Sample Paper-01
SUMMATIVE ASSESSMENT –II
MATHEMATICS
Class – X

Time allowed: 3 hours Maximum Marks: 90

General Instructions:

a) All questions are compulsory.
b) The question paper consists of 31 questions divided into four sections – A, B, C and D.
c) Section A contains 4 questions of 1 mark each, Section B contains 6 questions of 2 marks each, Section C contains 10 questions of 3 marks each and Section D contains 11 questions of 4 marks each.
d) Use of calculator is not permitted.

Section A

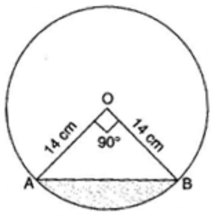
1. It is given that in a group of 3 students, the probability of 2 students not having the same birthday is 0.992. What is the probability that the 2 students have the same birthday?
2. Find a relation between x and y if the points $(x, y), (1, 2)$ and $(7, 0)$ are collinear.
3. For what value of k: 2k, k+10 and 3k+2 are in AP?
4. A man is standing on the deck of a ship which is 25 m above water level. He observes the angle of elevation of the top of a lighthouse as $60°$ and the angle of depression of the base of the lighthouse as $45°$. Calculate the height of the lighthouse.

Section B

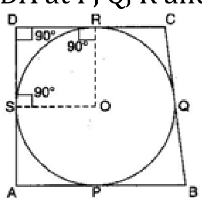
5. The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having area equal to the sum of the areas of the two circles.
6. A heap of rice is in the form of a cone of radius 3 m and height 3 m. Find the volume of the rice. How much cloth is required to just cover the heap?
7. A solid metallic hemisphere of radius 6 cm is melted and re-casted into a right circular cone of base radius 3 cm. Determine the height of the cone.
8. For what value of k, are the roots of the equation $3x^2 + 2kx + 27 = 0$ are real and equal?
9. Which term of the AP: 3, 8, 13, 18 ... is 78?
10. The tangent at a point C of a circle and a diameter AB when extended intersect at P. If $\angle$PCA = 100°, then find $\angle$CBA.

Section C

11. Find the point on the x–axis which is equidistant from (2, –5) and (–2, 9).
12. If A and B are (–2, –2) and (2, –4) respectively, find the coordinates of P such that AP = $\frac{3}{7}$AB and P lies on the line segment AB.
13. A chord AB of a circle of radius 14 cm makes a right angle at the centre (O) of the circle. Find the area of the minor segment. $\left(\text{Use } \pi = \frac{22}{7}\right)$



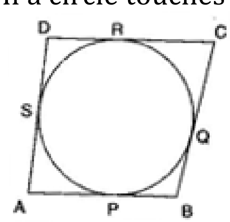
14. The inner circumference of a circular track is 440 m. The track is 14 m wide. Find the diameter of the outer circle of the track. $\left(\text{Use } \pi = \frac{22}{7}\right)$
15. Water flows out through a circular pipe whose internal radius is 1 cm, at the rate of 80 cm/second into an empty cylindrical tank, the radius of whose base is 40 cm. By how much will the level of water rise in the tank in half an hour?
16. Find the value of k for which the roots of the quadratic equation $kx^2 - 10x + 5 = 0$ are equal.
17. Find the 31st term of an AP whose 11th term is 38 and 16th term is 73.
18. ABCD is a quadrilateral such that $\angle$ D = 90°. A circle C (O, r) touches the sides AB, BC, CD and DA at P, Q, R and S respectively. If BC = 38 cm, CD = 25 cm and BP = 27 cm, then find r.



19. A girl who is 1.2 m tall, spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eye of the girl at any instant is $60°$. After sometime, the angle of elevation reduces to $30°$. Find the distance travelled by the balloon during the interval.
20. A game consists of tossing a one rupee coin 3 times and noting its outcome each time. Hanif wins if all the tosses give the same result, i.e., three heads or three tails and loses otherwise. Calculate the probability that Hanif will lose the game.

Section D

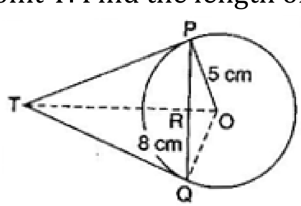
21. Prove that the length of tangents drawn from an external point to a circle are equal.
Using the above result, prove the following:
If a circle touches all the four sides of a quadrilateral ABCD, then prove that: AB + CD = BC + DA



22. Draw a right triangle ABC, in which $\angle$B = 90°, AB = 5 cm, BC = 4 cm. Then construct another triangle A'BC' whose sides are $\frac{5}{3}$ times the corresponding sides of Δ ABC.
23. A card is drawn at random from a well shuffled deck of playing cards. Find the probability that the card drawn is
(i) a card of spades of an ace
(ii) a red king

(iii) neither a king nor a queen
(iv) either a king or a queen
(v) a face card
(vi) cards which is neither king nor a red card.

24. If, Q (0, 1) is equidistant from P (5, −3) and R (x, 6), find the values of x. Also, find the distances QR and PR.
25. A cylindrical vessel with internal diameter 10 cm and height 10.5 cm is full of water. A solid cone of base diameter 7 cm and height 6 cm is completely immersed in water. Find the volume of:
    (i) water displaced out of the cylindrical vessel.
    (ii) water left in the cylindrical vessel. $\left(\text{Use } \pi = \frac{22}{7}\right)$
26. From a solid cylinder whose height is 8 cm and radius 6 cm, a conical cavity of height 8 cm and of base radius 6 cm, is hollowed out. Find the volume of the remaining solid correct to two places of decimals. Also find the total surface area of the remaining solid. $(\text{Use } \pi = 3.1416)$
27. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If, the average speed of the express train is 11 km/h more than that of the passenger train, find the average speed of two trains.
28. There are two windows in a house. A window of the house is at a height of 1.5 m above the ground and the other window is 3 m vertically above the lower window. Ram and Shyam are sitting inside the two windows. At an instant, the angle of elevation of a balloon from these windows are observed as 45° and 30° respectively.
    Read the above passage and answer the following questions:
    (a) Find the height of the balloon from the ground.
    (b) Among Ram and Shyam, who is more closer to the balloon?
    (c) Why windows are essential in any construction, commercial or residential?
    (d) If the balloon is moving towards the building, then both angles of elevation will remain same or not?
29. Sum of the areas of two squares is 468 m^2. If the difference of their perimeters is 24 m, then find the sides of two squares.
30. A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If, each prize is Rs 20 less than its preceding term, find the value of each of the prizes.
31. PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the length of TP.



Sample Paper-01
SUMMATIVE ASSESSMENT –II
MATHEMATICS
Class – X

(Solutions)

SECTION-A

1. Let E be the event of having the same birthday

$\Rightarrow$ P(E) = 0.992

But P(E) + P$\left(\overline{E}\right)$ = 1

$\therefore$ P$\left(\overline{E}\right)$ = 1 – P(E) = 1 – 0.992 = 0.008

2. The points A(x, y), B (1, 2) and C (7, 0) will be collinear if

Area of triangle = 0

$\Rightarrow \frac{1}{2}\left[x(2-0)+1(0-y)+7(y-2)\right]=0$

$\Rightarrow 2x-y+7y-14=0$

$\Rightarrow 2x+6y-14=0$

$\Rightarrow x+3y-7=0$

3. Given numbers are in AP

$\therefore$ (k+10)-2k=(3k+2)-(k+10)

$\Rightarrow$ -k+10=2k-8 or 3k=18 or k = 6

4. H = Height of lighthouse = h + 25 ……. (i)

In right $\Delta ADC, \frac{x}{25} = \cot 45° = 1$

$\Rightarrow x = 25\ m$

In right $\Delta ADE, \frac{x}{h} = \cot 60° = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{25}{h} = \frac{1}{\sqrt{3}} \Rightarrow h = 25\sqrt{3}$

Now $H = h + 25 = 25\sqrt{3} + 25$

$= 25\left(\sqrt{3}+1\right)m$

5. Let R be the radius of the circle which has area equal to the sum of areas of the two circles, then According to the question,

$\pi R^2 = \pi(8)^2 + \pi(6)^2$

$\Rightarrow R^2 = (8)^2 + (6)^2$

$\Rightarrow R^2 = 64 + 36$

$\Rightarrow R^2 = 100$

$\Rightarrow$ R = 10 cm

6. Volume of rice = $\frac{1}{3}\pi r^2 h$ = $\frac{1}{3}\times\frac{22}{7}\times(3)^2(3) = \frac{198}{7}$ = 28.29 m³ (approx.)

Cloth required = $\pi r\sqrt{r^2+h^2}$ = $\frac{22}{7}\times 3\sqrt{3^2+3^2}$ = $\frac{198\sqrt{2}}{7}$ m²

7. Let the height of the cone be H cm. Then

$\frac{1}{3}\pi(3)^2\ \text{H} = \frac{2}{3}\pi(6)^3 \quad\Rightarrow\quad \text{H} = 48 \text{ cm}$

8. Here, $a = 3, b = 2k, c = 27$

For real and equal roots, $b^2 - 4ac = 0$

$\Rightarrow \quad (2k)^2 - 4(3)(27) = 0$

$\Rightarrow \quad 4k^2 = 324 \quad \Rightarrow \quad k^2 = 81$

$\Rightarrow \quad k = \pm 9$

9. First term = a=3, Common difference = d = 8 – 3=13 – 8=5 and $a_n = 78$

Using formula $a_n=a+(n-1)d$,to find nth term of arithmetic progression,

$a_n = 3+(n-1)5, \quad \Rightarrow \quad 78=3+(n-1)5 \Rightarrow \quad 75=5n-5$

$\Rightarrow \quad 80=5n \quad \Rightarrow \quad n=16$

It means 16th term of the given AP is equal to 78.

10. $\angle$PCA = 100° and $\angle$BCA = 90°

$\therefore \quad \angle$PCB = 100° – 90° = 10°

$\angle$OCP = 90°

$\Rightarrow \quad \angle$OCB = $\angle$PCB = 90°

$\Rightarrow \quad \angle$OCB + 10° = 90°

$\Rightarrow \quad \angle$OCB = 80°

$\because \quad$ OB = OC

$\therefore \quad \angle$OBC = $\angle$OCB = 80°

$\therefore \quad \angle$CBA = 80°

11. Let the point be (x, 0) on x–axis which is equidistant from (2, –5) and (–2, 9).

Using Distance Formula and according to given conditions we have:

$\sqrt{[x-2]^2+[0-(-5)]^2} = \sqrt{[x-(-2)]^2+[(0-9)]^2}$

$\Rightarrow \sqrt{x^2+4-4x+25} = \sqrt{x^2+4+4x+81}$

Squaring both sides, we get

$\Rightarrow x^2 + 4 - 4x + 25 = x^2 + 4 + 4x + 81$

$\Rightarrow -4x + 29 = 4x + 85$

$\Rightarrow 8x = -56$

$\Rightarrow x = -7$

Therefore, point on the x–axis which is equidistant from (2, –5) and (–2, 9) is (–7, 0)

12. A = (–2, –2) and B=(2, –4)

It is given that AP=$\frac{3}{7}$AB

PB=AB – AP=AB−$\frac{3}{7}$AB=$\frac{4}{7}$AB

So, we have AP:PB = 3:4

Let coordinates of P be (x, y)

Using Section formula to find coordinates of P, we get

$$x = \frac{(-2)\times 4 + 2\times 3}{3+4} = \frac{6-8}{7} = \frac{-2}{7}$$

$$y = \frac{(-2)\times 4 + (-4)\times 3}{3+4} = \frac{-8-12}{7} = \frac{-20}{7}$$

Therefore, Coordinates of point P are $\left(\frac{-2}{7}, \frac{-20}{7}\right)$.

13. Area of the minor segment = Area of sector AOB – Area of Δ AOB

$$= \frac{\theta}{360°}\times \pi r^2 - \frac{1}{2}\times b\times h$$

$$= \frac{90^\circ}{360^\circ}\times\frac{22}{7}\times 14\times 14 - \frac{1}{2}\times 14\times 14$$

= 56 cm^2

14. $2\pi r = 440$

$\Rightarrow \quad 2\times\frac{22}{7}\times r = 440 \quad \Rightarrow \quad r = \frac{440\times 7}{2\times 22}$ = 70 m

Width of track = 14 m

$\therefore$ Radius of outer circle = 70 + 14 = 84 m

$\therefore$ Diameter of the outer circle = 2 x 84 = 168 m

15. Volume of water that flows out through the pipe in half an hour,

= $\pi(1)^2\times 80\times 60\times 30$

Let the water level rise by x cm. Then,

$$\pi(40)^2\times x = \pi(1)^2\times 80\times 60\times 30$$

$\Rightarrow \quad x$ = 90 cm

16. Here, $a = k, b = -10, c = 5$

For equal roots, $\quad b^2 - 4ac = 0 \quad \Rightarrow \quad (-10)^2 - 4.k.5 = 0$

$\Rightarrow \quad 100 - 20k = 0 \quad \Rightarrow \quad k = 5$

17. Here $a_{11} = 38$ and $a_{16} = 73$

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

38=a+(11−1)(d) and 73=a+(16−1)(d)

⇒ 38=a+10d and 73=a+15d

These are equations consisting of two variables.

We have, 38=a+10d

$\Rightarrow a = 38 - 10d$

Let us put value of a in equation ($73 = a + 15d$),

$73 = 38 - 10d + 15d$

$\Rightarrow 35 = 5d$

Therefore, Common difference $= d = 7$

Putting value of d in equation $38 = a + 10d$,

$38 = a + 70$

$\Rightarrow a = -32$

Therefore, common difference = d = 7 and First term = a = –32

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$a_{31} = -32 + (31-1)(7) = -32 + 210 = 178$

Therefore, 31^{st} term of AP is 178.

18. $\because$ Tangent is perpendicular to the radius through the point of contact.

$\therefore$ $\angle ORD = \angle OSD = 90^\circ$

Also, OR = OS [Radii of the same circle]

$\therefore$ ORDS is a square.

$\because$ Tangent segments from an external point to a circle are equal in length.

$\therefore$ BP = BQ, CQ = CR and DR = DS

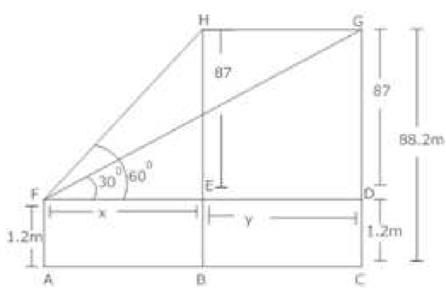
Now, BP = BQ = BC – CQ

$\Rightarrow$ 27 = 38 – CQ $\Rightarrow$ CQ = 11 cm $\Rightarrow$ CR = 11 cm

$\Rightarrow$ CD – DR = 11 $\Rightarrow$ 25 – DR = 11

$\Rightarrow$ DR = 14 cm $\Rightarrow$ OR = 14 cm [$\because$ ORDS is a square]

19. In right ΔHEF,



$$\frac{87}{x} = \tan 60^\circ$$

$$\Rightarrow \frac{87}{x} = \frac{\sqrt{3}}{1}$$

$$\Rightarrow x = \frac{87}{\sqrt{3}}$$

$$\Rightarrow x = \frac{87}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{87\sqrt{3}}{3} = 29\sqrt{3}m$$

In right ΔGDF,

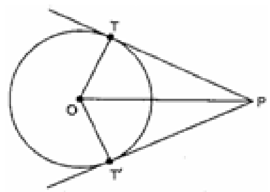
$$\frac{87}{x+y} = \tan 30°$$

$$\Rightarrow \frac{87}{29\sqrt{3}+y} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow y = 87\sqrt{3} - 29\sqrt{3} = 58\sqrt{3}m$$

20. The outcomes associated with the experiment in which a coin is tossed thrice:
HHH, HHT, HTH, THH, TTH, HTT, THT, TTT
Therefore, Total number of favourable outcomes = 8
Number of favourable outcomes = 6
Hence required probability = $\frac{6}{8} = \frac{3}{4}$

21. **First part**: Given : A circle with centre O and a point P outside the circle. PT and PT' are tangents from P to the circle.



To Prove : We need to prove thatPT = PT'
Construction: Joined OP, OT and OT'
Proof : $\because$ OT is a radius and PT is a tangent.

$\therefore$ $\angle OTP = 90^\circ$

Similarly, $\angle OT'P = 90^\circ$

Now in right triangles OTP and OT'P,

OT = OT' ……. (Radii of the same circle)

and OP = OP ……..(Common)

$\therefore$ $\Delta OTP \cong \Delta OT'P$ ……..(RHS congruency)

Hence, PT = PT'

Second part: Using the above, we get,

AP = AS BP = BQ

CR = CQ DR = DS

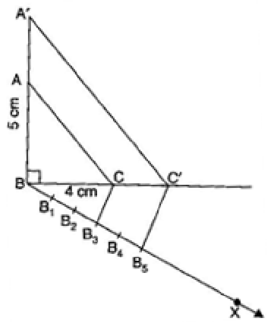
On adding, we get,

(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

$\Rightarrow$ AB + CD = DA + BC

22. **Steps of construction**:
(a) Draw a right angled triangle ABC with given measurements.
(b) Draw any ray BX making an acute angle with BC on the side opposite to the vertex A.
(c) Locate its points B_1, B_2, B_3, B_4, B_5 on BX so that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$.
(d) Join B_3 to C and draw a line through B_5 parallel to B_3C, intersecting the extended line segment BC at C'.

(e) Draw a line through C’ parallel to CA intersecting the extended line segment BA at A’. The A’BC’ is the required triangle.



23. Total possible outcomes = 52

(i) No. of spades = 13

No. of ace = 4

1 card is common [ace of spade]

Favourable outcomes = 13+4 – 1=16

$\therefore$ Required probability = $\frac{16}{52} = \frac{4}{13}$

(ii) No. of red kings = 2

Favourable outcomes = 2

$\therefore$ Required probability = $\frac{2}{52} = \frac{1}{26}$

(iii) No. of king and queen = 4+4=8

Favourable outcomes = 52 – 8=44

Required probability = $\frac{44}{52} = \frac{11}{13}$

(iv) No. of king and queen =4+4=8

Required probability = $\frac{8}{52} = \frac{2}{13}$

(v) No. of face cards = 4+4+4=12 [Jack, queen and king are face card]

Required probability = $\frac{12}{52} = \frac{3}{13}$

(vi) No. of cards which are neither red card nor king = 52 – (26+4 – 2)

=52 – 28=24

$\therefore$ Required probability = $\frac{24}{52} = \frac{6}{13}$

24. It is given that Q is equidistant from P and R. Using Distance Formula, we get

PQ = RQ $\Rightarrow PQ^2 = RQ^2$

$\Rightarrow \sqrt{(0-5)^2 + [1-(-3)^2]} = \sqrt{(0-x)^2 + (1-6)^2}$

$\Rightarrow \sqrt{(-5)^2 + [4^2]} = \sqrt{(x)^2 + (-5)^2} \Rightarrow \sqrt{25+16} = \sqrt{x^2 + 25}$

Squaring both sides, we get

$\Rightarrow 25+16=x^2+25 \Rightarrow x^2=16$

$\Rightarrow$ x=4,−4

Thus, Q is (4, 6) or (−4, 6).

Using Distance Formula to find QR, we get

Using value of x = 4

QR= $\sqrt{(4-0)^2+\left[6-1^2\right]}=\sqrt{16+25}=\sqrt{41}$

Using value of x = −4

QR= $\sqrt{(-4-0)^2+\left[6-1^2\right]}=\sqrt{16+25}=\sqrt{41}$

Therefore, QR= $\sqrt{41}$

Using Distance Formula to find PR, we get

Using value of x = 4

PR= $\sqrt{(4-5)^2+\left[6-(-3)^2\right]}=\sqrt{1+81}=\sqrt{82}$

Using value of x =−4

PR= $\sqrt{(-4-5)^2+\left[6-(-3)^2\right]}=\sqrt{81+81}=\sqrt{162}=9\sqrt{2}$

Therefore, x = 4, −4

QR= $\sqrt{41}$, PR= $\sqrt{82}, 9\sqrt{2}$

25. **For cylindrical vessel**

Internal diameter = 10 cm

Internal radius $(r)=\frac{10}{2}=$ 5 cm

Height (h) = 10.5 cm

Volume of water = Volume of cylindrical vessel = $\pi r^2 h$

$= \frac{22}{7}\times 5\times 5\times 10.5$ = 825 cm^3

For solid cone

Base diameter = 7 cm

Base radius (R) = $\frac{7}{2}$ cm

Volume of solid cone = $\frac{1}{3}\pi R^2 H$

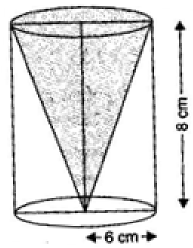
$= \frac{1}{3}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\times 6$

= 77 cm^3

(i) Water displaced out of the cylindrical vessel = Volume of the solid cone
= 77 cm^3

(ii) Water left in the cylindrical vessel
= Volume of cylindrical vessel – Volume of solid cone
= 825 – 77 = 748 cm^3

26. For cylinder



r = 6 cm

h = 8 cm

Volume = $\pi r^2 h = \pi(6)^2(8) = 288\pi$ cm³

For cone

R = 6 cm

H = 8 cm

Volume = $\frac{1}{3}\pi R^2 H = \frac{1}{3}\pi(6)^2(8) = 96\pi$ cm³

Volume of remaining solid = Volume of cylinder – Volume of cone

= $288\pi - 96\pi = 192\pi$

= 192 x 3.1416 = 603.1872 cm³

Curved Surface area of the cylinder = $2\pi rh$

= $2\pi(6)(8) = 96\pi$ cm²

Area of the base of cylinder = $\pi r^2 = \pi(6)^2 = 36\pi$ cm²

Slant height of the cone (L) = $\sqrt{R^2+H^2} = \sqrt{6^2+8^2} = \sqrt{36+64} = \sqrt{100}$ = 10 cm

Curved surface area of the cone = $\pi RL = \pi(6)(10) = 60\pi$ cm²

Now, total surface area of the remaining solid

= Curved surface area of the cylinder + Area of the base of the cylinder

+ Curved surface area of the cone

= $96\pi + 36\pi + 60\pi = 192\pi$

= 192 x 3.1416

= 603.1872 cm²

27. Let average speed of passenger train = x km/h

Let average speed of express train = (x+11) km/h

Time taken by passenger train to cover 132 km = $\frac{132}{x}$ hours

Time taken by express train to cover 132 km = $\left(\frac{132}{x+11}\right)$ hours

According to the given condition,

$$\frac{132}{x} = \frac{132}{x+11} + 1$$

$$\Rightarrow 132\left(\frac{1}{x} - \frac{1}{x+11}\right) = 1$$

$\Rightarrow 132\left(\frac{x+11-x}{x(x+11)}\right)=1$

$\Rightarrow 132(11)=x(x+11)$

$\Rightarrow 1452=x^2+11x$

$\Rightarrow x^2+11x-1452=0$

Comparing equation $x^2+11x-1452=0$ with general quadratic equation $ax^2+bx+c=0$, we get $a=1, b=11$ and $c=-1452$

Applying Quadratic Formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-11\pm\sqrt{(11)^2-4(1)(-1452)}}{2\times1}$

$\Rightarrow x=\frac{-11\pm\sqrt{121+5808}}{2}$

$\Rightarrow x=\frac{-11\pm\sqrt{5929}}{2}$

$\Rightarrow x=\frac{-11\pm77}{2}$

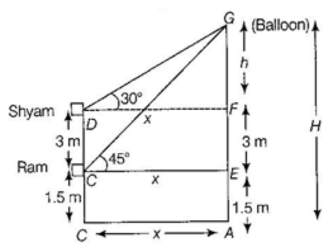
$\Rightarrow x=\frac{-11+77}{2},\frac{-11-77}{2}$

$\Rightarrow x=33,-44$

As speed cannot be in negative. Therefore, speed of passenger train = 33 km/h

And, speed of express train = $x+11=33+11=44$ km/h

28. (a) Let H be the height of the balloon from the ground and C and D be the position of the windows.



At C and G, angle of elevation are $\angle ECG = 45^\circ$ and $\angle FDG = 30^\circ$.

Let CE = DF = x m and FG = h m

In ΔCEG, we have

$$\tan 45^\circ = \frac{EG}{EC}$$

$$\Rightarrow \quad 1 = \frac{EF + FG}{EC}$$

$$\Rightarrow \quad 1 = \frac{3+h}{x}$$

$$\Rightarrow \quad x = 3+h \qquad \text{..........(i)}$$

In Δ DFG, we have

$$\tan 30^\circ = \frac{GF}{DF}$$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x} \qquad \Rightarrow x = \sqrt{3}h$ ....(ii)

Substituting $x = \sqrt{3}h$ in eq, (i), we get,

$\sqrt{3}h = 3 + h$

$\Rightarrow \quad \sqrt{3}h - h = 3 \qquad \Rightarrow \quad h(\sqrt{3}-1) = 3 \qquad \Rightarrow \quad h = \frac{3}{\sqrt{3}-1}$

$\Rightarrow \quad h = \frac{3}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} \qquad \Rightarrow \quad h = \frac{3(\sqrt{3}+1)}{3-1} \qquad \Rightarrow \quad h = \frac{3 \times (1.732+1)}{2}$

$\Rightarrow \quad h = 4.098$ m

Hence, the height of the balloon from the ground is

H = EA + FE + h = 1.5 + 3 + 4.098 = 8.598 m

(b) The person who makes small angle of elevation is more closer to the balloon. Hence, Shyam is more closer to the balloon.

(c) Windows are most important part of any building they add value to it.

They are useful for the proper ventilation, which is very much required as natural air, keeps the building freah and suffocation free.

(d) No, when the balloon is moving towards the building then the angle of elevation will automatically increase.

29. Let the side of the larger square be x m. Then its perimeter = $4x$ m

Perimeter of the larger square – Perimeter of the smaller square = 24 m

$\Rightarrow \quad 4x -$ Perimeter of the smaller square = 24

$\Rightarrow \quad$ Perimeter of the smaller square = $(4x - 24)$ m

$\Rightarrow \quad$ Side of the smaller square = $\frac{4x-24}{4} = (x-6)$ m

According to the question,

Area of the larger square + Area of the smaller square = 468 m²

$\Rightarrow \quad x^2 + (x-6)^2 = 468 \qquad \Rightarrow \quad x^2 + x^2 - 12x - 432 = 0$

$\Rightarrow \quad 2x^2 - 12x - 432 = 0 \qquad \Rightarrow \quad x^2 - 6x - 216 = 0$

$\Rightarrow \quad x^2 - 18x + 12x - 216 = 0 \qquad \Rightarrow \quad x(x-18) + 12(x-18) = 0$

$\Rightarrow \quad (x-18)(x+12) = 0 \qquad \Rightarrow \quad x = 18, -12$

$x = -12$ is inadmissible as x is the length of a side which cannot be negative.

$\therefore \quad x = 18$ and $x - 6 = 12$

Hence, the sides of the two squares are 18 m and 12 m.

30. It is given that sum of seven cash prizes is equal to Rs 700.

And, each prize is R.s 20 less than its preceding term.

Let value of first prize = Rs. a

Let value of second prize =Rs (a−20)
Let value of third prize = Rs (a−40)
So, we have sequence of the form:
a,a−20,a−40,a – 60...
It is an arithmetic progression because the difference between consecutive terms is constant.
First term = a, Common difference = d = (a – 20) – a= –20
n = 7 (Because there are total of seven prizes)
S_7 = Rs 700 {given}

Applying formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ to find sum of n terms of AP, we get

$S_7 = \frac{7}{2}\left[2a + (7-1)(-20)\right]$ ⇒ $700 = \frac{7}{2}\left[2a - 120\right]$

⇒ 200=2a–120 ⇒ 320=2a ⇒ a=160
Therefore, value of first prize = Rs 160
Value of second prize = 160 – 20= Rs 140
Value of third prize = 140 – 20= Rs 120
Value of fourth prize = 120 – 20 = Rs 100
Value of fifth prize = 100 – 20 = Rs 80
Value of sixth prize = 80 – 20 = Rs 60
Value of seventh prize = 60 – 20 = Rs 40

31. In right triangles OPT and OQT,

OP = OQ [Radii of the same circle]
OT = OT [Common]
∴ Δ OPT $\cong \Delta$ OQT [RHS congruence axiom]
∴ $\angle$PTO = $\angle$QTO [C.P.C.T.]
⇒ $\angle$PTR = $\angle$QTR ……….(i)

In Δ PTR and Δ QTR,

TP = TQ [Tangents segments from an external point T]
$\angle$PTR = $\angle$QTR [From eq. (i)]
TR = TR [Common]
∴ Δ PTR $\cong \Delta$ QTR [SAS]
∴ PR = QR = $\frac{1}{2}$ PQ = $\frac{1}{2} \times 8$ = 4 cm ……….(ii)
And $\angle$PRT = $\angle$QRT [C.P.C.T.]
But $\angle$PRT + $\angle$QRT = 180°
∴ $\angle$PRT = $\angle$QRT = 90°

In right angled triangle OPR,

$OP^2 = OR^2 + PR^2$ [By Pythagoras theorem]
⇒ $5^2 = OR^2 + 4^2$
⇒ OR = 3 cm ……….(iii)

∴ In right angles triangle TRP,

$TP^2 = TR^2 + PR^2$ [By Pythagoras theorem]

$\Rightarrow$ $TP^2 = TR^2 + 16$ [From eq. (ii)] ……….(iv)

In right angles triangle OPT,

$OP^2 = PT^2 + OT^2$ [By Pythagoras theorem]

$\Rightarrow$ $5^2 + PT^2 = (TR + 3)^2$

$\Rightarrow$ $25 + PT^2 = TR^2 + 6TR + 9$ ……….(v)

Subtracting eq. (iv) from eq. (v),

$25 = 6TR - 7$ $\Rightarrow$ $TR = \frac{16}{3}$ cm

$\therefore$ From eq. (iv), $TP^2 = \left(\frac{16}{3}\right)^2 + 16$

$\Rightarrow$ TR = 6.67 cm (approx.)