IMO INTERNATIONAL MATHEMATICS OLYMPIAD

Polynomials

MATHEMATICAL REASONING

1. The value of k for which $(x+2)$ is a factor of (x $(x+1)^7+(3x+k)^3$ is ____.

(a) –7 (b) 7

(c) –1 (d) $-6-3^{(7/3)}$

2. The remainder when x^4-y^4 is divided by $x-y$ is ____.

(a) 0 (b) $x+y$

(c) x^2-y^2 (d) $2y^4$

3. When $p(x)=x^3+ax^2+2x+a$ is divided by $(x+a)$. the remainder is ___.

(a) 0 (b) a

(c) – a (d) 2a

4. $x^{12}-y^{12}=$

(a) $(x-y)(x^2+xy+y^2)(x+y)(x^2-xy+y^2)(x^2+y^2)(x^4-x^2y^2+y^4)$

(b) $(x+y)(x^2-xy+y^2)(x+y)(x^2-xy+y^2)(x^2+y^2)(x^4-x^2y^2+y^4)$

(c) $(x+y)(x^2+xy-y^2)(x+y)(x^2-xy+y^2)(x^2+y^2)(x^4-x^2y^2+y^4)$

(d) $(x-y)(x^2-xy+y^2)(x+y)(x^2-xy+y^2)(x^2+y^2)(x^4-x^2y^2+y^4)$

5. If $X=\frac{a-b}{a+b}.y=\frac{b-c}{b+c},Z=\frac{c-a}{c+a}$, then the value of $\frac{(1+x)(1+y)(1+z)}{(1-x)(1-y)(1-z)}$ is ______.

(a) abc (b) $a^2b^2c^2$

(c) 1 (d) –1

6. If $(x+2)$ and $(x-1)$ are factors of (x^3+10x^2+3x+n), then the value of m, n respectively are ______.

(a) – 5, 5 (b) 7, 18

(c) 7, – 18 (d) – 5, – 18

7. Given that $x=2$ is a solution of $x^3-7x+6=0$. The other solutions are_____.

(a) – 1, 3 (b) 1, – 3

(c) 1, – 2 (d) –1, – 2

8. If $(x+k)$ is a common factor of $f(x)=(x^2+px+q)$ and $g(x)=(x^2+lx+m)$, then the value of k is ____.

(a) $l+p$ (b) $m-q$

(c) $\frac{l-p}{m-q}$ (d) $\frac{m-q}{l-p}$

9. The product $(a+b)(a-b)(a^2-ab+b^2)(a^2+ab+b^2)$ is equal to ___.

10. The value of $(x-a)^3+(x-b)^3+(x-c)^3-3(x-a)(x-b)(x-c)$, when $a+b+c=3x$ is ______.

(a) 3 (b) 2

(c) 1 (d) 0

11. Value of R, if $\frac{a^2-19a-25}{a-7}=a-12+\frac{R}{a-7}$ is _____.

(a) – 109 (b) – 88

(c) – 84 (d) – 64

12. When (x^3-2x^2+px-q) is divided by (x^2-2x-3), the remainder is $(x-6)$. The values of p and q respectively are

(a) – 2, – 6 (b) 2, – 6

(c) – 2, 6 (d) 2, 6

13. Find the remainder when the expression $3x^3+8x^2-6x+1$ is divided by $x+3$.

(a) 1 (b) 10

(c) 6 (d) 0

14. If x^2-1 is a factor of $ax^4+bx^3+ex^2+dx+e$, then

(a) $a+b+e=c+d$

(b) $a+b+c=d+e$

(c) $b+c+d=a+e$

(d) None of these

15. If a, b, c are all non-zeroes and $a+b+c=0$, then $\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}=$ _____.

(a) 0 (b) 1

(c) 2 (d) 3

EVERDAY MATHEMATICS

16. Length, breadth and height of a cuboidal tank are $(x-3y)m, (x+3y)m$ and $(x^2+9y^2)m$ respectively. Find the volume of the tank.
(a) $(x^3+3xy+27y^3)m^3$
(b) $(x^4+2x^2y^2+81y^4)m^3$
(c) $(x^2-81y^4)m^3$
(d) $(x^4+81y^4)m^3$

17. A rectangular field has an area $(35x^2+13x-12)m^2$. What could be the possible expression for length and breadth of the field?
(a) $(5x+4)$ and $(7x-3)m$
(b) $(3x+9)m$ and $(7x-12)m$
(c) Both (a) and (b)
(d) None of these

18. Santosh has ₹ $(x^3-3x^2+4x+50)$. He want to buy chocolates each of cost ₹ $(x-3)$. After buying maximum number of chocolates with his money, how much money is left with him?
(a) ₹ 50 (b) ₹ 40
(c) ₹ 62 (d) ₹ 20

19. Area of a rectangular field is $(2x^3-11x^2-4x+5)$ sq. units and side of a square field is $(2x^2+4)$ units. Find the difference between their areas (in sq. units).
(a) $4x^4-2x^3-4x+11$
(b) $4x^4-2x^3+27x^2+4x+11$
(c) $4x^4+27x^2+4x-11$
(d) $4x^4+2x^3+27x^2+4x+11$

20. Vikas has ₹ $(x^3+2ax+b)$, with this money he can buy exactly $(x-1)$ jeans or $(x+1)$ shirts with no money left. How much money Vikas has, if $x=4$?
(a) ₹ 80 (b) ₹ 120
(c) ₹ 30 (d) ₹ 60

ACHIEVERS SECTION (HOTS)

21. Which of the following statements is INCORRECT?
(a) Every non-zero constant polynomial has zero roots.
(b) Zero polynomial has zero root.
(c) Every linear polynomial has exactly one root.
(d) If $x-a$ is the root of $p(x)=0$, then $p(a)=0$,

22. If $(5x^2+14x+2)^2-(4x^2-5x+7)^2$ is divided by (x^2+x+1), then quotient 'q' and remainder '/-' respectively, are ____.
(a) $(x^2+19x-5), 0$ (b) $9(x^2+19x-5), 0$
(c) $(x^2+19x-5), 1$ (d) $9(x^2+19x-5), 1$

23. Select the CORRECT statement.
(a) If $x=\frac{\sqrt{3}+1}{\sqrt{3}-1}+\frac{\sqrt{3}-1}{\sqrt{3}+1}+\frac{\sqrt{3}-2}{\sqrt{3}+2}$, then value of $x^2+\left(\frac{39}{x}\right)^2$ is 110.
(b) Every integer is a whole number.
(c) Between two rational numbers, there exist infinite number of integers.
(d) None of these

24. Match the following.

Column - I	Column – II
(p) If $f(x)=x^3-6x^2+11x-6$, then $f(-1)=$ ____.	–210
(q) If $f(x)=2x^3-13x^2+17x+12$, then $f(-3)=$ ____.	1
(r) if $x=\frac{4}{3}$ is a root of $f(x)=6x^3-11x^2+kx-20$, then $k=$ ____.	(iii) –24
(s) If $x=-1$ is a root of $f(x)=x^{100}+2x^{99}+k$, then $k=$ ____.	

(a) (p) → (iii); (q) → (iv); (r) → (i); (s) → (ii)
(b) (p) → (ii); (q) → (iv); (r) → (i); (s) → (iii)
(c) (p) → (iii); (q) → (i); (r) → (iv); (s) → (ii)
(d) (p) → (iii); (q) → (ii); (r) → (i); (s) → (iv)

25. Study the given statements.
Statement-I:
$\frac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}{(a+b)^3(b+c)^3+(c+a)^3}$
Statement-II: $a^2+b^2+c^2-ab-bc-ca$
$=\frac{1}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$
Which of the following options holds?
(a) Both Statement-I and Statement-II are true.
(b) Statement-I is true but Statement-II is false.
(c) Statement-1 is false but Statement-II is true.
(d) Both Statement-1 and Statement-II are false.

HINTS & EXPLANATIONS

1. (b) : Let $f(x) = (x+1)^7 + (3x+k)^3$

Since, $(x+2)$ is a factor of $f(x)$. Therefore, by factor theorem, $f(-2) = 0$

$\Rightarrow (-2+1)^7 + (3x(-2)+k)3 = 0$

$\Rightarrow (-1)^7 + (-6+k)^3 = 0 \Rightarrow (-6+k)^3 = 1$

$\Rightarrow -6+k = 1 \Rightarrow k = 7$

2. (a) : Let $f(x) = x^4 - y^4$ and $g(x) = x - y$

Now, $g(x) = 0 \Rightarrow x - y = 0 \Rightarrow x = y$

$\therefore$ By remainder theorem, we know that when $f(x)$ is divided by $g(x)$. then the remainder is $f(y)$.

Now, $f(y) = y^4 - y^4 = 0$

3. (c) : We have, $p(x) = x^3 + ax^2 + 2x +$

By remainder theorem, we know that when $p(x)$ is divided by $(x+a)$. then the remainder is $(p-a)$. Now, $p(-a) = (-a)^3 + a(-a)^2 + 2(-a) + a$

$= -a^3 + a^3 - 2a + a = -a$

4. (a) : We have, $x^{12} - y^{12}$

$= (x^6)^2 - (y^6)^2 = (x^6 - y^6)(x^6 + y^6)$

$= [(x^3)^2 - (y^3)^2][(x^2)^3 + (y^2)^3]$

$= (x^3 + y^3)(x^3 - y^3)[(x^2 + y^2)(x^4 + y^4 - x^2y^2)]$

$= (x+y)(x^2 + y^2 - xy)(x-y)(x^2 + y^2 + xy)$

$(x^2 + y^2)(x^4 + y^4 - x^2y^2)$

5. (c) : We have,

$x = \frac{a-b}{a+b}, y = \frac{b-c}{b+c}, z = \frac{c-a}{c+a}$

Now, $1 + x = 1 + \frac{a-b}{a+b} = \frac{a+b+a-b}{a+b} = \frac{2a}{a+b}$

$1 - x = 1 - \frac{(a-b)}{a+b} = \frac{a+b-a+b}{a+b} = \frac{2b}{a+b}$

Similarly, $1 + y = \frac{2b}{b+c}, 1 + z = \frac{2c}{a+c}$

$1 - y = \frac{2c}{b+c}, 1 - z = \frac{2a}{a+c}$

Now, $\frac{(1+x)(1+y)(1+z)}{(1-x)(1-y)(1-z)}$

$$= \frac{\left(\frac{2a}{a+b}\right)\left(\frac{2b}{b+c}\right)\left(\frac{2c}{a+c}\right)}{\left(\frac{2b}{a+b}\right)\left(\frac{2c}{b+c}\right)\left(\frac{2a}{a+c}\right)} = 1$$

6. (c) : Let, $f(x) = x^3 + 10x^2 + mx + n$

Since, $(x+2)$ and $(x-1)$ are factors of $f(x)$.

Therefore, by factor theorem.

$f(-2) = 0$ and $f(1) = 0$

$\Rightarrow (-2)^3 + 10(-2)^2 + m(-2) + n = 0$

and $(1)^3 + 10(1)^2 + m(1) + n = 0$

$\Rightarrow -8 + 40 - 2m + n = 0$

and $1 + 10 + m + n = 0$

$\Rightarrow -2m + n = -32$ ...(i)

and $m + n = -11$ ...(ii)

Subtracting (i) form (ii), we get

$3m = 21 \Rightarrow m = 7$

From (ii), $7 + n = -11 \Rightarrow n = -18$

7. (b) : Let, $f(x) = x^3 - 7x + 6$ and $g(x) = x - 2$ M

Now, by long division method, we have

$$\begin{array}{r|l} & x^2 + 2x - 3 \\ \hline x-2 & x^3 - 7x + 6 \\ & x^3 - 2x^2 \\ & - \quad + \\ \hline & 2x^2 - 7x + 6 \\ & 2x^2 - 4x \\ & - \quad + \\ \hline & -3x + 6 \\ & -3x + 6 \\ & + \quad - \\ \hline & 0 \end{array}$$

$\therefore (x-2)(x^2 + 2x - 3) = 0$

For other solutions, $x^2 + 2x - 3 = 0$

$\Rightarrow x^2 + 3x - x - 3 = 0 \Rightarrow (x+3)(x-1) = 0$

$\Rightarrow x = -3$ or 1

8. (d) : Since $(x+k)$ is a common factor of $f(x) = x^2 + px + q$ and $g(x) = x^2 + lx + m$

Then, $f(-k) = 0$ and $g(-k) = 0$

$\Rightarrow k^2 - kp + q = 0$ and $k^2 - kl + m = 0$

$\Rightarrow k^2 = kp - q$ ...(i)

and $k^2 = kl - m$

From (i) and (ii), we have

$kp - q = kl - m$

$\Rightarrow k = \frac{q-m}{p-l} \Rightarrow k = \frac{m-q}{l-p}$

9. (b) : We have,

$(a+b)(a-b)(a^2 - ab + b^2)(a^2 + ab + b^2)$

$= (a+b)(a^2 - ab + b^2)(a-b)(a^2 + ab + b^2)$

$= (a^3 + b^3)(a^3 - b^3) = (a^3)^2 - (b^3)^2 = a^6 - b^6$

10. (d) : It is given that $a + b + c = 3x$. Then,

$(x-a)^3 + (x-b)^3 + (x-c)^3 - 3(x-a)(x-b)(x-c)$

$= (x - a + x - b + x - c)[(x-a)^2 + (x-b)^2$

$+(x-c)^2-(x-a)(x-b)-(x-b)(x-c)$
$-(x-c)(x-a)]=0$

11. (a) : We have, $\dfrac{a^2-19a-25}{a-7}=a-12+\dfrac{R}{a-7}$

$\Rightarrow \dfrac{a^2-19a-25}{a-7}=\dfrac{(a-12)(a-7)+R}{a-7}$

$\Rightarrow a^2-19a-25=a^2-19a+84+R$

$\Rightarrow -84-25=R \Rightarrow R=-109$

12. (c) : Let $f(x)=x^3-2x^2+px-q$

and $g(x)=x^2-2x-3=(x-3)(x+1)$

Now, when $g(x)$, leaves a remainder $(x-6)$.

Then, we have, $f(3)=x-6$ and $f(-1)=x-6$

$\Rightarrow 27-18+3p-q=3-6$

and $-1-2-p-q=-1-6$

$\Rightarrow 3p-q=-12$ ...(i)

and $p+q=4$ ...(ii)

Adding (i) and (ii), we get

$4p=-8 \Rightarrow p=-2$

and $q=3(-2)+12=-6+12=6$

13. (b) : Let $f(x)=3x^3+8x^2-6x+1$

We know that when $f(x)$ is divided by $x+3$. the remainder is $f(-3)$.

Now, $f(-3)=3(-3)3+8(-3)2-6(-3)+1$

$=-81+72+18+1=10$

14. (d) : Let, $f(x)=ax^4+bx^3+cx^2+dx+e$

Since, (x^2-1) i.e., $(x-1)(x+1)$ is a factor of $f(x)$. Therefore, by factor theorem,

$f(1)=0$ and $f(-1)=0$

$\Rightarrow a+b+c+d+e=0$ and $a-b+c-d+e=0$

$\Rightarrow a+b+c=-(d+e)$ and $a+c+e=b+d$

15. (d) : We have, $a+b+c=0$

$\therefore a^3+b^3+c^3=3abc$ ...(i)

Now, $\dfrac{a^2}{bc}+\dfrac{b^2}{ca}+\dfrac{c^2}{ab}=\dfrac{a^3+b^3+c^3}{abc}$

$\dfrac{1}{abc}=(3abc)=3$ [Using (i)]

16. (c) : Volume of cuboidal tank

$=$Length$\times$breadth$\times$height

$(x-3y)(x+3y)(x^2+9y^2)$

$=(x^2-9y^2)(x^2+9y^2)=(x^4-81y^4)m^3$

17. (a) : Area of rectangular field $=35x^2+13x-12$

$=35x^2+28x-15x-12$

$=7x(5x+4)-3(5x+4)$

$=(5x+4)(7x-3)$

So, possible expression for length and breadth of the field are $(5x+4)m$ and $(7x-3)m$

18. (c) : Amount of money Santosh has

$=$ ₹ $(x^3-3x^2+4x+50)$

Cost of each chocolate $=$ ₹$(x-3)$

By long division method, we have

$$\begin{array}{r|l} & x^2+4 \\ \hline x-3 & x^3-3x^2+4x+50 \\ & x^3-3x^2 \\ & -\quad + \\ \hline & 4x+50 \\ & 4x-12 \\ & -\quad + \\ \hline & 62 \\ \hline \end{array}$$

$\therefore x^3-3x^2+4x+50=(x-3)(x^2+4)+62$

So, Santosh bought (x^2+4) chocolates and ₹ 62 left with him.

19. (b) : Area of rectangular field

$=(2x^3-11x^2-4x+5)$ sq. units

Side of square field $=(2x^2+4)$ units

$\therefore$ Areal of square field $=(2x^2+4)^2$

$=(4x^4+16+16x^2)$ sq. units

$\therefore$ Required difference

$=4x^4+16+16x^2-2x^3+11x^2+4x-5$

$=(4x^4-2x^3+27x^2+4x+11)$ sq. units

20. (d) : Amount of money Vikas has

$=$ ₹$(x^2+2ax+b)$

Now, he can buy exactly $(x-1)$ Jeans or $(x+1)$ shirts.

$\therefore (x-1)$ and $(x+1)$ are factors of $x^3+2ax+b$.

$\therefore (1)^3+2a(1)+b=0 \Rightarrow 2a+b=-1$ ...(i)

and $(-1)^3-2a+b=0 \Rightarrow 2a-6b=-1$ ...(ii)

Adding (i) and (ii), we get

$4a=-2 \Rightarrow a=\dfrac{-1}{2}$,

$\therefore -1+b=-1 \Rightarrow b=0$

So, amount of money he has $=$ ₹(x^3-x)

$=$ ₹ $(4^3-4)=$ ₹$(64-4)=$ ₹ 60

21. (b) : Zero polynomial has one root.

22. (b) : Let $f(x)=(5x^2+14x+2)^2-(4x^2-5x+7)^2$

$=25x^4+196x^2+4+140x^3+56x+20x^2$

$-16x^4 - 25x^2 - 49 + 40x^3 + 70x - 56x^2$
$= 9x^4 + 180x^3 + 135x^2 + 126x - 45$
and $g(x) = x^2 + x + 1$
By long division method, we have

$$\begin{array}{r|l} & \quad 9x^2 + 171x - 45 \\ \hline x^2 + x + 1 & 9x^4 + 180x^3 + 135x^2 + 126x - 45 \\ & 9x^4 + 9x^3 + 9x^2 \\ & - \quad - \quad - \\ \hline & 171x^3 + 126x^2 + 126x - 45 \\ & 171x^3 + 171x^2 + 171x \\ & - \quad - \quad - \\ \hline & -45x^2 - 45x - 5 \\ & -45x^2 - 45x - 45 \\ & + \quad + \quad + \\ \hline & \qquad 0 \end{array}$$

$\therefore r = 0$ and $q = 9x^2 + 171x - 45$
i.e., $9(x^2 + 19x - 5)$.

23. (d) :

24. (c) :

25. (c) : Statement – I: We have,

$$\frac{(a^2-b^2)^3+(b^2-c^2)+(c^2-a^2)^3}{(a+b)^3+(b+c)^3+(c+a)^3}$$

$$\frac{(a^2-b^2+b^2-c^2+c^2-a^2)[(a^2-b^2)^2+(b^2-c^2)^2+(c^2-a^2)^2-(a^2-b^2)(b^2-c^2)-(b^2-c^2)(c^2-a^2)-(c^2-a^2)^2(a^2-b^2)]+3(a^2-b^2)(b^2-c^2)(c^2-a^2)}{(a+b+b+c+c+a)[(a+b)^2+(b+c)^2+(c+a)^2-(a+b)(b+c)-(b+c)(c+a)-(c+a)(a+b)]+3(a+b)(b+c)(c+a)}$$

Statement – II: We have,

$a^2 + b^2 + c^2 - ab - bc - ac = \frac{1}{2}\{2a - 2b^2$
$+2c^2 - 2ab - 2bc - 2ac\}$
$= \frac{1}{2}[(a-b)^2 + (b-c)^2 + (c-a)^2]$
$\therefore$ Statement – I is false but Statement –II is true.