

# 1. Relation

## Exercise 1A

### 1. Question

Find the domain and range of the relation

$$R = \{(-1, 1), (1, 1), (-2, 4), (2, 4)\}.$$

### Answer

$$\text{dom}(R) = \{-1, 1, -2, 2\} \text{ and } \text{range}(R) = \{1, 4\}$$

### 2. Question

Let  $R = \{(a, a^3) : a \text{ is a prime number less than } 5\}$ .

Find the range of  $R$ .

### Answer

$$\text{range}(R) = \{8, 27\}$$

### 3. Question

Let  $R = \{(a, a^3) : a \text{ is a prime number less than } 10\}$ .

Find (i)  $R$  (ii)  $\text{dom}(R)$  (iii)  $\text{range}(R)$ .

### Answer

$$(i) R = \{(2, 8), (3, 27), (5, 125), (7, 343)\}$$

$$(ii) \text{dom}(R) = \{2, 3, 5, 7\}$$

$$(iii) \text{range}(R) = \{8, 27, 125, 343\}$$

### 4. Question

Let  $R = (x, y) : x + 2y = \text{be a relation on } \mathbb{N}$ .

Write the range of  $R$ .

### Answer

$$\{3, 2, 1\}$$

### 5. Question

Let  $R = \{(a, b) : a, b \in \mathbb{N} \text{ and } a + 3b = 12\}$ .

Find the domain and range of  $R$ .

### Answer

$$\text{dom}(R) = \{3, 6, 9\} \text{ and } \text{range}(R) = \{3, 2, 1\}$$

### 6. Question

Let  $R = \{(a, b) : b = |a - 1|, a \in \mathbb{Z} \text{ and } |a| < 3\}$ .

Find the domain and range of  $R$ .

### Answer

$$\text{dom}(R) = \{-2, -1, 0, 1, 2\} \text{ and } \text{range}(R) = \{3, 2, 1, 0\}$$

### 7. Question

$$\text{Let } R = \left\{ \left( a, \frac{1}{a} \right) : a \in \mathbb{N} \text{ and } 1 < a < 5 \right\}.$$

Find the domain and range of R.

**Answer**

$$\text{dom}(R) = \{2, 3, 4\} \text{ and range}(R) = \left\{ \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \right\}$$

### 8. Question

Let  $R = \{(a, b) : a, b \in \mathbb{N} \text{ and } b = a + 5, a < 4\}$ .

Find the domain and range of R.

**Answer**

$$\text{dom}(R) = \{1, 2, 3\} \text{ and range}(R) = \{6, 7, 8\}$$

### 9. Question

Let S be the set of all sets and let  $R = \{(A, B) : A \subset B\}$ , i.e., A is a proper subset of B. Show that R is (i) transitive (ii) not reflexive (iii) not symmetric.

**Answer**

Let  $R = \{(A, B) : A \subset B\}$ , i.e., A is a proper subset of B, be a relation defined on S.

Now,

Any set is a subset of itself, but not a proper subset.

$$\Rightarrow (A, A) \notin R \quad \forall A \in S$$

$\Rightarrow R$  is not reflexive.

Let  $(A, B) \in R \quad \forall A, B \in S$

$\Rightarrow A$  is a proper subset of B

$\Rightarrow$  all elements of A are in B, but B contains at least one element that is not in A.

$\Rightarrow B$  cannot be a proper subset of A

$$\Rightarrow (B, A) \notin R$$

For e.g., if  $B = \{1, 2, 5\}$  then  $A = \{1, 5\}$  is a proper subset of B. we observe that B is not a proper subset of A.

$\Rightarrow R$  is not symmetric

Let  $(A, B) \in R$  and  $(B, C) \in R \quad \forall A, B, C \in S$

$\Rightarrow A$  is a proper subset of B and B is a proper subset of C

$\Rightarrow A$  is a proper subset of C

$$\Rightarrow (A, C) \in R$$

For e.g., if  $B = \{1, 2, 5\}$  then  $A = \{1, 5\}$  is a proper subset of B.

And if  $C = \{1, 2, 5, 7\}$  then  $B = \{1, 2, 5\}$  is a proper subset of C.

We observe that  $A = \{1, 5\}$  is a proper subset of C also.

$\Rightarrow R$  is transitive.

Thus, R is transitive but not reflexive and not symmetric.

### 10. Question

Let A be the set of all points in a plane and let O be the origin. Show that the relation  $R = \{(P, Q) : P, Q \in A \text{ and } OP = OQ\}$  is an equivalence relation.

### Answer

In order to show R is an equivalence relation, we need to show R is Reflexive, Symmetric and Transitive.

Given that, A be the set of all points in a plane and O be the origin. Then,  $R = \{(P, Q) : P, Q \in A \text{ and } OP = OQ\}$

Now,

R is Reflexive if  $(P, P) \in R \forall P \in A$

$\forall P \in A$ , we have

$$OP = OP$$

$$\Rightarrow (P, P) \in R$$

Thus, R is reflexive.

R is Symmetric if  $(P, Q) \in R \Rightarrow (Q, P) \in R \forall P, Q \in A$

Let  $P, Q \in A$  such that,

$$(P, Q) \in R$$

$$\Rightarrow OP = OQ$$

$$\Rightarrow OQ = OP$$

$$\Rightarrow (Q, P) \in R$$

Thus, R is symmetric.

R is Transitive if  $(P, Q) \in R \text{ and } (Q, S) \in R \Rightarrow (P, S) \in R \forall P, Q, S \in A$

Let  $(P, Q) \in R$  and  $(Q, S) \in R \forall P, Q, S \in A$

$$\Rightarrow OP = OQ \text{ and } OQ = OS$$

$$\Rightarrow OP = OS$$

$$\Rightarrow (P, S) \in R$$

Thus, R is transitive.

Since R is reflexive, symmetric and transitive it is an equivalence relation on A.

### 11. Question

On the set S of all real numbers, define a relation  $R = \{(a, b) : a \leq b\}$ .

Show that R is (i) reflexive (ii) transitive (iii) not symmetric.

### Answer

Let  $R = \{(a, b) : a \leq b\}$  be a relation defined on S.

Now,

We observe that any element  $x \in S$  is less than or equal to itself.

$$\Rightarrow (x, x) \in R \forall x \in S$$

$\Rightarrow R$  is reflexive.

Let  $(x, y) \in R \forall x, y \in S$

$\Rightarrow x$  is less than or equal to  $y$

But  $y$  cannot be less than or equal to  $x$  if  $x$  is less than or equal to  $y$ .

$\Rightarrow (y,x) \notin R$

For e.g. , we observe that  $(2,5) \in R$  i.e.  $2 < 5$  but 5 is not less than or equal to 2  $\Rightarrow (5,2) \notin R$

$\Rightarrow R$  is not symmetric

Let  $(x,y) \in R$  and  $(y,z) \in R \forall x, y, z \in S$

$\Rightarrow x \leq y$  and  $y \leq z$

$\Rightarrow x \leq z$

$\Rightarrow (x,z) \in R$

For e.g. , we observe that

$(4,5) \in R \Rightarrow 4 \leq 5$  and  $(5,6) \in R \Rightarrow 5 \leq 6$

And we know that  $4 \leq 6 \therefore (4,6) \in R$

$\Rightarrow R$  is transitive.

Thus,  $R$  is reflexive and transitive but not symmetric.

## 12. Question

Let  $A = \{1, 2, 3, 4, 5, 6\}$  and let  $R = \{(a, b) : a, b \in A \text{ and } b = a + 1\}$ .

Show that  $R$  is (i) not reflexive, (ii) not symmetric and (iii) not transitive.

## Answer

Given that,

$A = \{1, 2, 3, 4, 5, 6\}$  and  $R = \{(a, b) : a, b \in A \text{ and } b = a + 1\}$ .

$\therefore R = \{(1,2),(2,3),(3,4),(4,5),(5,6)\}$

Now,

$R$  is Reflexive if  $(a,a) \in R \forall a \in A$

Since,  $(1,1),(2,2),(3,3),(4,4),(5,5),(6,6) \notin R$

Thus,  $R$  is not reflexive .

$R$  is Symmetric if  $(a,b) \in R \Rightarrow (b,a) \in R \forall a,b \in A$

We observe that  $(1,2) \in R$  but  $(2,1) \notin R$  .

Thus,  $R$  is not symmetric .

$R$  is Transitive if  $(a,b) \in R$  and  $(b,c) \in R \Rightarrow (a,c) \in R \forall a,b,c \in A$

We observe that  $(1,2) \in R$  and  $(2,3) \in R$  but  $(1,3) \notin R$

Thus,  $R$  is not transitive.

## Exercise 1B

### 1. Question

Define a relation on a set. What do you mean by the domain and range of a relation? Give an example.

## Answer

**Relation:** Let  $A$  and  $B$  be two sets. Then a relation  $R$  from set  $A$  to set  $B$  is a subset of  $A \times B$ . Thus,  $R$  is a relation to  $A$  to  $B \Leftrightarrow R \subseteq A \times B$ .

If  $R$  is a relation from a non-void set  $B$  and if  $(a,b) \in R$ , then we write  $a R b$  which is read as 'a is related to b by the relation  $R$ '. if  $(a,b) \notin R$ , then we write  $a \not R b$ , and we say that  $a$  is not related to  $b$  by the relation  $R$ .

**Domain:** Let  $R$  be a relation from a set  $A$  to a set  $B$ . Then the set of all first components or coordinates of the ordered pairs belonging to  $R$  is called the domain of  $R$ .

Thus, domain of  $R = \{a : (a,b) \in R\}$ . The domain of  $R \subseteq A$ .

**Range:** let  $R$  be a relation from a set  $A$  to a set  $B$ . then the set of all second component or coordinates of the ordered pairs belonging to  $R$  is called the range of  $R$ .

Example 1:  $R = \{(-1, 1), (1, 1), (-2, 4), (2, 4)\}$ .

$\text{dom}(R) = \{-1, 1, -2, 2\}$  and  $\text{range}(R) = \{1, 4\}$

Example 2:  $R = \{(a, b) : a, b \in \mathbb{N} \text{ and } a + 3b = 12\}$

$\text{dom}(R) = \{3, 6, 9\}$  and  $\text{range}(R) = \{3, 2, 1\}$

## 2. Question

Let  $A$  be the set of all triangles in a plane. Show that the relation

$R = \{(\Delta_1, \Delta_2) : \Delta_1 \sim \Delta_2\}$  is an equivalence relation on  $A$ .

### Answer

Let  $R = \{(\Delta_1, \Delta_2) : \Delta_1 \sim \Delta_2\}$  be a relation defined on  $A$ .

Now,

$R$  is Reflexive if  $(\Delta, \Delta) \in R \forall \Delta \in A$

We observe that for each  $\Delta \in A$  we have,

$\Delta \sim \Delta$  since, every triangle is similar to itself.

$\Rightarrow (\Delta, \Delta) \in R \forall \Delta \in A$

$\Rightarrow R$  is reflexive.

$R$  is Symmetric if  $(\Delta_1, \Delta_2) \in R \Rightarrow (\Delta_2, \Delta_1) \in R \forall \Delta_1, \Delta_2 \in A$

Let  $(\Delta_1, \Delta_2) \in R \forall \Delta_1, \Delta_2 \in A$

$\Rightarrow \Delta_1 \sim \Delta_2$

$\Rightarrow \Delta_2 \sim \Delta_1$

$\Rightarrow (\Delta_2, \Delta_1) \in R$

$\Rightarrow R$  is symmetric

$R$  is Transitive if  $(\Delta_1, \Delta_2) \in R$  and  $(\Delta_2, \Delta_3) \in R \Rightarrow (\Delta_1, \Delta_3) \in R \forall \Delta_1, \Delta_2, \Delta_3 \in A$

Let  $(\Delta_1, \Delta_2) \in R$  and  $(\Delta_2, \Delta_3) \in R \forall \Delta_1, \Delta_2, \Delta_3 \in A$

$\Rightarrow \Delta_1 \sim \Delta_2$  and  $\Delta_2 \sim \Delta_3$

$\Rightarrow \Delta_1 \sim \Delta_3$

$\Rightarrow (\Delta_1, \Delta_3) \in R$

$\Rightarrow R$  is transitive.

Since  $R$  is reflexive, symmetric and transitive, it is an equivalence relation on  $A$ .

## 3. Question

Let  $R = \{(a, b) : a, b \in \mathbb{Z} \text{ and } (a + b) \text{ is even}\}$ .

Show that  $R$  is an equivalence relation on  $\mathbb{Z}$ .

### Answer

In order to show  $R$  is an equivalence relation, we need to show  $R$  is Reflexive, Symmetric and Transitive.

Given that,  $\forall a, b \in \mathbb{Z}$ ,  $R = \{(a, b) : (a + b) \text{ is even}\}$ .

Now,

R is Reflexive if  $(a,a) \in R \forall a \in \mathbb{Z}$

For any  $a \in \mathbb{Z}$ , we have

$a+a = 2a$ , which is even.

$\Rightarrow (a,a) \in R$

Thus, R is reflexive.

R is Symmetric if  $(a,b) \in R \Rightarrow (b,a) \in R \forall a,b \in \mathbb{Z}$

$(a,b) \in R$

$\Rightarrow a+b$  is even.

$\Rightarrow b+a$  is even.

$\Rightarrow (b,a) \in R$

Thus, R is symmetric .

R is Transitive if  $(a,b) \in R$  and  $(b,c) \in R \Rightarrow (a,c) \in R \forall a,b,c \in \mathbb{Z}$

Let  $(a,b) \in R$  and  $(b,c) \in R \forall a, b,c \in \mathbb{Z}$

$\Rightarrow a+b = 2P$  and  $b+c = 2Q$

Adding both, we get

$a+c+2b = 2(P+Q)$

$\Rightarrow a+c = 2(P+Q)-2b$

$\Rightarrow a+c$  is an even number

$\Rightarrow (a, c) \in R$

Thus, R is transitive on  $\mathbb{Z}$ .

Since R is reflexive, symmetric and transitive it is an equivalence relation on  $\mathbb{Z}$ .

#### 4. Question

Let  $R = \{(a, b) : a, b \in \mathbb{Z} \text{ and } (a - b) \text{ is divisible by } 5\}$ .

Show that R is an equivalence relation on  $\mathbb{Z}$ .

#### Answer

In order to show R is an equivalence relation, we need to show R is Reflexive, Symmetric and Transitive.

Given that,  $\forall a, b \in \mathbb{Z}$ ,  $aRb$  if and only if  $a - b$  is divisible by 5.

Now,

R is Reflexive if  $(a,a) \in R \forall a \in \mathbb{Z}$

$aRa \Rightarrow (a-a)$  is divisible by 5.

$a-a = 0 = 0 \times 5$  [since 0 is multiple of 5 it is divisible by 5]

$\Rightarrow a-a$  is divisible by 5

$\Rightarrow (a,a) \in R$

Thus, R is reflexive on  $\mathbb{Z}$ .

R is Symmetric if  $(a,b) \in R \Rightarrow (b,a) \in R \forall a,b \in \mathbb{Z}$

$(a,b) \in R \Rightarrow (a-b)$  is divisible by 5

$$\Rightarrow (a-b) = 5z \text{ for some } z \in \mathbb{Z}$$

$$\Rightarrow -(b-a) = 5z$$

$$\Rightarrow b-a = 5(-z) [\because z \in \mathbb{Z} \Rightarrow -z \in \mathbb{Z}]$$

$$\Rightarrow (b-a) \text{ is divisible by } 5$$

$$\Rightarrow (b,a) \in R$$

Thus, R is symmetric on  $\mathbb{Z}$ .

R is Transitive if  $(a,b) \in R$  and  $(b,c) \in R \Rightarrow (a,c) \in R \forall a,b,c \in \mathbb{Z}$

$$(a,b) \in R \Rightarrow (a-b) \text{ is divisible by } 5$$

$$\Rightarrow a-b = 5z_1 \text{ for some } z_1 \in \mathbb{Z}$$

$$(b,c) \in R \Rightarrow (b-c) \text{ is divisible by } 5$$

$$\Rightarrow b-c = 5z_2 \text{ for some } z_2 \in \mathbb{Z}$$

Now,

$$a-b = 5z_1 \text{ and } b-c = 5z_2$$

$$\Rightarrow (a-b) + (b-c) = 5z_1 + 5z_2$$

$$\Rightarrow a-c = 5(z_1 + z_2) = 5z_3 \text{ where } z_1 + z_2 = z_3$$

$$\Rightarrow a-c = 5z_3 [\because z_1, z_2 \in \mathbb{Z} \Rightarrow z_3 \in \mathbb{Z}]$$

$$\Rightarrow (a-c) \text{ is divisible by } 5.$$

$$\Rightarrow (a, c) \in R$$

Thus, R is transitive on  $\mathbb{Z}$ .

Since R is reflexive, symmetric and transitive it is an equivalence relation on  $\mathbb{Z}$ .

## 5. Question

Show that the relation R defined on the set  $A = \{1, 2, 3, 4, 5\}$ , given by

$R = \{(a, b) : |a - b| \text{ is even}\}$  is an equivalence relation.

## Answer

In order to show R is an equivalence relation we need to show R is Reflexive, Symmetric and Transitive.

Given that,  $\forall a, b \in A, R = \{(a, b) : |a - b| \text{ is even}\}$ .

Now,

R is Reflexive if  $(a,a) \in R \forall a \in A$

For any  $a \in A$ , we have

$$|a-a| = 0, \text{ which is even.}$$

$$\Rightarrow (a,a) \in R$$

Thus, R is reflexive.

R is Symmetric if  $(a,b) \in R \Rightarrow (b,a) \in R \forall a,b \in A$

$$(a,b) \in R$$

$$\Rightarrow |a-b| \text{ is even.}$$

$$\Rightarrow |b-a| \text{ is even.}$$

$$\Rightarrow (b,a) \in R$$

Thus, R is symmetric .

R is Transitive if  $(a,b) \in R$  and  $(b,c) \in R \Rightarrow (a,c) \in R \forall a,b,c \in A$

Let  $(a,b) \in R$  and  $(b,c) \in R \forall a, b,c \in A$

$\Rightarrow |a - b|$  is even and  $|b - c|$  is even

$\Rightarrow$  (a and b both are even or both odd) and (b and c both are even or both odd)

Now two cases arise:

Case 1 : when b is even

Let  $(a,b) \in R$  and  $(b,c) \in R$

$\Rightarrow |a - b|$  is even and  $|b - c|$  is even

$\Rightarrow$  a is even and c is even [ $\because$  b is even]

$\Rightarrow |a - c|$  is even [ $\because$  difference of any two even natural numbers is even]

$\Rightarrow (a, c) \in R$

Case 2 : when b is odd

Let  $(a,b) \in R$  and  $(b,c) \in R$

$\Rightarrow |a - b|$  is even and  $|b - c|$  is even

$\Rightarrow$  a is odd and c is odd [ $\because$  b is odd]

$\Rightarrow |a - c|$  is even [ $\because$  difference of any two odd natural numbers is even]

$\Rightarrow (a, c) \in R$

Thus, R is transitive on Z.

Since R is reflexive, symmetric and transitive it is an equivalence relation on Z.

## 6. Question

Show that the relation R on  $N \times N$ , defined by

$$(a, b) R (c, d) \Leftrightarrow a + d = b + c$$

is an equivalent relation.

## Answer

In order to show R is an equivalence relation we need to show R is Reflexive, Symmetric and Transitive.

Given that, R be the relation in  $N \times N$  defined by  $(a, b) R (c, d)$  if  $a + d = b + c$  for  $(a, b), (c, d)$  in  $N \times N$ .

R is Reflexive if  $(a, b) R (a, b)$  for  $(a, b)$  in  $N \times N$

Let  $(a,b) R (a,b)$

$$\Rightarrow a+b = b+a$$

which is true since addition is commutative on N.

$\Rightarrow$  R is reflexive.

R is Symmetric if  $(a,b) R (c,d) \Rightarrow (c,d) R (a,b)$  for  $(a, b), (c, d)$  in  $N \times N$

Let  $(a,b) R (c,d)$

$$\Rightarrow a+d = b+c$$

$$\Rightarrow b+c = a+d$$



$\Rightarrow c+b = d+a$  [since addition is commutative on  $\mathbb{N}$ ]

$\Rightarrow (c,d) R (a,b)$

$\Rightarrow R$  is symmetric.

$R$  is Transitive if  $(a,b) R (c,d)$  and  $(c,d) R (e,f) \Rightarrow (a,b) R (e,f)$  for  $(a, b), (c, d), (e,f)$  in  $\mathbb{N} \times \mathbb{N}$

Let  $(a,b) R (c,d)$  and  $(c,d) R (e,f)$

$\Rightarrow a+d = b+c$  and  $c+f = d+e$

$\Rightarrow (a+d) - (d+e) = (b+c) - (c+f)$

$\Rightarrow a-e = b-f$

$\Rightarrow a+f = b+e$

$\Rightarrow (a,b) R (e,f)$

$\Rightarrow R$  is transitive.

Hence,  $R$  is an equivalence relation.

### 7. Question

Let  $S$  be the set of all real numbers and let

$R = \{(a, b) : a, b \in S \text{ and } a = \pm b\}$ .

Show that  $R$  is an equivalence relation on  $S$ .

### Answer

In order to show  $R$  is an equivalence relation we need to show  $R$  is Reflexive, Symmetric and Transitive.

Given that,  $\forall a, b \in S, R = \{(a, b) : a = \pm b\}$

Now,

$R$  is Reflexive if  $(a,a) \in R \forall a \in S$

For any  $a \in S$ , we have

$a = \pm a$

$\Rightarrow (a,a) \in R$

Thus,  $R$  is reflexive.

$R$  is Symmetric if  $(a,b) \in R \Rightarrow (b,a) \in R \forall a,b \in S$

$(a,b) \in R$

$\Rightarrow a = \pm b$

$\Rightarrow b = \pm a$

$\Rightarrow (b,a) \in R$

Thus,  $R$  is symmetric .

$R$  is Transitive if  $(a,b) \in R$  and  $(b,c) \in R \Rightarrow (a,c) \in R \forall a,b,c \in S$

Let  $(a,b) \in R$  and  $(b,c) \in R \forall a, b, c \in S$

$\Rightarrow a = \pm b$  and  $b = \pm c$

$\Rightarrow a = \pm c$

$\Rightarrow (a, c) \in R$

Thus,  $R$  is transitive.

Hence, R is an equivalence relation.

### 8. Question

Let S be the set of all points in a plane and let R be a relation in S defined by  $R = \{(A, B) : d(A, B) < 2 \text{ units}\}$ , where  $d(A, B)$  is the distance between the points A and B.

Show that R is reflexive and symmetric but not transitive.

#### Answer

Given that,  $\forall A, B \in S, R = \{(A, B) : d(A, B) < 2 \text{ units}\}$ .

Now,

R is Reflexive if  $(A, A) \in R \forall A \in S$

For any  $A \in S$ , we have

$d(A, A) = 0$ , which is less than 2 units

$\Rightarrow (A, A) \in R$

Thus, R is reflexive.

R is Symmetric if  $(A, B) \in R \Rightarrow (B, A) \in R \forall A, B \in S$

$(A, B) \in R$

$\Rightarrow d(A, B) < 2 \text{ units}$

$\Rightarrow d(B, A) < 2 \text{ units}$

$\Rightarrow (B, A) \in R$

Thus, R is symmetric .

R is Transitive if  $(A, B) \in R$  and  $(B, C) \in R \Rightarrow (A, C) \in R \forall A, B, C \in S$

Consider points  $A(0,0), B(1.5,0)$  and  $C(3.2,0)$ .

$d(A, B) = 1.5 \text{ units} < 2 \text{ units}$  and  $d(B, C) = 1.7 \text{ units} < 2 \text{ units}$

$d(A, C) = 3.2 \not< 2$

$\Rightarrow (A, B) \in R$  and  $(B, C) \in R \Rightarrow (A, C) \notin R$

Thus, R is not transitive.

Thus, R is reflexive, symmetric but not transitive.

### 9. Question

Let S be the set of all real numbers. Show that the relation  $R = \{(a, b) : a^2 + b^2 = 1\}$  is symmetric but neither reflexive nor transitive.

#### Answer

Given that,  $\forall a, b \in S, R = \{(a, b) : a^2 + b^2 = 1\}$

Now,

R is Reflexive if  $(a, a) \in R \forall a \in S$

For any  $a \in S$ , we have

$a^2 + a^2 = 2a^2 \neq 1$

$\Rightarrow (a, a) \notin R$

Thus, R is not reflexive.

R is Symmetric if  $(a, b) \in R \Rightarrow (b, a) \in R \forall a, b \in S$

$$(a,b) \in R$$

$$\Rightarrow a^2 + b^2 = 1$$

$$\Rightarrow b^2 + a^2 = 1$$

$$\Rightarrow (b,a) \in R$$

Thus, R is symmetric .

R is Transitive if  $(a,b) \in R$  and  $(b,c) \in R \Rightarrow (a,c) \in R \forall a,b,c \in S$

Let  $(a,b) \in R$  and  $(b,c) \in R \forall a, b,c \in S$

$$\Rightarrow a^2 + b^2 = 1 \text{ and } b^2 + c^2 = 1$$

Adding both, we get

$$a^2 + c^2 + 2b^2 = 2$$

$$\Rightarrow a^2 + c^2 = 2 - 2b^2 \neq 1$$

$$\Rightarrow (a, c) \notin R$$

Thus, R is not transitive.

Thus, R is symmetric but neither reflexive nor transitive.

### 10. Question

Let  $R = \{(a, b) : a = b^2\}$  for all  $a, b \in \mathbb{N}$ .

Show that R satisfies none of reflexivity, symmetry and transitivity.

### Answer

We have,  $R = \{(a, b) : a = b^2\}$  relation defined on  $\mathbb{N}$ .

Now,

We observe that, any element  $a \in \mathbb{N}$  cannot be equal to its square except 1.

$$\Rightarrow (a,a) \notin R \forall a \in \mathbb{N}$$

$$\text{For e.g. } (2,2) \notin R \because 2 \neq 2^2$$

$\Rightarrow R$  is not reflexive.

Let  $(a,b) \in R \forall a, b \in \mathbb{N}$

$$\Rightarrow a = b^2$$

But b cannot be equal to square of a if a is equal to square of b.

$$\Rightarrow (b,a) \notin R$$

For e.g., we observe that  $(4,2) \in R$  i.e  $4 = 2^2$  but  $2 \neq 4^2 \Rightarrow (2,4) \notin R$

$\Rightarrow R$  is not symmetric

Let  $(a,b) \in R$  and  $(b,c) \in R \forall a, b,c \in \mathbb{N}$

$$\Rightarrow a = b^2 \text{ and } b = c^2$$

$$\Rightarrow a \neq c^2$$

$$\Rightarrow (a,c) \notin R$$

For e.g., we observe that

$$(16,4) \in R \Rightarrow 16 = 4^2 \text{ and } (4,2) \in R \Rightarrow 4 = 2^2$$

But  $16 \neq 2^2$

$\Rightarrow (16,2) \notin R$

$\Rightarrow R$  is not transitive.

Thus,  $R$  is neither reflexive nor symmetric nor transitive.

### 11. Question

Show that the relation  $R = \{(a, b) : a > b\}$  on  $N$  is transitive but neither reflexive nor symmetric.

#### Answer

We have,  $R = \{(a, b) : a > b\}$  relation defined on  $N$ .

Now,

We observe that, any element  $a \in N$  cannot be greater than itself.

$\Rightarrow (a,a) \notin R \forall a \in N$

$\Rightarrow R$  is not reflexive.

Let  $(a,b) \in R \forall a, b \in N$

$\Rightarrow a$  is greater than  $b$

But  $b$  cannot be greater than  $a$  if  $a$  is greater than  $b$ .

$\Rightarrow (b,a) \notin R$

For e.g., we observe that  $(5,2) \in R$  i.e  $5 > 2$  but  $2 \ngtr 5 \Rightarrow (2,5) \notin R$

$\Rightarrow R$  is not symmetric

Let  $(a,b) \in R$  and  $(b,c) \in R \forall a, b, c \in N$

$\Rightarrow a > b$  and  $b > c$

$\Rightarrow a > c$

$\Rightarrow (a,c) \in R$

For e.g., we observe that

$(5,4) \in R \Rightarrow 5 > 4$  and  $(4,3) \in R \Rightarrow 4 > 3$

And we know that  $5 > 3 \therefore (5,3) \in R$

$\Rightarrow R$  is transitive.

Thus,  $R$  is transitive but not reflexive not symmetric.

### 12. Question

Let  $A = \{1, 2, 3\}$  and  $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$ .

Show that  $R$  is reflexive but neither symmetric nor transitive.

#### Answer

Given that,  $A = \{1, 2, 3\}$  and  $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$ .

Now,

$R$  is reflexive  $\because (1,1), (2,2), (3,3) \in R$

$R$  is not symmetric  $\because (1,2), (2,3) \in R$  but  $(2,1), (3,2) \notin R$

$R$  is not transitive  $\because (1,2) \in R$  and  $(2,3) \in R \Rightarrow (1,3) \notin R$

Thus,  $R$  is reflexive but neither symmetric nor transitive.

### 13. Question

Let  $A = \{1, 2, 3, 4\}$  and  $R = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (1, 3), (3, 2)\}$ . Show that  $R$  is reflexive and transitive but not symmetric.

#### Answer

Given that,  $A = \{1, 2, 3\}$  and  $R = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (1, 3), (3, 2)\}$ .

Now,

$R$  is reflexive  $\because (1,1), (2,2), (3,3), (4,4) \in R$

$R$  is not symmetric  $\because (1,2), (1,3), (3,2) \in R$  but  $(2,1), (3,1), (2,3) \notin R$

$R$  is transitive  $\because (1,3) \in R$  and  $(3,2) \in R \Rightarrow (1,2) \in R$

Thus,  $R$  is reflexive and transitive but not symmetric.

### Objective Questions

#### 1. Question

Mark the tick against the correct answer in the following:

Let  $A = \{1, 2, 3\}$  and let  $R = \{(1, 1), (2, 2), (3, 3), (1, 3), (3, 2), (1, 2)\}$ . Then,  $R$  is

- A. reflexive and symmetric but not transitive
- B. reflexive and transitive but not symmetric
- C. symmetric and transitive but not reflexive
- D. an equivalence relation

#### Answer

Given set  $A = \{1, 2, 3\}$

And  $R = \{(1, 1), (2, 2), (3, 3), (1, 3), (3, 2), (1, 2)\}$

#### Formula

For a relation  $R$  in set  $A$

Reflexive

The relation is reflexive if  $(a, a) \in R$  for every  $a \in A$

Symmetric

The relation is Symmetric if  $(a, b) \in R$ , then  $(b, a) \in R$

Transitive

Relation is Transitive if  $(a, b) \in R$  &  $(b, c) \in R$ , then  $(a, c) \in R$

Equivalence

If the relation is reflexive, symmetric and transitive, it is an equivalence relation.

Check for reflexive

Since,  $(1,1) \in R$ ,  $(2,2) \in R$ ,  $(3,3) \in R$

Therefore,  $R$  is reflexive ..... (1)

Check for symmetric

Since  $(1,3) \in R$  but  $(3,1) \notin R$

Therefore,  $R$  is not symmetric ..... (2)

Check for transitive

Here ,  $(1,3) \in R$  and  $(3,2) \in R$  and  $(1,2) \in R$

Therefore ,  $R$  is transitive ..... (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (B)

## 2. Question

Mark the tick against the correct answer in the following:

Let  $A = \{a, b, c\}$  and let  $R = \{(a, a), (a, b), (b, a)\}$ . Then,  $R$  is

A. reflexive and symmetric but not transitive

B. reflexive and transitive but not symmetric

C. symmetric and transitive but not reflexive

D. an equivalence relation

## Answer

Given set  $A = \{a, b, c\}$

And  $R = \{(a, a), (a, b), (b, a)\}$

### Formula

For a relation  $R$  in set  $A$

Reflexive

The relation is reflexive if  $(a, a) \in R$  for every  $a \in A$

Symmetric

The relation is Symmetric if  $(a, b) \in R$  , then  $(b, a) \in R$

Transitive

Relation is Transitive if  $(a, b) \in R$  &  $(b, c) \in R$  , then  $(a, c) \in R$

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Since ,  $(b,b) \notin R$  and  $(c,c) \notin R$

Therefore ,  $R$  is not reflexive ..... (1)

Check for symmetric

Since ,  $(a,b) \in R$  and  $(b,a) \in R$

Therefore ,  $R$  is symmetric ..... (2)

Check for transitive

Here ,  $(a,b) \in R$  and  $(b,a) \in R$  and  $(a,a) \in R$

Therefore ,  $R$  is transitive ..... (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (C)

## 3. Question

Mark the tick against the correct answer in the following:

Let  $A = \{1, 2, 3\}$  and let  $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2)\}$ . Then,  $R$  is

- A. reflexive and symmetric but not transitive
- B. symmetric and transitive but not reflexive
- C. reflexive and transitive but not symmetric
- D. an equivalence relation

**Answer**

Given set  $A = \{1, 2, 3\}$

And  $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2)\}$

Formula

For a relation  $R$  in set  $A$

Reflexive

The relation is reflexive if  $(a, a) \in R$  for every  $a \in A$

Symmetric

The relation is Symmetric if  $(a, b) \in R$ , then  $(b, a) \in R$

Transitive

Relation is Transitive if  $(a, b) \in R$  &  $(b, c) \in R$ , then  $(a, c) \in R$

Equivalence

If the relation is reflexive, symmetric and transitive, it is an equivalence relation.

Check for reflexive

Since,  $(1,1) \in R$ ,  $(2,2) \in R$ ,  $(3,3) \in R$

Therefore,  $R$  is reflexive ..... (1)

Check for symmetric

Since,  $(1,2) \in R$  and  $(2,1) \in R$

$(2,3) \in R$  and  $(3,2) \in R$

Therefore,  $R$  is symmetric ..... (2)

Check for transitive

Here,  $(1,2) \in R$  and  $(2,3) \in R$  but  $(1,3) \notin R$

Therefore,  $R$  is not transitive ..... (3)

Now, according to the equations (1), (2), (3)

Correct option will be (A)

**4. Question**

Mark the tick against the correct answer in the following:

Let  $S$  be the set of all straight lines in a plane. Let  $R$  be a relation on  $S$  defined by  $a R b \Leftrightarrow a \perp b$ . Then,  $R$  is

- A. reflexive but neither symmetric nor transitive
- B. symmetric but neither reflexive nor transitive
- C. transitive but neither reflexive nor symmetric
- D. an equivalence relation

## Answer

According to the question ,

Given set  $S = \{x, y, z\}$

And  $R = \{(x, y), (y, z), (x, z), (y, x), (z, y), (z, x)\}$

### Formula

For a relation  $R$  in set  $A$

Reflexive

The relation is reflexive if  $(a, a) \in R$  for every  $a \in A$

Symmetric

The relation is Symmetric if  $(a, b) \in R$  , then  $(b, a) \in R$

Transitive

Relation is Transitive if  $(a, b) \in R$  &  $(b, c) \in R$  , then  $(a, c) \in R$

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Since ,  $(x,x) \notin R$  ,  $(y,y) \notin R$  ,  $(z,z) \notin R$

Therefore ,  $R$  is not reflexive ..... (1)

Check for symmetric

Since ,  $(x,y) \in R$  and  $(y,x) \in R$

$(z,y) \in R$  and  $(y,z) \in R$

$(x,z) \in R$  and  $(z,x) \in R$

Therefore ,  $R$  is symmetric ..... (2)

Check for transitive

Here ,  $(x,y) \in R$  and  $(y,x) \in R$  but  $(x,x) \notin R$

Therefore ,  $R$  is not transitive ..... (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (B)

## 5. Question

Mark the tick against the correct answer in the following:

Let  $S$  be the set of all straight lines in a plane. Let  $R$  be a relation on  $S$  defined by  $a R b \Leftrightarrow a \parallel b$ . Then,  $R$  is

- A. reflexive and symmetric but not transitive
- B. reflexive and transitive but not symmetric
- C. symmetric and transitive but not reflexive
- D. an equivalence relation

## Answer

According to the question ,

Given set  $S = \{x, y, z\}$



And  $R = \{(x, x), (y, y), (z, z)\}$

### Formula

For a relation  $R$  in set  $A$

Reflexive

The relation is reflexive if  $(a, a) \in R$  for every  $a \in A$

Symmetric

The relation is Symmetric if  $(a, b) \in R$ , then  $(b, a) \in R$

Transitive

Relation is Transitive if  $(a, b) \in R$  &  $(b, c) \in R$ , then  $(a, c) \in R$

Equivalence

If the relation is reflexive, symmetric and transitive, it is an equivalence relation.

Check for reflexive

Since,  $(x, x) \in R$ ,  $(y, y) \in R$ ,  $(z, z) \in R$

Therefore,  $R$  is reflexive ..... (1)

Check for symmetric

Since,  $(x, x) \in R$  and  $(x, x) \in R$

$(y, y) \in R$  and  $(y, y) \in R$

$(z, z) \in R$  and  $(z, z) \in R$

Therefore,  $R$  is symmetric ..... (2)

Check for transitive

Here,  $(x, x) \in R$  and  $(y, y) \in R$  and  $(z, z) \in R$

Therefore,  $R$  is transitive ..... (3)

Now, according to the equations (1), (2), (3)

Correct option will be (D)

### **6. Question**

Mark the tick against the correct answer in the following:

Let  $Z$  be the set of all integers and let  $R$  be a relation on  $Z$  defined by  $a R b \Leftrightarrow (a - b)$  is divisible by 3. Then,  $R$  is

- A. reflexive and symmetric but not transitive
- B. reflexive and transitive but not symmetric
- C. symmetric and transitive but not reflexive
- D. an equivalence relation

### **Answer**

According to the question,

Given set  $Z = \{1, 2, 3, 4, \dots\}$

And  $R = \{(a, b) : a, b \in Z \text{ and } (a - b) \text{ is divisible by } 3\}$

### Formula

For a relation  $R$  in set  $A$

Reflexive

The relation is reflexive if  $(a, a) \in R$  for every  $a \in A$

Symmetric

The relation is Symmetric if  $(a, b) \in R$ , then  $(b, a) \in R$

Transitive

Relation is Transitive if  $(a, b) \in R$  &  $(b, c) \in R$ , then  $(a, c) \in R$

Equivalence

If the relation is reflexive, symmetric and transitive, it is an equivalence relation.

Check for reflexive

Consider,  $(a, a)$

$(a - a) = 0$  which is divisible by 3

$(a, a) \in R$  where  $a \in \mathbb{Z}$

Therefore,  $R$  is reflexive ..... (1)

Check for symmetric

Consider,  $(a, b) \in R$

$\therefore (a - b)$  which is divisible by 3

$-(a - b)$  which is divisible by 3

(since if 6 is divisible by 3 then -6 will also be divisible by 3)

$\therefore (b - a)$  which is divisible by 3  $\Rightarrow (b, a) \in R$

For any  $(a, b) \in R$ ;  $(b, a) \in R$

Therefore,  $R$  is symmetric ..... (2)

Check for transitive

Consider,  $(a, b) \in R$  and  $(b, c) \in R$

$\therefore (a - b)$  which is divisible by 3

and  $(b - c)$  which is divisible by 3

$[(a - b) + (b - c)]$  is divisible by 3 [if 6 is divisible by 3 and 9 is divisible by 3 then 6+9 will also be divisible by 3]

$\therefore (a - c)$  which is divisible by 3  $\Rightarrow (a, c) \in R$

Therefore  $(a, b) \in R$  and  $(b, c) \in R$  then  $(a, c) \in R$

Therefore,  $R$  is transitive ..... (3)

Now, according to the equations (1), (2), (3)

Correct option will be (D)

## 7. Question

Mark the tick against the correct answer in the following:

Let  $R$  be a relation on the set  $\mathbb{N}$  of all natural numbers, defined by  $a R b \Leftrightarrow a$  is a factor of  $b$ . Then,  $R$  is

A. reflexive and symmetric but not transitive

B. reflexive and transitive but not symmetric

C. symmetric and transitive but not reflexive

D. an equivalence relation

### Answer

According to the question ,

Given set  $N = \{1, 2, 3, 4, \dots\}$

And  $R = \{(a, b) : a, b \in N \text{ and } a \text{ is a factor of } b\}$

#### Formula

For a relation  $R$  in set  $A$

Reflexive

The relation is reflexive if  $(a, a) \in R$  for every  $a \in A$

Symmetric

The relation is Symmetric if  $(a, b) \in R$  , then  $(b, a) \in R$

Transitive

Relation is Transitive if  $(a, b) \in R$  &  $(b, c) \in R$  , then  $(a, c) \in R$

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Consider ,  $(a, a)$

$a$  is a factor of  $a$

$(2, 2)$  ,  $(3, 3)$ ...  $(a, a)$  where  $a \in N$

Therefore ,  $R$  is reflexive ..... (1)

Check for symmetric

$a R b \Rightarrow a$  is factor of  $b$

$b R a \Rightarrow b$  is factor of  $a$  as well

Ex \_  $(2, 6) \in R$

But  $(6, 2) \notin R$

Therefore ,  $R$  is not symmetric ..... (2)

Check for transitive

$a R b \Rightarrow a$  is factor of  $b$

$b R c \Rightarrow b$  is a factor of  $c$

$a R c \Rightarrow a$  is a factor of  $c$  also

Ex \_  $(2, 6)$  ,  $(6, 18)$

$\therefore (2, 18) \in R$

Therefore ,  $R$  is transitive ..... (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (B)

### 8. Question

Mark the tick against the correct answer in the following:

Let  $Z$  be the set of all integers and let  $R$  be a relation on  $Z$  defined by  $a R b \Leftrightarrow a \geq b$ . Then,  $R$  is

- A. symmetric and transitive but not reflexive
- B. reflexive and symmetric but not transitive
- C. reflexive and transitive but not symmetric
- D. an equivalence relation

**Answer**

According to the question ,

Given set  $Z = \{1, 2, 3, 4, \dots\}$

And  $R = \{(a, b) : a, b \in Z \text{ and } a \geq b\}$

Formula

For a relation  $R$  in set  $A$

Reflexive

The relation is reflexive if  $(a, a) \in R$  for every  $a \in A$

Symmetric

The relation is Symmetric if  $(a, b) \in R$  , then  $(b, a) \in R$

Transitive

Relation is Transitive if  $(a, b) \in R$  &  $(b, c) \in R$  , then  $(a, c) \in R$

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Consider ,  $(a,a)$   $(b,b)$

$\therefore a \geq a$  and  $b \geq b$  which is always true.

Therefore ,  $R$  is reflexive ..... (1)

Check for symmetric

$a R b \Rightarrow a \geq b$

$b R a \Rightarrow b \geq a$

Both cannot be true.

Ex \_ If  $a=2$  and  $b=1$

$\therefore 2 \geq 1$  is true but  $1 \geq 2$  which is false.

Therefore ,  $R$  is not symmetric ..... (2)

Check for transitive

$a R b \Rightarrow a \geq b$

$b R c \Rightarrow b \geq c$

$\therefore a \geq c$

Ex \_  $a=5$  ,  $b=4$  and  $c=2$

$\therefore 5 \geq 4$  ,  $4 \geq 2$  and hence  $5 \geq 2$

Therefore ,  $R$  is transitive ..... (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (C)

### 9. Question

Mark the tick against the correct answer in the following:

Let S be the set of all real numbers and let R be a relation on S defined by  $a R b \Leftrightarrow |a| \leq b$ . Then, R is

- A. reflexive but neither symmetric nor transitive
- B. symmetric but neither reflexive nor transitive
- C. transitive but neither reflexive nor symmetric
- D. none of these

### Answer

According to the question ,

Given set  $S = \{ \dots, -2, -1, 0, 1, 2, \dots \}$

And  $R = \{ (a, b) : a, b \in S \text{ and } |a| \leq b \}$

#### Formula

For a relation R in set A

Reflexive

The relation is reflexive if  $(a, a) \in R$  for every  $a \in A$

Symmetric

The relation is Symmetric if  $(a, b) \in R$  , then  $(b, a) \in R$

Transitive

Relation is Transitive if  $(a, b) \in R$  &  $(b, c) \in R$  , then  $(a, c) \in R$

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Consider ,  $(a, a)$

$\therefore |a| \leq a$  and which is not always true.

Ex\_ if  $a = -2$

$\therefore |-2| \leq -2 \Rightarrow 2 \leq -2$  which is false.

Therefore , R is not reflexive ..... (1)

Check for symmetric

$a R b \Rightarrow |a| \leq b$

$b R a \Rightarrow |b| \leq a$

Both cannot be true.

Ex \_ If  $a = -2$  and  $b = -1$

$\therefore 2 \leq -1$  is false and  $1 \leq -2$  which is also false.

Therefore , R is not symmetric ..... (2)

Check for transitive

$$a R b \Rightarrow |a| \leq b$$

$$b R c \Rightarrow |b| \leq c$$

$$\therefore |a| \leq c$$

Ex \_a=-5 , b= 7 and c=9

$$\therefore 5 \leq 7 , 7 \leq 9 \text{ and hence } 5 \leq 9$$

Therefore , R is transitive ..... (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (C)

### 10. Question

Mark the tick against the correct answer in the following:

Let S be the set of all real numbers and let R be a relation on S, defined by  $a R b \Leftrightarrow |a - b| \leq 1$ . Then, R is

- A. reflexive and symmetric but not transitive
- B. reflexive and transitive but not symmetric
- C. symmetric and transitive but not reflexive
- D. an equivalence relation

### Answer

According to the question ,

Given set  $S = \{\dots, -2, -1, 0, 1, 2, \dots\}$

And  $R = \{(a, b) : a, b \in S \text{ and } |a - b| \leq 1\}$

#### Formula

For a relation R in set A

Reflexive

The relation is reflexive if  $(a, a) \in R$  for every  $a \in A$

Symmetric

The relation is Symmetric if  $(a, b) \in R$  , then  $(b, a) \in R$

Transitive

Relation is Transitive if  $(a, b) \in R$  &  $(b, c) \in R$  , then  $(a, c) \in R$

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Consider ,  $(a, a)$

$$\therefore |a - a| \leq 1 \text{ and which is always true.}$$

Ex \_if a=2

$$\therefore |2-2| \leq 1 \Rightarrow 0 \leq 1 \text{ which is true.}$$

Therefore , R is reflexive ..... (1)

Check for symmetric

$$a R b \Rightarrow |a - b| \leq 1$$

$$b R a \Rightarrow |b - a| \leq 1$$

Both can be true.

Ex \_ If  $a=2$  and  $b=1$

$\therefore |2 - 1| \leq 1$  is true and  $|1-2| \leq 1$  which is also true.

Therefore , R is symmetric ..... (2)

Check for transitive

$$a R b \Rightarrow |a - b| \leq 1$$

$$b R c \Rightarrow |b - c| \leq 1$$

$\therefore |a - c| \leq 1$  will not always be true

Ex \_  $a=-5$  ,  $b= -6$  and  $c= -7$

$\therefore |6-5| \leq 1$  ,  $|7 - 6| \leq 1$  are true But  $|7 - 5| \leq 1$  is false.

Therefore , R is not transitive ..... (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (A)

### 11. Question

Mark the tick against the correct answer in the following:

Let S be the set of all real numbers and let R be a relation on S, defined by  $a R b \Leftrightarrow (1 + ab) > 0$ . Then, R is

- A. reflexive and symmetric but not transitive
- B. reflexive and transitive but not symmetric
- C. symmetric and transitive but not reflexive
- D. none of these

### Answer

According to the question ,

Given set  $S = \{.....,-2,-1,0,1,2 .....\}$

And  $R = \{(a, b) : a,b \in S \text{ and } (1 + ab) > 0 \}$

#### Formula

For a relation R in set A

Reflexive

The relation is reflexive if  $(a, a) \in R$  for every  $a \in A$

Symmetric

The relation is Symmetric if  $(a, b) \in R$  , then  $(b, a) \in R$

Transitive

Relation is Transitive if  $(a, b) \in R$  &  $(b, c) \in R$  , then  $(a, c) \in R$

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Consider ,  $(a,a)$

$\therefore (1 + a \times a) > 0$  which is always true because  $a \times a$  will always be positive.

Ex\_ if  $a=2$

$\therefore (1 + 4) > 0 \Rightarrow (5) > 0$  which is true.

Therefore , R is reflexive ..... (1)

Check for symmetric

$a R b \Rightarrow (1 + ab) > 0$

$b R a \Rightarrow (1 + ba) > 0$

Both the equation are the same and therefore will always be true.

Ex \_ If  $a=2$  and  $b=1$

$\therefore (1 + 2 \times 1) > 0$  is true and  $(1+1 \times 2) > 0$  which is also true.

Therefore , R is symmetric ..... (2)

Check for transitive

$a R b \Rightarrow (1 + ab) > 0$

$b R c \Rightarrow (1 + bc) > 0$

$\therefore (1 + ac) > 0$  will not always be true

Ex \_  $a=-1$  ,  $b= 0$  and  $c= 2$

$\therefore (1 + -1 \times 0) > 0$  ,  $(1 + 0 \times 2) > 0$  are true

But  $(1 + -1 \times 2) > 0$  is false.

Therefore , R is not transitive ..... (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (A)

## 12. Question

Mark the tick against the correct answer in the following:

Let S be the set of all triangles in a plane and let R be a relation on S defined by  $\Delta_1 S \Delta_2 \Leftrightarrow \Delta_1 \equiv \Delta_2$ . Then, R is

- A. reflexive and symmetric but not transitive
- B. reflexive and transitive but not symmetric
- C. symmetric and transitive but not reflexive
- D. an equivalence relation

## Answer

According to the question ,

Given set  $S = \{\dots \text{All triangles in plane} \dots\}$

And  $R = \{(\Delta_1 , \Delta_2) : \Delta_1 , \Delta_2 \in S \text{ and } \Delta_1 \equiv \Delta_2\}$

### Formula

For a relation R in set A

Reflexive

The relation is reflexive if  $(a , a) \in R$  for every  $a \in A$

Symmetric



The relation is Symmetric if  $(a, b) \in R$ , then  $(b, a) \in R$

Transitive

Relation is Transitive if  $(a, b) \in R$  &  $(b, c) \in R$ , then  $(a, c) \in R$

Equivalence

If the relation is reflexive, symmetric and transitive, it is an equivalence relation.

Check for reflexive

Consider,  $(\Delta_1, \Delta_1)$

$\therefore$  We know every triangle is congruent to itself.

$(\Delta_1, \Delta_1) \in R$  all  $\Delta_1 \in S$

Therefore,  $R$  is reflexive ..... (1)

Check for symmetric

$(\Delta_1, \Delta_2) \in R$  then  $\Delta_1$  is congruent to  $\Delta_2$

$(\Delta_2, \Delta_1) \in R$  then  $\Delta_2$  is congruent to  $\Delta_1$

Both the equations are the same and therefore will always be true.

Therefore,  $R$  is symmetric ..... (2)

Check for transitive

Let  $\Delta_1, \Delta_2, \Delta_3 \in S$  such that  $(\Delta_1, \Delta_2) \in R$  and  $(\Delta_2, \Delta_3) \in R$

Then  $(\Delta_1, \Delta_2) \in R$  and  $(\Delta_2, \Delta_3) \in R$

$\Rightarrow \Delta_1$  is congruent to  $\Delta_2$ , and  $\Delta_2$  is congruent to  $\Delta_3$

$\Rightarrow \Delta_1$  is congruent to  $\Delta_3$

$\therefore (\Delta_1, \Delta_3) \in R$

Therefore,  $R$  is transitive ..... (3)

Now, according to the equations (1), (2), (3)

Correct option will be (D)

### 13. Question

Mark the tick against the correct answer in the following:

Let  $S$  be the set of all real numbers and let  $R$  be a relation on  $S$  defined by  $a R b \Leftrightarrow a^2 + b^2 = 1$ . Then,  $R$  is

- A. symmetric but neither reflexive nor transitive
- B. reflexive but neither symmetric nor transitive
- C. transitive but neither reflexive nor symmetric
- D. none of these

### Answer

According to the question,

Given set  $S = \{\dots, -2, -1, 0, 1, 2, \dots\}$

And  $R = \{(a, b) : a, b \in S \text{ and } a^2 + b^2 = 1\}$

Formula

For a relation R in set A

Reflexive

The relation is reflexive if  $(a, a) \in R$  for every  $a \in A$

Symmetric

The relation is Symmetric if  $(a, b) \in R$ , then  $(b, a) \in R$

Transitive

Relation is Transitive if  $(a, b) \in R$  &  $(b, c) \in R$ , then  $(a, c) \in R$

Equivalence

If the relation is reflexive, symmetric and transitive, it is an equivalence relation.

Check for reflexive

Consider,  $(a, a)$

$\therefore a^2 + a^2 = 1$  which is not always true

Ex\_ if  $a=2$

$\therefore 2^2 + 2^2 = 1 \Rightarrow 4 + 4 = 1$  which is false.

Therefore, R is not reflexive ..... (1)

Check for symmetric

$a R b \Rightarrow a^2 + b^2 = 1$

$b R a \Rightarrow b^2 + a^2 = 1$

Both the equations are the same and therefore will always be true.

Therefore, R is symmetric ..... (2)

Check for transitive

$a R b \Rightarrow a^2 + b^2 = 1$

$b R c \Rightarrow b^2 + c^2 = 1$

$\therefore a^2 + c^2 = 1$  will not always be true

Ex\_  $a=-1$ ,  $b=0$  and  $c=1$

$\therefore (-1)^2 + 0^2 = 1$ ,  $0^2 + 1^2 = 1$  are true

But  $(-1)^2 + 1^2 = 1$  is false.

Therefore, R is not transitive ..... (3)

Now, according to the equations (1), (2), (3)

Correct option will be (A)

#### 14. Question

Mark the tick against the correct answer in the following:

Let R be a relation on  $N \times N$ , defined by  $(a, b) R (c, d) \Leftrightarrow a + d = b + c$ . Then, R is

- A. reflexive and symmetric but not transitive
- B. reflexive and transitive but not symmetric
- C. symmetric and transitive but not reflexive
- D. an equivalence relation

## Answer

According to the question ,

$$R = \{(a, b) , (c, d) : a + d = b + c \}$$

### Formula

For a relation R in set A

Reflexive

The relation is reflexive if  $(a, a) \in R$  for every  $a \in A$

Symmetric

The relation is Symmetric if  $(a, b) \in R$  , then  $(b, a) \in R$

Transitive

Relation is Transitive if  $(a, b) \in R$  &  $(b, c) \in R$  , then  $(a, c) \in R$

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Consider ,  $(a, b) R (a, b)$

$$(a, b) R (a, b) \Leftrightarrow a + b = a + b$$

which is always true .

Therefore , R is reflexive ..... (1)

Check for symmetric

$$(a, b) R (c, d) \Leftrightarrow a + d = b + c$$

$$(c, d) R (a, b) \Leftrightarrow c + b = d + a$$

Both the equation are the same and therefore will always be true.

Therefore , R is symmetric ..... (2)

Check for transitive

$$(a, b) R (c, d) \Leftrightarrow a + d = b + c$$

$$(c, d) R (e, f) \Leftrightarrow c + f = d + e$$

On adding these both equations we get ,  $a + f = b + e$

Also,

$$(a, b) R (e, f) \Leftrightarrow a + f = b + e$$

$\therefore$  It will always be true

Therefore , R is transitive ..... (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (D)

## 15. Question

Mark the tick against the correct answer in the following:

Let A be the set of all points in a plane and let O be the origin. Let  $R = \{(P, Q) : OP = OQ\}$ . Then, R is

A. reflexive and symmetric but not transitive

- B. reflexive and transitive but not symmetric
- C. symmetric and transitive but not reflexive
- D. an equivalence relation

There is printing mistake in the question...

R should be  $R = \{(P, Q) : OP = OQ\}$

Instead of  $R = \{(P, Q) : OP = QQ\}$

### **Answer**

According to the question ,

O is the origin

$R = \{(P, Q) : OP = OQ\}$

### Formula

For a relation R in set A

Reflexive

The relation is reflexive if  $(a, a) \in R$  for every  $a \in A$

Symmetric

The relation is Symmetric if  $(a, b) \in R$  , then  $(b, a) \in R$

Transitive

Relation is Transitive if  $(a, b) \in R$  &  $(b, c) \in R$  , then  $(a, c) \in R$

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Consider ,  $(P, P) \in R \Leftrightarrow OP = OP$

which is always true .

Therefore , R is reflexive ..... (1)

Check for symmetric

$(P, Q) \in R \Leftrightarrow OP = OQ$

$(Q, P) \in R \Leftrightarrow OQ = OP$

Both the equation are the same and therefore will always be true.

Therefore , R is symmetric ..... (2)

Check for transitive

$(P, Q) \in R \Leftrightarrow OP = OQ$

$(Q, R) \in R \Leftrightarrow OQ = OR$

On adding these both equations, we get ,  $OP = OR$

Also,

$(P, R) \in R \Leftrightarrow OP = OR$

$\therefore$  It will always be true

Therefore , R is transitive ..... (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (D)

### 16. Question

Mark the tick against the correct answer in the following:

Let  $Q$  be the set of all rational numbers, and  $*$  be the binary operation, defined by  $a * b = a + 2b$ , then

- A.  $*$  is commutative but not associative
- B.  $*$  is associative but not commutative
- C.  $*$  is neither commutative nor associative
- D.  $*$  is both commutative and associative

### Answer

According to the question ,

$Q$  is set of all rational numbers

$$R = \{(a, b) : a * b = a + 2b\}$$

#### Formula

$*$  is commutative if  $a * b = b * a$

$*$  is associative if  $(a * b) * c = a * (b * c)$

Check for commutative

Consider ,  $a * b = a + 2b$

And ,  $b * a = b + 2a$

Both equations will not always be true .

Therefore ,  $*$  is not commutative ..... (1)

Check for associative

Consider ,  $(a * b) * c = (a + 2b) * c = a + 2b + 2c$

And ,  $a * (b * c) = a * (b + 2c) = a + 2(b + 2c) = a + 2b + 4c$

Both the equation are not the same and therefore will not always be true.

Therefore ,  $*$  is not associative ..... (2)

Now , according to the equations (1) , (2)

Correct option will be (C)

### 17. Question

Mark the tick against the correct answer in the following:

Let  $a * b = a + ab$  for all  $a, b \in Q$ . Then,

- A.  $*$  is not a binary composition
- B.  $*$  is not commutative
- C.  $*$  is commutative but not associative
- D.  $*$  is both commutative and associative

### Answer

According to the question ,

$$Q = \{ a, b \}$$

$$R = \{(a, b) : a * b = a + ab\}$$

### Formula

\* is commutative if  $a * b = b * a$

\* is associative if  $(a * b) * c = a * (b * c)$

Check for commutative

Consider ,  $a * b = a + ab$

And ,  $b * a = b + ba$

Both equations will not always be true .

Therefore , \* is not commutative ..... (1)

Check for associative

Consider ,  $(a * b) * c = (a + ab) * c = a + ab + (a + ab)c = a + ab + ac + abc$

And ,  $a * (b * c) = a * (b + bc) = a + a(b + bc) = a + ab + abc$

Both the equation are not the same and therefore will not always be true.

Therefore , \* is not associative ..... (2)

Now , according to the equations (1) , (2)

Correct option will be (B)

### **18. Question**

Mark the tick against the correct answer in the following:

Let  $Q^+$  be the set of all positive rationals. Then, the operation \* on  $Q^+$  defined by  $a * b = \frac{ab}{2}$  for all  $a, b \in Q^+$  is

- A. commutative but not associative
- B. associative but not commutative
- C. neither commutative nor associative
- D. both commutative and associative

### **Answer**

According to the question ,

$$Q = \{ \text{Positive rationals} \}$$

$$R = \{(a, b) : a * b = ab/2\}$$

### Formula

\* is commutative if  $a * b = b * a$

\* is associative if  $(a * b) * c = a * (b * c)$

Check for commutative

Consider ,  $a * b = ab/2$

And ,  $b * a = ba/2$

Both equations are the same and will always be true .

Therefore , \* is commutative ..... (1)

Check for associative

$$\text{Consider , } (a * b) * c = (ab/2) * c = \frac{\frac{ab}{2} \times c}{2} = abc/4$$

$$\text{And , } a * (b * c) = a * (bc/2) = \frac{a \times \frac{bc}{2}}{2} = abc/4$$

Both the equation are the same and therefore will always be true.

Therefore , \* is associative ..... (2)

Now , according to the equations (1) , (2)

Correct option will be (D)

### 19. Question

Mark the tick against the correct answer in the following:

let Z be the set of all integers and let  $a * b = a - b + ab$ . Then, \* is

- A. commutative but not associative
- B. associative but not commutative
- C. neither commutative nor associative
- D. both commutative and associative

### Answer

According to the question ,

$$Q = \{ \text{All integers} \}$$

$$R = \{(a, b) : a * b = a - b + ab \}$$

#### Formula

\* is commutative if  $a * b = b * a$

\* is associative if  $(a * b) * c = a * (b * c)$

Check for commutative

$$\text{Consider , } a * b = a - b + ab$$

$$\text{And , } b * a = b - a + ba$$

Both equations are not the same and will not always be true .

Therefore , \* is not commutative ..... (1)

Check for associative

$$\text{Consider , } (a * b) * c = (a - b + ab) * c$$

$$= a - b + ab - c + (a - b + ab)c$$

$$= a - b + ab - c + ac - bc + abc$$

$$\text{And , } a * (b * c) = a * (b - c + bc)$$

$$= a - (b - c + bc) + a(b - c + bc)$$

$$= a - b + c - bc + ab - ac + abc$$

Both the equation are not the same and therefore will not always be true.

Therefore , \* is not associative ..... (2)

Now , according to the equations (1) , (2)

Correct option will be (C)

## 20. Question

Mark the tick against the correct answer in the following:

Let  $Z$  be the set of all integers. Then, the operation  $*$  on  $Z$  defined by

$$a * b = a + b - ab \text{ is}$$

- A. commutative but not associative
- B. associative but not commutative
- C. neither commutative nor associative
- D. both commutative and associative

## Answer

According to the question ,

$$Q = \{ \text{All integers} \}$$

$$R = \{(a, b) : a * b = a + b - ab \}$$

### Formula

$*$  is commutative if  $a * b = b * a$

$*$  is associative if  $(a * b) * c = a * (b * c)$

Check for commutative

$$\text{Consider , } a * b = a + b - ab$$

$$\text{And , } b * a = b + a - ba$$

Both equations are the same and will always be true .

Therefore ,  $*$  is commutative ..... (1)

Check for associative

$$\text{Consider , } (a * b) * c = (a + b - ab) * c$$

$$= a + b - ab + c - (a + b - ab)c$$

$$= a + b - ab + c - ac - bc + abc$$

$$\text{And , } a * (b * c) = a * (b + c - bc)$$

$$= a + (b + c - bc) - a(b + c - bc)$$

$$= a + b + c - bc - ab - ac + abc$$

Both the equation are the same and therefore will always be true.

Therefore ,  $*$  is associative ..... (2)

Now , according to the equations (1) , (2)

Correct option will be (D)

## 21. Question

Mark the tick against the correct answer in the following:

Let  $Z^+$  be the set of all positive integers. Then, the operation  $*$  on  $Z^+$  defined by  $a * b = a^b$  is

- A. commutative but not associative
- B. associative but not commutative



- C. neither commutative nor associative
- D. both commutative and associative

**Answer**

According to the question ,

$$Q = \{ \text{All integers} \}$$

$$R = \{(a, b) : a * b = a^b\}$$

Formula

\* is commutative if  $a * b = b * a$

\* is associative if  $(a * b) * c = a * (b * c)$

Check for commutative

$$\text{Consider , } a * b = a^b$$

$$\text{And , } b * a = b^a$$

Both equations are not the same and will not always be true .

Therefore , \* is not commutative ..... (1)

Check for associative

$$\text{Consider , } (a * b) * c = (a^b) * c = (a^b)^c$$

$$\text{And , } a * (b * c) = a * (b^c) = a^{(b^c)}$$

$$\text{Ex } a=2 \ b=3 \ c=4$$

$$(a * b) * c = (2^3) * c = (8)^4$$

$$a * (b * c) = 2 * (3^4) = 2^{(81)}$$

Both the equation are not the same and therefore will not always be true.

Therefore , \* is not associative ..... (2)

Now , according to the equations (1) , (2)

Correct option will be (C)

**22. Question**

Mark the tick against the correct answer in the following:

Define \* on  $Q - \{-1\}$  by  $a * b = a + b + ab$ . Then, \* on  $Q - \{-1\}$  is

- A. commutative but not associative
- B. associative but not commutative
- C. neither commutative nor associative
- D. both commutative and associative

**Answer**

According to the question ,

$$R = \{(a, b) : a * b = a + b + ab\}$$

Formula

\* is commutative if  $a * b = b * a$

\* is associative if  $(a * b) * c = a * (b * c)$

Check for commutative

Consider ,  $a * b = a + b + ab$

And ,  $b * a = b + a + ba$

Both equations are same and will always be true .

Therefore , \* is commutative ..... (1)

Check for associative

Consider ,  $(a * b) * c = (a + b + ab) * c$

$= a + b + ab + c + (a + b + ab)c$

$= a + b + c + ab + ac + bc + abc$

And ,  $a * (b * c) = a * (b + c + bc)$

$= a + b + c + bc + a(b + c + bc)$

$= a + b + c + ab + bc + ac + abc$

Both the equation are same and therefore will always be true.

Therefore , \* is associative ..... (2)

Now , according to the equations (1) , (2)

Correct option will be (D)