CHAPTER

02

Linear Equations in Two Variables

Reema asked Salma:- If one-fourth portion of a pole is painted blue, one third red and the remaining 10 meters are painted black, what is the height of the pole?

Salma said:- We learnt about linear equations in one variable in a previous class. We know that in such situations we make linear equations in one variable and solve them to find the values of the variables (unknowns).

Reema:- Good! So, how will we determine the length of the pole?

Salma:- If we assume that the total length of the pole is x meters, then

length of the blue part = $\frac{x}{4}$ meter

length of the red part = $\frac{x}{3}$ meter

length of the black part = 10 meter.

Therefore, totals length of the pole = length of the blue part + length of the red part + length of the black part

$$x = \frac{x}{4} + \frac{x}{3} + 10$$

$$x = \frac{3x + 4x + 120}{12}$$

$$12x = 7x + 120$$

$$12x - 7x = 120$$

$$5x = 120$$

$$x = \frac{120}{5}$$

$$x = 24 \text{ meters}$$

So, total length of the pole is 24 meters.

Can you tell what will be the length of the blue and red parts of the pole?

Salma and Reema discussed many other problems with each-other and tried to solve them.

Example-1. Salma said to Reema:- My number is two more than your number and the sum of our numbers is 14. Can you find our numbers?

Solution: Reema;- Let us assume that my number = x

Then, your number will be $x + 2$

$\because$ The sum of the two numbers is 14

$\therefore$ $x + x + 2 = 14$

$\Rightarrow$ $2x + 2 = 14$

$\Rightarrow$ $2x = 14 - 2$

$\Rightarrow$ $2x = 12$

$\Rightarrow$ $x = \dfrac{12}{2}$

$\Rightarrow$ $x = 6$

This means that my number is 6 and your number is 8.

Example-2. What is the value of each of the internal angles of the triangle given below?

Solution: $\because$ The sum of internal angles of a triangle is 180°

$\therefore \angle A + \angle B + \angle C = 180°$

$x° + (x + 40)° + (x + 20)° = 180°$

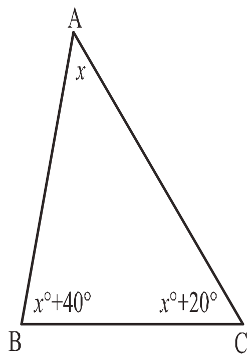
$\Rightarrow$ $3x° + 60° = 180°$

$\Rightarrow$ $3x° = 180° - 60°$

$\Rightarrow$ $3x° = 120°$

$\Rightarrow$ $x° = \dfrac{120°}{3}$

$\Rightarrow$ $x° = 40°$



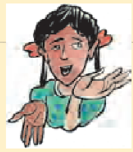
$\therefore$ the value of the internal angles of the given triangles are as follows:

$\angle A = x° = 40°$,

$\angle B = x° + 20° = 40° + 20° = 60°$

$\angle C = x° + 40° = 40° + 40° = 80°$

Try this



1. A bag contains 50 paisa coins. Find the number of coins in the bag if the total money in the bag is Rs. 50.
2. If one of the internal angles of a right angle triangle is 60° then find the value of the other angles.
3. If father's age is twice the age of his son, then what are their current ages.

Forming equations

Salma and Reema also discussed some other types of questions.

I have Rs. 1 and 50 paisa coins in my bag. If there are 100 coins in all, then find the number of 50 paisa coins and the number of Rs. 1 coins?

Here, we have two different types of coins and there numbers are different. We don't know the number of any of the coins therefore both have to be shown by unknowns. So, we will say that the number of 50 paisa coins is x and the number of Rs. 1 coins is y. We know that the total number of coins in the bag is 100 which means that

$$x + y = 100$$

But we still can't say how many Rs. 1 and 50 paisa coins are there.

Let us see some more examples where we are able to form equations but are not able to find their solutions.

Example-3. There are some deer and some cranes in the forest. The total number of legs is 180. How many deer and how many cranes are there?

Solution: Let the number of deer = x

Let the number of cranes = y

Since, one deer has 4 legs

Therefore, the number of legs of deer = $4x$

Since, one crane bird has 2 legs

Therefore, the number of legs of cranes = $2y$

According to the statement,

Legs of deer + Legs of cranes = 180

That is, $4x + 2y = 180$

Example-4. The cost of one copy and two pencils is Rs.45.

Solution: Let the cost of one copy = Rs. x

Let the cost of one pencil = Rs. y

Then, the cost of one copy + the cost of two pencils = Rs. 45

$$x + 2y = 45$$

Try this

Try to form equations for the following statements

1. The sum of any two numbers is 8.
2. The difference in ages of Shashank and his father is 30 years.
3. A bag has of Rs. 1 and Rs. 5 coins. The total number of coins is 100.
4. The cost of three pens and four copies is Rs. 105.
5. A farm has some hens and some cows and the number of legs is 60.

Simultaneous equations

In the above examples, we were able to form equations but not able to find their solutions.

Let us now discuss the following situations.

A father distributed Rs. 8 among his son, Saurabh and daughter, Santosh. Can we find out how much Saurabh got and how much Santosh got?

If Saurabh got Rs. x and Santosh got Rs. y then the equation will be as follows:

$x + y = 8$ ....(1)

On the basis of this equation, we can say that if Saurabh got Rs. 1 then Santosh got Rs. 7, if Saurabh got Rs. 2 then Santosh got Rs. 6 and so on. We see that if Saurabh got Rs. 7 then Santosh gets Rs. 1. We can summarize the ways of distributing Rs. 8 between Saurabh and Santosh as shown in the table below:

Rupees							
Saurabh	1	2	3	4	5	6	7
Santosh	7	6	5	4	3	2	1

We see that we can't tell how much money Saurabh and Santosh actually got. But suppose we find out that father gave Saurabh three times more money than Santosh then we can write

$x = 3y$ ....(2)

In equation (1), if we put $x = 3y$ then we get the following equation in one variable

$$3y + y = 8$$
$$4y = 8$$
$$y = \frac{8}{4}$$
$$y = 2$$

Putting the value of y in equation (2)

$$x = 3y$$

Then $x = 3 \times 2$

$$x = 6$$

This means that Santosh got Rs. 2 and Saurabh got Rs. 6 which is three times Santosh's money.

Similarly, in example-3, when the total number of legs of cranes and deer was 180 then, our equation was $4x + 2y = 180$ ....(1)

Suppose we know that the sum of eyes of deer and eyes of cranes is 120

That is: $2x + 2y = 120$......................(2) (each deer has 2 eyes and each crane has 2 eyes)

From equation (2)

$2y = 120 - 2x$

Putting this in equation (1)

$$\Rightarrow \quad 4x + 120 - 2x = 180$$
$$\Rightarrow \quad 2x = 60$$
$$\Rightarrow \quad x = \frac{60}{2}$$
$$\Rightarrow \quad x = 30$$

x, i.e. the number of deer is 30. Putting this value of x in equation (1)

$$\Rightarrow \quad 4(30) + 2y = 180$$

$$\Rightarrow \quad 120 + 2y = 180$$

$$\Rightarrow \quad 2y = 180-120$$

$$\Rightarrow \quad 2y = 60$$

$$\Rightarrow \quad y = \frac{60}{2}$$

$$\Rightarrow \quad y = 30$$

y, i.e. the number of crane birds is also 30.

In the above examples we saw that in situations where we had two variables but only one equation, we could only estimate the answers. Once we got the second condition and were able to form the second equation, we were able to arrive at the exact answer.

Think and discuss

Can we find the answers to the problems given below? If not, then why?

1. You are given a parallelogram where in the pair of adjacent angles the value of one angle is 4/5 times that of the other. Find the angles.
2. Some cuckoos and sparrows are sitting on a tree. If the sum of their legs is 36, find the number of cuckoos and the number of sparrows.
3. A fruit-basket contains apples and mangoes. The number of fruits is 39. Another fruit-basket has some mangoes and oranges then find the number of mangoes in the second basket.

Solutions of equations

How can we find the solutions to equations describing different conditions? We need to solve the equations and there are many methods to do so. Let us learn about some of them.

Graphical method

In coordinate geometry or as part of drawing graphs, you have learnt how to depict graphs of equations in two variables. We can draw graphs of equations describing many different conditions and learn about their solutions.

Let us draw the graphs for the equation showing the relation between the legs of cranes and deer and for the equation showing the relation between their eyes. We will see if we can get the solution of the equations from the graph.

For the equation $4x + 2y = 180$ showing the relation between their legs, we will make a table.

$2y = 180 - 4x$

$$y = \frac{180 - 4x}{2}$$

$y = 90 - 2x$ ....(3)

Let us find the corresponding values of y by putting x = 10, 20, 30….etc. in equation (3).

Table-1

x	10	20	30	40
y	70	50	30	10

Similarly, for the equation $2x + 2y$ for eyes

$2y = 120 - 2x$

$$y = \frac{120 - 2x}{2}$$

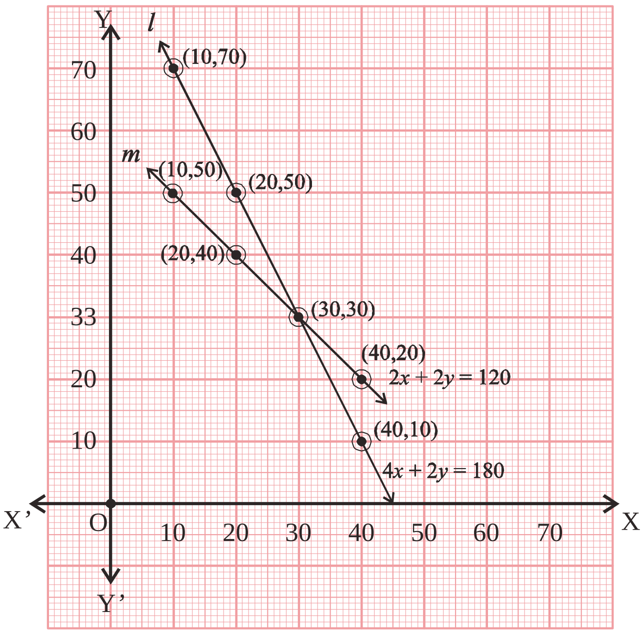
$y = 60 - x$ ....(4)

The values of y obtained by putting x = 10,20, 30…. in equation (4) are given in table-2.

Table-2

x	10	20	30	40
y	50	40	30	20

Now, let us draw a graph using the values given in table-1 and table-2.



Graph-1

We find that the lines in the graph intersect at the point (30, 30). These are the values for the number of deer and number of cranes, as we had previously calculated.

Try this

Draw graphs for the equations $x + y = 8$ and $x = 3y$ and find the solutions.

The questions given below have also been solved using the same method.

Example:-5. 10 students of class X took part in a quiz. The number of girls taking part in the quiz was four more than the boys. Find the number of girls and the number of boys who took part in the quiz.

Solution: Suppose that the number of boys taking part in the quiz was x and the number of girls was y. Then,

Total number of students = number of boys + number of girls

$$10 = x + y$$

$$\text{Or} \quad x + y = 10 \quad \text{..............(1)}$$

Since the number of girls was four more than the boys, we also get the following equation

$$y = x + 4 \quad \text{...............(2)}$$

Now, to draw the graphs for equations (1) and (2), we will make a table of corresponding values of x and y and use these to draw the graph.

By putting $x = 1,2,3\ldots$in equation (1) we will get the corresponding values of y and these are shown in table-1.

Table - 1

(For $x + y = 10$)

x	1	2	3	4	5	6
y	9	8	7	6	5	4

Similarly, by putting x = 1,2,3….in equation (2) we will get the corresponding values of y and these are shown in table-2.

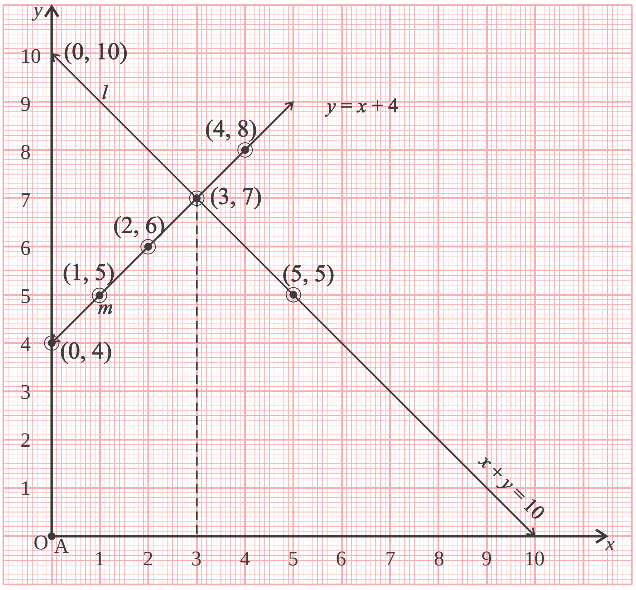
Table - 2

(For $y = x + 4$)

x	1	2	3	4	5	6
y	5	6	7	8	9	10

If we depict the values from table -1 and 2 on graph paper then we get two straight lines l and m. From the graph we can see that these two lines l and m cut or intersect each other at point (3, 7). This point is situated on the straight lines for the two equations.

On this point x = 3 and y = 7 and the values satisfy both the equations. Hence, this is the solution to our problem.



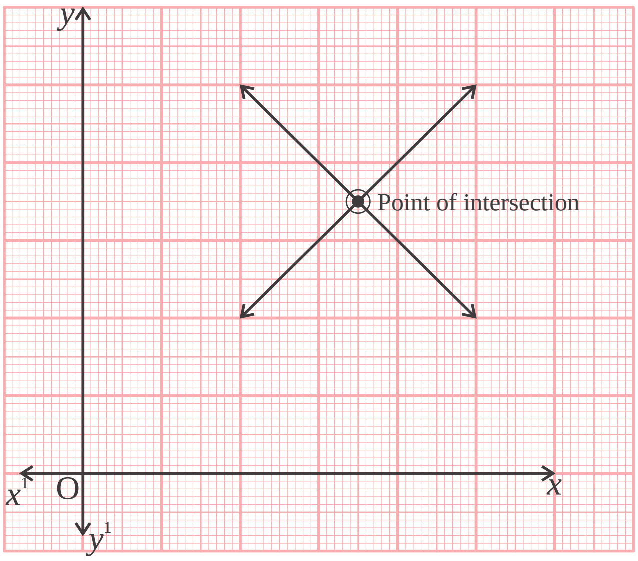
Graph-2

Thus, the number of boys is 3 and the number of girls is 7.

The intersection point of the straight lines depicting the equations is the solution for the equations.

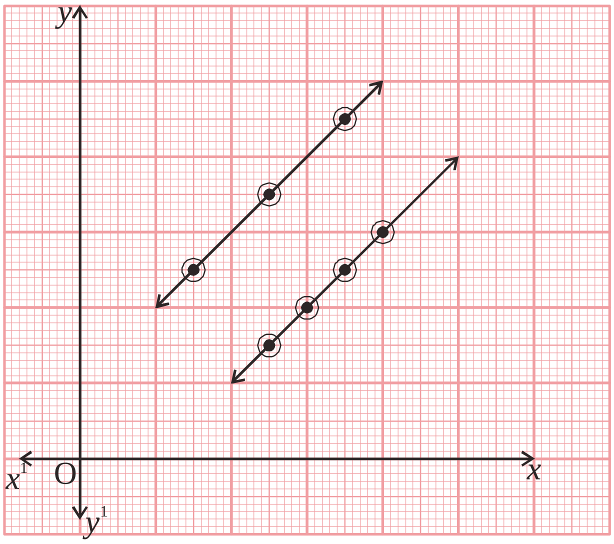
Can we find intersecting lines in all situations? In fact, lines on graphs for equations describing different conditions look very different. Let us understand what this means.

1. When the lines for equations intersect at a point then we say that the equations have a unique solution. The values of x and y at the intersection point is the solution of the equations.



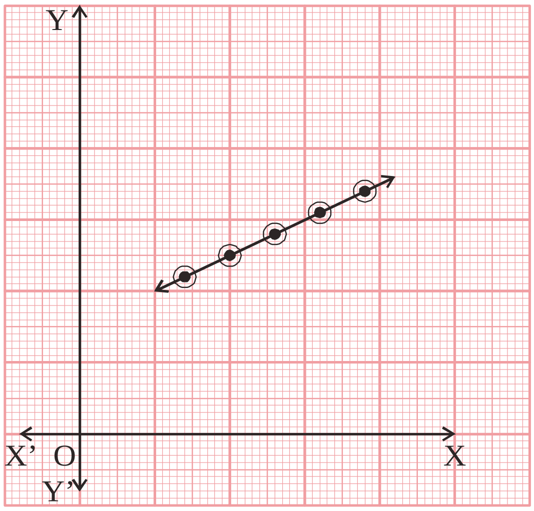
Graph-3

2. When the lines for equations are parallel then the two equations do not have any solution because the equations have no point in common.



Graph-4

3. When the lines for the two equations are co-incident, i.e. they lie over one another, then the equations have infinite number of solutions because in this situation there are infinite points common to the two lines.



Graph-5

The properties of the lines depicting different equations help us understand day to day problems better.

Let us understand through some examples how the properties of lines and graphs described above are helpful in daily-life problems.

Example:6. Kavita purchased 1 pencil and 2 erasers for Rs.4 and Savita bought 2 pencils and 4 erasers for Rs. 16. Can we use this information to find the price paid by Kavita and Savita for one pencil and one eraser?

Solution: Let us assume that the price of one pencil is Rs. x and the price of one eraser is Rs. y. Since, Kavita paid Rs. 4 for one pencil and two erasers we can show this using the following equation:

$$1 \times x + 2 \times y = 4$$

$$x + 2y = 4 \qquad \text{..........(1)}$$

Similarly, Savita paid Rs. 16 to buy 2 pencils and 4 erasers and this can be written in the form of an equation as shown below:

$$2 \times x + 4 \times y = 16$$

$$2x + 4y = 16 \qquad \text{..........(2)}$$

From equation (1)

$$x + 2y = 4$$

Or $\quad 2y = 4 - x$

$$y = \frac{4-x}{2} \qquad \text{...........(3)}$$

Putting x = 0,1,2,3,… in equation (3) will give us corresponding values for y and we will write them in table - 1.

Table-1

x	0	1	2	3	4	5	6
y	2	1.5	1	0.5	0	–0.5	-1

Equation (2) can be re-written as follows:

$$\Rightarrow \quad 2x + 4y = 16$$

$$\Rightarrow \quad 2(x + 2y) = 16$$

$$\Rightarrow \quad x + 2y = 8$$

$$\Rightarrow \quad 2y = 8 - x$$

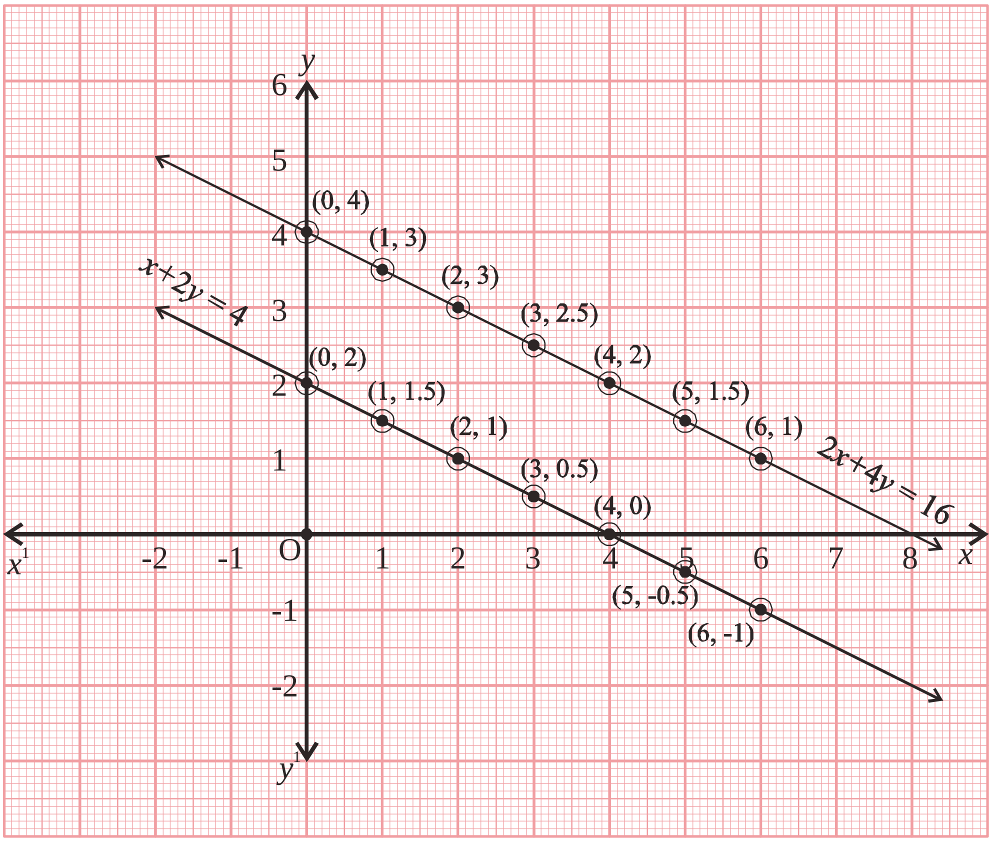
$$\Rightarrow \quad y = \frac{8-x}{2}$$

Putting x = 0,1,2,3,4,5 and 6 in equation (4) will give us corresponding values 4,3.5,3,2.5,2,1.5,1 respectively for y and we will write them in table - 2.

Table-2

x	0	1	2	3	4	5	6
y	4	3.5	3	2.5	2	1.5	1

Using the values in table - 1 and table - 2, we can draw the following graph:



Graph-6

We see that we are getting two parallel lines for the equations then what will be the values for x and y?

We can see that there is no point of intersection therefore the two equations do not have a unique solution. This means that the costs of pencil and eraser purchased by Savita and Kavita are different.

Example:7. A person purchased three chairs and two tables for Rs. 1200 and then paid Rs. 2400 for six chairs and two tables. Then, what is cost of one table and one chair?

Solution: Let the cost of one chair be Rs. x

And the cost of one table be Rs. y.

Then, the cost of three chairs and two tables is $3x + 2y$

According to the problem $3x + 2y = 1200$..............(1)

Similarly, the cost of six chairs and 4 tables is Rs. 2400.

$\Rightarrow \quad 6x + 4y = 2400$

$\Rightarrow \quad 2(3x + 2y) = 2400$

$$\Rightarrow \quad (3x + 2y) = \frac{2400}{2}$$

$$\Rightarrow \quad 3x + 2y = 1200 \quad ....(2)$$

Both the equations are identical and if we draw their graphs we will get coincident lines.

In both equations (1) and (2),

If $x = 100$ then $y = \dfrac{1200 - 3x}{2} = \dfrac{1200 - 3(100)}{2}$

$$y = \frac{900}{2} = 450$$

$x = 200$ then $y = \dfrac{1200 - 3(200)}{2} = \dfrac{1200 - 600}{2}$

$$y = \frac{600}{2} = 300$$

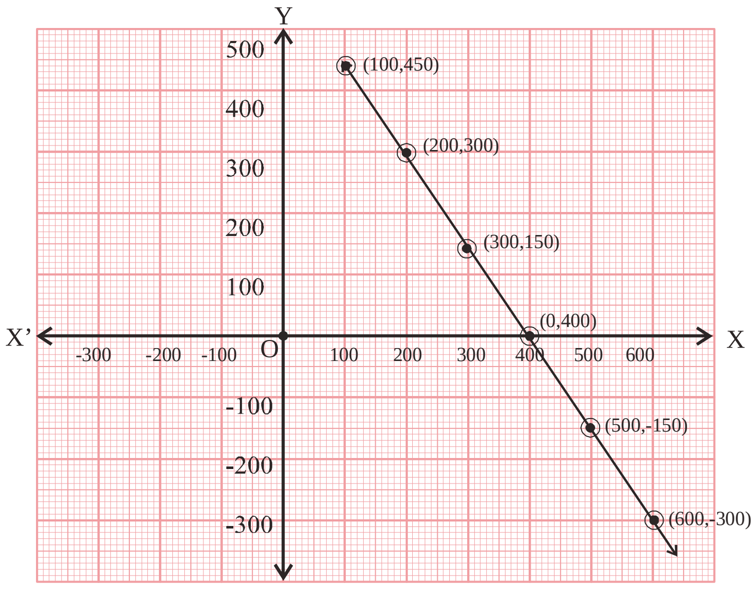
Similarly, we will find the corresponding values of y for different values of x and write then in the form of a table:

x	100	200	300	400	500	600
y	450	300	150	0	–150	–300

This table is common for both the equations therefore if we draw a graph using these values the two lines which we get will be coincident.

Clearly, the values of x and y in the two equations can be called the solutions of the equation system. Since x and y can gave infinite values therefore the given system of equations can have infinite solution.

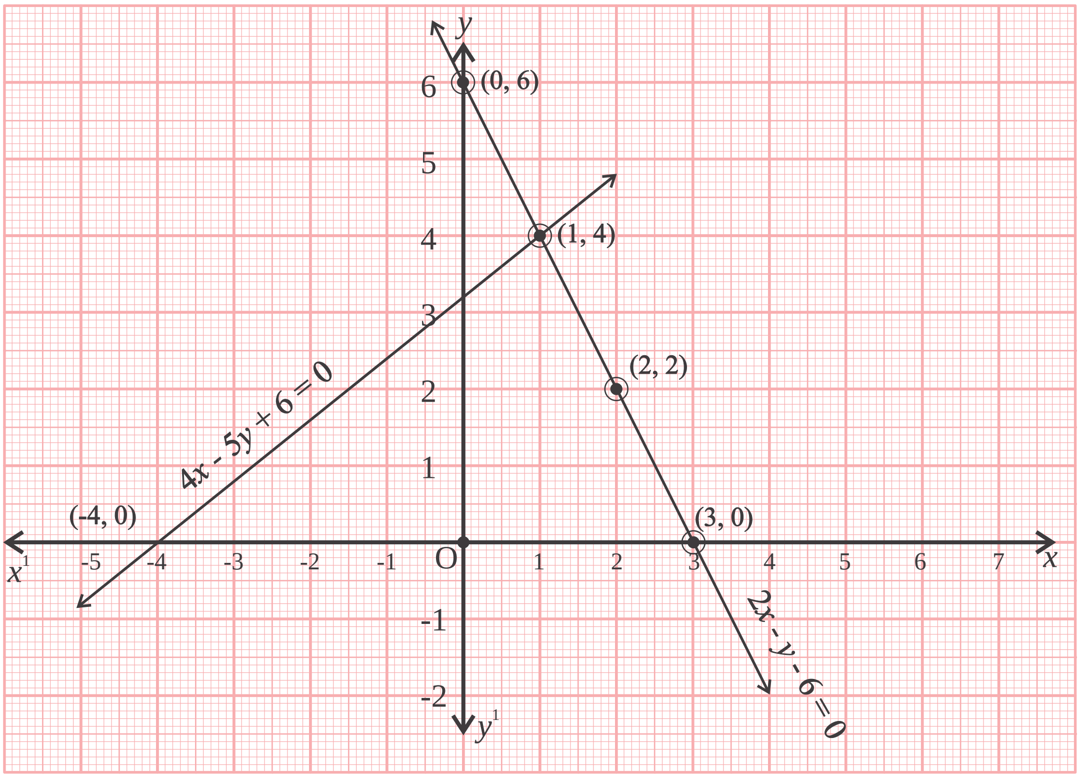
Since in the equation x and y stand for price of one chair and one table respectively therefore it can be said that there can be many possible prices for a chair and a table.



Graph-7

Example 8. Find the values of x and y for the system of equations depicted in the graph given below.

Solution:



Graph-8

From the graph it can be seen clearly that the lines for the two equations intersect each other at point (1,4) and thus, the solution for this system of equation will be $x = 1, y = 4$.

Exercise-1

1. Write the following statements in the form of equations:

   a. The cricket coach in a school purchased 3 bats and 6 balls for Rs. 3900. He bought a bat and two balls from the same shop for Rs. 1300.

   b. The sum of two numbers is 16 and their difference is 8.

   c. The cost of 2 kg apples and 1 kg grapes is Rs. 160 in a fruit shop. In the same shop, the cost of 4 kg apples and 2 kg grapes is Rs. 300.

   d. Naresh said to his daughter that seven years ago my age was seven times your age and three years from now I will be three times as old as you.

   e. A person travels 90 km by train and taxi to reach his office. The distance travelled by train is twice the distance travelled by taxi.

2. Look at the graphs for the given equations and try to find their solutions.

a. In Equations

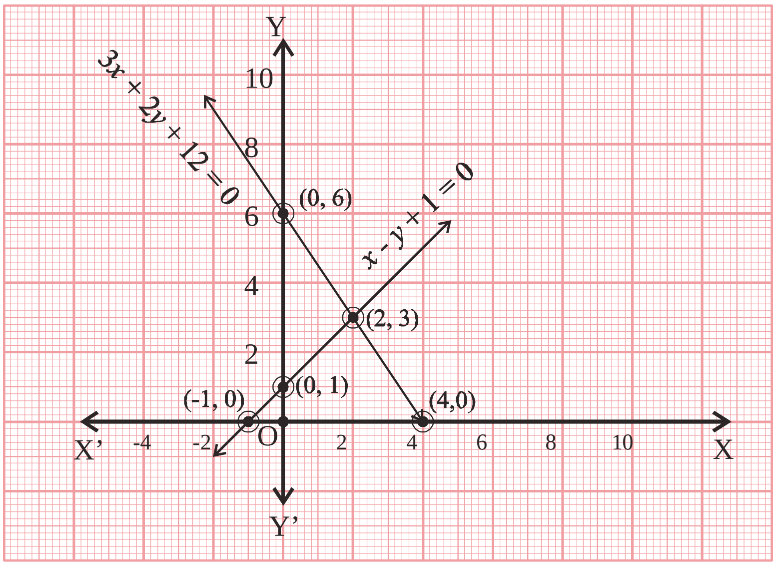
$3x + 2y - 12 = 0$

$x - y + 1 = 0$

The solution is..................

And, therefore the values of x and y are

....................., ...,



Graph-9

b. In Equations

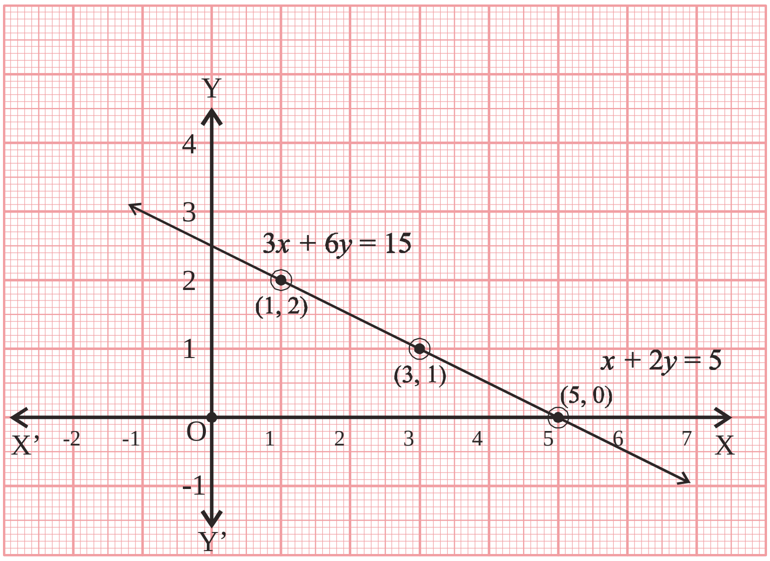
$3x + 6y = 15$

$x + 2y = 5$

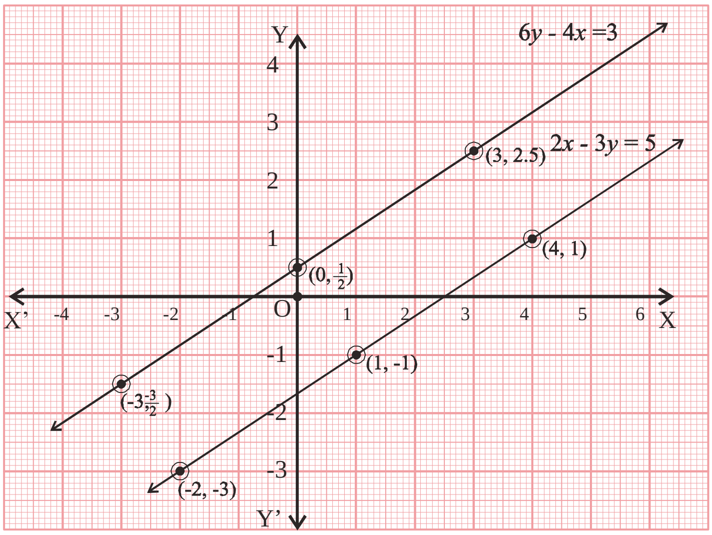
The solution is..................

And, therefore the values of x and y are

....................., ...,



Graph-10



Graph-11

c. In Equations

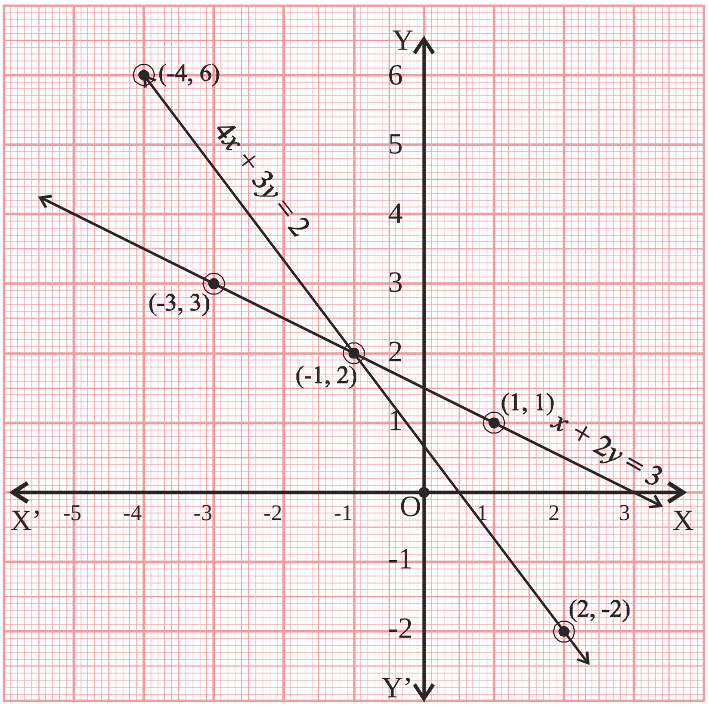
$-4x + 6y = 3$

$2x - 3y = 5$

The solution is.................

And, therefore the values of x and y are

....................., ...,



Graph-12

d. In Equations

$x + 2y = 3$

$4x + 3y = 2$ में

The solution is.................

And, therefore the values of x and y are

....................., ...,

Algebraic methods for solving equations

1. Substitution method

We have learnt how to find the solutions of equations in two variables using graphs. Now we will discuss some more methods to find solutions of linear equations in two variables. In one method, we put the values of one equation in the second equation to convert it into an equation in one variable and then find its solution. We can understand this method through some examples.

Example-9. There are some rabbits and some birds in a small cave and the number of heads is 35 and the number of feet is 98. Find the number of birds and rabbits.

Solution: Let the number of rabbits = x

And, the number of birds = y

Number of heads of rabbits + number of heads of birds = 35

$$\therefore \quad x + y = 35 \quad \text{................(1)}$$

Number of feet of rabbits + number of feet of birds = 98

$$\therefore \quad 4x + 2y = 98$$

$$2(2x + y) = 98$$

$$2x + y = \frac{98}{2}$$

$$2x + y = 49$$

$$y = 49 - 2x \quad \text{................(2)}$$

Placing $y = 49 - 2x$ in equation (1)

$$x + 49 - 2x = 35$$

$$\Rightarrow \quad -x + 49 = 35$$

$$\Rightarrow \quad -x = 35 - 49$$

$$\Rightarrow \quad -x = -14$$

$$\Rightarrow \quad x = 14$$

Now, placing $x = 14$ in equation (2)

$$\Rightarrow \quad y = 49 - 2x$$

$$\Rightarrow \quad y = 49 - 2(14)$$

$$\Rightarrow \quad y = 49 - 28$$

$$\Rightarrow \quad y = 21$$

Clearly, the number of rabbits is 14 and the number of birds is 21.

2. Elimination method

In another method for solving equations, we sometimes add (or sometimes subtract) the two equations to reduce them to equations in one variables which can be solved. Let us see some examples where this method is used.

Example 10. Richa and Naina had some toffees. If Richa gives 30 toffees to Naina then Naina will have two times as many toffees as Richa. If Naina gives 10 toffees to Richa then Richa will have three times as many toffees as Naina. Find out how many toffees each of them has.

Solution: Let the number of toffees with Richa = x

Let the number of toffees with Naina = y

Now, if Richa gives 30 toffees to Naina

Then, new number of toffees with Richa = x - 30

And, new number of toffees with Naina = y + 30

According to the question $2(x-30) = y + 30$

$\Rightarrow \quad 2x - 60 = y + 30$

$\Rightarrow \quad 2x - y = 30 + 60$

$\Rightarrow \quad 2x - y = 90 \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$

But when Naina gives 10 toffees to Richa

Then, number of toffees with Richa = x + 10

And number of toffees with Naina = y -10

According to the question $x + 10 = 3(y-10)$

$\Rightarrow \quad x + 10 = 3y - 30$

$\Rightarrow \quad x - 3y = -30 - 10$

$\Rightarrow \quad x - 3y = -40 \ldots\ldots\ldots\ldots\ldots\ldots(2)$

Now

$$2x - y = 90 \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$x - 3y = -40 \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Are there any common coefficients for x and y in equations (1) and (2)?

No, x and y do not have any common coefficients. Then, let us see if adding or subtracting equations (1) from (2), will eliminate either x or y? If there are some common coefficients for x or y then it is possible to eliminate one of them.

To get common coefficients, we will multiply both sides of equation (2) with 2, which is the coefficient of x in equation (1).

$$2(x-3y) = -40 \times 2$$

$$2x - 6y = -80 \quad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

On subtracting equation (3) from equation (1),

$$\Rightarrow \quad 2x - y - (2x - 6y) = 90 - (-80)$$
$$\Rightarrow \quad 2x - y - 2x + 6y = 90 + 80$$
$$\Rightarrow \quad 5y = 170$$
$$\Rightarrow \quad y = \frac{170}{5}$$
$$\Rightarrow \quad y = 34$$

On putting the value of y in equation (1),

$$2x - 34 = 90$$
$$\Rightarrow \quad 2x = 90 + 34$$
$$\Rightarrow \quad 2x = 124$$
$$\Rightarrow \quad x = \frac{124}{2}$$
$$\Rightarrow \quad x = 62$$

Clearly, Richa has 62 and Naina has 34 toffees.

Example:-11. The students of a class are standing in rows. If we remove 4 students from each row then we get one extra row but if 4 more students stand in each row then two less rows formed. Find the number of students in the class.

Solution: Let the number of rows = x

And the number of students in each row = y

Then total number of students = number of rows X number of students in each row

$= xy$

Now, if there are 4 less students in each row

Then, new number of students in each row = $y - 4$

And, new number rows = $x + 4$

Then, total number of students = $(x + 4)(y - 4)$

$$\Rightarrow \quad xy = xy - 4x + 4y - 16$$
$$\Rightarrow \quad xy - xy = -4x + 4y - 16$$
$$\Rightarrow \quad 0 = 4(-x + y - 4)$$
$$\Rightarrow \quad -x + y - 4 = 0$$
$$\Rightarrow \quad -x + y = -4 \qquad \text{................(1)}$$

But when four additional students are standing in each row

Then, number of students in each row = $y + 4$

And, number of rows = $x - 2$

Then, total number of students = $(x-2)(y+4)$

$$\Rightarrow \quad xy = (x-2)(y+4)$$
$$\Rightarrow \quad xy = xy + 4x - 2y - 8$$
$$\Rightarrow \quad xy - xy = 4x - 2y - 8$$
$$\Rightarrow \quad 0 = 4x - 2y - 8$$
$$\Rightarrow \quad 2(2x - y - 4) = 0$$
$$\Rightarrow \quad 2x - y - 4 = 0$$
$$\Rightarrow \quad 2x - y = 4 \qquad \text{...............(2)}$$

The system of equations is:

$$-x + y = 4 \qquad \text{................(1)}$$
$$2x - y = 4 \qquad \text{................(2)}$$

Since, the coefficient of y is common and the signs are opposite in the system of equations, therefore y will be eliminated on adding the two equations.

$$-x + y + 2x - y = 4 + 4$$
$$\Rightarrow \quad x = 8$$

On putting value of x in equation (1)

$$\Rightarrow \quad -8 + y = 4$$
$$\Rightarrow \quad y = 4 + 8$$
$$\Rightarrow \quad y = 12$$

The total number of students = xy = $8 \times 12 = 96$.

We have discussed several methods to solve equations. You can use any method which you find easy and convenient.

Example 12. Ten years ago the sum of ages of Sunil and Vinay was one third the age of their father. If Sunil is 2 years younger than Vinay and the sum of their current ages is 14 less than their father's age. Find the present ages of Vinay, Sunil and their father.

Solution. Let Vinay's present age be = x years

Sunil's present age = $x - 2$ years

Their father's present age = y years

Ten years ago, Vinay's age = $x - 10$

And, Sunil's age = $(x - 2 - 10)$

Their father's age = $(y - 10)$ years

$$\therefore \quad (x-10)+(x-2-10) = \frac{1}{3} \times (y-10)$$

$$\Rightarrow \quad x-10+x-2-10 = \frac{1}{3}(y-10)$$

$$\Rightarrow \quad 2x-22 = \frac{1}{3}(y-10)$$

$$\Rightarrow \quad 3(2x-22) = y-10$$

$$\Rightarrow \quad 6x-66 = y-10$$

$$\Rightarrow \quad 6x-y = -10+66$$

$$\Rightarrow \quad 6x-y = 56 \quad \text{.....................(1)}$$

The sum of present ages of Sunil and Vinay is 14 less than their father's age.

$$\therefore \quad x+x-2 = y-14$$

$$\Rightarrow \quad 2x-2 = y-14$$

$$\Rightarrow \quad 2x-y = -14+2$$

$$\Rightarrow \quad 2x-y = -12 \quad \text{...................(2)}$$

On subtracting equation (2) from equation (1),

$$6x-y-(2x-y) = 56-(-12)$$

$$\Rightarrow \quad 6x-y-2x+y = 68$$

$$\Rightarrow \quad 4x = 68$$

$$\Rightarrow \quad x = \frac{68}{4}$$

$$\Rightarrow \quad x = 17$$

On putting value of x in equation (1),

$$6\times17-y = 56$$

$$102-y = 56$$

$$y = 102-56$$

$$y = 46$$

$\therefore$ Present age of Vinay = 17 years

Present age of Sunil = 17 – 2 = 15 years

Father's present age = y years = 46 years

A variety of questions are presented below where the different methods discussed have been used.

Example:13. Seven times of a two digit number is equal to 4 times of the number formed by reversing the digits. The sum of the two digits is 3. Find the numbers.

Solution. For the given two digit number, let the digit at the unit's place be x and that at the ten's place be y. Then the number is $(10x + y)$.

$\therefore$ According to the question $7(10x + y) = 4(10y + x)$

$$\Rightarrow \quad 70x + 7y = 40y + 4x$$
$$\Rightarrow \quad 70x - 4x - 40y + 7y = 0$$
$$\Rightarrow \quad 66x - 33y = 0$$
$$\Rightarrow \quad 33(2x - y) = 0$$
$$\Rightarrow \quad 2x - y = 0 \quad ....(1)$$
$$\text{and} \quad x + y = 3 \quad ....(2)$$

On adding equations (1) and (2),

$$2x - y + x + y = 0 + 3$$
$$\Rightarrow \quad 3x = 3$$
$$\Rightarrow \quad x = \frac{3}{3}$$
$$\Rightarrow \quad x = 1$$

On putting the value of x in equation (2)

$$\Rightarrow \quad x + y = 3$$
$$\Rightarrow \quad 1 + y = 3$$
$$\Rightarrow \quad y = 3-1$$
$$\Rightarrow \quad y = 2$$

Therefore, the number is 12.

Example: 14. Find the values of the variables in the given equations.

$2x - 5y = -8$;

$x - 4y = -7$

Solution. Equation $2x - 5y = -8$ ....(1)

$x - 4y = -7$ ....(2)

From equation (2) $x = -7 + 4y$ ....(3)

On putting this value of x in equation (1),

$$\Rightarrow \quad 2(-7 + 4y) - 5y = -8$$
$$\Rightarrow \quad -14 + 8y - 5y = -8$$
$$\Rightarrow \quad 3y = -8 + 14$$
$$\Rightarrow \quad 3y = 6$$
$$\Rightarrow \quad y = \frac{6}{3} = 2$$

On putting this value of y in equation (3),

$$\Rightarrow \quad x = -7 + 4y$$
$$\Rightarrow \quad x = -7 + 4(2)$$
$$\Rightarrow \quad x = -7 + 8$$
$$\Rightarrow \quad x = 1$$

Therefore, $x = 1, y = 2$

Example:15. Solve the given equations.

$41x - 17y = 99$;

$17x - 41y = 75$.

Solution. Equation $41x - 17y = 99$ ....(1)

$17x - 41y = 75$ ....(2)

On adding equations (1) and (2)

$$41x - 17y + 17x - 41y = 99 + 75$$
$$\Rightarrow \quad 58x - 58y = 174$$
$$\Rightarrow \quad 58(x - y) = 174$$
$$\Rightarrow \quad x - y = \frac{174}{58}$$
$$\Rightarrow \quad x - y = 3 \qquad \text{................(3)}$$

On subtracting equation (2) from equation (1)

$$41x - 17y - (17x - 41y) = 99 - 75$$
$$\Rightarrow \quad 41x - 17y - 17x + 41y = 24$$
$$\Rightarrow \quad 24x + 24y = 24$$
$$\Rightarrow \quad x + y = \frac{24}{24}$$
$$\Rightarrow \quad x + y = 1 \qquad \text{..................(4)}$$

On adding equations (3) and (4),

$$\Rightarrow \quad x - y + x + y = 3 + 1$$
$$\Rightarrow \quad 2x = 4$$
$$\Rightarrow \quad x = \frac{4}{2}$$
$$\Rightarrow \quad x = 2$$

When in a system of equations, the coefficients of different variables are same, then we get new equations by first adding the two equations and then subtracting them.

Putting $x = 2$ in equation (3)

$$\Rightarrow \quad x - y = 3$$
$$\Rightarrow \quad 2 - y = 3$$
$$\Rightarrow \quad 2 - 3 = y$$
$$\Rightarrow \quad y = -1$$

Here, $x = 2$ and $y = -1$.

Example:16. In ΔABC, $\angle A = x°$, $\angle B = 3x°$ and $\angle C = y°$. If $3y° - 5x° = 30°$ then prove that this is a right triangle.

Solution. The values of three angles in ΔABC are respectively, $\angle A = x°$, $\angle B = 3x°$ and $\angle C = y°$

$\because$ The sum of interior angles of a triangle is 180°

$\therefore$ $\angle A + \angle B + \angle C = 180°$

On putting the values of $\angle A$, $\angle B$, $\angle C$

$$x° + 3x° + y° = 180°$$
$$4x° + y° = 180°$$
$$y° = 180° - 4x° \quad \text{..................(1)}$$

Given $\quad 3y° - 5x° = 30° \quad$ ....(2)

Putting the value of y from equation (1) in equation (2)

$$\Rightarrow \quad 3\,[180° - 4x°] - 5x° = 30°$$
$$\Rightarrow \quad 3 \times 180° - 3 \times 4x° - 5x° = 30°$$
$$\Rightarrow \quad 540° - 12x° - 5x° = 30°$$
$$\Rightarrow \quad -17x° = 30° - 540°$$
$$\Rightarrow \quad -17x° = -510°$$
$$\Rightarrow \quad x° = \frac{510°}{17°}$$
$$x° = 30°$$

Putting the value of x in equation (1)

$$y° = 180° - 4(30°)$$

$$\Rightarrow \quad y° = 180° - 120°$$

$$\Rightarrow \quad y° = 60°$$

$$\therefore \quad \angle A = x° = 30°$$

$$\angle B = 3x° = 3 \times 30° = 90°$$

$$\angle C = y° = 60°$$

Clearly, of the three angles in ABC, one is 90° and the other two are acute angles measuring 30° and 60°.

Therefore, the given ΔABC is a right angle triangle.

Example:17. A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km down-stream. Determine the speed of the stream and that of the boat in still water.

Solution: Let the speed of boat downstream (in the direction of flow of water) = x km/h

And let the speed of boat upstream (opposite to the direction of flow of water) = y km/h

Time taken to travel 44 km downstream = distance/speed

$$= \frac{44}{x} \text{ hours}$$

Time taken to travel 30 km upstream = $\frac{30}{y}$ hours

$\therefore$ According to the question,

Time taken to go 30 km upstream and 44 km downstream = 10 hours

$$\therefore \quad \frac{44}{x} + \frac{30}{y} = 10 \quad \text{................(1)}$$

Since the time taken to go 40 km upstream and 55 km downstream = 13 hours

$$\therefore \quad \frac{55}{x} + \frac{40}{y} = 13 \quad \text{...................(2)}$$

Putting $(1/x)$ = u and $(1/y)$ = v in equations (1) and (2), gives us equations (3) and (4).

$$44u + 30v = 10 \qquad \left\{\because \text{Speed} = \frac{\text{Distance}}{\text{Time}}\right\}$$

$$22u + 15v = 5 \quad \text{.................(3)}$$

$$\text{and} \qquad 55u + 40v = 13 \qquad \text{..................(4)}$$

Now, on multiplying equation (3) by 55 and equation (4) by 22

$$1210u + 825v = 275$$
$$-1210u - 880v = -286$$
$$-55v = -11$$
$$v = \frac{-11}{-55}$$
$$v = \frac{1}{5} \qquad \therefore \qquad y = 5$$

On putting value of v in equation (3)

$$22u + 15 \times \frac{1}{5} = 5$$
$$22u + 3 = 5$$
$$22u = 5 - 3$$
$$u = \frac{2}{22}$$
$$u = \frac{1}{11} \qquad \therefore \qquad x = 11$$
$$\Rightarrow \quad x = 11 \qquad \Rightarrow \quad y = 5$$

Therefore, the speed of boat downstream = 11 km/h and the speed of boat upstream = 5 km/h.

Think and Discuss

Solve the above system of equations using different methods. Discuss whether the solutions obtained using the different methods are same or not?

$2x + 5y = 1$

$2x + 3y = 3$

Exercises 2

1. Check whether values of x and y given in 'A' and 'B' are solutions of given equations.

(A)	$x = 2, y = 5$	(B)	$x = -1, y = 3$
(i)	$x + y = 7$	(ii)	$2x + 5y = 13$
(iii)	$2x - 3y = -11$	(iv)	$5x + 3y = 4$

2. Check which one of "A" or "B" is a solution for given equations.

(A)	$x = 3, y = -1$	(B)	$x = \frac{1}{2},\ y = \frac{1}{3}$
(i)	$2x + 5y = 1;$ $2x + 3y = 3.$	(ii)	$x + y = 5xy;$ $3x + 2y = 13xy.$
(iii)	$2x - \frac{3}{y} = 9;$ $3x + \frac{7}{y} = 2$	(iv)	$2x + 5y = \frac{8}{3};$ $3x - 2y = \frac{5}{6}$

3. Solve the given equations. You can use whichever method you want.

(i)	$x - y = -1;$ $3x - 2y = 12.$	(ii)	$x - 2y = 5;$ $2x - 4y = 6$
(iii)	$x + y = 6;$ $x = y + 2.$	(iv)	$5x - 8y = -1$ $3x - \frac{24}{5}y + \frac{3}{5} = 0$
(v)	$3x - 4y - 1 = 0;$ $2x - \frac{8}{3}y + 5 = 0$	(vi)	$x + 2y = 8;$ $2x + 4y = 16$

4. Solve the following system of equations for the given variables.

(i)	$x + y = 7;$ $x - y = -1.$	(ii)	$2x + y = 8;$ $x - 2y = -1$
(iii)	$4x + 3y = 5;$ $2x - y = 2$	(iv)	$\sqrt{7}x + \sqrt{11}y = 0;$ $\sqrt{3}x - \sqrt{5}y = 0$

5. The cost of 15 kg tea and 17 kg coffee is Rs. 183 and the cost of 25 kg tea and 13 kg coffee is Rs. 213. What will be the cost of 7 kg tea and 1 kg coffee?

6. A person owns some pigeons and cows. The number of legs of these animals is

180 and the total number of eyes is 120. How many cows and pigeons does the person own?

7. A bag has 50 paisa and 25 paisa coins. The total number of coins is 94 and their value is Rs. 29.75. Find the number of 50 paisa and 25 paisa coins.

8. The sum of two numbers is 25. The sum of their reciprocals is ¼. Find the two numbers.

[Hint: (x - y)2= (x + y)2 - 4xy]

9. The difference of two numbers is 14 and the difference in their squares is 448. Find the numbers.

[Hint: x2 - y2= (x + y)(x - y)]

10. The product of two numbers is 45 and their sum is 14. Find the two numbers.

11. 5 years ago I was three times as old as my daughter. Ten years hence I will be twice as old as my daughter. Find my present age and my daughter's present age.

12. The distance between two cities, A and B, is 70 km. Two cars start moving from A and B respectively. If they move in the same direction, they will meet in 7 hours and if they move towards each other they will meet in 1 hour. Find the speeds of both the cars.

13. Some students are sitting in classroom A and B in a school. When 10 students from room A are sent to room B, the number of students in both rooms becomes equal. When 20 students from room B are sent to room A, then the number of students in room A becomes twice that in room B. Find the number of students in each room.

14. If the length of a rectangle is decreased by 5 units and its breadth increased by 2 units then the area of the rectangle decreases by 80 square units. If its length is increased by 10 units and its breadth decreased by 5 units then the area of the rectangle increases by 50 square units. Find the original length and breadth of the rectangle.

Determining the type of solution by looking at the system of equations

Can you tell whether we can solve a system of equations or not simply by looking at it? Yes, it is possible but to do this we will need to find the relation between the coefficients of the variable terms and the fixed terms in the system of equations.

Look at the give system of equations:

$2x + 3y = 7$ ....(1)

$6x + 9y = 11$ ....(2)

In equation (1), the coefficient of x is 2, the coefficient of y is 3 and 7 is the constant term. If 2, 3 and 7 are denoted by a_1, b_1, and c_1 respectively and the coefficients and constant terms in equation (2) are denoted by a_2, b_2, and c_2 respectively then the system of equation can be written as follows:

$a_1x + b_1y = c_1$

$a_2x + b_2y = c_2$

We can write other systems of equations in the same way.

The table below shows the relation between the ratio of the coefficients of like variables and constants in a system of equations. We can draw some conclusions by looking at these ratios.

S.No.	Relation between coefficients of like terms (condition)	Solution of system of equations	Geometrical meaning
1.	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	A unique solution is obtained	The system of equations is depicted by two intersecting lines.
2.	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	There is no solution	The system of equations is depicted by two parallel lines.
3.	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	There are infinite solutions	The system of equations is depicted by coincident lines.

Let us understand about solutions of equations using these relations.

Example:18. What type of solutions do the following equations have? Find out.

$3x + 5y = 12$ ....(1)

$4x + 2y = 5$ ....(2)

Solution: In $3x + 5y = 12$, $a_1 = 3$, $b_1 = 5$, $c_1 = 12$

$4x + 2y = 5$ $\quad a_2 = 4$, $b_2 = 2$, $c_2 = 5$

$\because$ Here $\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}$ $\quad\therefore$ The equations have a unique solution.

Example:19. Find the solutions of $5x + 3y = 12$

and $15x + 9y = 15$

Solution: In $5x + 3y = 12$, $a_1 = 5$, $b_1 = 3$, $c_1 = 12$

and in $5x + 9y = 15$, $a_2 = 15$, $b_2 = 9$, $c_2 = 15$

$$\frac{a_1}{a_2} = \frac{5}{15}, \frac{b_1}{b_2} = \frac{3}{9}, \frac{c_1}{c_2} = \frac{12}{15}$$

$$\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{1}{3}, \frac{c_1}{c_2} = \frac{4}{5}$$

We find that $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$

$\therefore$ The equations have no solution.

Finding the value of unknown coefficient of variables

Example:20. Find the solution of $15x - 3y = 14$

and $\quad 60x - 12y = 56$

Solution: In $15x - 3y = 14$, $a_1 = 15$, $b_1 = -3$, $c_1 = 14$

In $\quad 60x - 12y = 56$, $a_2 = 60$, $b_2 = -12$, $c_2 = 56$

Here, $\dfrac{a_1}{a_2} = \dfrac{15}{60} = \dfrac{1}{4}$, $\quad \dfrac{b_1}{b_2} = \dfrac{-3}{-12} = \dfrac{1}{4}$, $\quad \dfrac{c_1}{c_2} = \dfrac{14}{56} = \dfrac{1}{4}$

Clearly $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$

$\therefore$ the equations have infinite solutions.

Do this

Complete the following table:

S.No.	System of equations	Ratio of coefficients of x	Ratio of coefficients of y	Ratio of constant terms	Relation between ratios	Solution of equations	Geometrical interpretations
1.	$2x-3y=1$ $2x-4y=-4$	$\frac{a_1}{a_2}=\frac{2}{2}=1$	$\frac{b_1}{b_2}=\frac{-3}{-4}=\frac{3}{4}$	$\frac{c_1}{c_2}=\frac{1}{-4}=-\frac{1}{4}$	$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$	Unique solution	Two intersecting lines
2.	$x+2y=5$ $2x-3y=-4$	....	....	....	....	....	....
3.	$4x-5y=3$ $5x-4y=5$	....	....	....	....	....	....
4.	$2x+3y=5$ $-4x-6y=8$	....	....	....	....	....	....
5.	$3x-4y=1$ $6x-8y=-15$	....	....	....	....	....	....
6.	$x+y=4$ $2x+2y=8$	....	....	....	....	....	....
7.	$5x-6y=4$ $10x-12y=8$	....	....	....	....	....	....

Finding the value of unknown coefficient

We saw different situations and different methods for solving systems of equations. However, we knew the coefficients of the variables in these cases. Can we find the solution if one of the coefficients is unknown and the system is as follows:

$$2x + 3y - 5 = 0;$$

$$kx - 6y - 8 = 0.$$

Can we still find the values of x, y and k in such a situation?

Example:21. For what value of k will the system of equations have a unique solution?

$x - ky = 2,\ 3x + 2y = -5$

Solution: In the given system of equations:

$x - ky - 2 = 0,\ 3x + 2y + 5 = 0$

Here, $a_1 = 1,\quad b_1 = -k,\quad c_1 = -2$

$a_2 = 3,\quad b_2 = 2,\quad c_2 = 5$

We are told that the equations have a unique solution.

Therefore, $$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{1}{3} \neq \frac{-k}{2}$$

$$\Rightarrow \quad \frac{2}{-3} \neq k$$

$$\Rightarrow \quad k \neq -\frac{2}{3}$$

Except for $k = \frac{-2}{3}$, the system will have a unique solution for all real values of k.

Example:22. Find the value of k for which the given system of equations has infinite solutions.

$(k - 3)\,x + 3y = k; \qquad kx + ky = 12$

Solution. System of equations is

$(k–3) + 3y = \text{k}; \qquad kx + ky = 12$

$a_1 = \text{k}–3,\quad b_1 = 3,\quad c_1 = \text{k}$

$a_2 = \text{k}\quad b_2 = \text{k}\quad c_2 = 12$

Since the system of equations has infinite solutions

Therefore, from $$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{k-3}{k}=\frac{3}{k}=\frac{+k}{+12}$$

$\Rightarrow \quad \frac{k-3}{k}=\frac{3}{k}$ ....(1) $\quad \frac{3}{k}=\frac{k}{12}$ ......(2) $\quad \frac{k-3}{k}=\frac{k}{12}$ ......(3)

$\Rightarrow \quad k-3=3$ $\qquad \Rightarrow k^2=36$ $\qquad \Rightarrow \quad k^2=12k-36$

$\Rightarrow \quad k=3+3$ $\qquad \Rightarrow k=\sqrt{36}$ $\qquad \Rightarrow \quad k^2-12k+36=0$

$\Rightarrow \quad k=6$ $\qquad \Rightarrow k=\pm\ 6$ $\qquad \Rightarrow \quad k-6k-6k+36=0$

$\Rightarrow \quad k(k-6)-6(k-6)=0$

$\Rightarrow \quad (k-6)\ (k-6)=0$

$k=6$

Only that value of k is valid which satisfies all equations. Here, 6 is such a value therefore $k=6$.

Exercise-3

1. Show that the given system of equations has a unique solution.
   $3x+5y=12$
   $5x+3y=4$
2. Show that the given system of equations has infinite solutions.
   $2x-3y=5;$
   $6x-9y=15$
3. Find the value of k for which the given systems of equations have no solution.
   (i) $8x+5y=9;\quad kx+10y=15$
   (ii) $kx+3y=3;\quad 12x+ky=6$
   (iii) $kx-5y=2;\quad 6x+2y=7$
4. Find the value of k for which the given systems of equations have a unique solution.
   (i) $kx+2y=5;\quad 3x+y=1$
   (ii) $x-2y=3;\quad 3x+ky=1$
   (iii) $kx+3y=k-3;\quad 12x+ky=k$
   (iv) $4x-5y=k;\quad 2x-3y=12$
5. Find the value of k for which the given systems of equations have infinite solutions.
   (i) $2x+3y=7;$
   $(k-1)\ x+(k+2)\ y=3k$

(ii) $kx + 2y - 4 = 0$; $5x - 3y + 6 = 0$

(iii) $3x + ky = 7$; $2x - 5y = 1$

(iv) $kx - 5y = 2$; $2x - 3y = 12$

6. If $x = 2$; $y = 4$ then find the value of p in $7x - 4y = p$.
7. If the straight line $2x - ky = 9$ passes through (1,–1), then find the value of k.
8. Test whether the following system of equations has a unique solution, infinite solution, or no solutions. If it has a unique solution then find the values of x and y.

$4x + 7y = 18$

$2x + y = 4$

Making statements from equations

So far we formulated equations for conditions derived from different statements and then found the values of variables. But can we write statements to describe given equations?

Come, let us write the following equations in the form of statements.

$x + y = 45$ ....(1)

$x - y = 13$ ....(2)

If we say that x is one number and y is another number then equations (1) and (2) can be written as follows:

The sum of two numbers is 45 and their difference is 13. Find the two numbers. Or, the cost of one book is x and the cost of one copy is y. The sum of their costs is 45 and the difference in their costs is 13. Can you find the cost of one book and one copy?

Can we form more statements from these equations? Make two more such statements.

Example 23 Write the following in the form of a statement.

$$\frac{x-1}{y} = \frac{1}{2} \quad ....(1)$$

$$\frac{x}{y+3} = \frac{3}{2} \quad ....(2)$$

Solution: If x/y is a fraction where the numerator is x and the denominator is y then the equations given above can be written in the form of the following statement:

"If 1 is subtracted from the numerator of a fraction then it becomes equal to ½ and if 3 is added to the denominator, we get 3/2."

There are many ways possible to write a given system of equations in the form of statements. The system given above can also be written in the form of other statements.

Do this

Write the given system of equations in the form of statements.

(i) $x + y = 60$
$x = 3y$

(ii) $x + y = 5$
$xy = 6$

What we have learnt

1. The graphs of equations in first degree in two variables are always straight lines and therefore equations in first degree in two variables are known as linear equations.
2. The graph of a pair of linear equations in two variables is represented by two lines. If the lines intersect at a point, then that point gives the unique solution of the twoequations.
3. If the graph of a pair of linear equations in two variables is represented by two parallel lines, then the pair of equations has no solution.
4. If the graph of a pair of linear equations in two variables is represented by two coincidentlines, then the pair of equations has infinite solutions.
5. Linear equations in the same two variables can be written as follows:

$a_1x + b_1y = c_1$

$a_2x + b_2y = c_2$

In the above equations, if

(i) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ Then lines are parallel and the pair of equations has no solution.

(ii) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ Then lines are intersecting and the pair of equations has unique solution.

(iii) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ Then lines are coincident and the pair of equations has infinite solutions.

ANSWER KEY

Exercise-1

1. (i) $3x + 6y = 3900$, $x + 2y = 1300$ (ii) $x + y = 16$, $x - y = 8$ (iii) $2x + y = 160$, $4x + 2y = 300$
   (iv) $x - 7y + 42 = 0$, $x - 3y - 6 = 0$ (v) $x + y = 90$, $x = 2y$
2. (a) $x = 2, y = 3$ (b) infinite solutions
   (c) no solution (d) $x = -1, y = 2$

Exercise-2

1. (i) (a) (ii) (b) (iii) (a), (b) (iv) (b)
2. (i) (a) (ii) (b) (iii) (a) (iv) (b)
3. (i) $x = 14,\ y = 15$ unique solution
   (ii) no solution, parallel lines
   (iii) $x = 4, y = 2$, a unique solution
   (iv) infinite solutions, coincident lines
   (v) no solution, parallel lines
   (vi) infinite solutions, coincident lines
4. (i) $x = 3,$ $y = 4$ (ii) $x = 3, y = 2$
   (iii) $x = 1.1, y = 0.2$ (iv) $x = 0,\ y = 0$
5. ₹43.80
6. Number of cows = 30, Number of pigeons = 30
7. Number of 25 paisa coins = 69, Number of 50 paisa coins = 25
8. Numbers = 20, 5
9. Numbers = 23, 9
10. Numbers = 9, 5
11. My age = 50 years, son's age = 20 years
12. 40 km/h, 30 km/h,
13. 100, 40
14. 40 units, 30 units

Exercise-3

3. (i) $k = 16$ (ii) $k = -6$ (iii) $k = -15$
4. (i) $k \neq 6$ (ii) $k \neq -6$ (iii) $k \neq \pm 6$
   (iv) $k \neq \frac{10}{3}$
5. (i) $k = 7$ (ii) $k = \frac{-10}{3}$
   (iii) k has no value (iv) $k = 24$
6. $p = -2$
7. $k = 7$
8. has a unique solution $x = 1, y = 2$