CBSE Test Paper 01

CH-04 Principle of Mathematical Induction

1. $\frac{3}{4}+\frac{15}{16}+\frac{63}{64}+\ldots\ldots\ldots\ldots$ to n terms is equal to
    a. $n+\frac{4^n}{3}-\frac{1}{3}$
    b. $n+\frac{4^{-n}}{3}-\frac{1}{3}$
    c. $n-\frac{4^n}{3}-\frac{1}{3}$
    d. $n+\frac{4^{-n}}{3}+\frac{1}{3}$
2. The greatest positive integer , which divides n (n + 1) (n + 2) (n + 3) for all n $\in$ N , is
    a. 120
    b. 6
    c. 24
    d. 2
3. For all positive integers n, the number $4^n+15n-1$ is divisible by :
    a. 16
    b. 24
    c. 9
    d. 36
4. If $49^n+16n+\lambda$ is divisible by 64 for all n $\in$ N , then the least negative integral value of λ is
    a. -1
    b. -3

c. -4

d. -2

5. For $n \in N, x^{n+1} + (x+1)^{2n-1}$ is divisible by :

a. $x^2 + x + 1$

b. $x^2 + x - 1$

c. $x + 1$

d. x

6. Fill in the blanks:

If $a_1 = 2$ and $a_n = 5\, a_{n-1}$, then the value of a_3 in the sequence is _______.

7. Fill in the blanks:

If $x^n - 1$ is divisible by x - k, then the least positive integral value of k is ________.

8. Prove by the principle of mathematical induction that for all n $\in$ N, 3^{2n} when divided by 8, the remainder is always 1.

9. Prove by Mathematical Induction that the sum of first n odd natural numbers is n^2.

10. Let $U_1 = 1$, $U_2 = 1$ and $U_{n+2} = U_{n+1} + U_n$ for $n \geq 1$. Use mathematical induction to show that:

$U_n = \frac{1}{\sqrt{5}}\left\{\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right\}$ for all $n \geq 1$.

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Solution

1. (b) $n+\frac{4^{-n}}{3}-\frac{1}{3}$

   Explanation: When n = 1 we get $3/4$, and the subsequent terms when n is replaced by 2,3,4...

2. (c) 24

   Explanation: If n = 1 then the statement becomes 1x2x3x4= 24 : the consecutive natural numbers when substituted will be multiples of 24.
3. (c) 9

   Explanation: Replace n = 1 we get 18 n = 2 we get 45.... By the principle of mathematical induction it is divisible by 9.
4. (a) -1

   Explanation: When n = 1 we have the value of the expression as 65 . Given that the expression is divisible be 64. Hence the value is -1.
5. (a) x^2+x+1

   Explanation: When n = 1 we get $x^2 + x + 1$

6. 50

7. 1

8. Let P(n) be the statement given by

   P(n) : 3^{2n} when divided by 8, the remainder is 1

   or, P(n) : $3^2 = 8\lambda + 1$ for some $\lambda \in N$

   P(1): $3^2 = 8\lambda + 1$ for some $\lambda \in N$.

   $\therefore 3^2 = 8 \times 1 + 1 = 8\lambda + 1$, where $\lambda = 1$

   P(1) is true

   Let P(m) be true. Then, $3^{2m} = 8\lambda + 1$ for some $\lambda \in N$ ...(i)

   We shall now show that P(m + 1) is true for which we have to show that $3^{2(m+1)}$ when divided by 8, the remainder is 1 i.e. $3^{2(m+1)} = 8\mu + 1$ for some $\mu \in$ N.

Now, $3^{2(m+1)} = 3^{2m} \times 3^2 = (8\lambda + 1) \times 9$ [Using (i)]

$= 72\lambda + 9 = 72\lambda + 8 + 1 = 8(9\lambda + 1) + 1 = 8\mu + 1$, where $\mu = 9\lambda + 1 \in$ N

$\Rightarrow$ P(m + 1) is true

Thus, P (m) is true $\Rightarrow$ P (m + 1) is true.

Hence, by the principle of mathematical induction P(n) is true for all n $\in$ N i.e.

3^{2n} when divided by 8 the remainder is always 1.

9. **Step I** Let P(n) denotes the given statement, i.e.,

$P(n) : 1 + 3 + 5 + \ldots n(terms) = n^2$

i.e.,$P(n) : 1 + 3 + 5 + \ldots + (2n - 1) = n^2$

Since,

First term = $2 \times 1 - 1 = 1$

Second term = $2 \times 2 - 1 = 3$

Third term = $2 \times 3 - 1 = 5$ ......

$\therefore$ n^{th} term$= 2n - 1$

Step II For n = 1, we have

LHS$= 2.1 - 1 = 1$

RHS $= 1^2 = 1 =$LHS

Thus, P(1) is true.

Step III For n = k, let us assume that P(k) is true,

i.e., $P(k) : 1 + 3 + 5 + \ldots + (2k - 1) = k^2$ ...(i)

Step IV For n = k + 1, we have to show that P(k + 1) is true, whenever P(k) is true i.e.,

P(k + 1) :$1 + 3 + 5 + \ldots + (2k - 1) + [2(k + 1) - 1] = (k + 1)^2$

LHS $= 1 + 3 + 5 + \ldots + (2k - 1) + [2(k + 1) - 1]$

$= k^2 + 2(k + 1) - 1$ [from Eq. (i)]

$= k^2 + 2k + 1 = (k + 1)^2 =$ RHS

So, P(k + 1) is true, whenever, P(k) is true.

Hence, by Principle of Mathematical Induction, P(n) is true for all $n \in N$.

10. Let P(n) be the statement given by

P(n) : $U_n = \frac{1}{\sqrt{5}}\left\{\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right\}$

We have,

$U_1 = \frac{1}{\sqrt{5}}\left\{\left(\frac{1+\sqrt{5}}{2}\right)^1 - \left(\frac{1-\sqrt{5}}{2}\right)^1\right\} = 1$

and,

$U_2 = \frac{1}{\sqrt{5}}\left\{\left(\frac{1+\sqrt{5}}{2}\right)^2 - \left(\frac{1-\sqrt{5}}{2}\right)^2\right\} = \frac{1}{\sqrt{5}}\left\{\left(\frac{1+5+2\sqrt{5}}{4}\right) - \left(\frac{1+5-2\sqrt{5}}{4}\right)\right\} = 1$

$\therefore$ P(1) and P(2) are true.

Let P(n) be true for all n $\leq$ m

i.e. $U_n = \frac{1}{\sqrt{5}}\left\{\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right\}$ for all n $\leq$ m ...(i)

We shall now show that P(n) is true for n = m + 1.

i.e. $U_{m+1} = \frac{1}{\sqrt{5}}\left\{\left(\frac{1+\sqrt{5}}{2}\right)^{m+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{m+1}\right\}$

We have,

$U_{n+2} = U_{n+1} + U_n$ for n $\geq$ 1

$\Rightarrow U_{m+1} = U_m + U_{m-1}$ for m $\geq$ 2 [On replacing n by (m-1)]

$\Rightarrow U_{m+1} = \frac{1}{\sqrt{5}}\left\{\left(\frac{1+\sqrt{5}}{2}\right)^m - \left(\frac{1-\sqrt{5}}{2}\right)^m\right\} +$

$\frac{1}{\sqrt{5}}\left\{\left(\frac{1+\sqrt{5}}{2}\right)^{m-1} - \left(\frac{1-\sqrt{5}}{2}\right)^{m-1}\right\}$[Using (i)]

$\Rightarrow U_{m+1} = \frac{1}{\sqrt{5}}\left[\left\{\left(\frac{1+\sqrt{5}}{2}\right)^m + \left(\frac{1+\sqrt{5}}{2}\right)^{m-1}\right\} - \left\{\left(\frac{1-\sqrt{5}}{2}\right)^m + \left(\frac{1-\sqrt{5}}{2}\right)^{m-1}\right\}\right.$

$\Rightarrow U_{m+1} = \frac{1}{\sqrt{5}}\left\{\left(\frac{1+\sqrt{5}}{2}\right)^{m-1}\left(\frac{1+\sqrt{5}}{2}+1\right) - \left(\frac{1-\sqrt{5}}{2}\right)^{m-1}\left(\frac{1-\sqrt{5}}{2}+1\right)\right\}$

$\Rightarrow U_{m+1} = \frac{1}{\sqrt{5}}\left\{\left(\frac{1+\sqrt{5}}{2}\right)^{m-1}\left(\frac{3+\sqrt{5}}{2}\right) - \left(\frac{1-\sqrt{5}}{2}\right)^{m-1}\left(\frac{3-\sqrt{5}}{2}\right)\right\}$

$\Rightarrow U_{m+1} = \frac{1}{\sqrt{5}}\left\{\left(\frac{1+\sqrt{5}}{2}\right)^{m-1}\left(\frac{6+2\sqrt{5}}{4}\right) - \left(\frac{1-\sqrt{5}}{2}\right)^{m-1}\left(\frac{6-2\sqrt{5}}{4}\right)\right\}$

$\Rightarrow U_{m+1} = \frac{1}{\sqrt{5}}\left\{\left(\frac{1+\sqrt{5}}{2}\right)^{m-1}\left(\frac{1+\sqrt{5}}{2}\right)^2 - \left(\frac{1-\sqrt{5}}{2}\right)^{m-1}\left(\frac{1-\sqrt{5}}{2}\right)^2\right\}$

$\Rightarrow U_{m+1} = \frac{1}{\sqrt{5}}\left\{\left(\frac{1+\sqrt{5}}{2}\right)^{m+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{m+1}\right\}$

$\therefore$ P(m + 1) is true.

Thus, P(n) is true for all n $\leq$ m $\Rightarrow$ P(n) is true for all n $\leq$ m + 1.

Hence, P(n) is true for all n $\in$ N.