Chapter - 14
Symmetry

J5E9R5

14.1 We have already learnt some concepts about symmetry in class VI. Let us recall the important aspects of symmetry we have already learnt.

Fig 14.1 is symmetrical figure, if and only the figure is folded such that the two parts of the figure will coincide. The line along which the figure is symmetrical is called line of symmetry. The qualitative feature which makes a figure symmetrical, is called its symmetry.

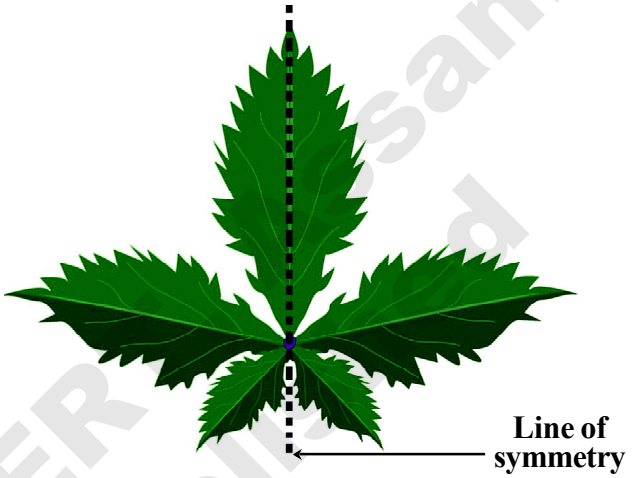


Figure - 14.1 symmetry of ladies finger leaf

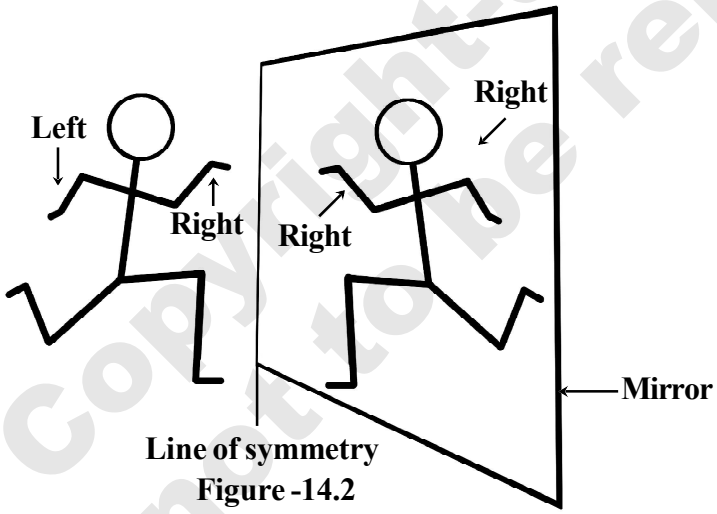


Figure -14.2

14.2 Linear symmetry :

The concept of Line of symmetry is related with a symmetrical figure. So such kind of symmetry is also called linear symmetry. The images on a plain miror is similar to linear symmetry. If a plain mirror is kept on the line of symmetry of a symmetrical figure the parts of the objects formed infront of the miror will be same with the images in the back side of the miror. Therefore linear symmetry may be called as reflection or mirror symmetry.

Characteristics of mirror symmetry is that the two parts divided by the line of symmetry mutually change themselves that is, the right part changes to the left part and vice versa.

Symmetry

We can see many examples of symmetry around us. Leaves, Venation of leaves, flowers, petals, fruit are some example of symmetry from plant kingdom

(i) Leaf of papaya tree

(ii) papaya

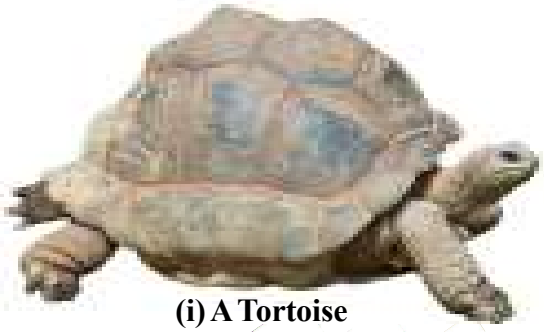
(iii) A jasmine tree leaf

(iv) A pipal tree leaf

(v) A Buhinia tree leaf

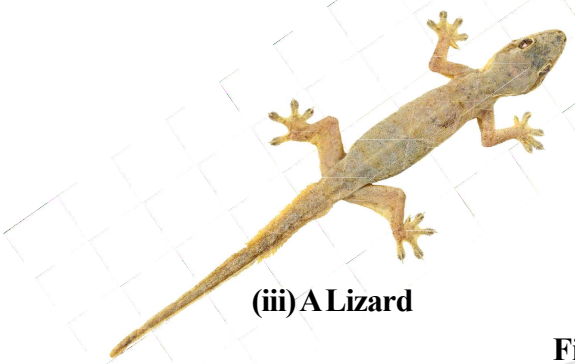
Figure - 14.3

There are many examples of symmetry in animal kingdom; symmetry is present in the physical structure of different kind of fish, tortoise, insects, ants etc.



(i) A Tortoise

(ii) A Spider



(iii) A Lizard

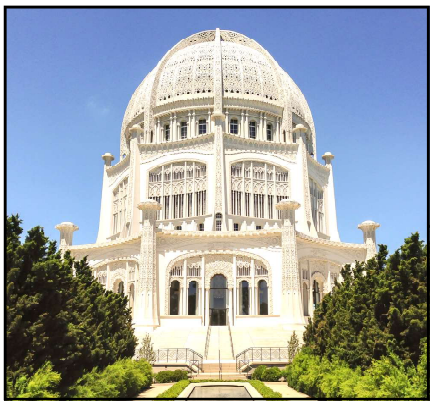
(iv) A Starfish

Figure - 14.4

In most of the ancient and modern Architecture symmetry is seen

(i) The Tajmahal

(ii) The Eiffel Tower



(iii) The Bahai Temple

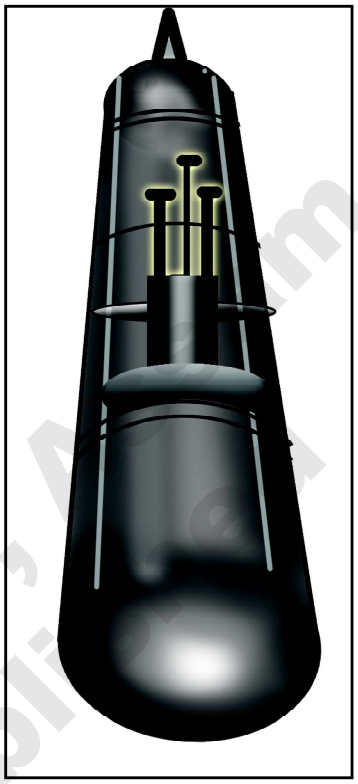
Figure -14.5

Symmetry

There is symmetry in the numerous instances of engineering discovery

(i) An Aeroplane

(ii) A Car

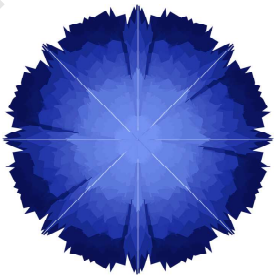


(iii) A Submarine

Figure - 14.6

We may make symmetrical shapes by inkblot method.

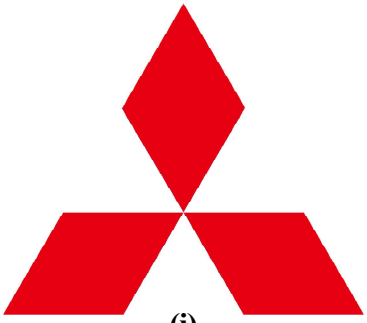
(i)



(ii)

Figure - 14.7

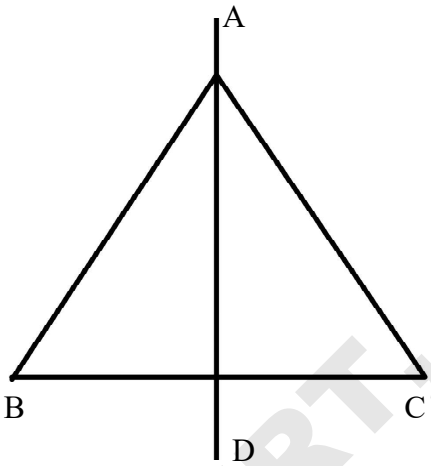
We can prepare different types of symmetrical figures by folding and cutting papers.



(i)

Figure - 14.8

We can draw symmetrical figures from asymmetrical figures with the help of graph paper. We can draw a symmetrical figure from any asymmetrical figure using tracing paper. In many cases for a symmetrical figure there may be many lines of symmetry. The line of symmetry in isosceles triangle is one.



Line of symmetry

Figure - 14.9 Line of symmetry in isosceles triangle

Line of symmetry in equilateral triangle is three (3)

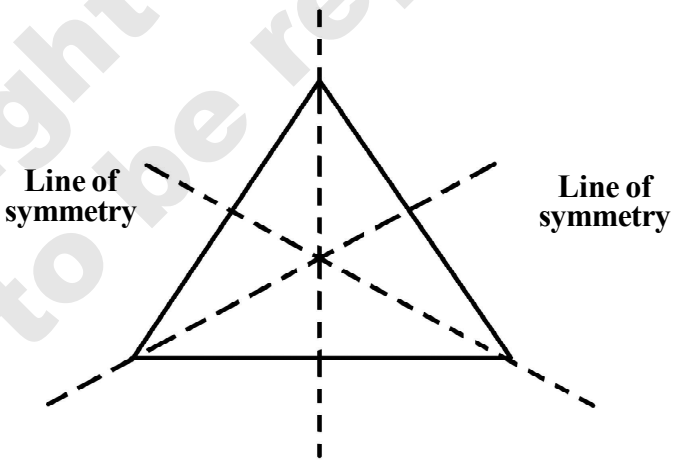


Figure - 14.10 Line of symmetry in equilateral triangle

There are two lines of symmetry in each of a rectangle and a rhombus.

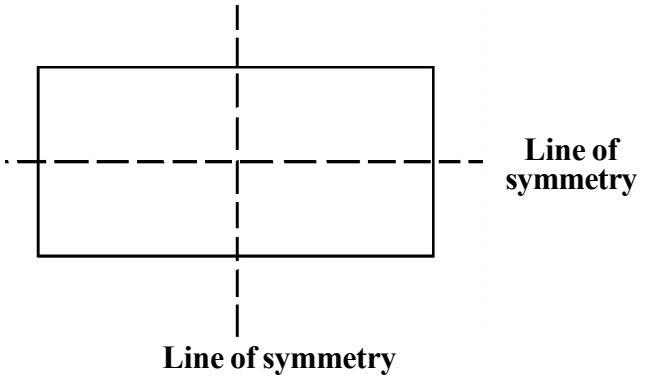


Figure - 14.11 Line of symmetry in rectangle

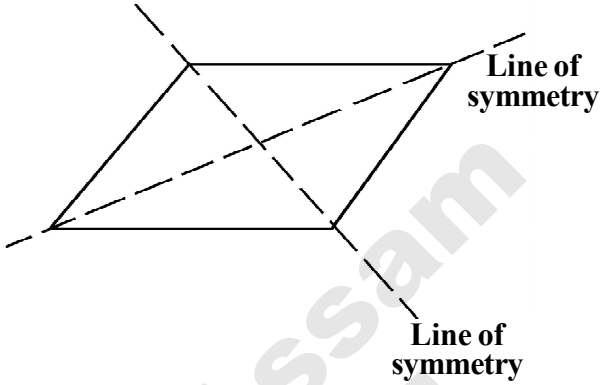


Figure - 14.12 Lines of symmetry rhombus

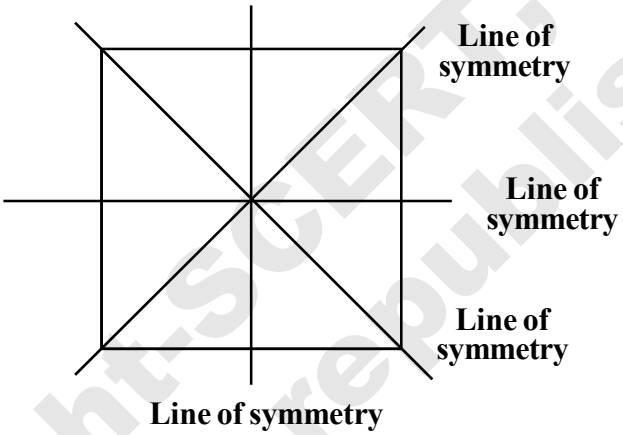


Figure - 14.13 Lines of symmetry in square

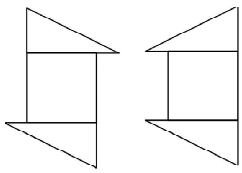
Similarly the lines of symmetry in Regular pentagon, hexagon, heptagon etc. are 5, 6, 7, ... respectively.

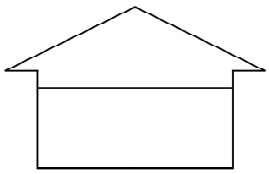
Exercise - 14.1

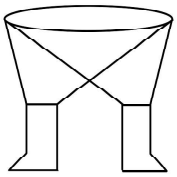
1. Give five examples of symmetry from plant kingdom.
2. Mention five objects of your house where symmetry is visible.
3. Draw the lines of symmetry of a parallelogram.
4. Give an example of geometric shape which has no line of symmetry.
5. How many lines of symmetry are there in a regular hexagon? Show the lines of symmetry of a regular hexagon.
6. How many lines of symmetry are there in a circle?

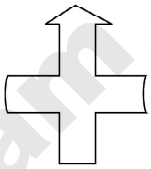
7.  How many lines of symmetry are there in the figure?

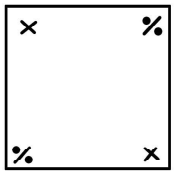
8. Identify the symmetrical and asymmetical figures from the following–

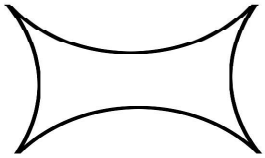

(i) (ii)


(iii)


(iv)


(v)


(vi)


(vii)


(viii)

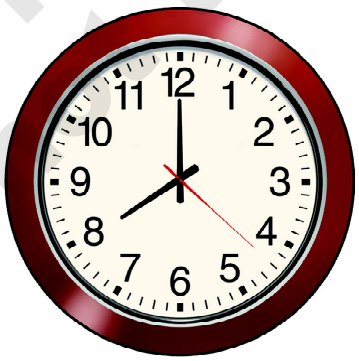

(ix)

14.3 Rotational Symmetry :

Often, when a shape is rotated about a fixed point, it looks similar in certain positions with is previous position as some of the shape when folded along a line both parts look similar. Therefore, like linear symmetry we can think of another type of symmetry which is depended up on rotation of shapes. In this case let us know first about rotation.

14.3.1 Rotation of hands of the clock :

We wear a watch to keep time. You have certainly seen wall clocks on the walls of homes and offices. A clock generally comprises of 2 hands – one for the hours and the other for minute. Both hands of a clock rotate from 1 to 12 or other such points marked on the dial to indicate time. Rotation of an objects is generally compared with rotation of a clock. When an object rotates in the same direction as a clock is refered as clockwise and when a object rotates in opposite of the hands of clock is refered as anticlockwise rotation.



(i) Diagram of a clock

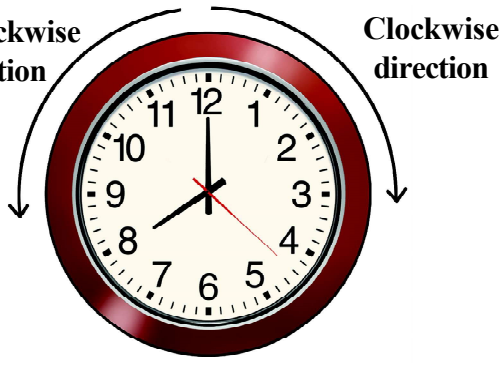


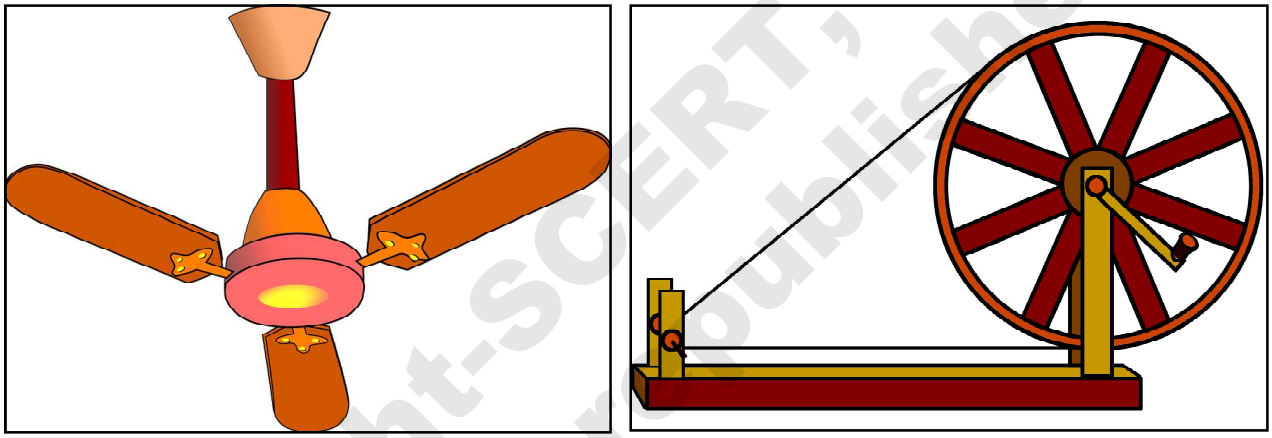
Figure -14.14

Let us now discuss the rotation of some objects used in our daily life.

The wheels, the pedals and the chain of a bicycle start to rotate when it start running. When a bicycle is running each of its two wheels, padel and chain are start to rotate.

What is the direction of these rotation ?

If you observe the running by cycle from your right hand side, you will see that the wheels, pedals and chain of the bicycle rotate clockwise. On the otherhand, if you observe the bicycle from your left hand side then you will see that each of these rotate in the anticlockwise direction. Observe a moving fan. If you look at the fan overhead, you will see it rotating in the anticlockwise direction. If however, it is possible to see from the top, you will see it rotating in the clockwise direction.



(i) Electric fan (ii) Spinning wheel (Charkha)

Figure -14.15

14.3.2 Centre of rotation and angle of rotation :

When an object rotates, its shape and size does not change, only the different places of the object is changed. Remember, that an object rotates about a fixed point. This fixed point is called the centre of rotation.

Again, when an object rotates about a fixed point, then all other points except the centre change their position. The respective angles made by the primary and final position of the points with the centre remains the same. This angle is called the angle of rotation.

Let us try to understand this by a simple activity.

Cut out any ΔABC from a card board. Now place ΔABC on a drawing sheet. Now put a pin in such a way so that you can rotate ΔABC with respect to a point 'O'. After that

mark with pencil on the drawing sheet according to the position of the vertex of ΔABC. Mark it as A, B, C respectively. Now rotate ΔABC in clockwise direction. For new position of the vertices of the ΔABC put make as A', B' and C' respectively. Now take out mark the ΔABC from the card board and join OA, OB, OC OA', OB', OC' with scale.

Measure $\angle AOA'$, $\angle BOB'$, $\angle COC'$ with protractor.

Are the measurement of the angles same?

You may understand that with respect to the centre of rotation of ΔABC the three vertices rotate making equal angles.

$\angle AOA' = \angle BOB' = \angle COC'$

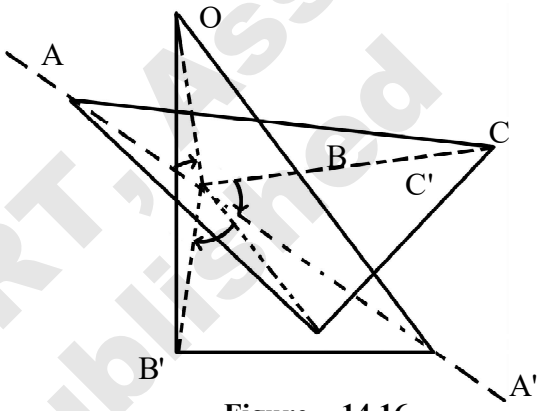


Figure - 14.16

Let us Observe :

Even if you can take any point other then O you can get same angle of measurement. Here 360^0 rotation of ΔABC means one complete rotation in clockwise direction with respect to the centre of rotation 'O'. Therefore vertices A, B, C and other point of ΔABC will rotate one complete rotation in clockwise direction with respect to O and again come to their own position i.e. after one complete rotation of ΔABC or rotation of 360^0 the triangle will look the same.

Therefore the order of rotational symmetry of the triangle is 1. Do you know any shape or a figure where angle of rotation is less than 360^0 but looks like same as its initial position.

Look at the figure 14.17. It is a model of an electric fan with two blades. Rotate the blades of the electric fan clockwise in different angles by putting a stick through the point 'O'.

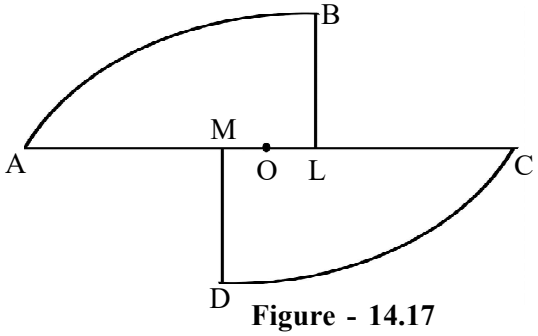


Figure - 14.17

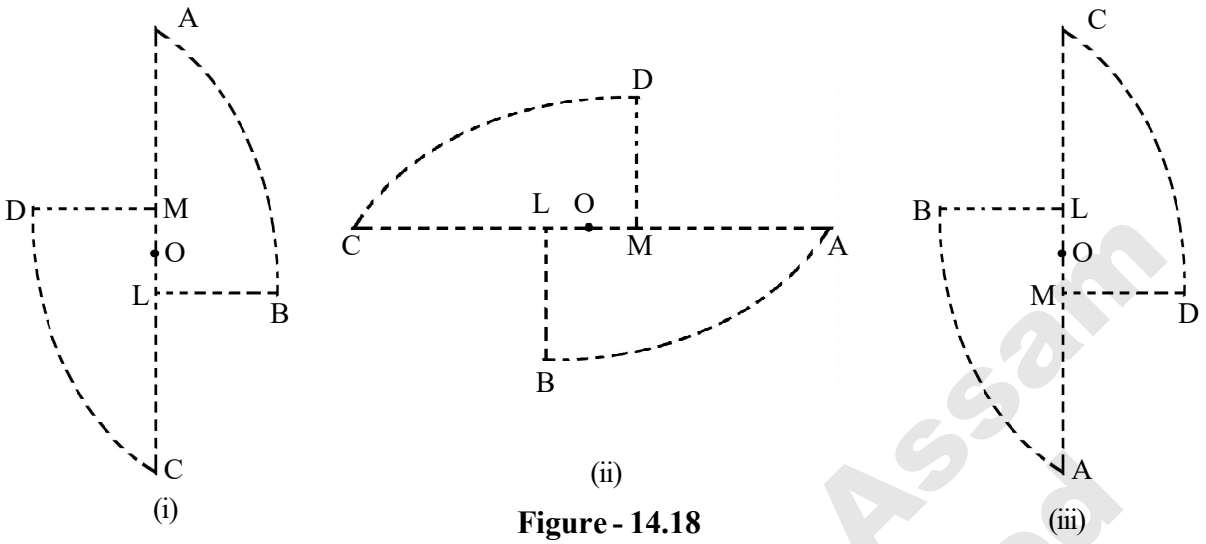


Figure - 14.18

If we rotate the fan by 90^0 or $\frac{1}{4}$ turn, then it looks as in Fig (i). Again if rotated the fan by 180^0 or $\frac{1}{2}$ turn then looks like as in Fig (ii). Similarly if you rotate it by 270^0 ($\frac{3}{4}$ turn) in the clockwise direction it will look like as in fig (iii). After that if the fan is rotated one complete turn i.e. 360^0 it will look like as in fig 14.17.

It is observed that if the fan with two blades is rotated in a clockwise direction making angles 180^0 and 360^0 then the fan looks like same as in original position.

If a shape is rotated by a definite angle with respect to any point and it appears similar to its riginal position, then the shape is said to have rotational symmetry. During one complete rotation of the shape, the number of times the shape looks like same is called the order of rotational symmetry.

As discussed above, the fan with two blades will have one complete rotation only a at the angles of 180^0 and 360^0. Therefore the orde with two blades is 2.

Is there any rotational symmetry in case of a fan with three blades ? If yes, what will be the order of its rotational symmetry.

Look at the figure 14.19. It is a model of a fan with three blades. The angles of the blades of the fan make an angle to each other.

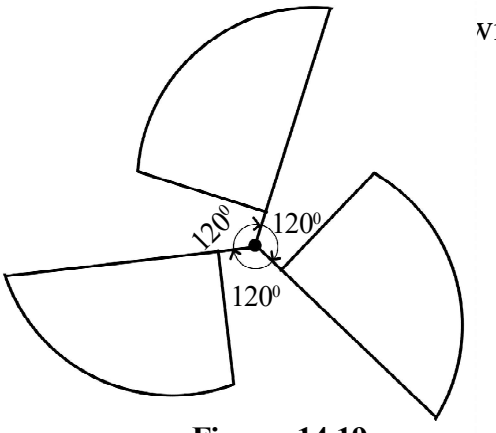


Figure -14.19

Consider an equilateral triangle ABC. It is a rectilinear symmetrical figure. The lines of symmetry AD, BE and CF meet at O. When measured with protractor you will find that the line segments OA, OB and OC make an angle of 120^0 with each other. (Fig. 14.20) but the position of the vertices of the triangle will change (Fig 14.21)

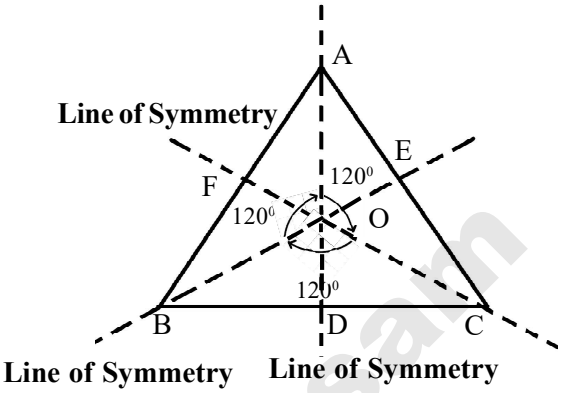


Figure - 14.20

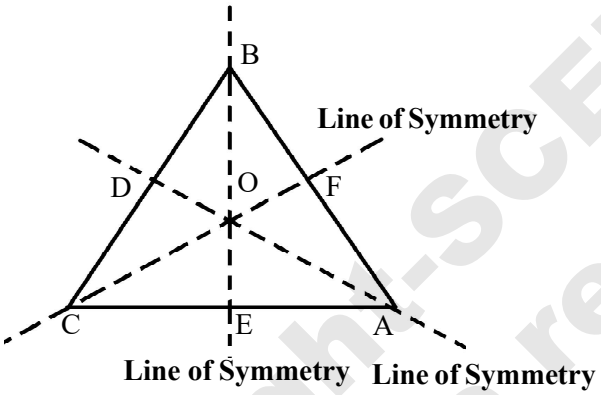


Figure - 14.21

If the triangle is rotated at an angle of 120^0, though it will look being like in its original position, in reallity the position of the vertices of the triangle had changed.

Again, rotate the triangle at 120^0 in clockwise direction, i.e. is total $(120^0 + 120^0) = 240^0$ from the original position. You must have noticed that their triangle looks same as fig 14.22. But in reality the position of the vertices have changed.

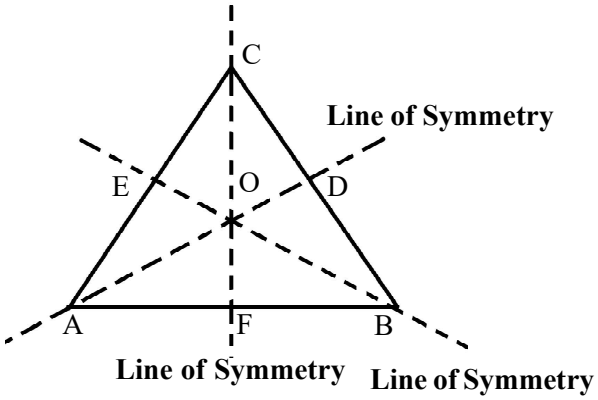


Figure - 14.22

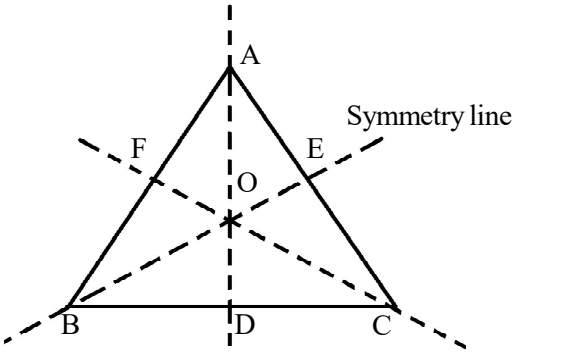


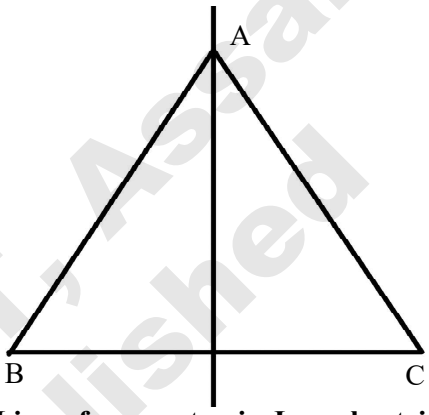
Figure - 14.23

Similarly, if you rotate the triangle at 120^0 in clockwise direction i.e. in total $(120^0 + 120^0 + 120^0)360^0$, i.e. one complete rotation, then the triangle will look like fig 14.23. This time the position of vertices of the triangle will remain same as in fig 14.20.

Therefore we observe that if, an equilateral triangle is rotated three times, that is, at 120^0 each time, it looks like Fig 14.20

So we can conclude that the equilateral triangle has rotational symmetry. 'O' is the centre of rotation, angles of rotation are $120^0, 240^0$ and 360^0. Therefore order of rotational symmetry is 3.

Let us find rotational symmetry for Isosceleus Triangle.

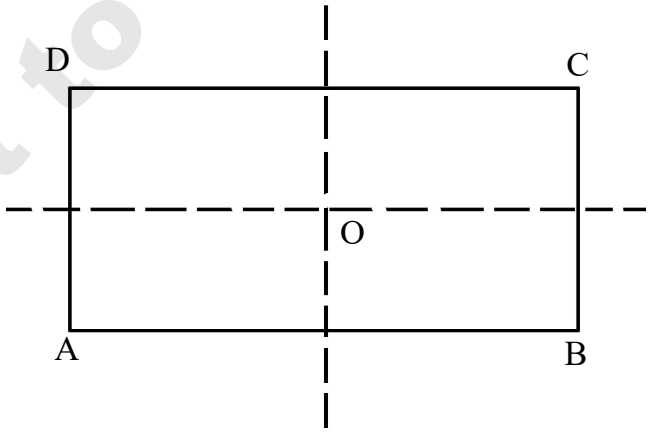


Line of symmetry in Isosceles triangle

Figure - 14.24

Draw a copy of the isoscelus triangle (fig ...) on card board. Now cut out the triangle.

We will know the rotational symmetry of a rectangle by adopting the same technique. You know that a rectangle is a rectlinear symmetrical figure. The number of lines of symmetry of a rectangle is 2 (Look at the following picture)



Symmetry line

Figure -14.25

How many times it will look the same at the centre of rotation of the rectangle with respect to O. (in one complete rotation).

How many times the centre of rotation of the rectangle with respect to O in one complete rotation (360^0) looks same as in initial position? Is it two times? When rotated at an angle of 180^0 and 360^0 then it will look same as in initial position.Therefore the order of rotational symmetry is 2.

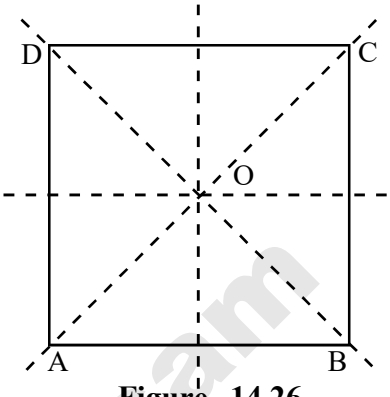


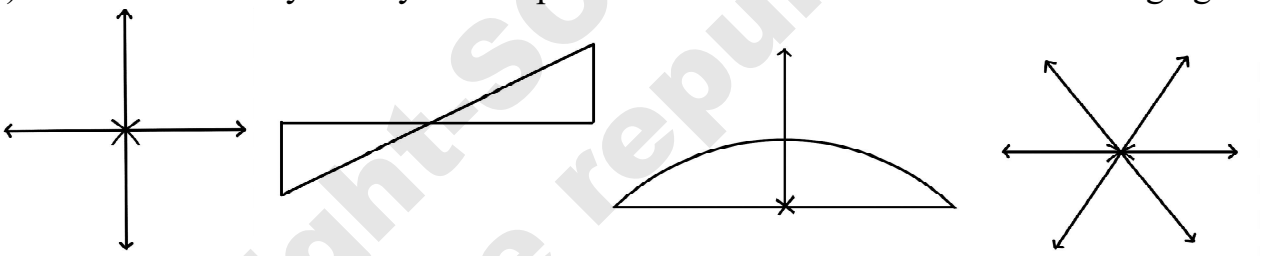
Figure - 14.26

Square is a rectilinear symmetrical figure. The number of lines of symmetry is 4 (look at the above figure). How many times the centre of rotation of the square with respect to O in one complete rotation (360^0) looks same as in intial position ? We shall observe that when there is a rotation at an angle of 90^0, 180^0, 270^0 and 360^0, then it will look same as in original position.

Therefore the order of rotational symmetry is 4. The angle of rotations are 90^0, 180^0, 270^0 and 360^0. The centre of rotation is O. Now discuss with your ... and try to solve the following problem.

(i) The number of rotational symmetry of parallelogram and rhombus.

(ii) Find the order of symmetry with respect to centre of rotation from the following figure

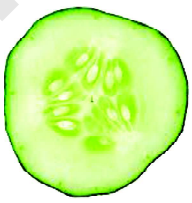


(i) (ii) (iii) (iv)

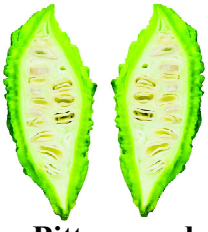
Figure - 14.27

We have seen many examples of rotational symmetry around us.

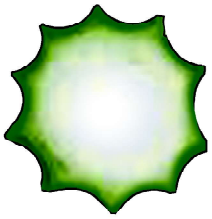
We have seen rotational symmetry common objects of daily use such as different types of porcelain dishes, plates etc. Even in some Japis, used as show pieces hung upon the walls, we find rotational symmetry you shall also notice rotational symmetry in the external layers of some vegetables such as cucumber, ridge gourds,bitter gourd etc. It is also visible in lemon, starfruit and sweet lime (decorated bamboo item of Assam originally used as subtitute of umbrella) we find rotational symmetry.



Cuccumber



Bitter gourd

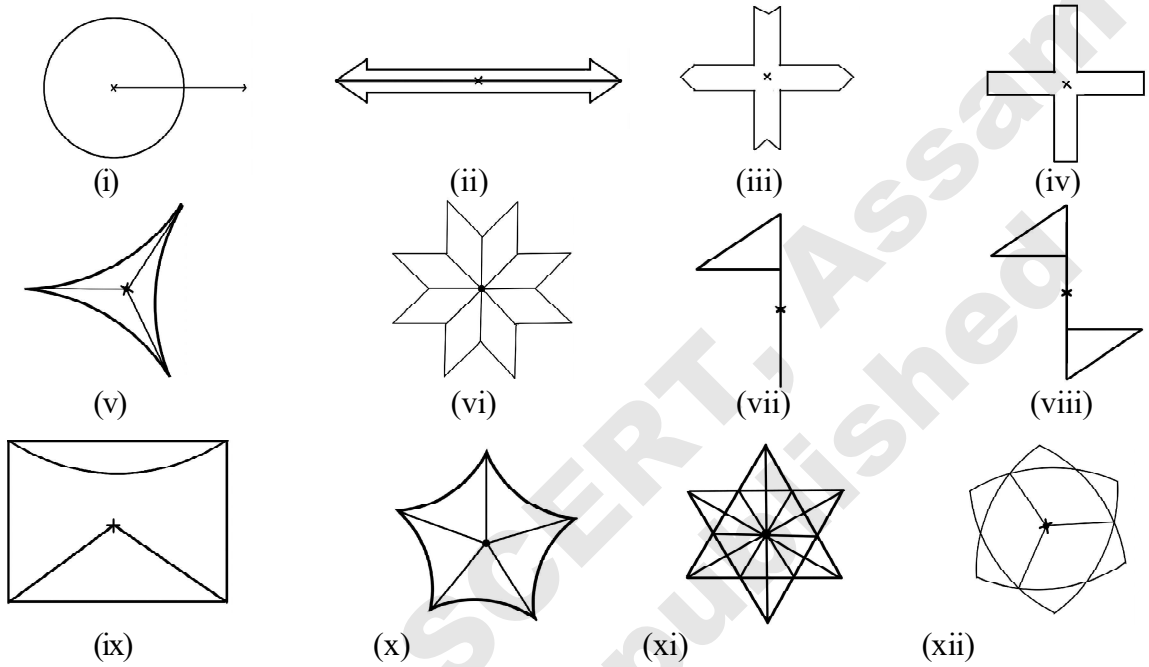


Ridge gourd

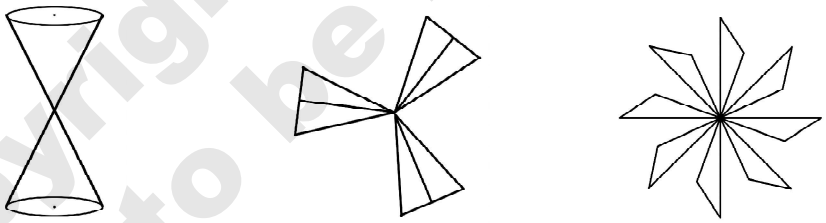
Figure - 14.28

Exercise -14.2

1. Look at the following shapes. Classify shapes having multiple order of rotation with respect to the marked centre of rotation. Mention the order of the rotational symmetry.



2. Verify the rotational symmetry of the following shapes. Mention the centre of rotation, the angle of rotation and order of rotational symmetry.



14.4 Rotational and linear Symmetry

We have observed from the discussion that the concept of symmetry can be put forwarded through 2 approaches– Line Symmetry and Rotational Symmetry

There is a relation between order of rotational symmetry and rotational symmetry. Like the relationship between line of symmetry and linear symmetry.

Is there any relation of rotational symmetry with the line symmetry of different shapes ?

Is there any relation of linear symmetry with rotational symmetry ?

Let us consider the case of a regular polygon. You must be aware that a regular polygon with smallest side is an equilateral triangle. You know that an equilateral triangle is a symmetrical figure and the lines of symmetry is 3.

Simalarly, there is a rotational symmetry in an equilateral triangle and the order of the rotational symmetry is 3.

Therefore, there is line symmetry as well as rotational symmetry in an equilateral triangle. The number of lines of symmetry and number of rotational symmetry is 3.

The next regular polygon after equilateral triangle is square. We know that the lines of symmetry and the order of rotational symmetry in a square is 4. The angles of rotation of a square are 90^0, 180^0, 270^0 and 360^0.

Therefore the number of lines of symmetry and the order of rotational symmetry is same. Similarly, the number of line symmetry and the order of line symmetry in a regular pentagon is same i.e. 5.

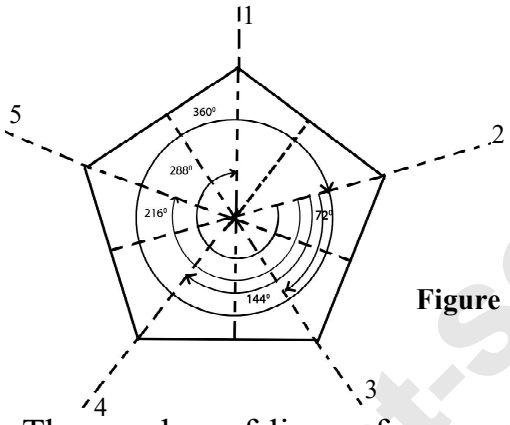


Figure - 14.29

In the figures 1, 2, 3, 4 and 5 the line of symmetry and rotational angle of 72^0, 144^0, 216^0, 288^0, 360^0 are shown.

The number of lines of symmetry and order of rotational symmetry in case of regular hexagon is same i.e. 6.

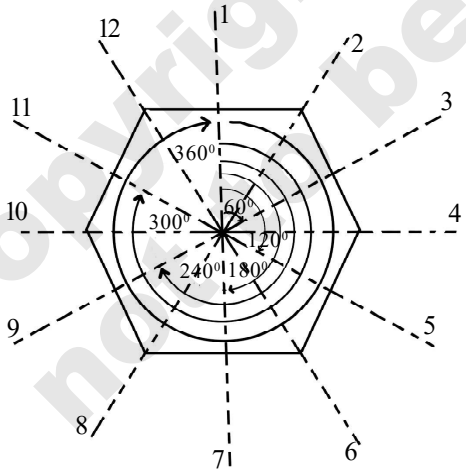


Figure - 14.30

In the figures 1, 2, 3, 4, 5 and 6 are lines of symmetry and rotational angles are 60^0, 120^0, 180^0, 240^0, 300^0 and 360^0 respectively.

Similarly we can come to the conclusion that the number of lines of symmetry and rotational symmetry is same as the number of the sides of the polygon.

In case of irregular polygon the lines of symmetry and rotational symmetry is not

same. Some polygons may not be symmetrical in shape. For example the shapes of a trapezium and a parallelogram are not symmetrical. Therefore both the shapes have no line of symmetry. But both of them have rotational symmetry. The order of rotational symmetry of a trapezium and a parallelogram are 1 and 2 repectively.

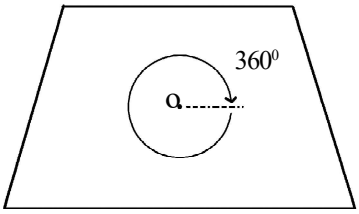


Figure - 14.31

A trapezium is not a symmetrical shape. There is no line of symmetry in a trapezium. But it has rotational symmetry. The angle of rotation of a trapezium is 360^0 and the order of rotation is 1.

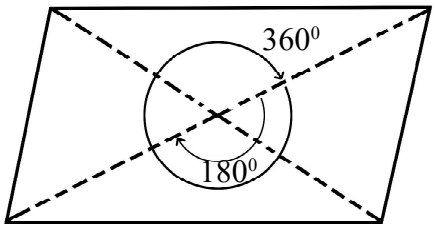


Figure - 14.32

A parallelogram is not a symmetrical shape. The number of lines of symmetry is 2. It has rotational symmetry. The order of its rotational symmetry is 2 and the angles of rotation is 180^0 and 360^0.

A rhombus is a symmetrical shape. The lines of symmetry of a rhombus is 2. It has rotational symmetry. The order of its rotational symmetry is 2. and the angles of rotation 180^0 and 360^0.

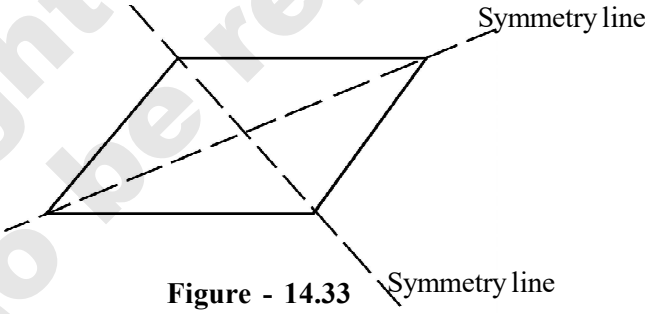


Figure - 14.33

A kite is a symmetrical shape. The number of line symmetry of a kite is 1. It has rotational symmetry also. The order of rotational symmetry of a kite is 1 and the angle of rotation is 360^0.

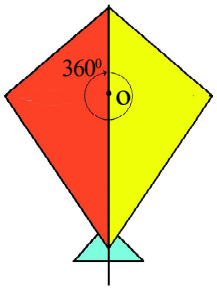


Figure - 14.34

Therefore it may be concluded that in case of irregular (not regular) polygon no definite relationship is seen between line of symmetry and order of rotation.

What will be the line symmetry and rotational symmetry in case of a circle?

A circle is a symmetrical shape with respect to its diameter. This means any diameter of a circle is a line of symmetry of the circle. Therefore a circle has infinite number of lines of symmetry.

On the other hand any angle produced at the centre of the circle looks same from any position of a circle in its angle of rotation. Since we can have any measurment of angle between 0^0 to 360^0, therefore we can say that a circle has lines of symmetry as well as rotational symmetry.

Try to understand line symmetry, lines of symmetry etc of different shapes seen in daily life.

Fill up the following table.

Shape	Line symmetry	Number of line symmetry	Rotational Symmetry	Order of Rotational symmetry
W	**Yes**	**1**	**Yes**	**1**
T				
O				
	**No**	**0**	**Yes**	**1**
A				

Exercise - 14.3

1. Name any two shapes that have both line symmetry and rotational symmetry.
2. Draw a sketch of triangle with line symmetry and rotational symmetry of order more than 1.
3. Draw the sketch of a quadrilateral which has no line of symmetry but has order of rotational symmetry is more than 1.
4. Draw the sketch of a quadrilateral which has both line of symmetry and a rotational symmetry.
5. Is there any triangle in which the order of rotational symmetry is 2?
6. What is the order of rotational symmetry in the Englishletter S? Does the letter S possess line symmetry?
7. What is the order of rotational symmetry of 3? Is there any line of symmetry in 3 ?
8. Write 6969 on a paper. Taking 6969 as geometrical shape, find the order of the rotational symmetry?

9. A figure is rotated at an angle of 120^0 in clockwise direction. The figure looks like in its initial position. Again for what other angles will the figure look same? What is the order of rotation of the figure?
10. Can we have a rotational symmetry of order more than one 1 whose angle of rotation is 72^0?
11. Can we have a rotational symmetry of order more than 1 whose angle rotation is 17^0?
12. Fill up the following table –

Shape	Centre of rotation	Angle of rotation	Order of rotational symmetry
Rectangle			
Square			
Rhombus			
Parallelogram			
Trapezium			
Regular Polygon			
Equilateral Triangle			
Isosceles Triangle			
Scalene Triangle			
Circle			
Semi Circle			

What we have learnt

1. A figure has line symmetry, if there is a line about which the figure may be folded so that two parts of the figure will coincide.
2. We have many objects around us like leaves, certain foods and physical structure of animal and plants which possess symmetry.
3. The artists, architects, potters etc. use the concept of symmetry in making car, ornaments etc.
4. Regular polygons e.g. equilateral triangle, square, pentagon have equal sides and equal angles. They have multiple (i.e. more than 1) lines of symmetry.
5. Each regular polygon has as many lines of symmetry as it has sides.
6. Rotation turns an object about a fixed point. The fixed point is the centre of rotation.
7. Rotation may be in clockwise or in anticlockwise direction.
8. If, after a rotation an object looks exactly the same as original then object is said to have rotational symmetry.
9. In a complete rotation or turn (360^0), the number of times an object looks exactly the same is called the order of rotational symmetry.