{3 \times 10^{-2}}{2 \times 10^{-3}}$$

$\therefore\ F = 2.32 \times 10^{-1}$ N

Illustration 7 : The velocities of cylindrical layers of liquid flowing through a tube, situated at distances 0.8 cm and 0.82 cm from the axis of the tube are 3 cm s^{-1} and 2.5 cm s^{-1} respectively. Find the viscous force acting between these layers if the length of the tube is 10 cm and the co-efficient of viscosity of the liquid is 8 poise.

Solution :

$r_1 = 0.8$ cm

$r_2 = 0.82$ cm

$\Delta v = 3 - 2.5 = 0.5$ cm s^{-1}

Δx = distance between the layers

= 0.02 cm

L = 10 cm

A = Area of contact of two layers

$$= 2\left(\frac{r_1 + r_2}{2}\right)L$$

η = 8 poise

$$F_v = \eta A \frac{\Delta v}{\Delta x}$$

$$= \eta \left[2\pi\left(\frac{r_1 + r_2}{2}\right)L \right] \frac{\Delta v}{\Delta x}$$

$$= 8 \left[2 \times 3.14 \left(\frac{0.8 + 0.82}{2}\right) 10 \right] \frac{0.5}{0.02}$$

$$= 16 \times 3.14 \times 0.81 \times 10 \times \frac{0.5}{0.02}$$

= 10173.6 dyne

5.10 Stokes' Law

When a body moves through a viscous medium, the layer of the medium in contact with the body drifts along with it. Hence, this layer moves with the velocity same as that of the body. But the distant layer remains stationary. Thus, laminar flow is produced between the body and distant stationary layer. Hence, viscous force acts between two adjacent layers of the medium and as a result a resistive force acts on the body moving through the medium.

The resistive force (viscous force) on a small, smooth, spherical, solid body of radius r moving with velocity v through a viscous medium, of large dimension, having co-efficient of viscosity η is given by,

$$F(v) = 6\pi\eta r v \quad (5.10.1)$$

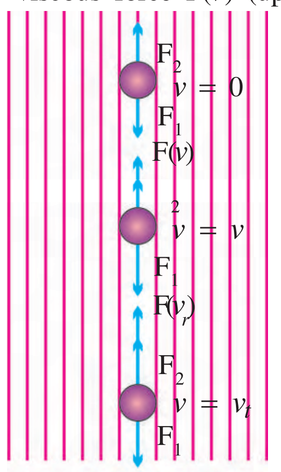
This equation is called Stokes' Law

Stokes' Law is an interesting example of a velocity dependent force in which the force acting on a body opposing its motion, is proportional to its velocity.

Motion of a sphere in a fluid and its terminal velocity :

Suppose, a small, smooth, solid sphere of radius r of material having density ρ starts motion with initial velocity zero in a fluid as shown in Figure 5.22. Let the density of the fluid be ρ_o and its co-efficient of viscosity be η. Here, $\rho > \rho_o$.

In Figure 5.22 forces acting on the sphere at three different instants are shown. These forces are : (1) weight of the sphere F_1 (downward) (2) buoyant force of the fluid F_2 (upward) (3) viscous force $F(v)$ (upward).



Free fall of a small, smooth, spherical body in a viscous medium

Figure 5.22

(1) Volume of the sphere, $V = \frac{4}{3}\pi r^3$

$\therefore$ Mass of the sphere, $m = V\rho = \frac{4}{3}\pi r^3 \rho$

$\therefore$ Weight of the sphere, $F_1 = mg = \frac{4}{3}\pi r^3 \rho g$

(2) The buoyant force exerted by the fluid is equal to the weight of the fluid displaced by the sphere. Volume of the fluid displaced by the sphere of volume

$$V = \frac{4}{3}\pi r^3$$

$\therefore$ Mass of the fluid displaced by the sphere,

$$m_o = V\rho_o = \frac{4}{3}\pi r^3 \rho_o$$

$\therefore$ Weight of the fluid displaced by the sphere, $m_o g = \frac{4}{3}\pi r^3 \rho_o g$

$\therefore$ The buoyant force, $F_2 = \frac{4}{3}\pi r^3 \rho_o g$

(5.10.3)

(3) Viscous force opposing the motion, according to Stokes' Law $F(v) = 6\pi\eta r v$

(5.10.4)

$\therefore$ The resultant force acting on fluid sphere is,

$$F = F_1 - F_2 - F(v)$$

$$\therefore F = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \rho_o g - 6\pi\eta r v \tag{5.10.5}$$

Equation 5.10.5 represents the equation of motion of the sphere in the fluid.

At $t = 0$, when the motion of the sphere starts in the fluid, its velocity $v = 0$.

Hence, the viscous force on the sphere $F(v) = 0$.

$$\therefore F = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \rho_o g = \frac{4}{3}\pi r^3 g(\rho - \rho_o) \tag{5.10.6}$$

If the acceleration of sphere is a_o at $t = 0$,

$$F = ma_o = \frac{4}{3}\pi r^3 \rho a_o \tag{5.10.7}$$

Comparing equations (5.10.6) and (5.10.7)

$$\frac{4}{3}\pi r^3 \rho a_o = \frac{4}{3}\pi r^3 g(\rho - \rho_o)$$

$$a_o = \frac{\rho - \rho_o}{\rho} \tag{5.10.8}$$

The sphere is accelerated in the fluid. As the velocity of the sphere increases gradually with time, the viscous force acting on the sphere in upward direction also increases. Thus the velocity of the sphere increases while its acceleration decreases.

When $F_1 = F_2 + F(v)$, the resultant force on the sphere becomes zero. Hence, the acceleration also becomes zero. Now onwards, the sphere travels with constant velocity. This velocity is known as the terminal velocity v_t of the sphere. When the sphere acquires terminal velocity, $F = 0$ and $v = v_t$ and from equation (5.10.8)

$$\therefore\ 0 = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \rho_o g - 6\pi\eta r v$$

$$\therefore\ 6\pi\eta r v_t = \frac{4}{3}\pi r^3 g(\rho - \rho_o)$$

$$\therefore\ v_t = \frac{2}{9}\frac{r^2 g}{\eta}(\rho - \rho_o) \qquad (5.10.9)$$

With the help of terminal velocity coefficient of viscosity of the fluid can be determined using equation. (5.10.9)

A bubble formed in a liquid may be considered to be a sphere of air. In this case $\rho_o > \rho$. Hence, from the beginning F_1 being less than F_2 the bubble is accelerated upward. Therefore, the bubble rises up and acquires terminal velocity after some time. This terminal velocity can be obtained using eqn. (5.10.9). Here, v_t is negative which shows that the bubble has terminal velocity in upward direction. You might have observed the bubble rising up in a bottle of soda water.

Illustration 8 : Two rain drops of equal volume, falling with terminal velocity 10 cm s^{-1}, merge while falling and forms a larger drop. Find the terminal velocity of the larger drop.

Solution :

Let the radius and volume of each drop be r and V respectively. When they merge and form a larger drop, its volume V' will be double the volume of each one of them. (As the mass and density remain constant).

Let the radius of the bigger drop so formed, be R.

Now, $V' = 2V$

$$\frac{4}{3}\pi R^3 = 2\left(\frac{4}{3}\pi r^3\right)$$

$$R^3 = 2r^3$$

$$\therefore\ R = (2^{\frac{1}{3}})r$$

Let the terminal velocity of the smaller drop be v and that of the larger drop be v',

$$v = \frac{2}{9}\frac{r^2 g}{\eta}(\rho - \rho_o) \text{ and}$$

$$v' = \frac{2}{9}\frac{R^2 g}{\eta}(\rho - \rho_o)$$

$$\therefore\ \frac{v'}{v} = \frac{R^2}{r^2}$$

$$\therefore\ v' = v\frac{R^2}{r^2} = 10(2^{\frac{1}{3}})^2 = 15.87 \text{ cm s}^{-1}$$

5.11 Reynold's Number and Critical Velocity

The flow of a fluid through a given tube may be streamline, turbulent or of mixed type. In almost all experiments designed for the measurement of the co-efficient of viscocity the flow must be streamline. Therefore, it becomes necessary to know the conditions in which the flow becomes streamline.

Osborne Reynolds (1842–1912), a British mathematician and physicist, has shown that the type of flow through a tube depends on (1) the co-efficient of viscosity (η) of fluid, (2) the density (ρ) of the fluid, (3) average velocity (v) of the fluid and (4) the diameter (D) of the tube.

The number (N_R) formed by the combination of these four physical quantities is called Reynold's number.

$$\text{Reynolds number, } N_R = \frac{\rho v D}{\eta} \qquad (5.11.1)$$

The magnitude of N_R depends on the type of the flow.

N_R is dimensionless. Experiment shows that, if $N_R < 2000$ the flow is streamline and if $N_R >$ 3000 the flow is turbulent. For $2000 < N_R < 3000$, the flow is unstable and its type keeps changing.

Critical Velocity : It is clear from equation 5.11.1 that with increase in velocity Reynolds number also increases. **The maximum velocity for which the flow remains streamline is called critical velocity.** The corresponding Reynolds Number is called critical Reynolds number.

It should be noted that if $\eta = 0$ (i. e. if fluid is non-viscous) N_R becomes infinite So, non-viscous flow can never be streamline.

Illustration 9 : As shown in Figure 5.23 laminar flow is obtained in a tube of internal radius r and length l. To maintain such flow, the force balancing the viscous force is obtained by producing the pressure difference

(p) across the ends of the tube. Derive the equation of velocity $v = \frac{p}{4\eta l}(r^2 - x^2)$ of a layer situated at distance 'x' from the axis of the tube.

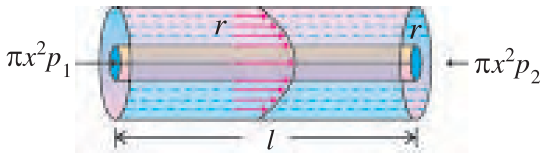


Figure 5.23

Solution : Consider a cylindrical layer of liquid of radius x as shown in Figure 5.23. The forces acting on it are as follows :

(1) The force due to the difference in pressure p is, $F_1 = \pi x^2 p$

(2) Viscous force, $F_2 = \eta A \frac{dv}{dx}$

$$= \eta(2\pi x l)\left(-\frac{dv}{dx}\right)$$

where, A = area of the curved surface of the cylinder = $2\pi x l$.

Here, v decreases with increase in x. Hence the velocity-gradient is taken negative. For the motion of the cylindrical layer with a constant velocity,

$$F_1 = F_2$$

$$\therefore \pi x^2 p = -\eta \cdot 2\pi x l \cdot \frac{dv}{dx}$$

$$\therefore -dv = \frac{p}{2\eta l} x \, dx$$

At $x = r$, $v = 0$ and at $x = x$, $v = v$ and so integrating the above equation in these limits we get,

$$-\int_v^0 dv = \int_x^r \frac{p}{2\eta l} x \, dx$$

$$\therefore -[v]_v^0 = \frac{p}{4\eta l}\left[x^2\right]_x^r$$

$$\therefore -[0 - v] = \frac{p}{4\eta l}[r^2 - x^2]$$

$$\therefore v = \frac{p}{4\eta l}(r^2 - x^2)$$

Illustration 10 : Find the volume of the liquid passing through the tube in one second in the above example. [**Hint :** Take the average of the velocities at the wall and the axis of the tube as the velocity of the flow.]

Solution :

$$v = \frac{p}{4\eta l}(r^2 - x^2)$$

$\therefore$ At the axis ($x = 0$), velocity $v = \frac{pr^2}{4\eta l}$

At the wall ($x = r$), velocity $v = 0$

$\therefore$ Average velocity $= \frac{pr^2}{8\eta l}$

Now, the volume of the liquid passing through the tube per second is,

V = (velocity) (area of cross-section)

$$= \left(\frac{pr^2}{8\eta l}\right)(\pi r^2)$$

$$\therefore V = \frac{\pi p r^4}{8\eta l}$$

[**Note :** This equation is called **Poiseiulle's Law**]

Illustration 11 : The radius of a pipe decreases according to $r = r_0 e^{-\alpha x}$; where $\alpha = 0.50 \text{ m}^{-1}$ and x is the distance of a cross-section from the first end ($x = 0$). Find the ratio of Reynolds number for two cross-sections lying at the distance of 2 m from each other. (take $e = 2.718$)

Solution : Reynolds number $N_R = \frac{\rho v D}{\eta}$

$\therefore$ For a given liquid $N_R \propto vD$

$$\therefore \frac{(N_R)_1}{(N_R)_2} = \frac{v_1}{v_2} \times \frac{D_1}{D_2} \quad (1)$$

According to the equation of continuity,

$$A_1 v_1 = A_2 v_2$$

$$\therefore \pi r_1^2 v_1 = \pi r_2^2 v_2$$

$$\therefore \frac{v_1}{v_2} = \left(\frac{r_2}{r_1}\right)^2 = \left(\frac{D_2}{D_1}\right)^2$$

From eqns. (1) and (2),

$$\frac{(N_R)_1}{(N_R)_2} = \left(\frac{D_2}{D_1}\right)^2 \times \frac{D_1}{D_2} = \frac{D_2}{D_1} = \frac{r_2}{r_1} = \frac{r_0 e^{-\propto x_2}}{r_0 e^{-\propto x_1}}$$

$$\frac{(N_R)_1}{(N_R)_2} = e^{-\propto(x_2 - x_1)} = e^{-(0.5)(2)} = e^{-1}$$

$$= 0.368$$

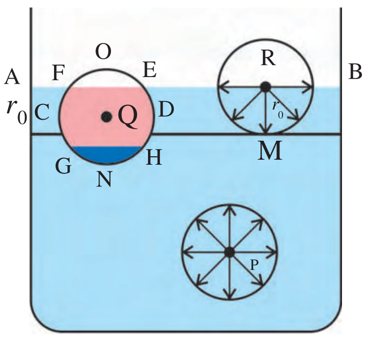
5.12 Surface Energy and Surface Tension

You must have noted that glass becomes wet due to water but lotus or lotus-leaves do not get wet by water. In lamp oil rises up against gravitational force. Some insects can walk on the water surface. If a needle is carefully placed horizontally, on the water surface, it floats. For such phenomena the property called surface tension of liquid is responsible. Due to surface tension liquid surface behaves like a stretched membrane. It is an exclusive property of liquids.

5.12.1 Surface energy :

The inter-molecular attractive force between the molecules of the same substance is called **cohesive force.** The attractive force between the molecules of different substances is known as **adhesive force.**

The maximum distance upto which two molecules can exert attractive force on each other is called the '**range of inter-molecular force (r_0).** An imaginary sphere of radius r_0, drawn by taking any molecule as the centre, is called the **sphere of the molecular action of the molecule at the centre.** Only the molecules inside this sphere can exert attractive forces on the molecule at the centre. The molecules outside this sphere will not exert forces of attraction on the molecule at the centre.



Spheres of molecular action

Figure 5.24

To understand the surface effect produced due to the inter-molecular forces, consider three molecules P, Q and R of a liquid along with their spheres of molecular action as shown in Figure 5.24.

Suppose the range of inter-molecular force is r_0. AB shows the free surface of the liquid. The sphere of action of molecule P is completely immersed in the liquid. Therefore, it is fully occupied uniformly with the molecules of the liquid. As a result P is acted upon by equal forces of attraction from all sides. The resultant force on P will thus be zero and it remains in equilibrium. All molecules at depths more than r_0 from the free surface of the liquid will be in similar situation.

The depth of molecule Q is less than r_0. Part FOEF of its sphere of action is outside the liquid and this part contains the molecules of both air and liquid vapour. The densities of air and vapour are much less than that of the liquid. Moreover, the adhesive forces acting between the molecules of air and liquid are comparatively feeble. Hence the resultant force due to the molecules in the GNHG part is more than the resultant upward force due to the molecules of air and vapour in the similar region FOEF. The number of molecules of the liquid in the regions CDHG and CDEF is equal. Hence the resultant force on Q due to molecules in these regions is zero. Thus, molecule Q is under the influence of resultant downward force. **A layer of thickness r_0 below the free surface of a liquid is called the surface of the liquid.** Thus, the resultant force on the molecules of the liquid in its surface is in vertically downward direction. As we move upwards in the surface, the magnitude of the downward resultant force keeps on increasing. The resultant force on the molecules on the free surface AB is maximum. Hence the molecules of liquid lying in the surface have a tendency to go inside the body of the liquid.

In these circumstances, some of the molecules do go down and as a result of this the density below he surface of the liquid increases. Hence, more than a certain number of molecules will not be able to go down. As a result the density of the liquid below the surface is more and it decreases gradually as we move upwards in the surface. In other words, the inter-molecular distances between the molecules are less below the surface while within the surface these distances are more. Taking the inter-molecular forces as a function of inter-molecular distances it can be proved that inter-molecular distance being more in the surface, the molecules lying in it experience force of tension parallel to the surfce.

Thus, the surface of a liquid has a tendency to contract like stretched elastic membrane and as a result tension prevails in the surface (parallel to the surface). The magnitude of this tension is given by a physical quantity known as surface tension.

"The force exerted by the moleculs lying on one side of an imaginary line of unit length, on the molecules lying on the other side of the line, which is perpendicular to the line and parallel to the surface is defined as the surface tension (T) of the liquid."

$$\therefore \text{ Surface tension } T = \frac{F}{L} \quad (5.12.1)$$

$$\therefore \; F = TL \quad (5.12.2)$$

The SI unit of surface tension is N m^{-1}.

It should be noted that the force of surface tension is not the resultant cohesive force between the molecules on the surface of a liquid. In fact, the resultant cohesive force on the molecules acts in a direction perpendicular to the surface and towards the inside of the liquid, while the force of surface tension is parallel to the surface.

For a line (imaginary) of unit length in the middle of the surface of a liquid, the molecules on both the sides of it exert forces which are equal in magnitude and opposite in direction. Hence, the effect of force of surface tension is not felt in the middle of the surface. At the edge of the surface there are no molecules on the other side of the edge. Hence, surface tension manifests there, parallel to the surface and perpendicular to the edge in the inward direction.

Surface tension in context of potential energy :

We have seen that the molecules in the surface of a liquid have a tendency to go down inside the liquid. This behaviour can be explained on the basis of potential energy of the molecules. If a molecule like P, in Fig 5.24 is to be brought up in the surface, work has to be done on it against the downward force acting on it during this. Hence, when such a molecule reaches the surface it acquires potential energy. This fact shows that the potential energy of the molecules in the surface is more than that of the molecules beneath the surface. **Now a system always tries to remain in such a state where its potential energy is minimum.** Therefore, molecules in the surface of a liquid have a tendency to reduce their potential energy and so the surface of a liquid has a tendency to contract in such a way that its area becomes minimum.

The magnitude of the surface tension can also be given in the context of the potential energy of the molecules. We have noted that work has to be done in bringing the molecules from within the liquid to the surface, which is stored in the form of its potential energy. An important point to be noted is that the molecule thus coming to the surface does not occupy a place between two molecules already present in the surface. The molecules reaching to the surface generate new surface, which means that the surface gets expanded. The whole surface of a liquid can be considered to have been generated in this way. Thus, the molecules in the surface of a liquid possesses potential energy equal to the work done on them in bringing them to the surface.

"The potential energy, stored in the surface of a liquid, per unit area, is known as surface tension (T) of the liquid."

According to this definition the unit of surface tension $T = \frac{E}{A}$

According to this definition the unit of surface tension is J m^{-2}.

$$\text{Now, } \frac{\text{joule}}{m^2} = \frac{\text{newton meter}}{\text{meter}^2} = \frac{\text{newton}}{\text{meter}}$$

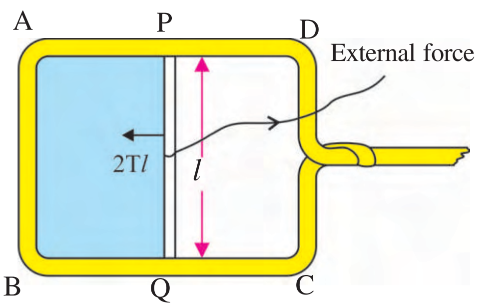
Thus, the unit obtained from both the definitions are the same. Surface tension of a liquid depends on the type of the liquid and its temperature. The surface tension decreases with increace in temperature and at critical temperature surface tension becomes zero. Also, the surface tension of a liquid depends on the type of the medium it is in contact with.

Surface-energy : Suppose, surface tension of a given liquid at a given temperature is T. If the surface of liquid is to be increased by unit at constant temperature, the work required to be done on it is T. But we know that when a surface expands, its temperature decreases. Hence, if surface is to be expanded at constant temperature, heat should be supplied from outside during its expansion. Thus, by increasing the surface of liquid by unit, the new surface so formed gets thermal energy over and above the potential energy (=T).

∴ Total surface-energy per unit area = Potential energy (surface tension) + Heat energy.

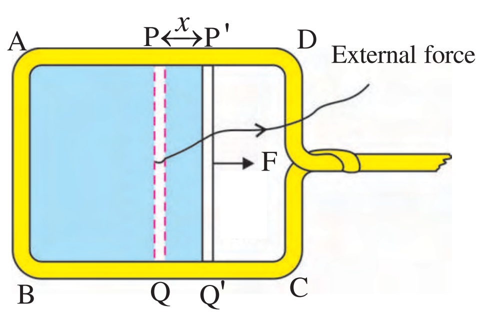
Thus, at any temperature the value of surface energy is more than surface tension. The surface tension and surface energy both decrease with increase in temperature and at critical temperature they become zero.

Our discussion so far have been phenomenological. Now, we experimentally verify the conclusions of this discussion. For this, concentrate on a rectangular frame ABCD made from a wire as shown in Fig. 5.25. The wire PQ is able to slide without friction over the sides AD and BC of the frame. A light string is tied to PQ.



(*a*)

A thin film of liquid formed on a rectangular frame



(*b*)

Expansion of the film

Figure 5.25

If the frame is dipped and taken out of the soap solution then, pulling the wire PQ properly with the help of the string, a thin film ABQP is formed on the frame. If the string is released, PQ is found to slide towards AB, and the film contracts.

This experiment shows that the surface tension manifests itself on the edge of the surface of the liquid, perpendicular to the edge and parallel to the surface.

Again prepare the film ABQP. Pull the wire (with the help of the string) by a force, which is slightly more than the force of surface tension acting on it and displace it by x. The work done for these can be calculated as under.

Suppose the surface tension of the solution is T and the length of wire PQ is l.

Hence, the force acting on the wire due to surface tension is = 2Tl.

As the film has two free surfaces, '2' appears in the equation of force.

Applied external force F = 2Tl (5.12.4)

Now, work W = external force × displacement.

∴ W = 2Tlx

But, increase in the area of the surface of the film = ΔA = 2lx. (5.12.5)

∴ W = TΔA

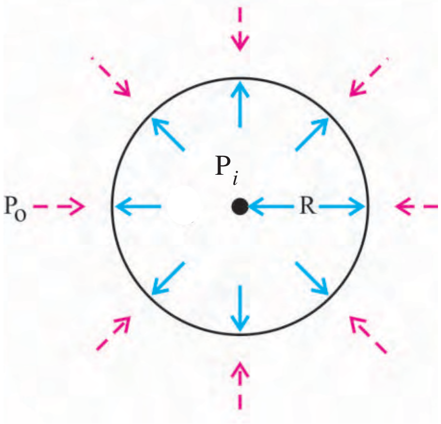
If ΔA = 1 unit, W = T

∴ Work done to increase the area of the surface by 1 unit is equal to the measure of surface tension.

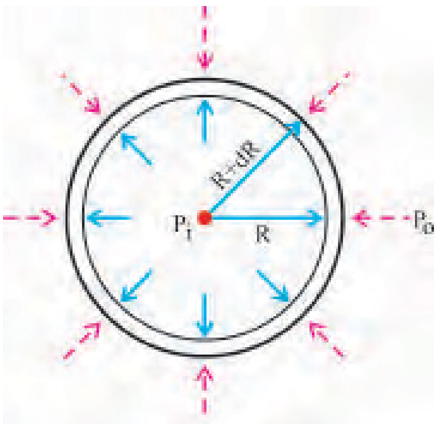
5.13 Drops and Bubbles

Small drops of liquid or bubbles are always spherical. Obvious question coming to mind is that why they should be of spherical in shape only ? Due to surface tension free surface of liquids have a tendency to make its surface area as small as possible. Since spherical surface has minimum area for a given volume, small drops of liquid are always spherical.

The surface of a drop or a bubble are curved. The pressure on a concave surface is always more than that on the convex surface. Hence, the pressure inside a drop or a bubble is always more than the pressure outside.



(a)



(b)

Bubble

Figure 5.26

Consider a bubble of radius R, in air as shown in Figure 5.26. The pressures inside and outside are P_i and P_0 respectively. Here, $P_i > P_o$. Let the surface tension of the liquid (solution) forming the wall of the bubble be T.

Suppose, by blowing the bubble its radius increases from R to (R + *dR)*. (Figure 5.26). Hence, the area of its free surface increases from S to S + *dS*. The work done in this process can be calculated in two different ways :

(1) While blowing a bubble, the force exerted on its surface of area $4\pi R^2$, due to the pressure difference $(P_i - P_0)$, is $(P_i - P_0)\ 4\pi R^2$. And the surface displaces by an amount dR under the influence of this force.

∴ Work done on the surface is

$$W = \text{force} \times \text{displacement}$$

$$= (P_i - P_0)\ 4\pi R^2 \cdot dR \quad (5.13.1)$$

(2) The surface area of the bubble of radius R is, $S = 4\pi R^2$

Now, when the radius becomes $(R + dR)$, the increase in the surface area is,

$$dS = 8\pi R dR$$

But the bubble in air has two free surfaces.

∴ Total increase in its area $= 2 \times 8\pi R dR$

$= 16\pi R dR$

Hence, the work required to be done on the surface is,

W = surface tension × total increase in area.

$$\therefore\ W = 16\pi T R dR \quad (5.13.2)$$

Comparing equation (5.13.1) with equation (5.13.2),

$$4\pi(P_i - P_0)R^2\ dR = 16\pi T R dR$$

$$\therefore\ P_i - P_0 = \frac{4T}{R} \quad (5.13.3)$$

If a bubble is formed in a liquid, it has only one free surface.

$$\therefore\ P_i - P_0 = \frac{2T}{R} \quad (5.13.4)$$

Note : A drop of liquid has one free surface and so the pressure difference for the drop can be obtained from equation (5.13.4).

Illustration 12 : Find the pressure in a bubble of radius 0.2 cm formed at the depth of 5 cm from the free surface of water. The surface tension of water is 70 dyn cm^{-1} and its density is 1 g cm^{-3}. Atmospheric pressure is 10^6 dyn cm^{-2}. The gravitational acceleration is 980 cm s^{-2}.

Solution :

h = 5 cm

R = 0.2 cm

T = 70 dyne cm^{-1}

ρ = 1 g cm^{-3}

P = atmospheric pressure

= 10^6 dyne cm^{-2}

g = 980 cm s^{-2}

If the pressures inside and outside of the air bubble formed in water are P_i and P_0 respectively,

$$P_i - P_0 = \frac{2T}{R}$$ (A bubble in water has one free surface only)

$$\therefore\ P_i = P_0 + \frac{2T}{R} \quad (1)$$

But P_0 = atmospheric pressure + pressure due to water column of height h.

$\therefore\ P_0 = P + h\rho g$ (2)

from equations (1) and (2),

$$P_i = P + h\rho g + \frac{2T}{R}$$

$$= 10^6 + (5 \times 1 \times 980) + \frac{2 \times 70}{0.2}$$

$$= 10^6 + 4900 + 700$$

$$P_i = 1.0056 \times 10^6 \text{ dyn cm}^{-2}$$

Illustration 13 : When a hollow sphere having a hole in it is taken to the depth of 40 cm from the surface of water, water starts entering into the sphere. If the surface tension of water is 70 dyn cm^{-1}, find the radius of the hole. Take g = 10 ms^{-2}.

Solution : Let the radius of the hole be r. Here, the depth of the sphere is h = 40 cm. At this depth pressure of water is = $h\rho g$ = 40 × 1 × 1000 = 40000 dyn cm^{-2}.

When water enters into the sphere, bubble having the radius same as the radius of the hole comes out of it. The excess pressure inside the bubble $= \frac{2T}{r} = \frac{2 \times 70}{r}$

$\therefore$ In the state of equilibrium, $h\rho g = \frac{2T}{r}$

$$\therefore\ 40000 = \frac{2 \times 70}{r}$$

$$\therefore\ r = 3.5 \times 10^{-3} \text{ cm}$$

Illustration 14 : n droplets, each of radius r, merge to form a bigger drop of radius R. If the surface tension of the liquid is T, find the energy released.

Solution : Total volume of n droplets of radius r = Volume of a drop of radius R.

$$\therefore\ \left(n\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3$$

$$\therefore\ nr^3 = R^3 \quad (1)$$

Total surface area of n drops, $A_1 = n(4\pi r^2)$

and the area of one large drop $A_2 = 4\pi R^2$

$\therefore$ The decrease in the area = ΔA

$$= A_1 - A_2 = n \cdot 4\pi r^2 - 4\pi R^2$$

$$= 4\pi(nr^2 - R^2)$$

$\therefore$ Energy released, $W = T\Delta A = 4\pi T (nr^2 - R^2)$ (2)

(To obtain result (2) it is not necessary to obtain result (1), but to represent the result (2) in a following specific form result (1) is necessary.)

$$W = T\Delta A = 4\pi TR^3\left(\frac{nr^2 - R^2}{R^3}\right)$$

$$= 4\pi TR^3\left(\frac{nr^2}{nr^3} - \frac{R^2}{R^3}\right)$$

$$= 4\pi TR^3\left(\frac{1}{r} - \frac{1}{R}\right) \quad (3)$$

Illustration 15 : Two soap bubbles of radii R_1 and R_2 merge to form a bubble of radius R. If the pressure of atmosphere is P and surface tension of the soap solution is T, prove that,

$$P(R_1^3 + R_2^3 - R^3) = 4T(R^2 - R_1^2 - R_2^2)$$

Assume that the temperature remains constant during this process.

Solution :

Pressure inside the first bubble $= P_1$

$$= P + \frac{4T}{R_1}$$

Pressure inside the second bubble $= P_2$

$$= P + \frac{4T}{R_2}$$

And the pressure inside the compound bubble $= P_3 = P + \frac{4T}{R}$

Here, P = pressure outside each bubble = atmospheric pressure (which is the same for all)

If the volumes of these bubbles are V_1, V_2 and V_3 respectively,

$$V_1 = \frac{4}{3}\pi R_1^{\,3};\ V_2 = \frac{4}{3}\pi R_2^{\,3};\ V_3 = \frac{4}{3}\pi R^3$$

As temperature is constant, according to Boyle's Law,

$$P_1V_1 + P_2V_2 = P_3V_3$$

$$\therefore \left(P + \frac{4T}{R_1}\right)\left(\frac{4}{3}\pi R_1^{\,3}\right) + \left(P + \frac{4T}{R_2}\right)\left(\frac{4}{3}\pi R_2^{\,3}\right)$$

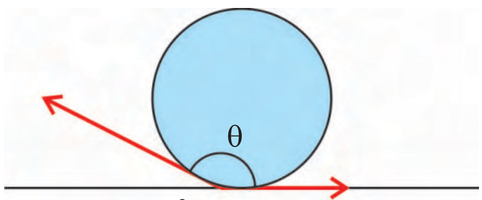
$$= \left(P + \frac{4T}{R}\right)\left(\frac{4}{3}\pi R^3\right)$$

$$\therefore \frac{4}{3}\pi P(R_1^{\,3} + R_2^{\,3} - R^3) = \frac{4}{3}\pi \times 4T(R^2 - R_1^{\,2} - R_2^{\,2})$$

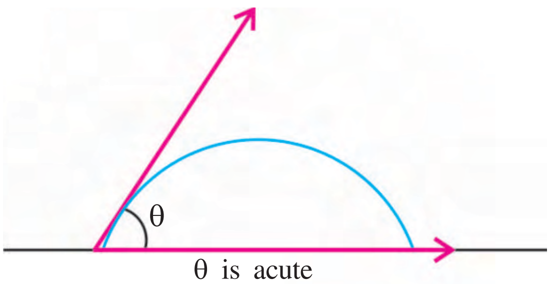
$$P\,(R_1^{\,3} + R_2^{\,3} - R^3) = 4T(R^2 - R_1^{\,2} - R_2^{\,2})$$

5.14 Angle of contact

You must have observed a dew-drop. It is spherical. Any liquid when comes in contact with another medium its surface is curved. Consider a liquid drop as shown in Figure 5.27(*a*) and 5.27(*b*) to understand it better.



(*a*)



(*b*)

Angle of contact

Figure 5.27

The angle between tangent to the liquid surface at the point of contact and solid surface inside the liquid is called the angle of contact. Angle of contact depends on types of liquid and solid which are in contact.

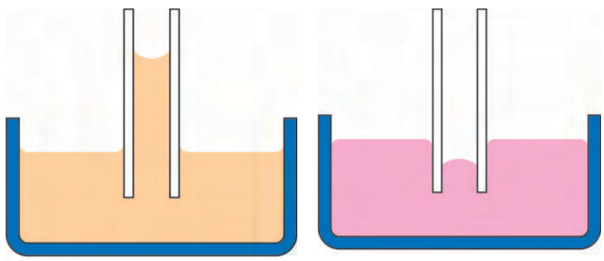
If the angle of contact is less than 90°, the liquids wet the solid, sticks to the solid and rises up in the capillary of that solid.

If the angle of contact is more than 90°, the liquid does not wet the solid, does not stick to the solid and falls in the capillary of that solid.

If a water droplet is in contact with lotus-leaf (Figure 5.27(*a*)) angle of contact is obtuse. If water is in contact with glass (Figure 5.27(*b*)) angle of contact is acute.

5.15 Capillarity

The phenomenon of rise or fall of a liquid in a capillary (held vertical in a liquid) is called capillarity. In this phenomenon, the surface tension of the liquid plays an important role.

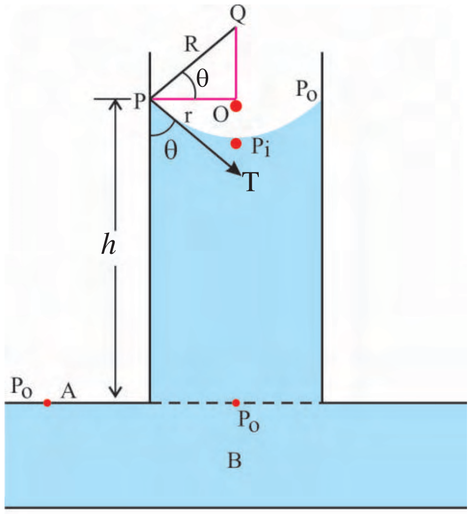


Water (*a*) Mercury (*b*)

Phenomenon of capillarity

Figure 5.28

As shown in Figure 5.28(*a*) when a glass capillary (having a small bore) is held vertical in water, water rises in the capillary. Whereas (as shown in Figure 5.28(*b*) when a capillary is held vertical in mercury, mercury falls in the capillary. Also, note that water wets the glass while mercury does not. If you observe attentively you will find that the free surface (meniscus) of the water rising in the capillary is concave, while the free surface of mercury falling in the capillary is convex.



Column of liquid in a capillary

Figure 5.29

As shown in Figure 5.29 suppose a capillary of radius r is held vertical in liquid and liquid rises to height h in the capillary. The radius of the concave meniscus of the liquid in capillary is R.

The relation between the radius of curvature of meniscus (R) and the radius of capillary (r) may be obtained as follows :

From the geometry of Figure 5.29 $\angle OPQ = \theta$ in ΔOPQ,

$$\therefore \cos\theta = \frac{OP}{PQ}$$

$$= \frac{\text{Radius of the capillary } (r)}{\text{Radius of the meniscus (R)}}$$

$$\therefore R = \frac{r}{\cos\theta} \qquad (5.15.1)$$

Now, the liquid shown in the figure is in equilibrium. Let the pressure on the concave surface of the meniscus be P_o and that on its convex surface be P_i. Here, $P_o > P_i$ and $P_o - P_i = \frac{2T}{R}$ ($\because$ The liquid has one free surface.) (5.15.2)

Note that P_o is atmospheric pressure. The same pressure acts on plain surface of the liquid at point A and also at point B in the same horizontal level, with A.

Pressure at point B is, $P_o = P_i + h\rho g$

Here, ρ is the density of the liquid and g is gravitational acceleration.

$$\therefore P_o - P_i = h\rho g \qquad (5.15.3)$$

Comparing eqns. (5.15.2) and (5.15.3)

$$\frac{2T}{R} = h\rho g$$

$$\therefore T = \frac{Rh\rho g}{2}$$

Substituting the value of R from eqn. (5.15.1),

$$T = \frac{rh\rho g}{2\cos\theta} \qquad (5.15.4)$$

Using this equation T can be found.

From this equation $h = \frac{2T\cos\theta}{r\rho g}$

(i) If $\theta < 90^o$, $\cos\theta$ is positive and this equation gives h as positive. ∴ The liquid rises up in the capillary (e.g. glass-water)

(ii) If $\theta < 90^o$, $\cos\theta$ is negative and this equation gives h as negative. ∴ The liquid falls in the capillary. (e.g. glass-mercury)

In this case meniscus is convex. Also, $P_i > P_o$. Thus in eqn. (5.15.2) $P_i - P_o = \frac{2T}{R}$ should be taken. As $P_i - P_o = h\rho g$, the final result in eqn. (5.15.4) will not change.

When a detergent or soap dissolves in water, surface tension of the solution becomes lesser than that of water. Due to this washing ability increases.

Illustration 16 : Radius of a glass capillary is 0.5 mm. Find the height of the column of water when it is held vertical in water. The density of water is 10^3 kg m^{-3} and the angle of contact between glass and water is 0^o, $g = 9.8$ ms^{-2} and the surface tension of water is T = 0.0727 Nm^{-1}.

Solution :

$r = 0.5$ mm $= 5 \times 10^{-4}$ m

$\rho = 103$ kg m^{-3}

$\theta = 0^o$ ∴ $\cos 0^o = 1$

$g = 9.8$ ms^{-2}

T = 0.0727 Nm^{-1}

$$T = \frac{rh\rho g}{2cos\theta}$$

$$\therefore \ h = \frac{2Tcos\theta}{r\rho g}$$

$$= \frac{2 \times 0.0727 \times 1}{5 \times 10^{-4} \times 10^{3} \times 9.8}$$

... $h = 0.0296$ m $= 2.96$ cm

Illustration 17 : Two rectangular slides of glass are kept 1 mm apart. They are partially immersed in water in such a way that air column (as well as that of water) may remain vertical between them as shown in Figure 5.30. What is the height of the water which rises between the plates ?

$T = 70$ dyn cm^{-1}.

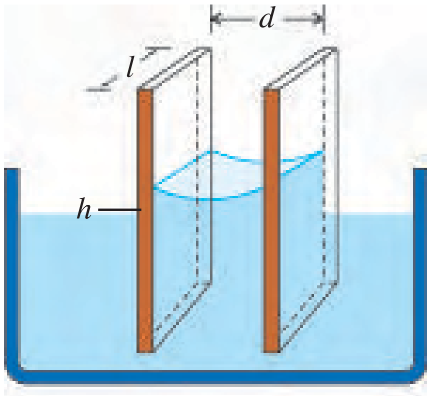


Figure 5.30

Solution : Suppose that the breadth of the glass slides is l. In this state total length at which water and glass are in contact is $2l$. The angle of contact between water and glass is zero. Suppose water rises to height h cm.

$\therefore$ Volume of the column of water rising up $= ldh$.

where d = distance between two plates.

If the density of water is ρ and gravitational acceleration is g, the weight of water in downward direction $= (ldh)\,\rho g$. This force should be equal to the force of surface tension acting on length $2l$.

$\therefore \ 2Tl = (ldg)h\rho$

$$h = \frac{2T}{dg\rho} = 1.43 \text{ cm}$$

SUMMARY

1. A substance that can flow is known as a fluid.
2. Magnitude of force acting perpendicularly to a surface of unit area is called pressure. It is a scalar quantity. Its unit is Nm^{-2} or (P_a).
3. If force acting on surface makes angle θ with the normal drawn to a surface, the component $Fcos\theta$ is taken into account for the pressure. Thus pressure $P = \frac{Fcos\theta}{A}$.
4. Ratio of mass of the body to its volume is called density. Its unit is kg m^{-3}.
5. Ratio of density of a substance and density of water at 277K is called specific density. It is dimensionless.
6. **Pascal's Law :** If the effect of gravity is neglected, pressure in fluid is same everywhere.
7. **Pascal's Law of transmission of pressure :** In an enclosed liquid, if pressure is changed in any part of the liquid, the change is transmitted equally to all the parts of the liquid.
8. Hydraulic lift, hydraulic brake, door closer and shockabsorbers of automobiles work on the Pascal's Law.
9. In a fluid rate of change in pressure with depth is ρg.

10. Pressure at the bottom of incompressible fluid column is $h\rho g$.

11. Pressure due to fluid column does not depend on shape and area of the container.

12. **Archimedes' Principle :** When a body is partially or completely immersed in a liquid, the buoyant force acting on it is equal to the weight of liquid displaced by it and it acts in the upward direction at the centre of mass of the displaced liquid.

13. **Law of Floatation :** When the weight of a body is equal to the weight of the liquid displaced by the part of the body immersed in it the body floats on the surface of the liquid.

14. **Steady Flow :** If in a fluid flow velocity of fluid particle remains constant with time, fluid flow is called steady flow.

15. **Turbulent flow :** If in a flow of fluid velocity of fluid particle changes in an irregular manner from time to time and from point to point, the flow is known as turbulent flow.

16. **Irrotational flow :** If an element of a fluid at each point has no net angular velocity about that point, the fluid flow is called irrotational.

17. **Incompressible flow :** If density of fluid remains constant with time everywhere, the flow is said to be incompressible.

18. **Non-viscous flow :** The flow of a fluid having small co-efficient of viscosity is known as non-viscous flow.

19. Flow of an ideal fluid is steady, irrotational, incompressible and non-viscous.

20. **Line of flow :** Path along which particle moves in a fluid is called a line of flow.

21. **Streamline :** The curve for which tangent drawn at any point shows the direction of velocity of fluid particle is known as streamline.

22. **Tube of flow :** An imaginary tube formed by a bundle of streamlines is called tube of flow.

23. **Volume flux :** Volume of fluid flowing through any cross section in unit time is called volume flux. It is equal to the product of area of cross section and velocity.

24. **Dynamic lift :** When an object undergoes relative motion with respect to fluid, a force arises which diverts the object from its original path. This phenomenon is known as dynamic lift.

25. **Aerofoil :** An object when travels horizontally experiences force in upward direction due to its shape is called an aerofoil.

26. **Force of viscosity :** In a laminar flow, any two consecutive layers of fluid have relative velocity between them. As a result, a resistive force is produced tangentially at the surfaces of the layers in contact. This force is known as viscous force.

27. **Velocity gradient :** In a laminar flow, the difference in velocity between two layers of liquid per unit perpendicular distance, in the direction perpendicular to the direction of flow, is called velocity gradient. Its unit is s^{-1}.