Algebra

- In our day to day life, we come across many examples of variables.

  For example, if patterns of  have to be made using match sticks, then we can obtain the following patterns.

(a) (b) (c)

  In this case, the number of match sticks required are given by the rule, 3*n*, where *n* is the number of  .

  If a pattern of 15  have to be made, then the number of match sticks required is 3 × 15 = 45

- Geometric formulae are written in general form by using variables.

  For example,
  (1) Perimeter of an equilateral triangle = 3 × length of one side
  Let the length of the side of the equilateral triangle be *l*.
  ∴ Perimeter of an equilateral triangle = 3*l*
  (2) Area of a square = $(\text{side})^2$
  Let the side of square be *s*.
  ∴ Area of a square = s^2

- Variables can be used to generalise properties as follows:
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- **Commutativity of addition of two numbers**

  For two variables, *a* and *b*, this rule can be expressed as $a + b = b + a$
  Here, the variables, *a* and *b*, can represent any number.
  Example: 9 + 2 = 2 + 9 (= 11)

- **Commutativity of multiplication of two numbers**

  For any two variables, *a* and *b*, this rule can be expressed as $a \times b = b \times a$
  Example: 6 × 11 = 11 × 6 = 66

- **Associativity of addition**

  For any three variables, *a*, *b* and *c*, this rule can be stated as $a + (b + c) = (a + b) + c$
  Example: 2 + (7 + 13) = 2 + 20 = 22
  Also, (2 + 7) + 13 = 9 + 13 = 22
  ∴ 2 + (7 + 13) = (2 + 7) + 13

- **Associativity of multiplication**

  For any three variables, *a*, *b* and *c*, this rule can be stated as $a \times (b \times c) = (a \times b) \times c$
  Example: 3 × (5 × 10) = 3 × 50 = 150
  Also, (3 × 5) × 10 = 15 × 10 = 150
  ∴ 3 × (5 × 10) = (3 × 5) × 10

- **Distributivity of numbers**

  For any three variables, *a*, *b*, and *c*, this rule can be stated as $a \times (b + c) = a \times b + a \times c$
  Example: 5 × (2 + 9) = 5 × 11 =55
  Also, 5 × 2 + 5 × 9 = 10 + 45 = 55
  ∴5 × (2 + 9) = (5 × 2) + (5 × 9)

- A **variable** is something that does not have a fixed value. The value of a variable varies.
- Variables are represented by English letters such as *x*, *y*, *z*, *a*, *b*, *c* etc.
- A combination of variables, numbers and operators (+, –, × and ÷) is known as **expression**.

- Using different operations on variables and numbers, expressions such as $\frac{1}{7} - 4y, 9x - 5$, can be formed.

  Example:

  Meena's age is 4 years less than 7 times the age of Ravi. Express it using variables.

  Solution:

  Let the age of Ravi be x years.

  7 times the age of Ravi can be expressed as $7x$.

  4 years less than 7 times the age of Ravi can be written as $7x - 4$.

  ∴ Age of Meena = $(7x - 4)$

- An **equation** is a condition to find the value of a variable.
- It is to be noted that an equation must have an 'equal sign' (=). The value of the right hand side (R.H.S.) and the left hand side (L.H.S.) of the 'equal sign' (=) must be equal.
- For example, $x + 3 = 9$ is an equation.
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- If L.H.S. is greater than R.H.S. or vice-versa, i.e., if L.H.S. > R.H.S. or L.H.S. < R.H.S., then it is not an equation.
- We can represent a situation in the form of equation.
- For example, Shalini is 7 years younger than Sandhya. If Shalini is 18 years old, then the equation for this situation is $y - 7 = 18$.

- The value of variable which satisfies the equation is known as the **solution** of the equation.
- For example, consider the equation $2x - 5 = 7$
- When $x = 6$, LHS = $2 \times 6 - 5 = 12 - 5 = 7$ = RHS.
- ∴ $x = 6$ is the solution of the equation, $2x - 5 = 7$.
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- The solution of an equation is calculated by a method known as **trial and error method**. In this method, some value of the variable is substituted and it is checked whether it satisfies the equation. Different values of variables are substituted until the right value is found, which satisfies the equation.

  For example, for the equation $7x - 2 = 19$
  Put $x = 1$,
  ∴ LHS = $7 \times 1 - 2 = 7 - 2 = 5 \neq 19$

$\Rightarrow$ LHS $\neq$ RHS
Therefore, $x = 1$ is not the solution.

Put $x = 2$,
LHS $= 7 \times 2 - 2 = 14 - 2 = 12 \neq 19$
$\therefore$ LHS $\neq$ RHS
Therefore, $x = 2$ is not the solution.

Put $x = 3$,
LHS $= 7 \times 3 - 2 = 21 - 2 = 19 =$ RHS
Therefore, $x = 3$ is the solution of the equation, $7x - 2 = 19$