Sample Question Paper - 26
Mathematics-Standard (041)
Class- X, Session: 2021-22
TERM II

Time Allowed : 2 hours   *Maximum Marks : 40*

General Instructions :

1. *The question paper consists of 14 questions divided into 3 sections A, B, C.*
2. *All questions are compulsory.*
3. *Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.*
4. *Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.*
5. *Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.*

SECTION - A

1. In a class test, 50 students obtained marks are as follows. Find the modal class and the median class.

Marks	0-20	20-40	40-60	60-80	80-100
Number	4	6	25	10	5

2. The sum of the first n terms of an A.P. is $4n^2 + 2n$. Find the nth term of this A.P.

OR

The fifth term of an A.P. is 20 and the sum of its seventh and eleventh terms is 64. Find the common difference of the A.P.

3. The dimensions of a metallic cuboid are 100 cm × 80 cm × 64 cm. It is melted and recast into a cube. Find the surface area of the cube.

4. The top of two poles are connected by a 50 m wire which makes an angle of 60° with the horizontal, then find the distance (in m) between the two poles.

5. In the given figure, AP and BP are tangents to a circle with centre O, such that $AP = 5$ cm and $\angle APB = 60°$. Find the length of chord AB.
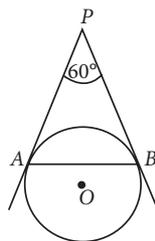

OR

In the given figure, if $AP = PB$, then show that $AC = BC$.
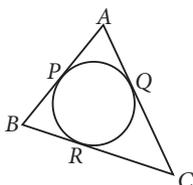
6. Find the angle of elevation of the top of a pole of height $40\sqrt{3}$ m from a point 40 m away from its foot.
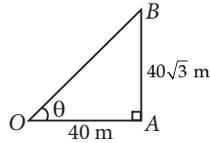
SECTION - B

7. Draw a circle of radius 1.8 cm. Take a point P on it. Draw a tangent to the circle at the point P, without using its centre.

8. Consider the following table:

Class interval	10-14	14-18	18-22	22-26	26-30
Frequency	5	11	16	25	19

Find the mode of the above data.

OR

Find the mean of the following data.

Class- interval	0-10	10-20	20-30	30-40	40-50
Frequency	3	5	9	5	3

9. From the top of a 60 m high building, the angles of depression of the top and the bottom of a tower are 45° and 60° respectively. Find the height of the tower. $\left[\text{Take } \sqrt{3} = 1.73\right]$

10. If the median of the following distribution is 46, find the missing frequencies p and q.

Class-interval	Frequency
10-20	12
20-30	30
30-40	p
40-50	65
50-60	q
60-70	25
70-80	18
Total	230

SECTION - C

11. Find the values of k for which the quadratic equation $(3k + 1)x^2 + 2(k + 1)x + 1 = 0$ has equal roots. Also find the roots.

OR

Adil and Aarav together have 45 silver coins. Both of them lost 5 silver coins each and the product of the number of silver coins they now have, is 124. Find out how many silver coins they had to start with?

12. If the sum of first 10 terms of an A.P. is 140 and the sum of first 16 terms is 320, then find the A.P. and hence find the sum of first m terms.

Case Study - 1

13. Smita always finds it confusing with the concept of tangent of a circle. But this time she has determined herself to get concepts easier. So, she started listing down about the concept of tangent of a circle. Here, some points in question form are listed by Smita in her notes. Try answering them to clear your concepts also.

(i) Find the length of the tangent drawn from a point 8 cm away from the centre of a circle of radius 6 cm.

(ii) If the angle between two radii of a circle is 140°, then find the angle between the tangents at the ends of the radii.

Case Study - 2

14. Meera and Dhara have 12 and 8 coins respectively each of radius 3.5 cm and thickness 0.5 cm. They place their coins one above the other to form solid cylinders.

Based on the above information, answer the following questions.

(i) Find the ratio of curved surface area of the cylinders made by Meera and Dhara.

(ii) Find the ratio of the volume of the cylinders made by Meera and Dhara.

Solution

MATHEMATICS STANDARD 041

Class 10 - Mathematics

1. The frequency distribution table for the given data can be drawn as :

Marks	Frequency (f_i)	Cumulative frequency (c.f.)
0-20	4	4
20-40	6	10
40-60	25	35
60-80	10	45
80-100	5	50
	$\Sigma f_i = 50$	

$\therefore$ 40-60 is the modal class as it has highest frequency.

Also, $\dfrac{N}{2} = \dfrac{50}{2} = 25$

The c.f. just greater than 25 lies in the interval 40-60. Hence the median class is 40-60.

2. We have, $S_n = 4n^2 + 2n$

$\therefore\ S_{n-1} = 4(n-1)^2 + 2(n-1)$
$= 4(n^2 + 1 - 2n) + 2n - 2$
$= 4n^2 + 4 - 8n + 2n - 2 = 4n^2 - 6n + 2$

Now, n^{th} term of the A.P., $a_n = S_n - S_{n-1}$
$= (4n^2 + 2n) - (4n^2 - 6n + 2) = 8n - 2$

OR

Let the first term be a and d be the common difference of the A.P.

Given, $a_5 = 20 \Rightarrow a + 4d = 20$ ...(i)

Also, $a_7 + a_{11} = 64$
$\Rightarrow a + 6d + a + 10d = 64 \Rightarrow 2a + 16d = 64$
$\Rightarrow a + 8d = 32$ ...(ii)

Subtracting (i) from (ii), we have
$4d = 12 \Rightarrow d = 3$

3. Volume of given cuboid = $100 \times 80 \times 64$
$= 512000 \text{ cm}^3$

Now, cuboid is melted and recast into a cube.
Let side of the cube = a cm
Also, volume of the cube = volume of the cuboid
$\Rightarrow a^3 = 512000 \Rightarrow a = 80$

$\therefore$ Surface area of cube $= 6a^2 = 6 \times (80)^2 = 38400 \text{ cm}^2$

4. Let AB and CD are the two poles and length of the wire attached to their tops i.e., AD = 50 m.
Now, draw $AE \perp CD$.

In right $\triangle ADE$,

$\cos 60° = \dfrac{AE}{AD}$
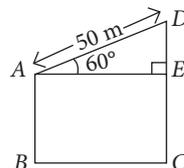
$\Rightarrow \dfrac{1}{2} = \dfrac{AE}{50}$

$\Rightarrow AE = \dfrac{50}{2} = 25$ m

Hence, distance between two poles is 25 m.

5. Given, PA and PB are tangents from an external point P.

$\therefore\ PA = PB = 5$ cm
$\Rightarrow \angle PAB = \angle ABP$ ($\because$ Angles opposite to equal sides are equal)

In $\triangle APB$, by angle sum property
$\angle APB = \angle PAB = \angle ABP = 60°$
$\Rightarrow \triangle PAB$ is an equilateral triangle.
$\therefore\ AB = PB = PA = 5$ cm

OR

Since, tangents drawn from an external point of a circle are equal.

$\therefore\ AP = AQ$; ...(i)
$BP = BR$; ...(ii)
$CR = CQ$ ...(iii)

Adding (i) and (iii), $AP + CR = AQ + CQ$
$\Rightarrow PB + CR = AC$ [$\because$ Given that $AP = PB$]
$\Rightarrow BR + CR = AC$ [Using (ii)]
$\Rightarrow BC = AC$

6. Let AB be the pole and O be the point of observation 40 m away from the foot of the pole AB.

Then, $AB = 40\sqrt{3}$ m and $OA = 40$ m

Let θ be the angle of elevation of the top of the pole.
i.e., $\angle BOA = \theta$

In right $\triangle OAB$, $\tan\theta = \dfrac{AB}{OA} = \dfrac{40\sqrt{3}}{40} = \sqrt{3}$

But we know that, $\tan 60° = \sqrt{3}$
$\Rightarrow \tan\theta = \tan 60° \Rightarrow \theta = 60°$

So, required angle of elevation is 60°.

7. Steps of construction :

Step-I : Draw a circle of radius 1.8 cm and take a point P on the circle.

Step-II : Draw a chord PQ through the point P on the circle.

Step-III : Take any point R in the major arc and join PR and RQ.

Step-IV : On taking PQ as base, construct $\angle QPY = \angle PRQ$.
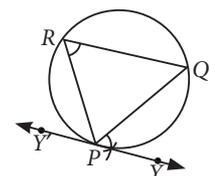
Step-V : Produce YP to Y'.

Then, $Y'PY$ is the required tangent at point P.

8. Here, the maximum frequency is 25 and the corresponding modal class is 22–26.

$\therefore$ $l = 22, h = 4, f_1 = 25, f_0 = 16$ and $f_2 = 19$

$\therefore$ Mode $= l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$= 22 + \left(\dfrac{25 - 16}{50 - 16 - 19}\right) \times 4 = 22 + \dfrac{9}{15} \times 4$

$= 22 + \dfrac{12}{5} = 24.4$

OR

Let the assumed mean $a = 25$.

Now, the frequency distribution table from the given data can be drawn as:

Class-interval	Class mark (x_i)	Frequency (f_i)	$d_i = x_i - a$	$f_i d_i$
0-10	5	3	−20	−60
10-20	15	5	−10	−50
20-30	25	9	0	0
30-40	35	5	10	50
40-50	45	3	20	60
		$\Sigma f_i = 25$		$\Sigma f_i d_i = 0$

$\therefore$ Mean, $\bar{x} = a + \dfrac{\Sigma f_i d_i}{\Sigma f_i} = 25 + \dfrac{0}{25} = 25$

9. Let AB be the building and DC be the tower of height h m.
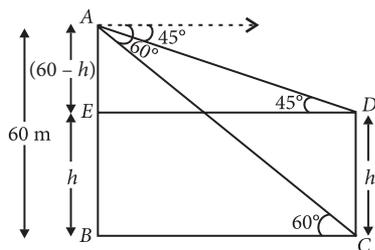
In $\triangle ABC$, $\tan 60° = \dfrac{AB}{BC}$

$\Rightarrow \sqrt{3} = \dfrac{60}{BC} \Rightarrow BC = \dfrac{60}{\sqrt{3}}$ m

In $\triangle AED$, $\tan 45° = \dfrac{AE}{ED}$

$\Rightarrow 1 = \dfrac{(60 - h)}{\dfrac{60}{\sqrt{3}}}$ $(\because BC = ED)$

$\Rightarrow \dfrac{60}{\sqrt{3}} = 60 - h \Rightarrow h = 60 - \dfrac{60}{\sqrt{3}} = 60\dfrac{(\sqrt{3} - 1)}{\sqrt{3}}$

$\Rightarrow h = 60\left(\dfrac{1.73 - 1}{1.73}\right) = 60\left(\dfrac{0.73}{1.73}\right) = 25.31$

$\therefore$ Height of tower = 25.31 m

10.

Class-interval	Frequency (f_i)	Cumulative frequency (c.f.)
10-20	12	12
20-30	30	42
30-40	p	$42 + p$
40-50	65	$107 + p$
50-60	q	$107 + p + q$
60-70	25	$132 + p + q$
70-80	18	$150 + p + q$
Total	$\Sigma f_i = 230$	

Since, median = 46, which lies in the interval 40-50, therefore its median class is 40-50.

So, $l = 40, n = 230, c.f. = 42 + p, f = 65, h = 10$

Now, median $= l + \left(\dfrac{\dfrac{n}{2} - c.f.}{f}\right) \times h$

$\therefore 46 = 40 + \left(\dfrac{\dfrac{230}{2} - (42 + p)}{65}\right) \times 10$

$\Rightarrow 6 = \dfrac{(115 - 42 - p)}{65} \times 10$

$\Rightarrow 39 = 73 - p \Rightarrow p = 34$ ...(i)

Also, $150 + p + q = 230$

$\Rightarrow 150 + 34 + q = 230$ (Using (i))

$\Rightarrow q = 230 - 184 \Rightarrow q = 46$

$\therefore$ Missing frequencies are 34 and 46.

11. We have, $(3k + 1)x^2 + 2(k + 1)x + 1 = 0$ ...(1)

Since roots are equal. $\therefore D = 0$

$\Rightarrow [2(k + 1)]^2 - 4(3k + 1) = 0$

$\Rightarrow [4(k^2 + 1 + 2k)] - 4(3k + 1) = 0$

$\Rightarrow k^2 + 2k + 1 - 3k - 1 = 0$

$\Rightarrow k^2 - k = 0 \Rightarrow k(k - 1) = 0 \Rightarrow k = 0$ or $k = 1$

When $k = 0$, (1) becomes $x^2 + 2x + 1 = 0$

$\Rightarrow (x + 1)^2 = 0 \Rightarrow x = -1, -1$

When $k = 1$, (1) becomes $4x^2 + 4x + 1 = 0$

$\Rightarrow (2x + 1)^2 = 0 \Rightarrow x = \dfrac{-1}{2}, \dfrac{-1}{2}$

Thus, equal roots of given equation are either -1 or $\dfrac{-1}{2}$.

OR

Given, Adil and Aarav together have 45 silver coins.

Let Adil has x silver coins.

Then, number of silver coins Aarav had = 45 − x

∵ Both of them lost 5 silver coins each.

∴ The number of silver coins Adil had = x − 5

and the number of silver coins Aarav had = 45 − x − 5
= 40 − x

Now, product of the number of silver coins = 124 [Given]

∴ $(x − 5)(40 − x) = 124$

⇒ $40x − x^2 − 200 + 5x = 124$

⇒ $−x^2 + 45x − 200 − 124 = 0$

⇒ $x^2 − 45x + 324 = 0$,   [Multiplying by (−1)]

which is the required quadratic equation.

Now, by factorisation method, we get

$x^2 − 36x − 9x + 324 = 0$

⇒ $x(x − 36) − 9(x − 36) = 0 ⇒ (x − 36)(x − 9)$

⇒ $x − 36 = 0$ or $x − 9 = 0 ⇒ x = 36$ or $x = 9$

When Adil has 36 silver coins, then Aarav has
45 − 36 = 9 silver coins

When Adil has 9 silver coins, then Aarav has
45 − 9 = 36 silver coins.

12. Let a be the first term and d be the common difference of the A.P.

Given, sum of first 10 terms, $S_{10} = 140$

⇒ $\frac{10}{2}[2a + (10 − 1)d] = 140 ⇒ 2a + 9d = \frac{140}{5}$

⇒ $2a + 9d = 28$ ...(i)

Also, given sum of first 16 terms, $S_{16} = 320$

⇒ $\frac{16}{2}[2a + (16 − 1)d] = 320$

⇒ $2a + 15d = \frac{320}{8} ⇒ 2a + 15d = 40$ ...(ii)

On subtracting (i) from (ii), we get $6d = 12 ⇒ d = 2$

Putting $d = 2$ in (i), we get

$2a + 9(2) = 28$

⇒ $2a = 28 − 18 ⇒ a = 10/2 = 5$

Thus, $a = 5$ and $d = 2$

∴ A.P. will be 5, 5 + 2, 5 + 2(2), 5 + 3(2),.....

i.e., 5, 7, 9, 11, ...

Also, sum of first m terms, $S_m = \frac{m}{2}[2a + (m − 1)d]$

$= \frac{m}{2}[2(5) + (m − 1)2]$

$= m(5 + m − 1) = m(m + 4) = m^2 + 4m$

13. (i) Let TP be the tangent to the circle and O be the centre of circle.

∠$OTP = 90°$ [∵ PT is tangent to the circle

∴ $PT \perp OT$]

In ΔOTP,

by Pythagoras theorem

$OP^2 = OT^2 + TP^2$

⇒ $8^2 = 6^2 + TP^2$
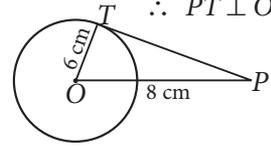
⇒ $TP^2 = 64 − 36 = 28 ⇒ TP = \sqrt{28} = 2\sqrt{7}$ cm

(ii) Let PA and QA be two tangents at the ends of radii OP and OQ respectively.

∠$POQ = 140°$ [Given]

∠$OPA = 90°$ and ∠$OQA = 90°$
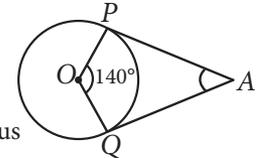
[∵ Angle between tangent and radius

through the point of contact is 90°]

In quadrilateral $OPAQ$,

∠POQ + ∠OPA + ∠OQA + ∠$PAQ = 360°$

[Angle sum property of a quadrilateral]

⇒ $140° + 90° + 90° + ∠PAQ = 360°$

⇒ ∠$PAQ = 360° − 320° = 40°$

14. We have, radius of each coin = 3.5 cm

$= \frac{35}{10}$ cm $= \frac{7}{2}$ cm

Thickness of each coin = 0.5 cm $= \frac{1}{2}$ cm

So, height of cylinder made by Meera(h_1) $= 12 \times \frac{1}{2} = 6$ cm

and height of cylinder made by Dhara (h_2) $= 8 \times \frac{1}{2} = 4$ cm

(i) Required ratio

$= \frac{\text{Curved surface area of cylinder made by Meera}}{\text{Curved surface area of cylinder made by Dhara}}$

$= \frac{2\pi r h_1}{2\pi r h_2} = \frac{h_1}{h_2} = \frac{6}{4} = \frac{3}{2}$ i.e., 3 : 2

(ii) Required ratio

$= \frac{\text{Volume of cylinder made by Meera}}{\text{Volume of cylinder made by Dhara}}$

$= \frac{\pi r^2 h_1}{\pi r^2 h_2} = \frac{h_1}{h_2} = \frac{6}{4} = \frac{3}{2}$ i.e., 3 : 2