10. Magnetic Fields due to Electric Current

Can you recall?

- Do you know that a magnetic field is produced around a current carrying wire?
- What is right hand rule?
- Can you suggest an experiment to draw magnetic field lines of the magnetic field around the current carrying wire?
- Do you know solenoid? Can you compare the magnetic field due to a current carrying solenoid with that due to a bar magnet?

Do you know?

You must have noticed high tension power transmission lines, the power lines on the big tall steel towers. Strong magnetic fields are created by these lines. Care has to be taken to reduce the exposure levels to less than 0.5 milligauss (mG).

10.1 Introduction:

In this Chapter you will be studying how magnetic fields are produced by an electric current. Important foundation for further developments will also be laid down.

Hans Christian Oersted first discovered that magnetic field is produced by an electric current passing through a wire. Later, Gauss, Henry, Faraday and others showed that magnetic field is an important partner of electric field. Maxwell's theoretical work highlighted the close relationship of electric and magnetic fields. This resulted into several practical applications in day today life, for example electrical motors, generators, communication systems and computers.

In electrostatics, we have considered static charges and the force exerted by them on other charge or test charge. We now consider forces between charges in motion.

Try this

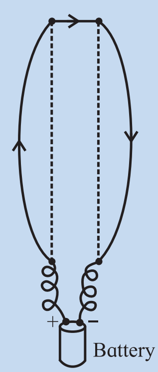


Fig. 10.1 (a)

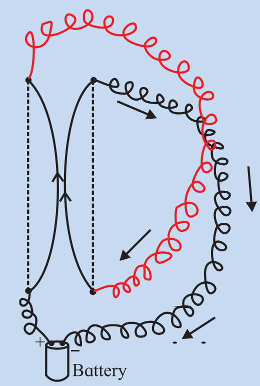


Fig. 10.1 (b)

You can show that wires having currents passing through them, (a) in opposite directions repel and (b) in the same direction attract.

Hang two conducting wires from an insulating support. Connect them to a cell first as shown in Fig. 10.1 (a) and later as shown Fig. 10.1 (b), with the help of binding posts. You will notice that the wires in (a) repel each other and those in (b) come closer, i.e., they attract each other as soon as the current starts. The force in this experiment is certainly not of electrostatic origin, even through the current is due to the electrons flowing in the wires. The overall charge neutrality is maintained throughout the wire, hence the electrostatic forces are ruled out.

You have learnt in Xth Std. that if a magnetic needle is held in close proximity of a current carrying wire, it shows the direction of magnetic field circling around the wire. Imagine that a current carrying wire is grabbed with your right hand with the thumb pointing in the direction of the current, then your fingers curl around in the direction of the magnetic field (Fig. 10.2).

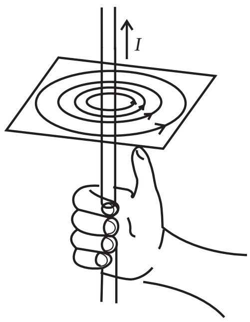


Fig. 10.2: Right hand thumb rule.

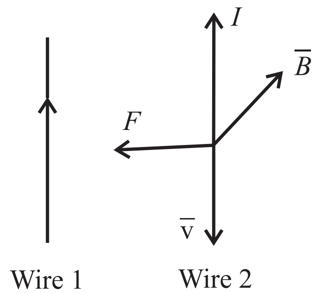


Fig. 10.3: Force on wire 2 due to current in wire 1.

How can one account for the force on the neighbouring current carrying wire? The magnetic field due to current in the wire 1 at any point on wire 2 is directed into the plane of the paper. The electrons flow in a direction opposite to the conventional current. Then the wire 2 experiences a force $\vec{F}$ towards wire 1.

10.2 Magnetic Force:

From the above discussion and Fig. 10.3, you must have realized that the directions of $\vec{v}$, $\vec{B}$ and $\vec{F}$ follow a vector cross product relationship. Actually the magnetic force F_m on an electron with a charge -e, moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ is

$$\vec{F}_m = -e(\vec{v} \times \vec{B}) \quad \text{--- (10.1)}$$

In general for a charge q, the magnetic force will be

$$\vec{F}_m = q(\vec{v} \times \vec{B}) \quad \text{--- (10.2)}$$

If both electric field $\vec{E}$ and the magnetic field $\vec{B}$ are present, the net force on charge q moving with the velocity $\vec{v}$ in

$$\vec{F} = q[\vec{E} + (\vec{v} \times \vec{B})] \quad \text{--- (10.3)}$$

$$= q\vec{E} + q(\vec{v} \times \vec{B}) = \vec{F}_e + \vec{F}_m \quad \text{--- (10.4)}$$

Justification for this law can be found in experiments such as the one described in Fig. 10.1 (a) and (b). The force described in Fig. (10.4) is known as Lorentz force. Here $\vec{F}_e$ is the force due to electric field and $\vec{F}_m$ is the force due to magnetic field.

There are interesting consequences of the Lorentz force law.

(i) If the velocity $\vec{v}$ of a charged particle is parallel to the magnetic field $\vec{B}$, the magnetic force is zero.

(ii) If the charge is stationary, $\vec{v}$ =0, the force = 0, even if $\vec{B} \neq 0$.

From Eq. (10.4) it may be observed that the force on the charge due to electric field depends on the strength of the electric field and the magnitude of the charge. However, the magnetic force depends on the velocity of the charge and the cross product of the velocity vector $\vec{v}$ the magnetic field vector $\vec{B}$, and the charge q.

Consider the vectors $\vec{v}$ and $\vec{B}$ with certain angle between them. Then $\vec{v} \times \vec{B}$ will be a vector perpendicular to the plane containing the vectors $\vec{v}$ and $\vec{B}$ (Fig. 10.4).

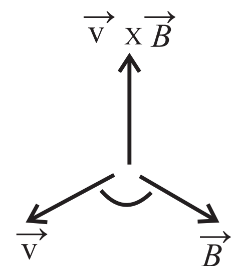


Fig. 10.4: The cross product is in the direction of the unit vector perpendicular to both $\vec{v}$ and $\vec{B}$.

Thus the vectors $\vec{v}$ and $\vec{F}$ are always perpendicular to each other. Hence. $\vec{F} \cdot \vec{v} = 0$, for any magnetic field $\vec{B}$. Magnetic force $\vec{F}_m$ is thus perpendicular to the displacement and hence the magnetic force never does any work on moving charges.

The magnetic forces may change the direction of motion of a charged particle but they can never affect the speed.

Interestingly, Eq. (10.2) leads to the definition of units of $\vec{B}$. From Eq. (10.2),

$$\vec{F} = q\,|\vec{v} \times \vec{B}|\,\hat{n} = qvB \sin\theta\,\hat{n}, \quad \text{--- (10.5)}$$

where θ is the angle between $\vec{v}$ and $\vec{B}$ and $\hat{n}$ is unit vector in the direction of force .

If the force F is 1 N acting on the charge of 1 C moving with a speed of 1m s^{-1} perpendicular to $\vec{B}$, then we can define the unit of B.

$$\because B = \frac{F}{qv}$$

$\therefore$ unit of B is $\frac{N.s}{C.m}$.

Dimensionally,

[B] = [F/qv]

This SI unit is called tesla (T)

1 T = 10^4 gauss. Gauss is not an SI unit, but is used as a convenient unit.

Can you recall?

Electromagnetic crane: How does it work?

Do you know?

Magnetic Resonance Imaging (MRI) technique used for medical imaging requires a magnetic field with a strength of 1.5 T and even upto 7 T. Nuclear Magnetic Resonance experiments require a magnetic field upto 14 T. Such high magnetic fields can be produced using superconducting coil electromagnet. On the other hand, Earth's magnetic field on the surface of the Earth is about 3.6×10^{-5} T = 0.36 gauss.

Example 10.1: A charged particle travels with a velocity $\vec{v}$ through a uniform magnetic field $\vec{B}$ as shown in the following figure, in three different situations. What is the direction of the magnetic force $\vec{F}_m$ due to the magnetic field, on the particle?

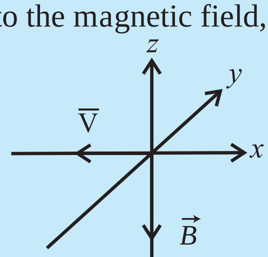


Fig. (a)

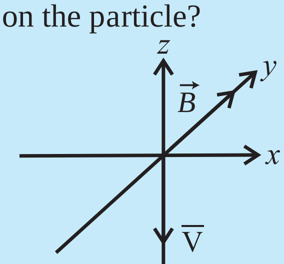


Fig. (b)

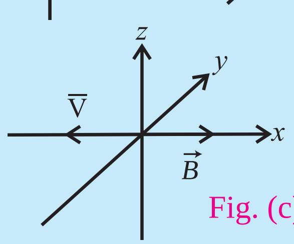


Fig. (c)

Solution: In Fig. (a), the direction of the vector $\vec{v} \times \vec{B}$ will be in the positive y direction. Hence $\vec{F}_m$ will be in the positive y direction. In Fig. (b) $\vec{v} \times \vec{B}$ will be in the positive x direction. Hence the force F_m will be in the same direction. In Fig. (c) $\vec{v}$ and $\vec{B}$ are antiparallel, the angle between them is 180° and because sin 180° = 0, $\vec{F}_m$ will be equal to zero.

10.3 Cyclotron Motion:

In a magnetic field, a charged particle typically undergoes circular motion. Figure 10.5 shows a uniform magnetic field directed perpendicularly into the plane of the paper (parallel to the -ve z axis).

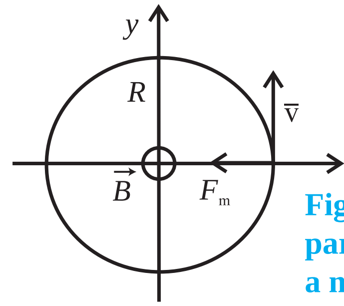


Fig. 10.5: Charged particle moving in a magnetic field.

Figure 10.5 shows a particle with charge q moving with a speed v, and a uniform magnetic field $\vec{B}$ is directed into the plane of the paper. According to the Lorentz force law, the magnetic force on the particle will act towards the centre of a circle of radius R, and this force will provide centripetal force to sustain a uniform circular motion.

Thus

$$qvB = \frac{mv^2}{R} \quad \text{--- (10.6)}$$

$$\therefore mv = p = qBR \quad \text{--- (10.7)}$$

Equation (10.7) represents what is known as cyclotron formula. It describes the circular motion of a charged particle in a particle accelerator, the cyclotron.

Do you know?

Let us look at a charged particle which is moving in a circle with a constant speed. This is uniform circular motion that you have studied earlier. Thus, there must be a net force acting on the particle, directed towards the centre of the circle. As the speed is constant, the force also must be constant, always perpendicular to the velocity of the particle at any given instant of time. Such a force is provided by the uniform magnetic field $\vec{B}$ perpendicular to the plane of the circle along which the charged particle moves.

Remember this

Field penetrating into the paper is represented as ⊗, while that coming out of the paper is shown by ⊙.

10.3.1 Cyclotron Accelerator:

Particle accelerators have played a key role in providing high energy (MeV to GeV) particle beams useful in studying particle-matter interactions and some of these are also useful in medical treatment of certain tumors/diseases.

The Cyclotron is a charged particle accelerator, accelerating charged particles to high energies. It was invented by Lawrence and Livingston in the year 1934 for the purpose of studying nuclear structure.

Both electric as well as magnetic fields are used in a Cyclotron, in combination. These are applied in directions perpendicular to each other and hence they are called crossed fields. The magnetic field puts the particle (ion) into circular path and a high frequency electric field accelerates it. Frequency of revolution of a charged particle is independent of its energy, in a magnetic field. This fact is used in this machine. Cyclotron consists of two semicircular disc-like metal chambers, D_1 and D_2, called the dees (Ds). Figure 10.6 shows a schematic diagram of a cyclotron. A uniform magnetic field B is applied perpendicular to plane of the Ds. This magnetic field is produced using an electromagnet producing a field upto 1.5 *T*. An alternating voltage upto 10000 V at high frequency, 10 MH_z (f_a), is applied between the two Ds. Positive ions are produced by a gas ionizing source kept at the point O in between the two Ds. The electric field provides acceleration to the charged particle (ion).

Once the ion in emitted, it accelerates due to the negative voltage of a D and performs a semi circular motion within the D. Whenever the ion moves from one D to the other D, it accelerates due to the potential difference between the two Ds and again performs semicircular motion in the other D. Thus the ion is acted upon by the electric field every time it moves from one D to the other D. As the electric field is alternating, its sign is changed in accordance with the circular motion of the ion. Hence the ion is always accelerated, its energy increases and the radius of its circular path also increases, making the entire path a spiral (See Fig. 10.6).

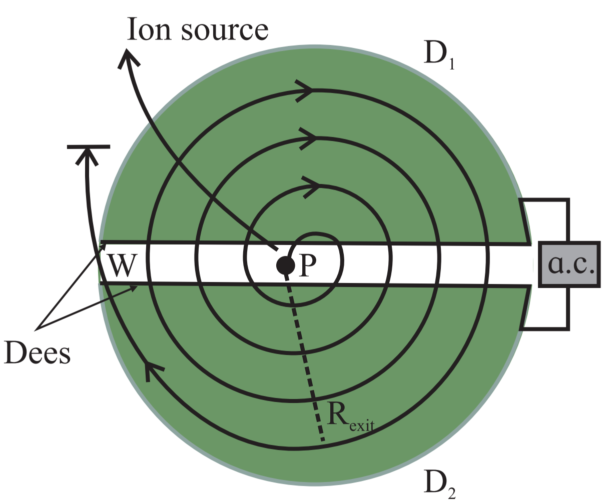


Fig 10.6: Schematic diagram of a Cyclotron with the two Ds. A uniform magnetic field $\vec{B}$ is perpendicular to the plane of the paper, coming out. The ions are injected into the D at point P. An alternating voltage is supplied to the Ds. The entire assembly is placed in a vacuum chamber.

Consider an ion source placed at P. An ion moves in a semi circular path in one of the Ds and reaches the gap between the two Ds in a time interval *T/2*, *T* being the period of a full revolution. Using the Cyclotron formula Eq. (10.7),

$mv = qBR$,

where q is the change on the ion.

$$\therefore T = \frac{2\pi R}{v} = \frac{m 2\pi R}{qBR}$$

$$= \frac{2\pi m}{qB} \quad \text{--- (10.8)}$$

The frequency of revolution (Cyclotron frequency) is

$$f_c = \frac{1}{T} = \frac{qB}{2\pi m} \quad \text{--- (10.9)}$$

The frequency of the applied voltage (f_a) between the two Ds is adjusted so that polarity of the two Ds is reversed as the ion arrives at the gap after completing one semi circle. This condition $f_a = f_c$ is the resonance condition.

The ions do not experience any electric field while they travel within the D. Their kinetic energy increases by eV every time they cross over from one D to the other. Here V is the voltage difference across the gap. The ions move in circular path with successively larger and larger radius to a maximum radius at which they are deflected by a magnetic field so that they can be extracted through an exit slit.

From Eq. (10.7),

$$v = \frac{qBR_{exit}}{m},$$

where R_{exit} is the radius of the path at the exit.

The kinetic energy of the ions/ protons will be

$$K.E. = \frac{1}{2} mv^2 = \frac{q^2 B^2 R_{exit}^2}{2m} \quad \text{--- (10.10)}$$

Thus the final energy is proportional to the square of the radius of the outermost circular path (R_{exit}).

10.4 Helical Motion:

So far it has been assumed that the charged particle moves in a plane perpendicular to magnetic field $\vec{B}$. If such a particle has some component of velocity parallel to $\vec{B}$, ($\vec{v}_{//}$) then it leads to helical motion. Since a component $\vec{v}_{//}$ is parallel to $\vec{B}$, the magnetic force $\vec{F}_m$ will be:

$$\vec{F}_m = \vec{v}_{//} \times \vec{B} = v \cdot B \sin(0°) = 0 \quad \text{--- (10.11)}$$

Thus, $\vec{v}_{//}$ will not be affected and the particle will move along the direction of $\vec{B}$. At the same time the perpendicular component of the velocity ($\vec{v}_{\perp}$) leads to circular motion as stated above. As a result, the particle moves parallel to the field $\vec{B}$ while moving along a circular path perpendicular to $\vec{B}$. Thus the path becomes a helix (Fig. 10.7).

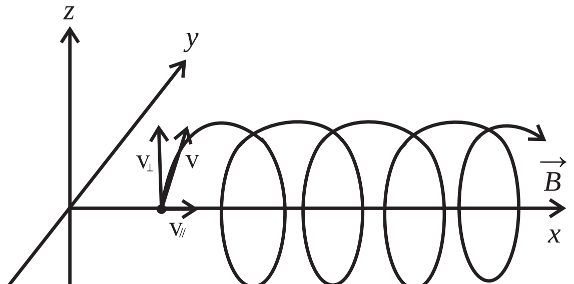


Fig. 10.7: Helical Motion of a charged particle in a magnetic field $\vec{B}$.

Do you know?

Particle accelerators are important for a variety of research purposes. Large accelerators are used in particle research. There have been several accelerators in India since 1953. The Department of Atomic Energy (DAE), Govt. of India, had taken initiative in setting up accelerators for research. Apart from ion accelerators, the DAE has developed and commissioned a 2 GeV electron accelerator which is a radiation source for research in science. This accelerator, 'Synchrotron', is fully functional at Raja Ramanna Centre for Advanced Technology, Indore. An electron accelerator, Microtron with electron energy 8-10 MeV is functioning at Physics Department, Savitribai Phule Pune University, Pune.

Internet my friend

(i) Existing and upcoming particle accelerators in India http://www.researchgate.net

(ii) Search the internet for particle accelerators and get more information.

10.5 Magnetic Force on a Wire Carrying a Current:

We have seen earlier the Lorentz force law (Eq. (10.4)). From this equation, we can obtain the force on a current carrying wire.

(i) Straight wire:

Consider a straight wire of length L as shown in Fig. 10.8. An external magnetic field $\vec{B}$ is applied perpendicular to the wire, coming out of the plane of the paper. Let a current I flow through the wire under an applied potential difference. If $\vec{v}_d$ is the drift velocity of conduction electrons in the part of length L of the wire, the charge q flowing across the plane pp in time t will be

$$q = I\,t$$

$$q = \frac{IL}{v_d} \qquad \text{--- (10.12)}$$

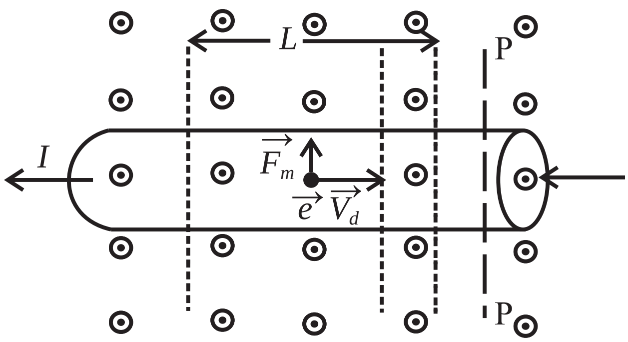


Fig. 10.8 Electrons in the wire having drift velocity $\vec{v}_d$ experience a magnetic force $\vec{F}_m$ upwards as the applied magnetic field lines come out of the plane of the paper.

The magnetic force $\vec{F}_m$ on this charge, according to Eq. (10.2), due to the applied magnetic field $\vec{B}$ is given by

$$\vec{F}_m = q(\vec{v}_d \times \vec{B})$$

$$= \frac{IL}{v_d} B \cdot v_d \sin\theta\,\hat{n}$$

$$= IL.B.\sin 90^\circ \hat{n} \;,$$

where $\hat{n}$ is a unit vector perpendicular to both $\vec{B}$ and $\vec{v}_d$, in the direction of $\vec{F}_m$

$$\vec{F}_m = ILB\hat{n} \qquad \text{--- (10.13)}$$

This is, therefore, the magnetic force acting on the portion of the straight wire having length L.

If $\vec{B}$ is not perpendicular to the wire, then the above Eq. (10.13) takes the form

$$\vec{F}_m = I\vec{L} \times \vec{B}, \qquad \text{--- (10.14)}$$

where $\vec{L}$ is the length vector directed along the portion of the wire of length L.

(ii) Arbitrarily shaped wire:

In the previous section we considered a straight wire. Equation (10.14) can be extended to a wire of arbitrary shape as shown in Fig. 10.9.

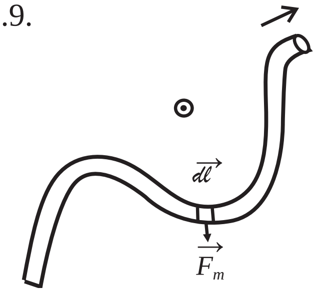


Fig. 10.9: Wire with arbitrary shape.

Consider a segment of infinitesimal length dl along the wire. If I in the current flowing, using Eq. (10.14), the magnetic force due to perpendicular magnetic field $\vec{B}$ (coming out of the plane of the paper) is given by

$$d\vec{F}_m = I\,d\vec{l} \times \vec{B} \qquad \text{--- (10.15)}$$

The force on the total length of wire is thus

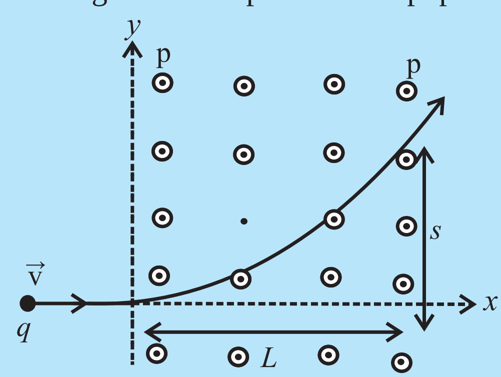
$$\vec{F}_m = \int d\vec{F}_m = I\int d\vec{l} \times \vec{B} \qquad \text{--- (10.16)}$$

If $\vec{B}$ is uniform over the whole wire,

$$\vec{F}_m = I\left[\int d\vec{l}\right] \times \vec{B} \qquad \text{--- (10.17)}$$

Example 10.2: A particle of charge q follows a trajectory as shown in the figure. Obtain the type of the charge (positive or negatively charged). Obtain the momentum p of the particle in terms of B, L, s, q, s being the distance travelled by the particle.

Particle trajectory: A uniform magnetic field $\vec{B}$ is applied in the region pp, perpendicular to the plane of the paper, coming out of the plane of the paper.



Solution: $\vec{B}$ is coming out of the paper. Since the particle moves upwards, there must be a force in that direction. The velocity is in the positive x direction.

$\therefore\ \vec{v} \times \vec{B}$ is in -ve y direction. As the force is in $+y$ direction, i.e., opposite, the charge must be negative. According to Eq (10.5), Force = Bqv in the y direction.

$\therefore$ acceleration = $\dfrac{Bqv}{m}$,

where m is the mass of the particle.

Using Newton's equation of motion, the distance travelled in the y direction is given by

$$s = ut + \frac{1}{2}\,a\,t^2$$

$$= 0 + \frac{1}{2}\,\frac{Bqv}{m}\,t^2$$ as the initial velocity in the y direction is zero. But in the same time t, the particle travels the distance L along the x direction, with uniform velocity $\vec{v}$.

$\therefore L = v.t$

$\therefore s = \dfrac{1}{2}\dfrac{BqL^2}{mv}$

$\therefore$ momentum $p = mv = \dfrac{1}{2}\dfrac{Bq}{s}L^2$

10.6 Force on a Closed Circuit in a Magnetic Field $\vec{B}$:

Equation (10.17) can be extended to a closed wire circuit C

$$\vec{F}_m = \oint_C I\, d\vec{l} \times \vec{B} \qquad \text{--- (10.18)}$$

Here, the integral is over the closed circuit C.

For uniform $\vec{B}$,

$$\vec{F}_m = I\left[\oint_C d\vec{l}\right] \times \vec{B} \qquad \text{--- (10.19)}$$

The term in the bracket in Eq. (10.19) is the sum of vectors along a closed circuit. Hence it must be zero.

$$\therefore\ \vec{F}_m = 0\ (\vec{B}\ \text{uniform}) \qquad \text{--- (10.20)}$$

Example 10.3: Consider a square loop of wire loaded with a glass bulb of mass m hanging vertically, suspended in air with its one part in a uniform magnetic field $\vec{B}$ with its direction coming out of the plane of the paper (⊙). Due to the current I flowing through the loop, there is a magnetic force in upward direction. Calculate the current I in the loop for which the magnetic force would be exactly balanced by the force on mass m due to gravity.

Solution: The current I in the loop with its part in the magnetic field B causes an upward force F_m in the horizontal part of the loop, given by

$F_m = IBa$,

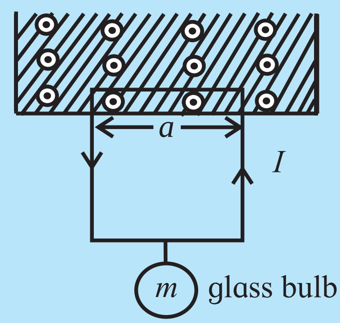
where a is the length of one arm of the loop.

This force is balanced by the force due to gravity.

$\therefore F_m = I\,Ba = mg$

$\therefore I = \dfrac{mg}{Ba}$

For this current, the wire loop will hang in air.



10.7 Torque on a Current Loop:

It will be very interesting to apply the results of the above sections to a current carrying loop of a wire. You have learnt about an electric motor in Xth Std. An electric motor works on the principle you have studied in the preceding sections, i.e., the magnetic force on a current carrying wire due to a magnetic field. Figure 10.10 shows a current carrying loop (abcd) in a uniform magnetic field. There will, therefore, be the magnetic forces $\vec{F}_m$ acting in opposite directions on the segments of the loop ab and cd. This results into rotation of the loop about its central axis.

Without going into the details of contact carbon brushes and external circuit, we can visualize the rotating action of a motor.

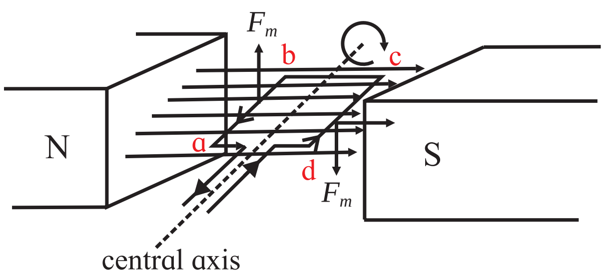


Fig. 10.10: A current loop in a magnetic field: principle of a motor.

The current carrying wire loop is of rectangular shape and is placed in the uniform magnetic field in such a way that the segments *ab* and *cd* of the loop are perpendicular to the field $\vec{B}$. We can use the right hand rule (Fig. 10.11) to find out the direction of the magnetic force $\vec{F}_m$. Let the pointing finger of the right hand show the direction of the current, let the middle finger show the direction of the magnetic field $\vec{B}$, then the stretched thumb shows the direction of the force.

Let us now look at the action of rotation in detail. For this purpose, consider Fig. 10.12 a, showing the rectangular loop abcd placed in a uniform magnetic field $\vec{B}$ such that the sides ab and cd are perpendicular to the magnetic field $\vec{B}$ but the sides bc and da are not.

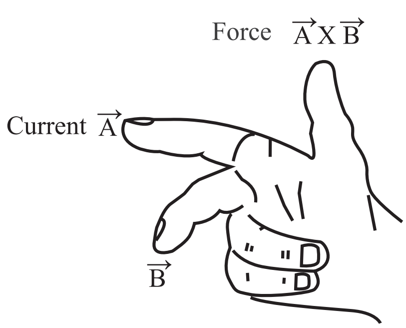


Fig. 10.11: The right hand rule.

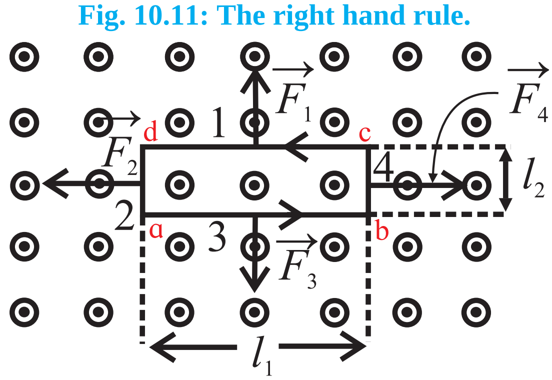


Fig. 10.12 (a): Loop abcd placed in a uniform magnetic field emerging out of the paper. Electric connections are not shown.

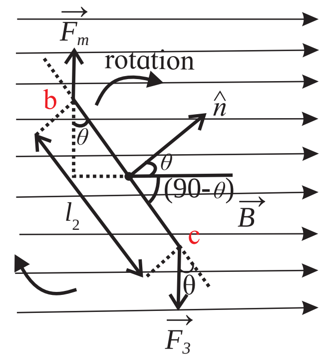


Fig. 10.12 (b): Side view of the loop abcd at an angle θ.

Now we can calculate the net force and the net torque on the loop in a situation depicted in Fig. 10.12 (a) and (b). Let us obtain the forces on all sides of the loop. The force $\vec{F}_4$ on side 4 (bc) will be

$$\vec{F}_4 = Il_2 B \sin(90-\theta) \qquad \text{--- (10.21)}$$

The force $\vec{F}_2$ on side 2 (da) will be equal and opposite to $\vec{F}_4$ and both act along the same line. Thus, $\vec{F}_2$ and $\vec{F}_4$ will cancel out each other.

The magnitudes of forces $\vec{F}_1$ and $\vec{F}_3$ on sides 3 (ab) and 1 (cd) will be $Il_1 B \sin 90°$ i.e., $Il_1 B$. These two forces do not act along the same line and hence they produce a net torque. This torque results into rotation of the loop so that the loop is perpendicular to the direction of $\vec{B}$, the magnetic field. The moment arm is $\frac{1}{2}(l_2 \sin\theta)$ about the central axis of the loop. The torque τ due to forces $\vec{F}_1$ and $\vec{F}_3$ will then be

$$\tau = (Il_1 B \frac{1}{2} l_2 \sin\theta) + (Il_1 B \frac{1}{2} l_2 \sin\theta)$$

$$= Il_1 l_2 B \sin\theta \qquad \text{--- (10.22)}$$

If the current carrying loop is made up of multiple turns *N*, in the form of a flat coil, the total torque will be

$$\tau' = N\tau = N I l_1 l_2 B \sin\theta$$

$$\tau' = (NIA)B \sin\theta \qquad \text{--- (10.23)}$$

$$A = l_1 l_2$$

Here *A* is the area enclosed by the coil. The above equation holds good for all flat (planar) coils irrespective of their shape, in a uniform magnetic field.

Can you recall?

How does the coil in a motor rotate by a full rotation? In a motor, we require continuous rotation of the current carrying coil. As the plane of the coil tends to become parallel to the magnetic field $\vec{B}$, the current in the coil is reversed externally. Referring to Fig. 10.10, the segment ab occupies the position cd. At this position of rotation, the current is reversed. Instead of from b to a, it flows from a to b, force $\vec{F}_m$ continues to act in the same direction so that the torque continues to rotate the coil. The reversal of the current is achieved by using a commutator which connects the wires of the power supply to the coil via carbon brush contacts.

10.7.1 Moving Coil Galvanometer:

A current in a circuit or a voltage of a battery can be measured in terms of a torque exerted by a magnetic field on a current carrying coil. Analog voltmeters and ammeters work on this principle. Figure 10.13 shows a cross sectional diagram of a galvanometer.

It consists of a coil of several turns mounted (suspended or pivoted) in such a way that it can freely rotate about a fixed axis, in a radial uniform magnetic field. A soft iron cylindrical core makes the field radial and strong. The coil rotates due to a torque acting on it as the current flows through it. This torque is given by (Eq. 10.23)

$\tau = N\ I\ A.B$, where A is the area of the coil, B the strength of the magnetic field, N the number of turns of the coil and I the current in the coil. Here, $\sin\theta = 1$ as the field is radial (plane of the coil will always be parallel to the field). However, this torque is counter balanced by a torque due to a spring fitted as shown in the Fig. 10.13.

This counter torque balances the magnetic torque, so that a fixed steady current I in the coil produces a steady angular deflection ϕ.

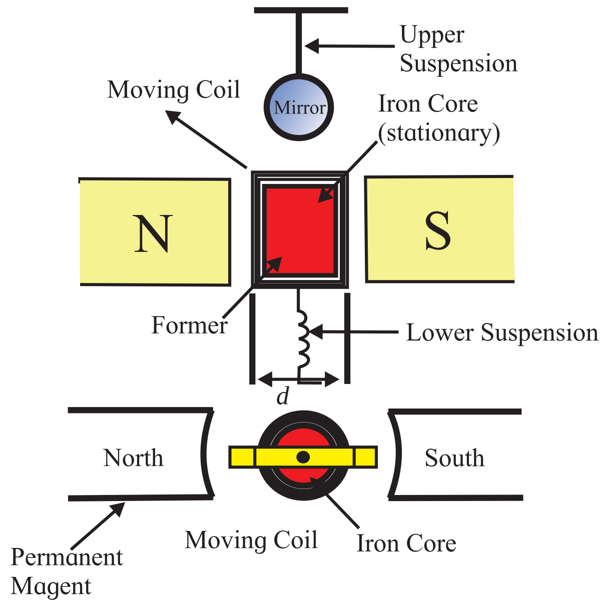


Fig. 10.13: Moving coil galvanometer.

Larger the current is, larger is the deflection and larger is the torque due to the spring. If the deflection is ϕ, the restoring torque due to the spring is equal to $K\ \phi$ where K is the torsional constant of the spring.

Thus, $K\ \phi = NIAB$,

and the deflection $\phi = \left(\frac{NAB}{K}\right) I$ --- (10.24)

Thus the deflection ϕ is proportional to the current I. Modern instruments use digital ammeters and voltmeters and do not use such a moving coil galvanometer.

10.8 Magnetic Dipole Moment:

In the preceding section, we have dealt with a current carrying coil. This current carrying coil can be described with a vector $\vec{\mu}$, its magnetic dipole moment. If $\hat{n}$ is a unit vector normal to the plane of the coil, the direction of $\vec{\mu}$ is the direction of $\hat{n}$ shown in Fig. 10.12 (b). We can then define the magnitude of $\vec{\mu}$ as

$\mu = NIA$, --- (10.25)

where N is the number of turns of the coil, I the current passing through the coil, A the area enclosed by each turn of the coil.

If held in uniform magnetic field $\vec{B}$, the torque responsible for the rotation of the coil, according to Eq. (10.23) will be

$\tau = \mu B \sin\theta$,

θ being an angle between $\vec{\mu}$ (i.e., $\hat{n}$) and $\vec{B}$.

$$\therefore \vec{\tau} = \vec{\mu} \times \vec{B} \quad \text{--- (10.26)}$$

You have learnt in XI[th] Std. about the torque on an electric dipole exerted by an electric field, $\vec{E}$.

$$\therefore \vec{\tau} = \vec{P} \times \vec{E} \quad \text{--- (10.27)}$$

Here $\vec{P}$ is the electric dipole moment.

The two expression Eq. (10.26) and Eq. (10.27) are analogous to each other.

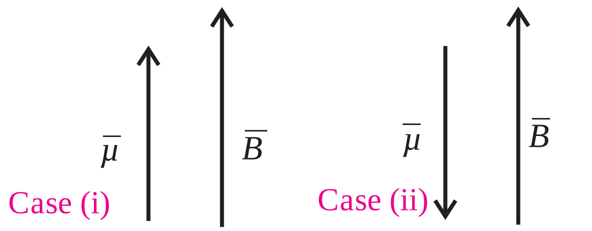


Fig. 10.14: Minimum and maximum magnetic potential energy of a magnetic dipole $\vec{\mu}$ in a magnetic field $\vec{B}$.

10.9 Magnetic Potential Energy of a Dipole:

A magnetic dipole freely suspended in a magnetic field possesses magnetic potential energy because of its orientation in the field.

You have learnt about an electric dipole in Chapter 8. Electrical Potential energy is associated with an electric dipole on account of its orientation in an electric field. It has been shown that the potential energy U of an electric dipole $\vec{P}$ in an electric field $\vec{E}$ is given by

$$U = -\vec{P} \cdot \vec{E} \quad \text{--- (10.28)}$$

Analogously, the magnetic potential energy of a magnetic dipole $\vec{\mu}$ in a magnetic field $\vec{B}$ is given by

$$U = -\vec{\mu} \cdot \vec{B} \quad \text{--- (10.29)}$$

$$= -\mu B.\cos\theta, \quad \text{--- (10.30)}$$

where θ is the angle between $\vec{\mu}$ and $\vec{B}$.

Case (i) : If $\theta = 0$, $U = -\mu B.\cos(0°) = -\mu B$

This is the minimum potential energy of a magnetic dipole in a magnetic field i.e., when and are parallel to each other.

Case (ii) : If = 180°, $U = -\mu.B.\cos(180°) = \mu B$.

This is the maximum potential energy of a magnetic dipole in a magnetic field, i.e., when and are antiparallel to each other.

Example 10.4: A circular coil of conducting wire has 500 turns and an area 1.26×10^{-4} m^2 is enclosed by the coil. A current 100 μA is passed through the coil. Calculate the magnetic moment of the coil.

Solution:

$\mu = NIA$

$= 500 \times 100 \times 10^{-6} \times 1.26 \times 10^{-4}$ Am^2

$= 630 \times 10^{-8} = 6.3 \times 10^{-6}$ Am^2 or J/T.

10.10 Magnetic Field due to a Current : Biot-Savart Law:

In sections 10.1 and 10.2, we have seen that magnetic field is produced by a current carrying wire. Can we calculate this magnetic field?

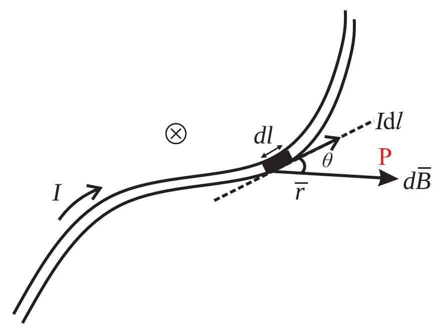


Figure 10.15: A current carrying wire of arbitrary shape, carrying a current I. The current in the differential length element dl produces differential magnetic field $d\vec{B}$ at a point P at a distance $\vec{r}$ from the element dl. The ⊗ indicates that $d\vec{B}$ is directed into the plane of the paper.

Figure 10.15 shows an arbitrarily shaped wire carrying a current *I*. *dl* is a length element along the wire. The current in this element is in the direction of the length vector $\vec{dl}$. Let us calculate the differential field $d\vec{B}$ at the point P, produced by the current *I* through the length element *dl*. Net magnetic field at the point P can be obtained by superimposition of magnetic fields $d\vec{B}$ at that point due to different length elements along the wire. This can be done by integrating i.e., summing up of magnetic fields $d\vec{B}$ from these length elements. Experimentally, the magnetic fields $d\vec{B}$ produced by current *I* in the length element $d\vec{l}$ is

$$dB = \frac{\mu_0}{4\pi} \frac{Idl \sin\theta}{r^2} \quad \text{--- (10.31)}$$

Here, θ is the angle between the directions of $\vec{dl}$ and $\vec{r}$. μ_0 is called permeability constant given by

$$\mu_0 = 4\pi \times 10^{-7} \text{ T. m/A} \quad \text{--- (10.32)}$$

$$\approx 1.26 \times 10^{-6} \text{ T. m/A} \quad \text{--- (10.33)}$$

The direction of $d\vec{B}$ is dictated by the cross product $\vec{dl} \times \vec{r}$. Vectorially,

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{l} \times \vec{r}}{r^3} \quad \text{--- (10.24)}$$

Equation (10.31) and Eq. (10.34) are known as the Biot and Savart law. This inverse square law is experimentally deduced. It may be noted that this is still inverse **square** law as $\vec{r}$ appears in the numerator and r^3 in the denominator. Using the Biot-Savart law, we can calculate the magnetic field produced by various distributions of currents as discussed below:

(i) Current in a straight, long wire:

You are aware of the right hand thumb rule which gives the direction of the magnetic field produced by a current flowing in a wire. Figure 10.16 shows a long wire of length *l*. We want to calculate magnetic field $\vec{B}$ at a point P which is at a perpendicular distance R from the wire. Let us consider a current length element (the infinitesimal length $d\vec{l}$ of the wire, multiplied by the current *I* passing through it) $I.\vec{dl}$ situated at a distance r from the point P. Using Eq. (10.31), the magnetic field $d\vec{B}$ produced at P due to the current length element $I.\vec{dl}$ becomes

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{l} \sin\theta}{r^2} \quad \text{--- (10.35)}$$

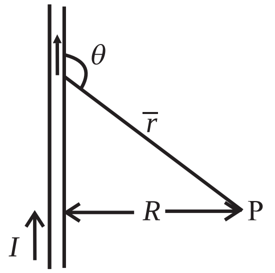


Fig. 10.16: The magnetic field $d\vec{B}$ at P going into the plane of the paper, due to current *I* through the wire.

Here, the direction of $d\vec{B}$ is given by the cross product $d\vec{l} \times \vec{r}$ (see Eq. (10.34)), hence into the plane of the paper.

(a) We now calculate the magnitude of the magnetic field produced at P by all current length elements in the upper half of the infinitely long wire. This we do by integrating Eq. (10.35) from o to ∞.

(b) Let us now calculate the magnitude of the magnetic field produced at P by a current length element in the lower half of the wire. By symmetry, this magnitude is the same as that from the upper half of the wire. The direction of this field is also the same as from the upper half of the wire, going into the plane of the paper.

Adding both the contributions (a) and (b), the total magnetic field *B* at point P is

$$B = 2\int_0^\infty dB = 2\frac{\mu_0}{4\pi}\int_0^\infty \frac{Idl \sin\theta}{r^2} \quad \text{--- (10.36)}$$

But $r = \sqrt{l^2 + R^2}$

and $\sin\theta = \sin(\pi - \theta) = \frac{R}{r} = \frac{R}{\sqrt{l^2 + R^2}}$ --(10.37)

$$\therefore B = \frac{\mu_0 I}{2\pi}\int_0^\infty \frac{Rdl}{\left(\sqrt{l^2+R^2}\right)\left(l^2+R^2\right)}$$

$$= \frac{\mu_0 I}{2\pi} R \int_0^\infty \frac{dl}{\left(l^2+R^2\right)^{3/2}}$$

$$\therefore B = \frac{\mu_0 I}{2\pi} R \frac{1}{R^2} = \frac{\mu_0 I}{2\pi R} \quad \text{--- (10.38)}$$

From Eq. (10.36), this is the magnetic field at a point P at a perpendicular distance *R* from the infinitely straight wire. This is due to both the upper semi-infinite part and the lower semi-infinite part of the wire. Thus, the magnetic field B due to semi-infinite straight wire is

$$\therefore B = \frac{\mu_0 I}{4\pi R} \quad \text{--- (10.39)}$$

In Eq. (10.38) and Eq. (10.39), the field is inversely proportional to the distance from the wire.

To solve $I=\int_0^{\infty}\frac{dl}{\left(l^2+R^2\right)^{3/2}}$,

we substitute $l = R\tan\theta$; $dl = r\sec^2\theta\ d\theta$

Now the limits of the integral also change.

$l = 0$, $\tan\theta = 0 \therefore \theta = 0$

$l = \infty$, $\tan\theta = \infty \therefore \theta = \pi/2$

$$\therefore I=\int_0^{\pi/2}\frac{R\sec^2\theta d\theta}{R^3(\tan^2\theta+1)^{3/2}}$$

$$=\frac{1}{R^2}\int_0^{\pi/2}\frac{(\cos^2\theta)^{3/2}\sec^2\theta d\theta}{(\sin^2\theta+\cos^2\theta)^{3/2}}$$

$$=\frac{1}{R^2}\int_0^{\pi/2}\cos\theta d\theta$$

$$=\frac{1}{R^2}\left[\sin\theta\right]_0^{\pi/2}=\frac{1}{R^2}[1-0]=\frac{1}{R^2}$$

10.11 Force of Attraction between two Long Parallel Wires:

As an application of the result obtained in the last section, let us obtain the force of attraction between two long, parallel wires separated by a distance d (Fig. 10.17). Let the currents in the two wires be I_1 and I_2.

The magnetic field at the second wire due to the current I_1 in the first one, according to Eq. (10.38),

$$B=\frac{\mu_0 I_1}{2\pi d} \quad \text{--- (10.40)}$$

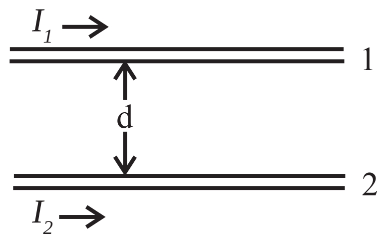


Fig. 10.17 : Two long parallel wires, distance d apart.

By the right hand rule, the direction of this field is into the plane of the paper. We now apply the Lorentz Force law. Accordingly, the force on the wire 2, because of the current I_2 and the magnetic field B due to current in wire 1, is given by (Eq. 10.13).

$$F=I_2\left(\frac{\mu_0 I_1}{2\pi d}\right)\int dl \quad \text{--- (10.41)}$$

The direction of this force is towards wire 1, i.e., it will be attractive force.

For infinitely long wires, this force will be infinite!

Force per unit length of the wire will be

$$F'=\frac{\mu_0}{2\pi}\frac{I_1 I_2}{d} \quad \text{--- (10.42)}$$

If the currents I_1 and I_2 are antiparallel, the force will be repulsive.

Let us consider a section of length L of the wire 2. The force on this section due to the current in wire 1 is given by

$$F = I_2 B.L \quad \text{--- (10.43)}$$

$$=\frac{\mu_0 I_1 I_2}{2\pi d}L=F_{21} \quad \text{--- (10.44)}$$

We will denote this force by F_{21} i.e., the force on a section of length L of wire 2 due to the current in wire 1. Similarly, the force on a section of the same length L of wire 1 will experience a force due to the current in wire 2.

This force we denote as F_{12}, which is equal and opposite to F_{21}

$$\therefore F_{21} = -F_{12} \quad \text{--- (10.44 A)}$$

The force of attraction per unit length is then, from Eq. (10.44),

$$\frac{F}{L}=\frac{\mu_0}{2\pi}\frac{I_1 I_2}{d} \quad \text{--- (10.45)}$$

If the currents I_1 and I_2 are flowing in opposite directions, then there is a force of repulsion on the sector of length L of each of the wires. The magnitude of the repulsive force per unit length of the wire is also given by

$$\frac{F}{L}=\frac{\mu_0}{2\pi}\frac{I_1 I_2}{d} \quad \text{--- (10.46)}$$

We can summarize these result as: **Parallel currents attract, antiparallel currents repel**.

The ampere: Definition of the unit of electrical current ampere, was adopted a few decades ago. Consider two parallel conducting wires having infinite length, have a separation of 1 m, and are placed in vacuum. The constant current through these wires producing a force on each other of magnitude 2×10^{-7} N per meter of their length, is 1 ampere (A).

It is a straight forward evaluation from Eq. (10.45).

$$\frac{F}{L} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}$$

$\frac{\mu_0}{4\pi}$ =10^{-7} Wb/m; d = 1 m

For $I_1 = I_2 = 1A$, $\frac{F}{L} = 2\times10^{-7}$ N per meter.

Here it is assumed that the wire diameter is very much less than 1 m.

10.12 Magnetic Field Produced by a Current in a Circular Arc of a Wire:

After considering straight parallel wires let us obtain the magnetic field at a point produced by a current in a circular arc of a wire. Figure 10.18 depicts a circular arc of a wire (AB), carrying a current *I*. We can first obtain the magnetic field produced by one current-length element of the arc and then integrate over the entire arc length. The circular arc AB subtends an angle θ at the centre O of the circle of which the arc is a part, and r is its radius. Using Biot-Savart law Eq. (10.34), the magnetic field produced at O is:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I.d\vec{l}\times\vec{r}}{r^3}$$

$$dB = \frac{\mu_0}{4\pi} I.\frac{dl\cdot r\cdot\sin 90^\circ}{r^3}$$

$$= \frac{\mu_0}{4\pi}\frac{Idl}{r^2} \quad \text{--- (10.47)}$$

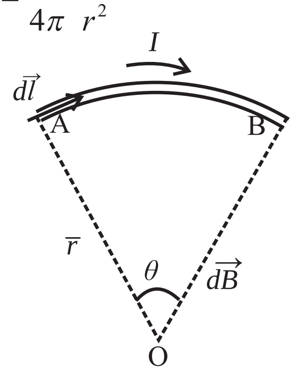


Fig. 10.18 : Current carrying wire of a shape of circular arc. The length element $d\vec{l}$ is always perpendicular to $\vec{r}$.

Equation (10.47) gives the magnitude of the field. The direction of the field is given by the right hand rule. Aligning the thumb in the direction of the current, the field direction is indicated by the curling fingers. Thus, the direction of each of the *dB* is into the plane of the paper. The total field at O is therefore,

$$B = \int dB = \frac{\mu_0}{4\pi} I \int_A^B \frac{dl}{r^2}$$

$$= \frac{\mu_0}{4\pi} I \int_0^\theta \frac{r}{r^2} d\theta = \frac{\mu_0}{4\pi}\frac{I}{r}\theta, \quad \text{--- (10.48)}$$

where the angle θ is in radians.

Magnetic field at the centre of a full circle of a wire, carrying a current *I* :

For a full circular wire carrying a current *I*, the magnetic field at the centre of the circle, using Eq. (10.48),

$$B = \frac{\mu_0}{4\pi}\frac{I}{r}2\pi$$

$$B = \frac{\mu_0 I}{2r} \quad \text{--- (10.49)}$$

Use your brain power

We have seen that in case of parallel conducting wires carrying steady currents, the Biot-Savart law and the Lorentz force law give the result in Eq. (10.44A):

$$F_{21} = - F_{12}$$

Is this consistent with Newton's third law? (Consider for example the gravitational pull experienced by the Earth towards the Sun and that by the Sun towards the Earth.)

Example 10.5: A wire has 2 straight sections and one arc as shown in the figure. Determine the direction and magnitude of the magnetic field produced at the centre O of the semicircle by the three sections individually and the total.

Solution: We apply the Biot-Savart law to the 3 sections of the wire.

For the section (i) and (iii) the angle between the current-length elements $I\,d\vec{l}$ and $\vec{R}$ is 180° and 0°, respectively.

$$\therefore dB = \frac{\mu_0}{4\pi}\frac{Idl\sin(180)^\circ}{R^2} = 0 = \frac{\mu_0}{4\pi}\frac{Idl\sin(0)^\circ}{R^2}$$

For section (ii), $d\vec{l}$ is always perpendicular to $\vec{R}$.

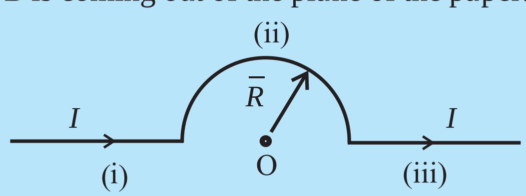
$$\therefore dB = \frac{\mu_0}{4\pi}\frac{Idl \sin(90°)}{R^2} = \frac{\mu_0}{4\pi}\frac{Idl}{R^2}$$

Integrating, $B = \frac{\mu_0}{4\pi}\frac{I}{R^2}\int_0^{\pi R} dl = \frac{\mu_0}{4\pi}\frac{I}{R^2}\pi R$

$$\therefore B = \frac{\mu_0}{4}\frac{I}{R}$$

$\therefore$ For the sections (i), and (iii), $B = 0$, for section (ii) $B = \frac{\mu_0 I}{4R}$ at the point O.

Total $B = 0 + \frac{\mu_0 I}{4R} + 0 = \frac{\mu_0 I}{4R}$; Direction of B is coming out of the plane of the paper.



10.13: Axial Magnetic Field Produced by Current in a Circular Loop:

Here we shall obtain the magnetic field, due to current in a circular loop, at different points along its axis. We assume that the current is steady.

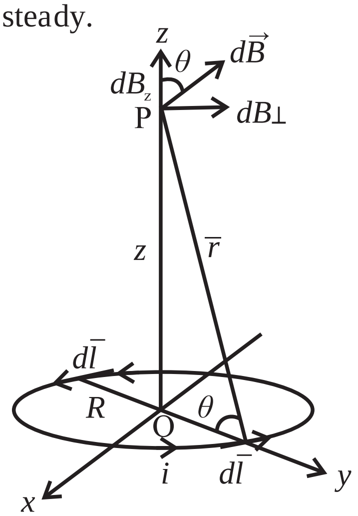


Fig. 10.19: Magnetic field on the axis of a circular current loop of radius R.

Figure 10.19 shows a circular loop of a wire carrying a current I. The loop itself is in the x-y plane with its centre at the origin O. The radius of the loop, carrying a steady current I, is R. We need to calculate the magnetic field at a point P on the Z-axis, at a distance $\vec{r}$ from the element $d\vec{l}$ on the loop. Using Biot-Savart law, the magnitude of the magnetic field dB is given by

$$dB = \frac{\mu_0}{4\pi} I \frac{\left|d\vec{l} \times \vec{r}\right|}{r^3} \quad \text{--- (10.50)}$$

We have $r^2 = R^2 + z^2$

Any element $d\vec{l}$ will always be perpendicular to the vector $\vec{r}$ from the element to the point P. The element $d\vec{l}$ is in the x-y plane, while the vector $\vec{r}$ is in the y-z plane. Hence $d\vec{l} \times \vec{r} = dl.r$

$$\therefore dB = \frac{\mu_0}{4\pi} I \frac{dl}{r^2} \quad \text{--- (10.51)}$$

$$= \frac{\mu_0}{4\pi} I \frac{dl}{(z^2 + R^2)} \quad \text{--- (10.52)}$$

The direction of $d\vec{B}$ is perpendicular to the plane formed by $d\vec{l}$ and $\vec{r}$. Its z component is dB_z and the component perpendicular to the z-axis is $dB_\perp$. The components $dB_\perp$ when summed over, yield zero as they cancel out due to symmetry. This can be easily seen from the diametrically opposite element $d\vec{l}$ giving $dB_\perp$ opposite to that due to $d\vec{l}$. Hence, only z component remains.

$\therefore$ The net contribution along the z axis is obtained by integrating $dB_z = dB \cos\theta$ over the entire loop.

From Fig. 10.19,

$$\cos\theta = \frac{R}{r} = \frac{R}{\sqrt{z^2 + R^2}}$$

$$\therefore B_z = \int dB_z = \frac{\mu_0}{4\pi} I \int \frac{dl}{(z^2 + R^2)} \cdot \cos\theta$$

$$= \frac{\mu_0}{4\pi} I \int \frac{Rdl}{(z^2 + R^2)^{3/2}}$$

$$= \frac{\mu_0}{4\pi} \frac{IR}{(z^2 + R^2)^{3/2}} \cdot 2\pi R$$

$$B_z = \frac{\mu_0}{2} \frac{IR^2}{(z^2 + R^2)^{3/2}} \quad \text{- (10.53)}$$

This is the magnitude of the magnetic field due to current I in the loop of radius R, on a point at P on the z axis of the loop.

Do you know?

So far we have used the constant μ_0 everywhere. This means in each such case, we have carried out the evaluation in free space (vacuum). μ_0 is the permeability of free space.

10.14 Magnetic Lines for a Current Loop:

We know that the magnetic field at a point P on the axis is given by Eq. (10.53) as

$$B_z = \frac{\mu_0 I R^2}{2(z^2 + R^2)^{3/2}}$$

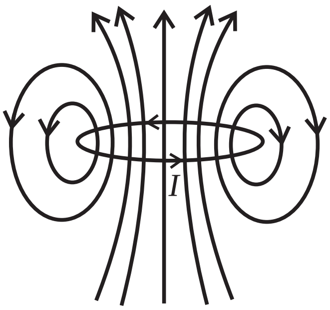


Fig. 10.20: Magnetic field lines for a current loop.

As a special case, the field at the centre of the loop is obtained from the above equation by letting z = 0:

$$B_0 = \frac{\mu_0 I}{2R} \quad \text{--- (10.54)}$$

For a coil of N turns,

$$B = \frac{\mu_0 N I}{2R} \quad \text{--- (10.55)}$$

The magnetic field lines from a circular loop are depicted in Fig. 10.20. The direction of the field is as per the right hand thumb rule: Curl the palm of your right hand along the circular wire with the fingers in the direction of the current. The stretched right hand thumb then gives the direction of the magnetic field (Fig. 10.21). Thus, the upper part of the loop seen in Fig. 10.20 may be regarded as the North pole and the lower part as the South pole of a bar magnet.

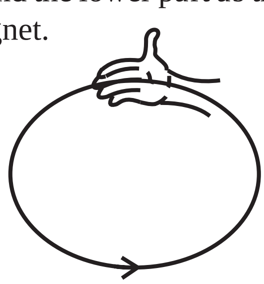


Fig. 10.21: The right hand thumb rule.

Circular loop carrying a current as a magnetic dipole: The behaviour of the magnetic field due to a circular current loop, at large distances is very similar to that due to electric field of an electric dipole. From the above equation for B_z, at large distance z from the loop along its axis,

$$z >> R$$

$$\therefore B_z = \frac{\mu_0 I R^2}{2z^3}$$

The area of the loop is $A = \pi R^2$

$$\therefore B_z = \frac{\mu_0 I A}{2\pi z^3} \text{ at } z >> R \quad \text{--- (10.56)}$$

Can you recall?

In XI[th] Std you have noted the analogy between the electrostatic quantities and magnetostatic quantities: The electrostatic analogue

The magnetic moment $\vec{m}$ of a circular loop is defined as $\vec{m} = I\vec{A}$, where $\vec{A}$ is a vector of magnitude A and direction perpendicular to A. Using Eq. (10.56),

$$\therefore B_z = \frac{\mu_0}{2\pi}\frac{m}{z^3}$$

$$\vec{B}_z = \frac{\mu_0}{4\pi}\frac{2\vec{m}}{z^3} \quad \text{--- (10.57)}$$

Note that $\vec{B}_z$ and $\vec{m}$ are in the same direction, perpendicular to the plane of the loop.

Using electrostatic analogue,

$$\vec{E} = \frac{2\vec{p}}{4\pi\varepsilon_0 z^3},$$

which is the electric field at an axial point of an electric dipole.

Use your brain power

- Using electrostatic analogue, obtain the magnetic field at a distance x on the perpendicular bisector of a magnetic dipole $\vec{m}$. For $x>>R$, verify that $\vec{B} = \frac{\mu_0}{4\pi}\frac{\vec{m}}{x^3}$
- What is the fundamental difference between an electric dipole and a magnetic dipole?

Example 10.6: Consider a closely wound 1000 turn coil, having radius of 1m. If a current of 10A passes through the coil, what will be the magnitude of the magnetic field at the centre?

Solution: N = 1000, R = 100 cm, I = 10A.

Using Eq. (10.50),

$$B = \frac{\mu_0 NI}{2R} = \frac{4\pi\times10^{-7}\times10^{3}\times10}{2\times1}$$

$$= 2\pi\times10^{-3} = 6.28\times10^{-3}\,T$$

10.15 Ampere's Law:

We know that if a distribution of charges is given, one can obtain the electric field by using the inverse square law. If the distribution of charges is planar, or has spherical or cylindrical symmetry, then with the help of Gauss' Law we can find the net electric field with relative ease. On similar note, we can obtain the magnetic field produced by a distribution of currents (not charges!).

Again, if the distribution of currents has some symmetry, then we can use Ampere's law to find out the magnetic field with fair ease, as you will see below. You have studied Biot-Savart law and its consequences. The Ampere's law can be derived from Biot-Savart Law. The law is due to Andre' Marie Ampere (1775-1836) after whom the SI unit of current is named.

The Ampere's law is:

$$\oint \vec{B}.d\vec{s} = \mu_0 I \qquad \text{--- (10.58)}$$

The sign $\oint$ indicates that the integral is to be evaluated over a closed loop called **Amperian loop**. The current I on the right hand side is the net current encircled by the Amperian loop. In an example shown in Fig. 10.22, cross-sections of four long straight wires carrying currents I_1, I_2, I_3, I_4 into or out of the plane of the paper are shown. An Amperian loop is drawn to encircle 3 of the current wires and not the fourth one. As the current goes perpendicular to the plane of the paper, $\vec{B}$ is in the plane of the paper even if its direction is unknown. The length element on the Amperian loop is $d\vec{l}$ (in the plane of the paper).

$\vec{B}.d\vec{l} = Bdl\cos\theta$, and from Eq. (10.58),

$$\oint \vec{B}.d\vec{l} = \oint B\cos\theta dl = \mu_0 I \qquad \text{--- (10.59)}$$

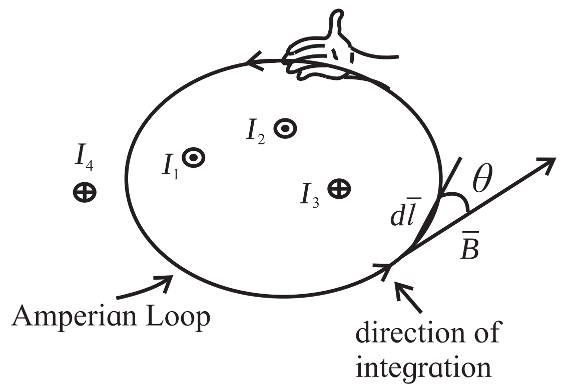


Fig. 10.22: Amperian Loop.

Thus the integration is over the product of length dl of the Amperian loop and the component of the magnetic field $B\cos\theta$, tangent to the loop.

We use the curled-palm right hand rule so that we can mark the currents with positive sign or negative sign. Curl the right hand palm along the Amperian loop, with fingers in the direction of integration. Then a current in the direction of the stretched thumb is assigned positive sign and the current in the direction opposite to the stretched thumb is assigned negative sign.

For the distribution of currents as shown in Fig. 10.22, I_1 and I_2 are coming out of the paper, (⊙) parallel to the stretched thumb. Hence these are positive. I_3, on the other hand is going into the plane of the paper (⊗). Thus, it is negative.

$$\therefore \oint B\cos\theta dl = \mu_0(I_1 + I_2 - I_3) \qquad \text{--- (10.60)}$$

The Current I_4 is not within the Amperian loop.

As the integration is over a full loop, contributions of I_4 to B cancel out.

Equation (10.58) represents Ampere's law or Ampere's circuital law.

An as application of Ampere's law let us consider a long straight wire carrying a current I (Fig. 10.23). $\vec{B}$ and $d\vec{l}$ are tangential to the Amperian loop which is a circle here.

$\therefore \ \vec{B}.d\vec{l} = B\,dl = B.rd\theta$

The field $\vec{B}$ at a distance r from the wire is given by

$$B = \frac{\mu_0}{2\pi}\frac{I}{r} \quad \text{--- (10.61)}$$

$$\therefore \oint_c \vec{B}\cdot d\vec{l} = \int_0^{2\pi} \frac{\mu_0 I}{2\pi r} r d\theta = \mu_0 I \quad \text{--- (10.62)}$$

This is in agreement with the Ampere's law. Equation (10.61) shows that the magnetic field B of an infinitely long wire is proportional to the current I but inversely proportional to the distance from the wire, as seen earlier.

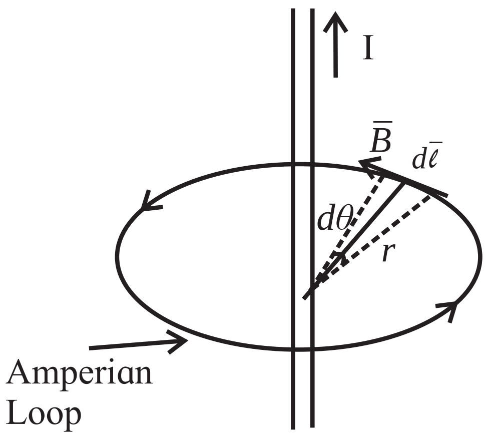
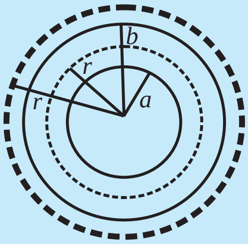


Fig. 10.23: Long straight current carrying wire.

You have studied Gauss' law in electrostatics as well as magnetism. The above example shows that if the distribution of currents has a high degree of symmetry such as cylindrical symmetry in case of a long wire, then the magnetic field for the given distribution of currents can be easily calculated. It will then become unnecessary to solve the integrals which appear in the Biot-Savart law.

We note here that the Biot-Savart law plays a role in magnetostatics that Coulomb's law plays in electrostatics. On parallel lines, we can say that what the role Gauss' law plays in electrostatics, plays the Ampere's law in magnetostatics.

Example 10.7: A coaxial cable consists of a central conducting core wire of radius a and a coaxial cylindrical outer conductor of radius b (see figure). The two conductors carry an equal current I in opposite directions in and out of the plane of the paper. What will be the magnitude of the magnetic field B for (i) $a < r < b$ and (ii) $b < r$? What will be its direction?



Solution: By symmetry, B will be tangent to any circle centred on the central conductor. In order to apply the Ampere's law, consider a circle of radius r such that $a < r < b$.

$$\therefore \oint \vec{B}.d\vec{l} = \mu_0 I$$

$$\therefore B.2\pi r = \mu_0 I$$

$$\therefore B = \frac{\mu_0}{2\pi}\frac{I}{r}, \ a < r < b$$

For r>b,

$\therefore \oint \vec{B}.d\vec{l} = \mu_0 (I - I) = 0$ ($\because$ The two current are equal and opposite)

$\therefore B.2\pi r = 0 \qquad r > b$

(Try to solve this using Biot-Savart Law !)

10.16 Magnetic Field of a Solenoid and a Toroid: (a) Solenoid:

You have learnt about a solenoid in XIth Std. qualitatively. Consider a long, closely wound helical coil of a conducting wire. We assume that the diameter of the coil is much smaller than its length. Figure 10.24 shows the schematic diagram of a cross section of a current carrying solenoid. The density of the magnetic field lines along the axis of the solenoid within the solenoid and at a certain distance away from the wire, is uniform. Hence the magnetic field B is parallel to the axis of the solenoid. The lines are widely spaced outside the solenoid and hence the magnetic field is weak there.

For a real solenoid of finite length, magnetic field is uniform and has a good strength at the centre and comparatively weak at the outside of the coil.

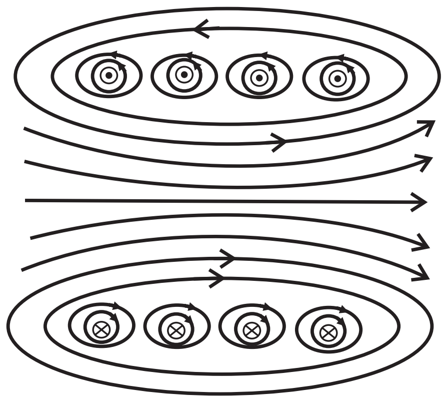


Fig. 10.24: Schematic diagram of a cross section of a current carrying Solenoid.

Let us consider an ideal solenoid as shown in Fig. 10.25.

For the application of the Ampere's law, an Amperian loop is drawn as shown in Fig. 10.25. From Eq. (10.58),

$$\oint \vec{B}.d\vec{l} = \mu_0 I$$

Over the rectangular loop abcd, the above integral takes the form

$$\int_a^b \vec{B}\cdot d\vec{l} + \int_b^c \vec{B}\cdot d\vec{l} + \int_c^d \vec{B}\cdot d\vec{l} + \int_d^a \vec{B}\cdot d\vec{l} = \mu_0 I \quad \text{--- (10.63)}$$

Do you know?

In an ideal solenoid, the length is infinite and the wire has a square cross section and is wound very closely (with a layer of insulating material in between these enamelled wires). The magnetic field inside the coil is then uniform and along the axis of the solenoid. Outside the solenoid, it is zero.

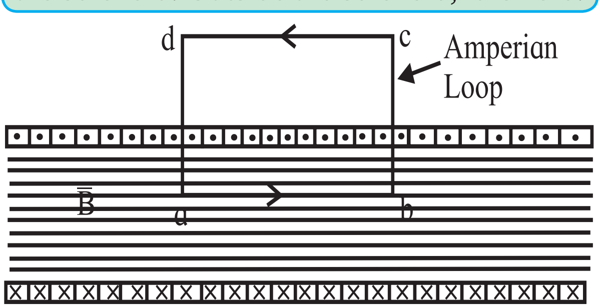


Fig. 10.25: Ampere's law applied to a part of a long ideal solenoid: The dots (.) show that the current is coming out of the plane of the paper and the crosses (x) show that the current is going into the plane of the paper, both in the coil of square cross section wire.

In the above equation, I is the net current encircled by the loop.

$$\therefore\ B.L + 0 + 0 + 0 = \mu_0 I \quad \text{--- (10.64)}$$

The second and fourth integrals are zero because $\vec{B}$ and $d\vec{l}$ are perpendicular to each other. The third integral is zero because outside the solenoid, $B = 0$. We can obtain the net current I.

If the number of turns is n per unit length of the solenoid and the current flowing through the wire is i, then the net current coming out of the plane of the paper is

$$I = nLi$$

$\therefore$ From Eq. (10.64),

$$BL = \mu_0 nLi$$

$$\therefore B = \mu_0 ni \quad \text{--- (10.65)}$$

Although the above result for B is obtained for an ideal solenoid, it is also valid for a realistic solenoid, particularly when applied to points in the middle of it but certainly not to points near the ends. Thus, a solenoid can be designed for a specific value of B by a choice of i and n.

(b) Toroid: A toroid is a solenoid of finite length bent into a hollow circular tube like structure similar to a pressurized rubber tube inside a tyre of vehicle. Schematic of a cross section of a toroid is shown in Fig. 10.26. By applying Ampere's law and taking into account the symmetry of this structure, we can obtain the magnetic field along the central axis of the tube in terms of the current. We construct a circular Amperian loop along the central axis of the tube, as shown in the figure.

The magnetic field lines are concentric circles in the toroid. The direction of the field is dictated by the direction of the current i in the coil around the toroid. Again, by the Ampere's law,

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I ,$$

where I is the net current encircled by the loop.

$$B.2\pi R = \mu_0 iN \quad \text{--- (10.66)}$$

Here N is the total number of turns in the toriod as the integration is over the full length of the loop, $2\pi R$.

$$\therefore B = \frac{\mu_0 iN}{2\pi R} \quad \text{--- (10.67)}$$

From the Eq. (10.67), B is inversely proportional to R. Thus, unlike the solenoid, magnetic field is not constant over the cross section of the toroid.

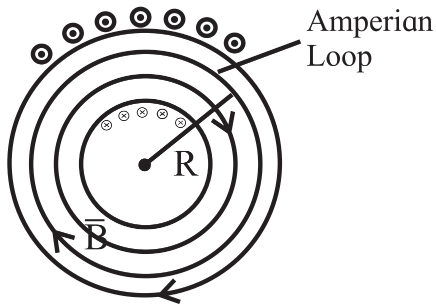


Fig. 10.26: Amperian loop along the central axis of the toroid.

Use your brain power

By making different choices for the Amperian loop, show that B = 0 for points outside an ideal toroid. What must be ideal toroid?

Example 10.8: A solenoid of length 25 cm has inner radius of 1 cm and is made up of 250 turns of copper wire. For a current of 3A in it, what will be the magnitude of the magnetic field inside the solenoid?

Solution: We use Eq. (10.65)

$$B = \mu_0 n i$$

$$B = 4\pi \times 10^{-7} \times \frac{250}{0.25} \times 3$$

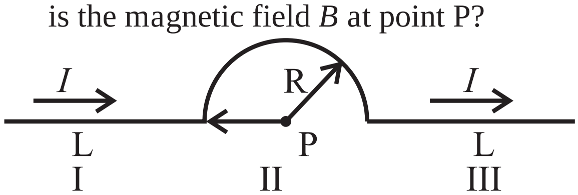
$$B = 4\pi \times 10^{-7} \times 10^{-3} \times 3$$

$$\therefore\ B = 3.77 \times 10^{-3} \text{T}$$

Exercises

1. Choose the correct option.

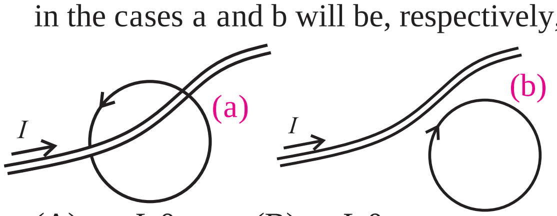
i) A conductor has 3 segments; two straight and of length L each and a semicircular with radius R. It carries a current I. What is the magnetic field B at point P?



(A) $\frac{\mu_0}{4\pi}\frac{I}{R}$ (B) $\frac{\mu_0}{4\pi}\frac{I}{R^2}$

(C) $\frac{\mu_0}{4}\frac{I}{R}$ (C) $\frac{\mu_0 I}{4\pi}$

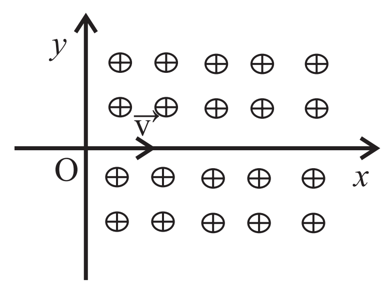
ii) Figure a, b show two Amperian loops associated with the conductors carrying current I in the sense shown. The $\oint \vec{B}.d\vec{l}$ in the cases a and b will be, respectively,



(A) $-\mu_0 I$, 0 (B) $\mu_0 I$, 0

(C) 0, $\mu_0 I$ (D) 0, $-\mu_0 I$

iii) A proton enters a perpendicular uniform magnetic field B at origin along the positive x axis with a velocity v as shown in the figure. Then it will follow the following path. [The magnetic field is directed into the paper].



(A) It will continue to move along positive x axis.

(B) It will move along a curved path, bending towards positive x axis.

(C) It will move along a curved path, bending towards negative y axis.

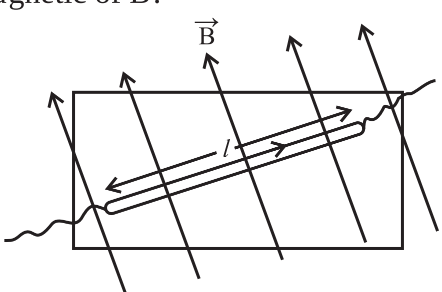
(D) It will move along a sinusoidal path along the positive x axis.

(iv) A conducting thick copper rod of length 1 m carries a current of 15 A and is located on the Earth's equator. There the magnetic flux lines of the Earth's magnetic field are horizontal, with the field of 1.3×10^{-4} T, south to north. The magnitude and direction of the force on the rod, when it is oriented so that current flows from west to east, are

(A) 14×10^{-4} N, downward.
(B) 20×10^{-4} N, downward.
(C) 14×10^{-4} N, upward.
(D) 20×10^{-4} N, upward.

v) A charged particle is in motion having initial velocity $\vec{v}$ when it enter into a region of uniform magnetic field perpendicular to $\vec{v}$. Because of the magnetic force the kinetic energy of the particle will
(A) remain uncharged.
(B) get reduced.
(C) increase.
(D) be reduced to zero.

2. A piece of straight wire has mass 20 g and length 1m. It is to be levitated using a current of 1 A flowing through it and a perpendicular magnetic field B in a horizontal direction. What must be the magnetic of B?



[Ans: 196 T]

3. Calculate the value of magnetic field at a distance of 2 cm from a very long straight wire carrying a current of 5 A (Given: $\mu_0 = 4\pi \times 10^{-7}$ Wb/Am).

[Ans: 2.5×10^{-5} T]

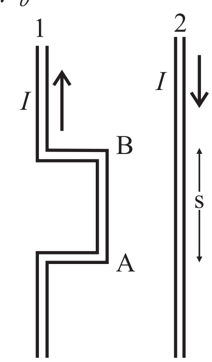
4. An electron is moving with a speed of 3×10^{-7} m/s in a magnetic field of 6×10^{-4} T perpendicular to its path. What will be the radium of the path? What will be frequency and the energy in keV ? [Given: mass of electron = 9×10^{-31} kg, charge $e = 1.6 \times 10^{-19}$ C, 1 eV = 1.6×10^{-19} J]

[Ans: 18.7 MHz, 2.53 keV T]

5. An alpha particle (the nucleus of helium atom) (with charge +2) is accelerated and moves in a vacuum tube with kinetic energy = 10 MeV. It passes through a uniform magnetic field of 1.88 T, and traces a circular path of radius 24.6 cm. Obtain the mass of the alpha particle. [l eV = 1.6×10^{-19} J, charge of electron = 1.6×10^{-19} C]

[Ans: 6.62×10^{-27} kg]

6. Two wires shown in the figure are connected in a series circuit and the same amount of current of 10 A passes through both, but in apposite directions. Separation between the two wires is 8 mm. The length AB is S = 22 cm. Obtain the direction and magnitude of the magnetic field due to current in wire 2 on the section AB of wire 1. Also obtain the magnitude and direction of the force on wire 1. [$\mu_0 = 4\pi \times 10^{-7}$ T.m/A]



[Ans: Repulsive, 2.75×10^{-4} kg]

7. A very long straight wire carries a current 5.2 A. What is the magnitude of the magnetic field at a distance 3.1 cm from the wire? [$\mu_0 = 4\pi \times 10^{-7}$ T.m/A]

[Ans: 8.35×10^{-5} T]

8. Current of equal magnitude flows through two long parallel wires having separation of 1.35 cm. If the force per unit length on each of the wires in 4.76×10^{-2} N, what must be I ?

[Ans: 56.7 A]

9. Magnetic field at a distance 2.4 cm from a long straight wire is 16 μT. What must be current through the wire?

[Ans: 1.92 A]

10. The magnetic field at the centre of a circular current carrying loop of radius 12.3 cm is 6.4×10^{-6} T. What will be the magnetic moment of the loop?

[Ans: 5.954×10^{-2} A.m^2]

11. A circular loop of radius 9.7 cm carries a current 2.3 A. Obtain the magnitude of the magnetic field (a) at the centre of the loop and (b) at a distance of 9.7 cm from the centre of the loop but on the axis.

[Ans: 1.49×10^{-5} T, 1.68×10^{-6} T]

12. A circular coil of wire is made up of 100 turns, each of radius 8.0 cm. If a current of 0.40 A passes through it, what be the magnetic field at the centre of the coil?

[Ans: 3.142×10^{-4} T]

13. For proton acceleration, a cyclotron is used in which a magnetic field of 1.4 Wb/m^2 is applied. Find the time period for reversing the electric field between the two Ds.

[Ans: 2.34×10^{-8} s]

14. A moving coil galvanometer has been fitted with a rectangular coil having 50 turns and dimensions 5 cm × 3 cm. The radial magnetic field in which the coil is suspended is of 0.05 Wb/m^2. The torsional constant of the spring is 1.5×10^{-9} Nm/degree. Obtain the current required to be passed through the galvanometer so as to produce a deflection of 30°.

[Ans: 1.2×10^{-5} A]

15. A solenoid of length π m and 5 cm in diameter has winding of 1000 turns and carries a current of 5 A. Calculate the magnetic field at its centre along the axis.

[Ans: 2×10^{-3}]

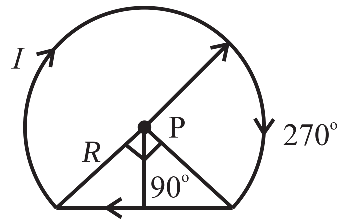
16. A toroid of narrow radius of 10 cm has 1000 turns of wire. For a magnetic field of 5×10^{-2} T along its axis, how much current is required to be passed through the wire?

[Ans: 25 A]

17. In a cyclotron protons are to be accelerated. Radius of its D is 60 cm. and its oscillator frequency is 10 MHz. What will be the kinetic energy of the proton thus accelerated?
(Proton mass = 1.67×10^{-27} kg,
e = 1.60×10^{-19} C, eV = 1.6×10^{-19} J)

[Ans: 7.515 MeV]

18. A wire loop of the form shown in the figure carries a current *I*. Obtain the magnitude and direction of the magnetic field at P.

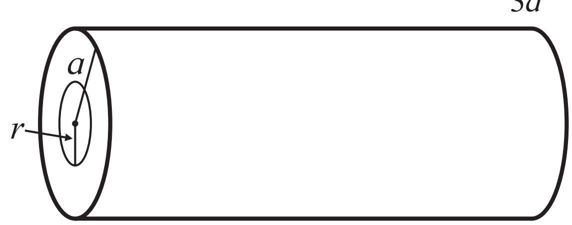


[Ans: $B = \frac{\mu_0}{4\pi}\frac{I}{R}\left[\frac{3\pi}{2} + \sqrt{2}\right]$]

19. Two long parallel wires going into the plane of the paper are separated by a distance *R*, and carry a current *I* each in the same direction. Show that the magnitude of the magnetic field at a point P equidistant from the wires and subtending angle θ from the plane containing the wires, is $B = \frac{\mu_0}{\pi}\frac{I}{R}\sin 2\theta$
What is the direction of the magnetic field?

20. Figure shows a cylindrical wire of diameter a, carrying a current *I*. The current density which is in the direction of the central axis of the wire varies linearly with radial distance r from the axis according to the relation $J = J_o r/a$. Obtain the magnetic field *B* inside the wire at a distance r from its centre.

[Ans: $B = \frac{J_0 \mu_0 r^2}{3a}$]



21. In the above problem, what will be the magnetic field *B* inside the wire at a distance r from its centre, if the current density *J* is uniform across the cross section of the wire?

[Ans: $B = \frac{\mu_0 J r}{\pi}$]

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