CBSE Test Paper 04

Chapter 4 Quadratic Equation

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1. The perimeter of a right triangle is 70cm and its hypotenuse is 29cm. The area of the triangle is **(1)**
   a. 210 sq.cm
   b. 200 sq.cm
   c. 180 sq.cm
   d. 250 sq.cm
2. $5x^2 + 8x + 4 = 2x^2 + 4x + 6$ is a **(1)**
   a. quadratic equation
   b. cubic equation
   c. constant
   d. linear equation
3. $x^2 - 30x + 225 = 0$ have **(1)**
   a. Real roots
   b. No real roots
   c. Real and Equal roots
   d. Real and Distinct roots
4. If the quadratic equation $bx^2 - 2\sqrt{ac}\,x + b = 0$ has equal roots, then **(1)**
   a. $b^2 = -ac$
   b. $2b^2 = ac$
   c. $b^2 = ac$
   d. $b^2 = 2ac$
5. A quadratic equation $ax^2 + bx + c = 0$ has real and distinct roots, if **(1)**
   a. $b^2 - 4ac > 0$
   b. $b^2 - 4ac < 0$
   c. None of these
   d. $b^2 - 4ac = 0$
6. Solve: $x^2 + 6x + 5 = 0$ **(1)**

7. Find the roots of the quadratic equation $2x^2 - x - 6 = 0$ **(1)**
8. Without solving, find the nature of the roots of the quadratic equations. $x^2 + x + 1 = 0$. **(1)**
9. Check whether it is quadratic equation: $(x + 1)^3 = x^3 + x + 6$ **(1)**
10. Find the discriminant of equation: $2x^2 - 7x + 6 = 0$. **(1)**
11. Solve the following problem: $x^2 - 45x + 324 = 0$ **(2)**
12. Find the roots of the equation, if they exist, by applying the quadratic formula: $x^2 + 5x - (a^2 + a - 6) = 0$. **(2)**
13. Use factorization method to solve the quadratic equation $ad^2x(\frac{a}{b}x + \frac{2c}{d}) + c^2b = 0$. **(2)**
14. A two-digit number is 4 times the sum of its digits and twice the product of the digits. Find the number. **(3)**
15. Sum of the areas of two squares is 400 cm^2. If the difference of their perimeters is 16 cm, find the sides of the two squares. **(3)**
16. Solve: $\frac{1}{(x+3)} + \frac{1}{(2x-1)} = \frac{11}{(7x+9)}, x \neq -3, \frac{1}{2}, \frac{-9}{7}$ **(3)**
17. Solve for x: $\frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3} (x \neq 2, 4)$ **(3)**
18. A man buys a number of pens for Rs. 180. If he had bought 3 more pens for the same amount, each pen would have cost him Rs. 3 less. How many pens did he buy? **(4)**
19. At t minutes past 2 p.m, the time needed by the minute hand of a clock to show 3 p.m. was found to be 3 minutes less than $\frac{t^2}{4}$ minutes. Find t. **(4)**
20. The hypotenuse of a right triangle is $3\sqrt{10}$ cm. If the smaller leg is tripled and the longer leg doubled, new hypotenuse will be $9\sqrt{5}$ cm. How long are the legs of the triangle? **(4)**

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Solution

1. a. 210 sq.cm

   Explanation: Let base of the right triangle be x cm.

   Given: Perpendicular = x + 29 = 70 $\Rightarrow$ Perpendicular = $(41 - x)$ cm

   Now, using Pythagoras theorem,

   $$(29)^2 = x^2 + (41 - x)^2$$
   $$\Rightarrow 841 = 1681 + x^2 - 82x + x^2$$
   $$\Rightarrow 2x^2 - 82x + 840 = 0$$
   $$\Rightarrow x^2 - 41x + 420 = 0$$
   $$\Rightarrow x^2 - 20x - 21x + 420 = 0$$
   $$\Rightarrow x\left(x - 20\right) - 21\left(x - 20\right) = 0$$
   $$\Rightarrow \left(x - 20\right)\left(x - 21\right) = 0$$
   $$\Rightarrow x - 20 = 0 \text{ and } x - 21 = 0$$
   $$\Rightarrow x = 20 \text{ and } x = 21$$

   Therefore, the two sides other than hypotenuse are of 20 cm and 21 cm.

   $\therefore$ Area of right triangle = $\frac{1}{2} \times$ Base $\times$ Perpendicular = $\frac{1}{2} \times 20 \times 21$ = 210 sq. cm

2. a. quadratic equation

   Explanation: Given: $5x^2 + 8x + 4 = 2x^2 + 4x + 6$

   $\Rightarrow 5x^2 - 2x^2 + 8x - 4x + 4 - 6 = 0$

   $\Rightarrow 3x^2 + 4x - 2 = 0$

   Here, the degree is 2, therefore it is a quadratic equation.

3. c. Real and Equal roots

   Explanation: D = $(-30)^2$ - 4 $\times$ 1 $\times$ 225

   D = 900 - 900

   D = 0. Hence Real and Equal roots.

4. c. $b^2 = ac$

   Explanation: If the quadratic equation $bx^2 - 2\sqrt{ac}x + b = 0$ has equal roots, then $b^2 - 4ac = 0$

$\Rightarrow (-2\sqrt{ac})^2 - 4 \times b \times b = 0$

$\Rightarrow 4ac - 4b^2 = 0$

$\Rightarrow b^2 = ac$

5. a. $b^2 - 4ac > 0$

   Explanation: A quadratic equation $ax^2 + bx + c = 0$ has real and distinct roots, if $b^2 - 4ac > 0$.

6. Given, $x^2 + 6x + 5 = 0$

   Splitting middle term,

   $\Rightarrow$ $x^2 + 5x + x + 5 = 0$

   $\Rightarrow$ $x(x + 5) + 1(x + 5) = 0$

   $\Rightarrow$ $(x + 5)(x + 1) = 0$

   $\Rightarrow$ $x + 5 = 0$ or $x + 1 = 0$

   Therefore, $x = -5$ or -1

7. Given, $2x^2 - x - 6 = 0$

   Splitting the middle term of the equation,

   $\Rightarrow$ $2x^2 - 4x + 3x - 6 = 0$

   $\Rightarrow$ $2x(x - 2) + 3(x - 2) = 0$

   $\Rightarrow$ $(x - 2)(2x + 3) = 0$

   $\Rightarrow$ $x - 2 = 0$ or $2x + 3 = 0$

   Therefore, $x = 2$ or $x = -\frac{3}{2}$

8. $x^2 + x + 1 = 0$. Here $a = 1,\ b = 1,\ c = 1$

   $D = (1)^2 - 4 \times 1 \times 1 = -3 < 0$

   $\therefore$ equation has no real roots.

9. We have the following equation,

   $(x + 1)^3 = x^3 + x + 6$

   $\Rightarrow$ $x^3 + 1 + 3x(x + 1) = x^3 + x + 6$

   $\Rightarrow$ $3x^2 + 2x - 5 = 0$.

   This is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

10. Given, $2x^2 - 7x + 6 = 0$

    a =2, b = -7 and c = 6

    $\therefore$ D = b^2 -4ac

    = $(-7)^2$ - 4(2)(6)

    = 49 - 48

    = 1

11. $x^2 - 45x + 324 = 0$

    ⇒ x^2 - 36x - 9x + 324 = 0 ⇒ x (x - 36) - 9(x - 36)=0

    ⇒ (x - 9)(x - 36) ⇒ x = 9, 36

12. The given equation is $x^2 + 5x - (a^2 + a - 6) = 0$
    Comparing it with $Ax^2 + Bx + C = 0$, we get
    $A = 1, B = 5 \ and \ C = -(a^2 + a - 6)$
    $\therefore D = B^2 - 4AC =$ $(5)^2$ - 4(1)(-(a^2 + a - 6))

    = 25 + $4a^2$ + 4a - 24 = $4a^2$ + 4a + 1 = $(2a + 1)^2$ > 0
    So, the given equation has real roots, given by
    $\alpha = \frac{-B+\sqrt{D}}{2A} = \frac{-5+\sqrt{(2a+1)^2}}{2\times1} = \frac{-5+(2a+1)}{2} = \frac{2a-4}{2}$ = a - 2
    $\beta = \frac{-B-\sqrt{D}}{2A} = \frac{-5-\sqrt{(2a+1)^2}}{2\times1} = \frac{-5-(2a+1)}{2} = \frac{-2a-6}{2}$ = -(a + 3)
    Hence, (a - 2) and -(a + 3) are the roots of the given equation.

13. We have, $ad^2x(\frac{a}{b}x + \frac{2c}{d}) + c^2b = 0$
    $\implies \frac{a^2d^2}{b}x^2 + 2acdx + c^2b = 0$
    $\implies \frac{a^2d^2}{b}x^2 + acdx + acdx + c^2b = 0$
    $\implies adx(\frac{ad}{b}x + c) + bc(\frac{ad}{b}x + c) = 0$
    $\implies (adx + bc)(\frac{ad}{b}x + c) = 0$
    Either adx + bc =0 or $(\frac{ad}{b}x + c) = 0$
    $\implies x = -\frac{bc}{ad}$
    Hence, $x = -\frac{bc}{ad}$ is the requirreed solution.

14. Let the ten's place digit be y and unit's place be x.

Therefore, number is 10y + x.

According to given condition,

10y + x = 4(x + y) and 10y + x = 2xy

$\Rightarrow$ x = 2y and 10y + x = 2xy

Putting x = 2y in 10y + x = 2xy

10y + 2y = 2.2y.y

$12y = 4y^2$

$4y^2 - 12y = 0 \Rightarrow 4y(y - 3) = 0$

$\Rightarrow$ y - 3 = 0 or y = 3

Hence, the ten's place digit is 3 and units digit is 6 (2y = x)

Hence the required number is 36.

15. Let the sides of two squares be a and b,

then $a^2 + b^2 = 400$ ...(i)

and 4(a -b) = 16

or, a -b = 4:

or, a =4 + b (ii)

From equations (i) and (ii), we get

$(4 + b)^2 + b^2 = 400$

or, $16 + b^2 + 8b + b^2 = 400$

or, $2b^2 + 8b - 384 = 0$

or, $b^2 + 4b - 192 = 0$

or, $b^2 + 16b - 12b - 192 = 0$

or, $b(b + 16) - 12(b + 16) = 0$

or, $(b + 16)(b - 12) = 0$

b = - 16 (Rejecting the negative value)

So, b=12 cm

then a=16 cm

16. Given,

$\frac{1}{(x+3)} + \frac{1}{(2x-1)} = \frac{11}{(7x+9)}$

Taking LCM, we get

$\Rightarrow \frac{(2x-1)+(x+3)}{(x+3)(2x-1)} = \frac{11}{(7x+9)} \Rightarrow \frac{(3x+2)}{2x^2+5x-3} = \frac{11}{(7x+9)}$

Now cross multiply

⇒ $(3x + 2)(7x + 9) = 11(2x^2 + 5x - 3)$

⇒ $21x^2 + 41x + 18 = 22x^2 + 55x - 33$

⇒ $x^2 + 14x - 51 = 0$

⇒ $x^2 + 17x - 3x - 51 = 0$

⇒ $x(x + 17) - 3(x + 17) = 0$

⇒ $(x + 17)(x - 3) = 0$

⇒ $x + 17 = 0$ or $x - 3 = 0$

⇒ $x = -17$ or $x = 3$.

Therefore, -17 and 3 are the roots of the given equation.

17. The given equation is

$\frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3}(x \neq 2, 4)$

$\Rightarrow \frac{(x-1)(x-4)+(x-3)(x-2)}{(x-2)(x-4)} = \frac{10}{3}$

$\Rightarrow \frac{x^2-4x-x+4+x^2-2x-3x+6}{x^2-4x-2x+8} = \frac{10}{3}$

$\Rightarrow \frac{2x^2-10x+10}{x^2-6x+8} = \frac{10}{3}$

$\Rightarrow 3\left(2x^2 - 10x + 10\right) = 10\left(x^2 - 6x + 8\right)$

$\Rightarrow 6x^2 - 30x + 30 = 10x^2 - 60x + 80$

$\Rightarrow 4x^2 - 30x + 50 = 0$

$\Rightarrow (2x)^2 - 2(2x)\left(\frac{15}{2}\right) + \left(\frac{15}{2}\right)^2 - \left(\frac{15}{2}\right)^2 + 50 = 0$

$\Rightarrow \left(2x - \frac{15}{2}\right)^2 - \frac{225}{4} + 50 = 0$

$\Rightarrow \left(2x - \frac{15}{2}\right)^2 - \frac{25}{4} = 0$

$\Rightarrow 2x - \frac{15}{2} = \pm\frac{5}{2} \Rightarrow 2x = \frac{15}{2} \pm \frac{5}{2}$

$\Rightarrow 2x = \frac{15}{2} + \frac{5}{2}, \frac{15}{2} - \frac{5}{2}$

$\Rightarrow 2x = 10, 5 \Rightarrow x = 5, \frac{5}{2}$

Hence, the solutions of the given equation and 5 and $\frac{5}{2}$.

18. Let the number of pens purchased be x.

Cost of 1 pen = Rs. $\frac{180}{x}$

If number of pens increase by 3. Then,

Cost of one pen = Rs. $\frac{180}{x+3}$

According to question,

$\frac{180}{x} - \frac{180}{x+3} = 3$

$\Rightarrow \frac{180x+540-180x}{x^2+3x} = 3$

$\Rightarrow$ 540 = $3x^2 + 9x$

$\Rightarrow$ $3x^2 + 9x - 540 = 0$

$\Rightarrow$ $x^2 + 3x - 180 = 0$

$\Rightarrow$ $x^2 + 15x - 12x - 180 = 0$

$\Rightarrow$ x(x + 15) - 12(x + 15) = 0

$\Rightarrow$ x + 15 = 0 or x - 12 = 0

$\Rightarrow$ x = -15 or x = 12

As number of pens can't be negative.

$\Rightarrow$ x = 12

Therefore, he bought 12 pens.

19. Total time taken by minute hand from 2 p.m. to 3 p.m. is 60 min.

According to question,

$t + \left(\frac{t^2}{4} - 3\right) = 60$

⇒ $4t + t^2 – 12 = 240$

⇒ $t^2 + 4t – 252 = 0$

⇒ $t^2 + 18t – 14t – 252 = 0$

⇒ t(t + 18) – 14(t +18) = 0

⇒ (t + 18) (t – 14) = 0

⇒ t + 18 = 0 or t – 14 = 0

⇒ t = –18 or t = 14 min.

As time can't be negative. Therefore, t = 14 min.

20. Suppose, the smaller side of the right triangle be x cm and the larger side be y cm.

Then,

$\therefore x^2 + y^2 = \left(3\sqrt{10}\right)^2$ [Using pythagoras theorem]

$\Rightarrow x^2 + y^2 = 90$ ....(i)

If the smaller side is tripled and the larger side be doubled, the new hypotenuse is $9\sqrt{5}$ cm.

$\therefore (3x)^2 + (2y)^2 = (9\sqrt{5})^2$ [Using pythagoras theorem]
$\Rightarrow 9x^2 + 4y^2 = 405$ ....(ii)
Putting $y^2 = 90 - x^2$ in equation (ii), we get
$9x^2 + 4(90 - x^2) = 405$
$\Rightarrow 9x^2 + 360 - 4x^2 = 405$
$\Rightarrow 5x^2 = 405 - 360$
$\Rightarrow 5x^2 = 45$
$\Rightarrow x^2 = 9$
$\Rightarrow$ x = ± 3
But, length of a side can not be negative. Therefore, $x = 3$
Putting $x = 3$ in (i), we get
$(3)^2 + y^2 = 90$
$\Rightarrow y^2 = 90 - 9$
$\Rightarrow y^2 = 81$
$\Rightarrow$ y = ± 9
But, length of a side can not be negative. Therefore, $y = 9$
Hence, the length of the smaller side is 3 cm and the length of the larger side is 9 cm.