Simultaneous Linear Equations

- **Substitution Method of Solving Pairs of Linear Equations**

  In this method, we have **substituted** the value of one variable by expressing it in terms of the other variable to solve the pair of linear equations. That is why this method is known as the **substitution method.**

  Example:
  Solve the following system of equations by substitution method.
  $x - 4y + 7 = 0$
  $3x + 2y = 0$

  Solution:
  The given equations are
  $x - 4y + 7 = 0$ … (1)
  $3x + 2y = 0$ … (2)
  From equation (2),
  $3x = -2$

  $$\Rightarrow x = -\frac{2}{3}y$$

  Put $x = -\frac{2}{3}y$ in equation (1)

  $$-\frac{2}{3}y - 4y + 7 = 0$$

  $$\Rightarrow \frac{-2y - 12y}{3} = -7$$

  $$\Rightarrow -14y = -21$$

  $$\Rightarrow y = \frac{-21}{-14} = \frac{3}{2}$$

  $$\therefore x = -\frac{2}{3}\left(\frac{3}{2}\right) = -1$$

  Therefore, the required solution is $\left(-1, \frac{3}{2}\right)$.

- **Elimination Method to Solve a Pair of Linear Equations**

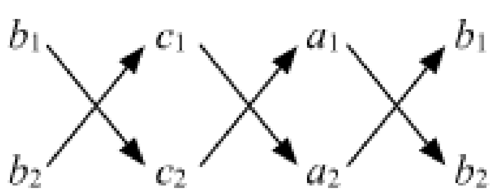
Example:
Solve the following pair of linear equations by elimination method.
$7x - 2y = 10$
$5x + 3y = 6$

Solution:
$7x - 2y = 10$ … (1)
$5x + 3y = 6$ … (2)
Multiplying equation (1) by 5 and equation (2) by 7, we get
$35x - 10y = 50$ … (3)
$35x + 21y = 42$ … (4)
Subtracting equation (4) from (3), we get
$-31y = 8 \Rightarrow y = -\frac{8}{31}$
Now, using equation (1):

$7x = 10 + 2y$
$\Rightarrow x = \frac{1}{7}\left\{10 + 2 \times \frac{-8}{31}\right\} = \frac{42}{31}$.
Required solution is $\left(\frac{42}{31}, -\frac{8}{31}\right)$.

- **Cross-Multiplication Method of Solving Pairs of Linear Equations**

The solution of the system of linear equations $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ can be determined by the following diagram.



That is,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\Rightarrow x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1} \qquad (a_1b_2 - a_2b_1 \neq 0)$$

Example:

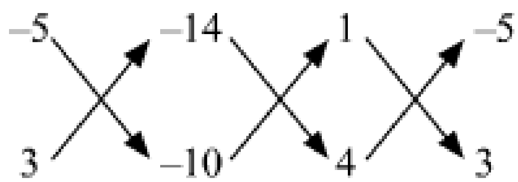
Solve the following pair of linear equations by the cross-multiplication method

$x - 5y = 14$, $4x + 3y = 10$

Solution:

$x - 5y - 14 = 0$

$4x + 3y - 10 = 0$



$$\frac{x}{(-5)\times(-10) - 3\times(-14)} = \frac{y}{(-14)\times 4 - (-10)\times 1} = \frac{1}{1\times 3 - 4\times(-5)}$$

$$\Rightarrow \frac{x}{50+42} = \frac{y}{-56+10} = \frac{1}{3+20}$$

$$\Rightarrow \frac{x}{92} = \frac{y}{-46} = \frac{1}{23}$$

$$\Rightarrow x = \frac{92}{23} = 4, y = -\frac{46}{23} = -2$$

∴ Required solution is (4, – 2).

Equations reducible to a pair of linear equations in two variables

Some pair of equations which are not linear can be reduced to linear form by suitable substitutions.

Example: Solve the following system of equations

$$\frac{2}{x-2}-\frac{1}{y-1}=1$$
$$\frac{5}{x-2}-\frac{6}{y-1}=20$$

Solution:

Let $\frac{1}{x-2}=u$ and $\frac{1}{y-1}=v$. Then, the given system of equations reduces to

$2u - v = 1$ … (1)

$5u - 6v = 20$ … (2)

Multiplying equation (1) by 6 and then subtracting from (2), we get

$$\begin{array}{l} 5u-6v=20 \\ 12u-6v=6 \\ - \quad + \quad - \\ \hline \quad -7u=14 \end{array}$$

$$\Rightarrow u=\frac{14}{-7}=-2$$

Equation (1) $\Rightarrow$ $v = 2u - 1$
$= 2(-2) - 1$
$= -4 - 1 = -5$

$$\therefore \frac{1}{x-2}=-2, \frac{1}{y-1}=-5$$

$$\Rightarrow \quad x-2=-\frac{1}{2} \quad \Rightarrow \quad y-1=-\frac{1}{5}$$
$$\Rightarrow \quad x=2-\frac{1}{2}=\frac{3}{2} \quad \Rightarrow \quad y=1-\frac{1}{5}=\frac{4}{5}$$

$\therefore \left(\frac{3}{2}, \frac{4}{5}\right)$ is the solution of the given system of equations.