CBSE Test Paper 02

CH-11 Conic Sections

1. The angle between the tangents drawn from the origin to the circle =

$$(x-7)^2 + (y+1)^2 = 25$$
 is

- a. $\frac{\pi}{3}$ b. $\frac{\pi}{2}$ c. $\frac{\pi}{6}$ d. $\frac{\pi}{8}$
- 2. The number of points on X-axis which are at a distance c units (c < 3) from (2, 3) is
 - a. 2
 - b. 0
 - c. 1
 - d. 3
- 3. The equations x = a cos θ , y = b sin θ , 0 < θ <2 π , a \neq b, represent
 - a. a parabola
 - b. an ellipse
 - c. a hyperbola
 - d. a circle
- 4. $x=rac{e^t+e^{-t}}{2}, y=rac{e^t-e^{-t}}{2}; t\in R$ represents
 - a. a circle
 - b. an ellipse
 - c. a parabola
 - d. a hyperbola
- 5. Two perpendicular tangents to the circle $x^2+y^2=r^2$ meet at P. The locus of P is
 - a. x + y = r
 - b. $x^2 + y^2 = 4 r^2$
 - c. $x^2 + y^2 = \frac{r^2}{2}$
 - d. $x^2 + y^2 = 2r^2$
- 6. Fill in the blanks:

The equation of the ellipse having foci (0, 1), (0, -1) and minor axis of length 1 is

7. Fill in the blanks:

The equation of the parabola whose focus is the point (2, 3) and directrix is the line x - 4y + 3 = 0 is _____.

- 8. Find the equation of the parabola with vertex at (0, 0) and focus at (0, 2).
- 9. Find the centre and radius of each of the following circle. $x^2 + (y + 2)^2 = 9$
- 10. Find the equation of the hyperbola whose directrix is 2x + y-1, focus (1, 2) and eccentricity $\sqrt{3}$.
- 11. Find the equation of a circle of radius 5 units, passing through the origin and having its centre on the Y-axis.
- 12. Find the centre and radius of the circle. $x^2 + y^2 + 6x 4y + 4 = 0$
- 13. Find the equation of the hyperbola, the length of whose latus rectum is 8 and eccentricity is $\frac{3}{\sqrt{5}}$.
- 14. Find the equation of the circle passing through the points (4, 1) and (6, 5) and whose centre is on the line 4x + y = 16.
- 15. Find the axes, eccentricity, latus-rectum and the coordinates of the foci of the hyperbola

$$25 x^2 - 36 y^2 = 225$$
.

CBSE Test Paper 02

CH-11 Conic Sections

Solution

- 1. (b) $\frac{\pi}{2}$ **Explanation:** The angle between the Radius and tangent is 90^0
- 2. (b) 0

Explanation: the shortest distance from x-axis to the point is 3.

3. (b) an ellipse

Explanation: parametric form of ellipse.

4. (d) a hyperbola

Explanation: $x=\frac{e^t+e^{-t}}{2}, y=\frac{e^t-e^{-t}}{2}; t\in R$ Squaring both sides of both the equation, we get $\mathbf{x}^2=\frac{(e^t+e^-t)^2}{4}$ and $\mathbf{y}^2=\frac{(e^t-e^-t)}{4}$ Subtracting one equation from another we get \mathbf{x}^2 - \mathbf{y}^2 = 1 which is nothing but equation of hyperbola

5. (d) $x^2+y^2=2$ r^2 **Explanation:** locus of P is a circle with centre at origin and radius $\sqrt{2r^2}$. This is known as the director circle of the circle $x^2+y^2=r^2$

$$6. \ \frac{4x^2}{1} + \frac{4y^2}{5} = 1$$

7.
$$16x^2 + y^2 + 8xy - 74x - 78y + 212 = 0$$

8. Given, the vertex is (0, 0) and focus is at (0, 2) which lies on Y-axis.

The Y axis is the axis of parabola.

Therefore, equation of parabola is of the form

$$x^2 = 4ay$$

$$x^2 = 4(2)y$$
 i.e., $x^2 = 8y$

9. Given equation of circle is $x^2 + (y + 2)^2 = 9$

$$\Rightarrow$$
 (x - 0)² + {y - (-2)}² = 3²

On comparing with $(x - h)^2 + (y - k)^2 = r^2$, we get

$$h = 0, k = -2 \text{ and } r = 3$$

Hence, centre of circle = (0, -2) and radius = 3.

10. Let S (1, 2) be the focus and P (x, y) be a point on the hyperbola. Draw PM perpendicular from P on the directrix. Then,

SP = ePM [By definition]

$$\Rightarrow \sqrt{(x-1)^2 + (y-2)^2} = \sqrt{3} \left| \frac{2x+y-1}{\sqrt{2^2+1^2}} \right|$$

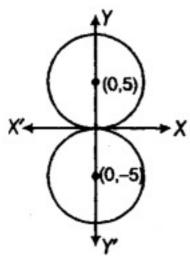
$$\Rightarrow (x-1)^2 + (y-2)^2 = \frac{3(2x+y-1)^2}{5}$$

$$\Rightarrow 5[(x-1)^2 + (y-2)^2] = 3(2x+y-1)^2$$

$$\Rightarrow 5x^2 + 5y^2 - 10x - 20y + 25 = 3(4x^2 + y^2 + 1 + 4xy - 4x - 2y)$$

$$\Rightarrow 7x^2 - 2y^2 + 12xy - 2x + 14y - 22 = 0, \text{ which is the required equation of the hyperbola.}$$

11. Radius of circle =5. The circle passes through the origin and its centre is on the Y-axis i.e., (0, 5) and (0, -5). Equations of circle are



$$(x - 0)^{2} + (y - 5)^{2} = 5^{2}$$

$$\Rightarrow x^{2} + y^{2} - 10y + 25 = 25$$

$$\Rightarrow x^{2} + y^{2} - 10y = 0$$
and $(x - 0)^{2} + (y + 5)^{2} = 5^{2}$

$$\Rightarrow x^{2} + y^{2} + 10y + 25 = 25$$

$$\Rightarrow x^{2} + y^{2} + 10y = 0$$

12. Given equation of circle is
$$x^2 + y^2 + 6x - 4y + 4 = 0$$

$$\Rightarrow (x^2 + 6x) + (y^2 - 4y) = -4$$

$$\Rightarrow (x^2 + 6x + 9 - 9) + (y^2 - 4y + 4 - 4) = -4$$

$$\Rightarrow (x^2 + 6x + 9) + (y^2 - 4y + 4) = -4 + 4 + 9$$

$$\Rightarrow (x + 3)^2 + (y - 2)^2 = 9$$

$$\Rightarrow \{x - (-3)\}^2 + \{y - 2\}^2 = 3^2$$

On comparing with $(x - h)^2 + (y - k)^2 = r^2$, we get

$$h = -3, k = 2 \text{ and } r = 3$$

Hence, centre of circle = (-3, 2) and radius = 3.

13. Let equation of the hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Now
$$\frac{2b^2}{a} = 8 \Rightarrow b^2 = 4a$$

But
$$b^2 = a^2(e^2 - 1)$$

$$\therefore a^2(e^2-1)=4a\Rightarrow a(e^2-1)=4\Rightarrow a\left(rac{a}{5}-1
ight)=4\Rightarrow$$
 a = 5

$$\therefore b^2 = 4 \times 5 = 20$$

Thus equation of required hyperbola is $rac{x^2}{25} - rac{y^2}{20} = 1$

14. The equation of the circle is

$$(x - h)^2 + (y - k)^2 = r^2 \dots (i)$$

Since the circle passes through point (4, 1)

$$(4 - h)^2 + (1 - k)^2 = r^2 \Rightarrow 16 + h^2 - 8h + 1 + k^2 - 2k = r^2$$

$$\Rightarrow$$
 h² + k² - 8h - 2k + 17 = r² (ii)

Also the circle passes through point (6, 5)

$$(6 - h)^2 + (5 - k)^2 = r^2 \Rightarrow 36 + h^2 - 12h + 25 + k^2 - 10k = r^2$$

$$\Rightarrow$$
 $h^2 + k^2 - 12h - 10k + 61 = r^2$

From (ii) and (iii), we have

$$h^2 + k^2 - 8h - 2k + 17 = h^2 + k^2 - 12h - 10k + 61$$

$$\Rightarrow$$
 4h + 8k = 44 \Rightarrow h + 2k = 11 (iv)

Since the centre (h, k) of the circle lies on the line 4x + y = 16

$$\therefore$$
 4h + k = 16 . . . (v)

Solving (iv) and (v), we have

$$h = 3 \text{ and } k = 4$$

Putting value of h and k in (ii), we have

$$(3)^2 + (4)^2 - 8 \times 3 - 2 \times 4 + 17 = r^2$$

$$r^2 = 10$$

Thus equation of required circle is

$$(x-3)^2 + (y-4)^2 = 10 \Rightarrow x^2 + 9 - 6x + y^2 + 16 - 8y = 10$$

 $\Rightarrow x^2 + y^2 - 6x - 8y + 15 = 0$

15. We have,

$$25x^{2} - 36y^{2} = 225$$

$$\Rightarrow \frac{25x^{2}}{225} - \frac{36y^{2}}{225} = 1$$

$$\Rightarrow \frac{x^{2}}{9} - \frac{4y^{2}}{25} = 1$$

$$\Rightarrow \frac{x^{2}}{9} - \frac{y^{2}}{\frac{25}{4}} = 1$$

$$\Rightarrow \frac{x^{2}}{(3)^{2}} - \frac{y^{2}}{\left(\frac{5}{2}\right)^{2}} = 1$$

This is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where a = 3 and b = $\frac{5}{2}$

The length of the transverse axis = 2a

$$= 2 \times 3 = 6$$

The length of the conjugate axis is $2b = 2 \times \frac{5}{2} = 5$

The eccentricity e is given by,

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

$$= \sqrt{1 + \frac{\frac{25}{4}}{9}}$$

$$= \sqrt{1 + \frac{25}{36}}$$

$$= \sqrt{\frac{61}{36}}$$

$$= \frac{\sqrt{61}}{6}$$

Length of Latusrectum = $\frac{2b^2}{a}=\frac{25}{6}$ and Foci ($\pm\frac{\sqrt{61}}{2}$,0)