SAMPLE QUESTION PAPER

BLUE PRINT

Time Allowed : 3 hours **Maximum Marks :** 80

S. No.	Chapter	VSA/Case based (1 mark)	SA-I (2 marks)	SA-II (3 marks)	LA (5 marks)	Total
1.	Relations and Functions	2(2)	–	1(3)	–	**3(5)**
2.	Inverse Trigonometric Functions	1(1)	1(2)	–	–	**2(3)**
3.	Matrices	2(2)	–	–	1(5)*	**3(7)**
4.	Determinants	1(1)	1(2)*	–	–	**2(3)**
5.	Continuity and Differentiability	1(1)*	1(2)	2(6)	–	**4(9)**
6.	Application of Derivatives	1(4)	1(2)	1(3)*	–	**3(9)**
7.	Integrals	1(1)*	1(2)*	1(3)	–	**3(6)**
8.	Application of Integrals	–	1(2)	1(3)	–	**2(5)**
9.	Differential Equations	1(1)*	1(2)	1(3)*	–	**3(6)**
10.	Vector Algebra	2(2)#	1(2)*	–	–	**3(4)**
11.	Three Dimensional Geometry	5(5)#	–	–	1(5)*	**6(10)**
12.	Linear Programming	–	–	–	1(5)*	**1(5)**
13.	Probability	1(4)	2(4)	–	–	**3(8)**
	Total	**18(24)**	**10(20)**	**7(21)**	**3(15)**	**38(80)**

*It is a choice based question.

#Out of the two or more questions, one/two question(s) is/are choice based.

Subject Code : **041**

MATHEMATICS

Time allowed : 3 hours ***Maximum marks : 80***

General Instructions :

1. *This question paper contains two parts A and B. Each part is compulsory. Part-A carries 24 marks and Part-B carries 56 marks.*
2. *Part-A has Objective Type Questions and Part-B has Descriptive Type Questions.*
3. *Both Part-A and Part-B have internal choices.*

Part - A :

1. *It consists of two Sections-I and II.*
2. *Section-I comprises of 16 very short answer type questions.*
3. *Section-II contains 2 case study-based questions.*

Part - B :

1. *It consists of three Sections-III, IV and V.*
2. *Section-III comprises of 10 questions of 2 marks each.*
3. *Section-IV comprises of 7 questions of 3 marks each.*
4. *Section-V comprises of 3 questions of 5 marks each.*
5. *Internal choice is provided in 3 questions of Section-III, 2 questions of Section-IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.*

PART - A

Section - I

1. If the function $f(x) = \begin{cases} kx^2, & \text{if} \quad x \le 2 \\ 3, & \text{if} \quad x > 2 \end{cases}$ is continuous at $x = 2$, then find the value of k.

OR

If $y = \log_7 (\log x)$, then find $\frac{dy}{dx}$.

2. If $\tan^{-1}(\cot\theta) = 2\theta$, then find the value of θ.

3. Find the value of $(\hat{i}+\hat{j})\times(\hat{j}+\hat{k})\cdot(\hat{k}+\hat{i})$.

OR

If lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-5}{1} = \frac{z-6}{-5}$ are mutually perpendicular, then find the value of k.

4. If a line makes angles 90°, 135°, 45° with the X, Y, Z axes respectively, then find its direction cosines.

5. Evaluate : $\int \frac{dx}{5-8x-x^2}$

OR

Evaluate : $\int_{-\pi/4}^{\pi/4} |\sin x|\, dx$

6. For matrix $A=\begin{bmatrix} 3 & 4 & -2 \\ -4 & 5 & -3 \\ 2 & 7 & 9 \end{bmatrix}$, find $\frac{1}{2}(A-A')$. (where A' is the transpose of the matrix A)

7. Find the direction cosines of the side AC of a ΔABC whose vertices are given by A (3, 5, 4), B (–2, –2, –2) and C (3, –5, 4).

OR

Show that three points $A(-2, 3, 5)$, $B(1, 2, 3)$ and $C(7, 0, -1)$ are collinear.

8. If $A = \{1, 5, 6\}$, $B = \{7, 9\}$ and $R = \{(a, b) \in A \times B : |a - b| \text{ is even}\}$. Then write the relation R.

9. Find the degree and order of the differential equation : $5x\left(\frac{dy}{dx}\right)^2 - \frac{d^2y}{dx^2} - 6y = \log x.$

OR

Solve the differential equation $(1 + x^2)\frac{dy}{dx} = e^y.$

10. If A and B are the points (– 3, 4, – 8) and (5, – 6, 4) respectively, then find the ratio in which yz-plane divides the line joining the points A and B.

11. If A is a square matrix such that $A^2 = A$, then find $(I + A)^3 - 7A$.

12. A line makes an angle of $\pi/4$ with each of X-axis and Y-axis. What angle does it make with Z-axis?

13. If $P=\begin{bmatrix} 10 & -2 \\ -5 & 1 \end{bmatrix}$, then check whether P^{-1} exists or not.

14. Write the projection of $\vec{b}+\vec{c}$ on $\vec{a}$, where $\vec{a}=2\hat{i}-2\hat{j}+\hat{k}, \vec{b}=\hat{i}+2\hat{j}-2\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$.

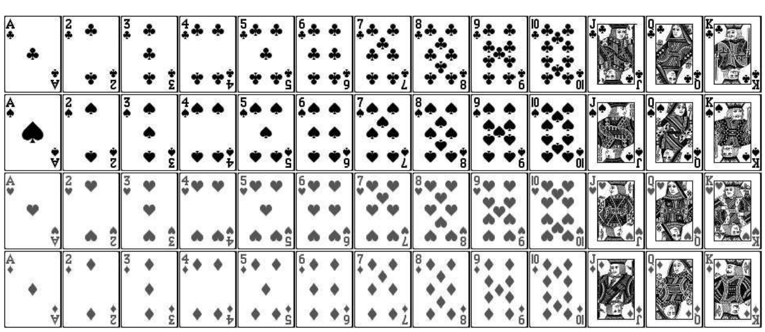
15. Let $n(A) = 4$ and $n(B) = 6$, then find the number of one-one functions from A to B.

16. A line makes 45° with OX, and equal angles with OY and OZ. Find the sum of these three angles.

Section - II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each sub-part carries 1 mark.

17. A card is lost from a pack of 52 cards. From the remaining cards of pack two cards are drawn and are found to be both spades.



Based on the above information, answer the following questions :

(i) The probability of drawing two spades, given that a card of spade is missing, is

(a) $\frac{21}{425}$ (b) $\frac{22}{425}$ (c) $\frac{23}{425}$ (d) $\frac{1}{425}$

(ii) The probability of drawing two spades, given that a card of club is missing, is

(a) $\frac{26}{425}$ (b) $\frac{22}{425}$ (c) $\frac{19}{425}$ (d) $\frac{23}{425}$

(iii) Let A be the event of drawing two spades from remaining 51 cards and E_1, E_2, E_3 and E_4 be the events that lost card is of spade, club, diamond and heart respectively, then the value of $\sum_{i=1}^{4} P(A/E_i)$ is

(a) 0.17 (b) 0.24 (c) 0.25 (d) 0.18

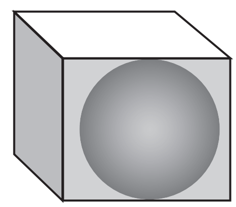
(iv) All of a sudden, missing card is found and, then two cards are drawn simultaneously without replacement. Probability that both drawn cards are aces is

(a) $\frac{1}{52}$ (b) $\frac{1}{221}$ (c) $\frac{1}{121}$ (d) $\frac{2}{221}$

(v) If two card are drawn from a well shuffled pack of 52 cards, with replacement, then probability of getting not a king in 1^{st} and 2^{nd} draw is

(a) $\frac{144}{169}$ (b) $\frac{12}{169}$ (c) $\frac{64}{169}$ (d) none of these

18. Arun got a rectangular parallelopiped shaped box and spherical ball inside it as his birthday present. Sides of the box are x, $2x$, and $x/3$, while radius of the ball is r cm.



Based on the above information, answer the following questions :

(i) If S represents the sum of volume of parallelopiped and sphere, then S can be written as

(a) $\frac{4x^3}{3}+\frac{2}{2}\pi r^2$ (b) $\frac{2x^2}{3}+\frac{4}{3}\pi r^2$

(c) $\frac{2x^3}{3}+\frac{4}{3}\pi r^3$ (d) $\frac{2}{3}x+\frac{4}{3}\pi r$

(ii) If sum of the surface areas of box and ball are given to be constant, then x is equal to

(a) $\sqrt{\frac{k^2-4\pi r^2}{6}}$ (b) $\sqrt{\frac{k^2-4\pi r}{6}}$ (c) $\sqrt{\frac{k^2-4\pi}{6}}$ (d) none of these

(iii) The radius of the ball, when S is minimum, is

(a) $\sqrt{\frac{k^2}{54+\pi}}$ (b) $\sqrt{\frac{k^2}{54+4\pi}}$ (c) $\sqrt{\frac{k^2}{64+3\pi}}$ (d) $\sqrt{\frac{k^2}{4\pi+3}}$

(iv) Relation between length of the box and radius of the ball can be represented as

(a) $x = 2r$ (b) $x=\frac{r}{2}$ (c) $x=\frac{r}{2}$ (d) $x = 3r$

(v) Minimum volume of the ball and box together is

(a) $\frac{k^2}{2(3\pi+54)^{2/3}}$ (b) $\frac{k}{(3\pi+54)^{3/2}}$ (c) $\frac{k^3}{3(4\pi+54)^{1/2}}$ (d) none of these

PART - B

Section - III

19. Find the intervals on which the function $f(x) = 2x^3 + 9x^2 + 12x + 20$ is increasing.

20. A vector $\vec{r}$ is inclined at equal angles to *OX*, *OY* and *OZ*. If the magnitude of $\vec{r}$ is 6 units, then find $\vec{r}$.

OR

Find the value of λ such that the vectors $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ are perpendicular to each other.

21. If *A* and *B* are two independent events, such that $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{5}$, then find the value of $P(A|A \cup B)$.

22. If $x \in [0, 1]$, then find the value of $\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$.

23. Evaluate : $\int \frac{\sqrt{16+(\log x)^2}}{x} dx$

OR

Evaluate : $\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x} dx$

24. Solve the differential equation : $\frac{dy}{dx} = \frac{3e^{2x} + 3e^{4x}}{e^x + e^{-x}}$

25. *A* and *B* are two events such that $P(A) \neq 0$. Find $P(B/A)$ if

(i) *A* is a subset of *B* (ii) $A \cap B = \phi$

26. Find the derivative of $[\sqrt{1-x^2}\sin^{-1}x - x]$ w.r.t. *x*.

27. Find the area bounded by the curve $x^2 + y^2 = 1$ in the first quadrant.

28. Compute the adjoint of the matrix $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 1 & 1 & 3 \end{bmatrix}$.

OR

If the matrix $\begin{bmatrix} 1 & a & 2 \\ 1 & 2 & 5 \\ 2 & 1 & 1 \end{bmatrix}$ is not invertible, then find the value of *a*.

Section - IV

29. Let $A = R - \{2\}$ and $B = R - \{1\}$. If $f: A \to B$ is a mapping defined by $f(x) = \frac{x-1}{x-2}$, then show that *f* is bijective.

30. Consider $f(x) = \begin{cases} \frac{\cos^2 x - \sin^2 x - 1}{\sqrt{x^2+1} - 1}, & \text{for } x \neq 0 \\ k, & \text{for } x = 0 \end{cases}$. If $f(x)$ is continuous at $x = 0$, then find the value of *k*.

31. Find the values of *x* for which $f(x) = (x(x-2))^2$ is an increasing function. Also, find the points on the curve, where the tangent is parallel to *x*-axis.

OR

An open box with a square base is to be made out of a given quantity of cardboard of area c^2 square units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units.

32. Evaluate : $\int_0^1 \{\tan^{-1}x + \tan^{-1}(1-x)\}\, dx$

33. If $y = x\log\left(\frac{x}{a+bx}\right)$, then prove that $x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$.

34. Solve the differential equation $\frac{dy}{dx} = \frac{e^x(\sin^2 x + \sin 2x)}{y(2\log y + 1)}$.

OR

Find the solution of the equation $\frac{dy}{dx} = \frac{y^2 - y - 2}{x^2 + 2x - 3}$.

35. Find the area enclosed between the curve $y = \log_e(x + e)$ and the coordinates axes.

Section - V

36. Find the image of the point having position vector $\hat{i} + 3\hat{j} + 4\hat{k}$ in the plane $\vec{r}\cdot(2\hat{i} - \hat{j} + \hat{k}) + 3 = 0$.

OR

Find the points on the line $\frac{x+2}{1} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of 2 units from the point (–2, –1, 3).

37. Solve the following linear programming problem (LPP) graphically.

Maximize $Z = 4x + 6y$

Subject to constraints:

$x + 2y \le 80,\ 3x + y \le 75;\ x, y \ge 0$

OR

Solve the following linear programming problem (LPP) graphically.

Minimize $Z = 30x + 20y$

Subject to constraints : $x + y \le 8,\ x + 4y \ge 12,\ 5x + 8y \ge 20;\ x, y \ge 0$

38. If $A = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$, then calculate AC, BC and $(A + B)C$. Also verify that $(A + B)C = AC + BC$.

OR

Find the matrix A satisfying the matrix equation $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} A \begin{bmatrix} 4 & 7 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

SOLUTIONS

1. Since, $f(x)$ is continuous at $x = 2$.

$\therefore \lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) \Rightarrow k(2)^2 = 3 \Rightarrow k = \frac{3}{4}$

OR

$y = \log_7 (\log x) = \frac{\log(\log x)}{\log 7}$

$\therefore \frac{dy}{dx} = \frac{1}{\log 7}\cdot\frac{1}{\log x}\cdot\frac{1}{x} \Rightarrow \frac{dy}{dx} = \frac{1}{x\log 7 \log x}$

2. We have, $\tan^{-1}(\cot\theta) = 2\theta \Rightarrow \cot\theta = \tan 2\theta$

$\Rightarrow \cot\theta = \cot\left(\frac{\pi}{2} - 2\theta\right)$

$\Rightarrow \theta = \frac{\pi}{2} - 2\theta \Rightarrow 3\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{6}$

3. We have,

$(\hat{i}+\hat{j})\times(\hat{j}+\hat{k})\cdot(\hat{k}+\hat{i}) = (\hat{i}\times\hat{j}+\hat{i}\times\hat{k}+\hat{j}\times\hat{k})\cdot(\hat{k}+\hat{i})$

$= (\hat{k}-\hat{j}+\hat{i})\cdot(\hat{k}+\hat{i}) = \hat{k}\cdot\hat{k}+\hat{i}\cdot\hat{i} \quad (\because \hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0)$

$= |\hat{k}|^2 + |\hat{i}|^2 = 1+1 = 2$

OR

Lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and

$\frac{x-1}{3k} = \frac{y-5}{1} = \frac{z-6}{-5}$ are perpendicular if

$a_1a_2 + b_1b_2 + c_1c_2 = 0$.

$\Rightarrow -3(3k) + 2k + 2(-5) = 0 \Rightarrow k = -\frac{10}{7}$

4. Here $\alpha = 90°, \beta = 135°, \gamma = 45°$

Direction cosines are $l = \cos\alpha = \cos 90° = 0$,

$m = \cos\beta = \cos 135° = \frac{-1}{\sqrt{2}}$, $n = \cos\gamma = \cos 45° = \frac{1}{\sqrt{2}}$

5. Let $I = \int\frac{dx}{5-8x-x^2} = \int\frac{dx}{21-(x+4)^2}$

$= \int\frac{dx}{(\sqrt{21})^2-(x+4)^2} = \frac{1}{2\sqrt{21}}\log\left|\frac{\sqrt{21}+x+4}{\sqrt{21}-x-4}\right| + C$

OR

Let $I = \int_{-\pi/4}^{\pi/4} |\sin x|\,dx = 2\int_0^{\pi/4}\sin x\,dx$

$= 2[-\cos x]_0^{\pi/4} = -2\left[\frac{1}{\sqrt{2}}-1\right] = 2-\sqrt{2}$

6. We have, $A = \begin{bmatrix} 3 & 4 & -2 \\ -4 & 5 & -3 \\ 2 & 7 & 9 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 3 & -4 & 2 \\ 4 & 5 & 7 \\ -2 & -3 & 9 \end{bmatrix}$

$\therefore \frac{1}{2}(A-A') = \frac{1}{2}\begin{bmatrix} 0 & 8 & -4 \\ -8 & 0 & -10 \\ 4 & 10 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 4 & -2 \\ -4 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$.

7. The direction cosines of the line AC are

$\frac{3-3}{\sqrt{0^2+(-10)^2+(0)^2}}, \frac{-5-5}{\sqrt{0^2+(-10)^2+0^2}},$

$\frac{4-4}{\sqrt{0^2+(-10)^2-0^2}}$

$= 0, -1, 0$

OR

Direction ratios of the line $AB = 3, -1, -2$,

Direction ratios of the line $BC = 6, -2, -4$

Now, $\frac{3}{6} = \frac{-1}{-2} = \frac{-2}{-4}$

Since the direction cosines of the line AB and BC are proportional and B is the common point. Hence, the points are collinear.

8. We have, $A \times B = \{(1, 7), (1, 9), (5, 7), (5, 9), (6, 7), (6, 9)\}$

$\therefore R = \{(1, 7), (1, 9), (5, 7), (5, 9)\}$

9. Here, highest order derivative is $\frac{d^2y}{dx^2}$, so its order is 2 and power of $\frac{d^2y}{dx^2}$ is one, so its degree is 1.

OR

We have, $(1 + x^2)\frac{dy}{dx} = e^y$

$\Rightarrow \frac{dy}{e^y} = \frac{dx}{1+x^2} \Rightarrow \int\frac{dy}{e^y} = \int\frac{dx}{1+x^2}$

$\Rightarrow -e^{-y} = \tan^{-1}x + C \Rightarrow e^{-y} + \tan^{-1}x + C_1 = 0.$

10. Let λ be the ratio in which yz-plane divides the line joining the points $(-3, 4, -8)$ and $(5, -6, 4)$. The co-ordinates of any point on the line joining the two points are $\left(\frac{5\lambda-3}{\lambda+1}, \frac{-6\lambda+4}{\lambda+1}, \frac{4\lambda-8}{\lambda+1}\right)$. If the point is in yz-plane, then its x-coordinate should be zero.

$\therefore \quad \dfrac{5\lambda-3}{\lambda+1}=0 \Rightarrow 5\lambda-3=0 \Rightarrow \lambda=\dfrac{3}{5}$

So, the required ratio is 3 : 5.

11. We have, $A^2 = A$ ...(i)

Now, $(I + A)^3 - 7A = I^3 + A^3 + 3A^2I + 3AI^2 - 7A$

$= I + A^2A + 3A^2I + 3AI - 7A$

$= I + AA + 3A + 3A - 7A$ [Using (i)]

$= I + A^2 - A = I + A - A$ [Using (i)]

$= I$

12. Let γ be the required angle. Then

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow \cos^2\gamma = 1-\dfrac{1}{2}-\dfrac{1}{2}=0 \Rightarrow \cos\gamma = 0$

$\Rightarrow \gamma = \dfrac{\pi}{2}$

13. Since $|P| = \begin{vmatrix} 10 & -2 \\ -5 & 1 \end{vmatrix} = 10-10=0$

$\therefore$ P^{-1} does not exist.

14. Here, $\vec{a} = 2\hat{i}-2\hat{j}+\hat{k}$

$\vec{b} = \hat{i}+2\hat{j}-2\hat{k}$ and $\vec{c} = 2\hat{i}-\hat{j}+4\hat{k}$

$\Rightarrow \vec{b}+\vec{c} = 3\hat{i}+\hat{j}+2\hat{k}$

$\therefore$ Projection of $\vec{b}+\vec{c}$ on $\vec{a}$

$= \dfrac{(\vec{b}+\vec{c})\cdot\vec{a}}{|\vec{a}|} = \dfrac{(3\hat{i}+\hat{j}+2\hat{k})\cdot(2\hat{i}-2\hat{j}+\hat{k})}{|2\hat{i}-2\hat{j}+\hat{k}|}$

$= \dfrac{3\times2+1\times(-2)+2\times1}{\sqrt{2^2+(-2)^2+1^2}} = \dfrac{6}{3} = 2$

15. Number of one-one functions from A to B

$= {}^6P_4 = 6\cdot5\cdot4\cdot3 = 360$

16. Here $\alpha = 45°$ and $\beta = \gamma$

$\therefore$ $\cos\alpha = \dfrac{1}{\sqrt{2}}$ and $\cos\beta = \cos\gamma$

Since, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow 1/2 + \cos^2\beta + \cos^2\beta = 1$

$\Rightarrow 2\cos^2\beta = 1/2 \Rightarrow \cos\beta = \dfrac{1}{2} \Rightarrow \beta = \gamma = 60°$

$\therefore$ $\alpha + \beta + \gamma = 45° + 60° + 60° = 165°$

17. (i) (b) : Required probability $= \dfrac{{}^{12}C_2}{{}^{51}C_2}$

$= \dfrac{12\times11}{51\times50} = \dfrac{22}{425}$

(ii) (a) : Required probability $= \dfrac{{}^{13}C_2}{{}^{51}C_2} = \dfrac{13\times12}{51\times50} = \dfrac{26}{425}$

(iii) (b) : We have, $P(E_1) = P(E_2) = P(E_3) = P(E_4)$

$= \dfrac{13}{52} = \dfrac{1}{4}$

$P(A/E_1) = \dfrac{{}^{12}C_2}{{}^{51}C_2} = \dfrac{22}{425}$

$P(A/E_2) = \dfrac{{}^{13}C_2}{{}^{51}C_2} = \dfrac{26}{425}$

$P(A/E_3) = P(A/E_4) = \dfrac{26}{425}$

$\therefore \sum_{i=1}^{4} P(A/E_i) = \dfrac{22}{425}+\dfrac{26}{425}+\dfrac{26}{425}+\dfrac{26}{425} = \dfrac{100}{425} = 0.24$

(iv) (b) : P(getting both aces) $= \dfrac{{}^4C_2}{{}^{52}C_2} = \dfrac{4\times3}{52\times51} = \dfrac{1}{221}$

(v) (a) : P(drawing a king) $= \dfrac{4}{52} = \dfrac{1}{13}$

$\therefore$ P(not drawing a king) $= 1-\dfrac{1}{13} = \dfrac{12}{13}$

$\therefore$ Required probability $= \dfrac{12}{13}\times\dfrac{12}{13} = \dfrac{144}{169}$

18. (i) (c) : Let S be the sum of volume of parallelopiped and sphere, then

$$S = x(2x)\left(\frac{x}{3}\right)+\frac{4}{3}\pi r^3 = \frac{2x^3}{3}+\frac{4}{3}\pi r^3 \quad \text{... (1)}$$

(ii) (a) : Since, sum of surface area of box and sphere is given to be constant.

$\therefore$ $2\left(x\times2x+2x\times\dfrac{x}{3}+\dfrac{x}{3}\times x\right)+4\pi r^2 = k^2$ (say)

$\Rightarrow 6x^2 + 4\pi r^2 = k^2$

$\Rightarrow x^2 = \dfrac{k^2-4\pi r^2}{6} \Rightarrow x = \sqrt{\dfrac{k^2-4\pi r^2}{6}}$ ... (2)

(iii) (b) : From (1) and (2), we get

$$S = \frac{2}{3}\left(\frac{k^2-4\pi r^2}{6}\right)^{3/2}+\frac{4}{3}\pi r^3$$

$$= \frac{2}{3\times6\sqrt{6}}(k^2-4\pi r^2)^{3/2}+\frac{4}{3}\pi r^3$$

$$\Rightarrow \frac{dS}{dr} = \frac{1}{9\sqrt{6}}\frac{3}{2}(k^2-4\pi r^2)^{1/2}(-8\pi r)+4\pi r^2$$

$$= 4\pi r\left[r-\frac{1}{3\sqrt{6}}\sqrt{k^2-4\pi r^2}\right]$$

For maximum/minimum, $\dfrac{dS}{dr} = 0$

$$\Rightarrow \frac{-4\pi r}{3\sqrt{6}}\sqrt{k^2-4\pi r^2} = -4\pi r^2$$

$\Rightarrow k^2 - 4\pi r^2 = 54r^2$

$$\Rightarrow r^2 = \frac{k^2}{54+4\pi} \Rightarrow r = \sqrt{\frac{k^2}{54+4\pi}} \quad \ldots (3)$$

(iv) (d): Since, $x^2 = \frac{k^2 - 4\pi r^2}{6} = \frac{1}{6}\left[k^2 - 4\pi\left(\frac{k^2}{54+4\pi}\right)\right]$ [From (2) and (3)]

$$= \frac{9k^2}{54+4\pi} = 9\left(\frac{k^2}{54+4\pi}\right) = 9r^2 = (3r)^2$$

$\Rightarrow x = 3r$

(v) (c) : Minimum volume is given by

$$V = \frac{2}{3}x^3 + \frac{4}{3}\pi r^3 = \frac{2}{3}(3r)^3 + \frac{4}{3}\pi r^3$$

$$= 18r^3 + \frac{4}{3}\pi r^3 = \left(18 + \frac{4}{3}\pi\right)r^3$$

$$= \left(18 + \frac{4}{3}\pi\right)\left(\frac{k^2}{54+4\pi}\right)^{3/2} \quad \text{[Using (3)]}$$

$$= \frac{1}{3}\frac{k^3}{(54+4\pi)^{1/2}}$$

19. Given, $f(x) = 2x^3 + 9x^2 + 12x + 20$

$\Rightarrow f'(x) = 6x^2 + 18x + 12$

$= 6(x^2 + 3x + 2) = 6(x + 1)(x + 2)$

For $f(x)$ to be increasing, $f'(x) > 0$

$\Rightarrow 6(x + 1)(x + 2) > 0$

$\Rightarrow (x + 1)(x + 2) > 0$

$\Rightarrow x + 1 > 0, x + 2 > 0$ or $x + 1 < 0, x + 2 < 0$

$\Rightarrow x > -1$ or $x < -2$

$\Rightarrow x \in (-1, \infty)$ or $x \in (-\infty, -2)$

$\therefore$ f is increasing in $(-\infty, -2) \cup (-1, \infty)$.

20. Suppose $\vec{r}$ makes an angle α with each of the axes OX, OY and OZ. Then, its direction cosines are $l = \cos\alpha$, $m = \cos\alpha$, $n = \cos\alpha \Rightarrow l = m = n$

Now, $l^2 + m^2 + n^2 = 1 \Rightarrow 3l^2 = 1 \Rightarrow l = \pm\frac{1}{\sqrt{3}}$

$\therefore \vec{r} = |\vec{r}|(l\hat{i} + m\hat{j} + n\hat{k})$

$$\Rightarrow \vec{r} = 6\left(\pm\frac{1}{\sqrt{3}}\hat{i} \pm \frac{1}{\sqrt{3}}\hat{j} \pm \frac{1}{\sqrt{3}}\hat{k}\right) = 2\sqrt{3}(\pm\hat{i} \pm \hat{j} \pm \hat{k}).$$

OR

If the vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other, then $\vec{a}\cdot\vec{b} = 0$.

$\Rightarrow (2\hat{i} + \lambda\hat{j} + \hat{k})\cdot(\hat{i} - 2\hat{j} + 3\hat{k}) = 0$

$\Rightarrow (2)(1) + \lambda(-2) + (1)(3) = 0$

$\Rightarrow -2\lambda + 5 = 0 \Rightarrow \lambda = \frac{5}{2}$

21. We have, $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{5}$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{2} + \frac{1}{5} - \left(\frac{1}{2}\right)\cdot\left(\frac{1}{5}\right)$ (A and B are independent events)

$= \frac{3}{5}$

$$\therefore P(A/A \cup B) = \frac{P(A \cap (A \cup B))}{P(A \cup B)}$$

$$= \frac{P(A)}{P(A \cup B)} = \frac{1/2}{3/5} = \frac{5}{6}$$

22. Let $x = \tan^2\theta \Rightarrow \sqrt{x} = \tan\theta \Rightarrow \theta = \tan^{-1}\sqrt{x}$

Now, $\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$= \frac{1}{2}\cos^{-1}(\cos 2\theta) = \frac{1}{2}(2\theta) = \theta = \tan^{-1}\sqrt{x}$

23. Let $I = \int \frac{\sqrt{16 + (\log x)^2}}{x}dx$

Put $\log x = t \Rightarrow \frac{1}{x}dx = dt$

$\therefore I = \int\sqrt{16 + t^2}\,dt$

$$= \frac{t}{2}\sqrt{16+t^2} + \frac{16}{2}\log|t + \sqrt{16+t^2}| + c$$

$$\therefore I = \frac{1}{2}\log x\sqrt{16 + (\log x)^2}$$

$$+ 8\log|\log x + \sqrt{16 + (\log x)^2}| + c$$

OR

Let $I = \int_0^{\pi/2} \frac{\sin x}{1 + \cos^2 x}dx$

Put $\cos x = t \Rightarrow -\sin x\,dx = dt$

When $x = 0$, $t = 1$ and when $x = \frac{\pi}{2}, t = 0$

$$\therefore I = -\int_1^0 \frac{dt}{1+t^2} = -\left[\tan^{-1}t\right]_1^0$$

$$= -[\tan^{-1}0 - \tan^{-1}1] = \frac{\pi}{4}$$

24. We have, $\frac{dy}{dx} = \frac{3e^{2x} + 3e^{4x}}{e^x + e^{-x}}$

$$\Rightarrow \int dy = \int\frac{3e^{2x}(1 + e^{2x})}{e^{-x}(e^{2x} + 1)}dx \quad \text{[Integrating both sides]}$$

$$\Rightarrow y = \int 3e^{3x}dx = \frac{3e^{3x}}{3} + c \Rightarrow y = e^{3x} + c$$

25. (i) Since, A is a subset of B. $\therefore\ A \subset B$

$\Rightarrow A \cap B = A$

$\therefore\ P(A \cap B) = P(A)$ ... (i)

Now, $P(B/A) = \dfrac{P(B \cap A)}{P(A)} = \dfrac{P(A)}{P(A)}$ [Using (i)]

$= 1$

(ii) If $A \cap B = \phi \Rightarrow P(A \cap B) = 0$

$\therefore\ P(B/A) = \dfrac{P(B \cap A)}{P(A)} = \dfrac{0}{P(A)} = 0$

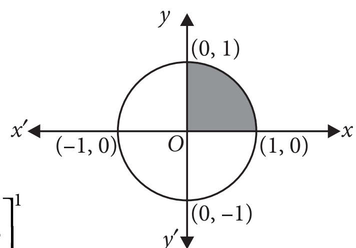
26. We have, $\dfrac{d}{dx}[(\sqrt{1-x^2})\sin^{-1}x - x]$

$= (\sqrt{1-x^2}) \cdot \dfrac{d}{dx}(\sin^{-1}x) + (\sin^{-1}x) \cdot \dfrac{d}{dx}(\sqrt{1-x^2}) - 1$

$= (\sqrt{1-x^2}) \cdot \dfrac{1}{(\sqrt{1-x^2})} + (\sin^{-1}x) \cdot \dfrac{1}{2}(1-x^2)^{-1/2} \cdot (-2x) - 1$

$= 1 - \dfrac{x\sin^{-1}x}{\sqrt{1-x^2}} - 1 = \dfrac{-x\sin^{-1}x}{\sqrt{1-x^2}}$

27. We have, $x^2 + y^2 = 1$, a circle with centre (0, 0) and radius = 1.



Required area

= area of shaded region

$A = \int_0^1 \sqrt{1-x^2}\,dx$

$= \left[\dfrac{x}{2}\sqrt{1-x^2} + \dfrac{1}{2}\sin^{-1}\dfrac{x}{1}\right]_0^1$

$= \left[\dfrac{1}{2}\sin^{-1}1\right] = \left(\dfrac{1}{2} \times \dfrac{\pi}{2}\right) = \dfrac{\pi}{4}$ sq. unit

28. Let $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 1 & 1 & 3 \end{bmatrix}$

$A_{11} = (-1)^{1+1}\begin{vmatrix} 1 & 0 \\ 1 & 3 \end{vmatrix} = 3;\ A_{12} = (-1)^{1+2}\begin{vmatrix} 5 & 0 \\ 1 & 3 \end{vmatrix} = -15$

$A_{13} = (-1)^{1+3}\begin{vmatrix} 5 & 1 \\ 1 & 1 \end{vmatrix} = 4;\ A_{21} = (-1)^{2+1}\begin{vmatrix} 0 & -1 \\ 1 & 3 \end{vmatrix} = -1$

$A_{22} = (-1)^{2+2}\begin{vmatrix} 2 & -1 \\ 1 & 3 \end{vmatrix} = 7;\ A_{23} = (-1)^{2+3}\begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} = -2;$

$A_{31} = (-1)^{3+1}\begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix} = 1;\ A_{32} = (-1)^{3+2}\begin{vmatrix} 2 & -1 \\ 5 & 0 \end{vmatrix} = -5;$

$A_{33} = (-1)^{3+3}\begin{vmatrix} 2 & 0 \\ 5 & 1 \end{vmatrix} = 2$

$\therefore\ \text{adj}\,A = \begin{bmatrix} 3 & -15 & 4 \\ -1 & 7 & -2 \\ 1 & -5 & 2 \end{bmatrix}' = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 7 & -5 \\ 4 & -2 & 2 \end{bmatrix}$

OR

The matrix is not invertible if $\begin{vmatrix} 1 & a & 2 \\ 1 & 2 & 5 \\ 2 & 1 & 1 \end{vmatrix} = 0$

$\Rightarrow 1(2 - 5) - a(1 - 10) + 2(1 - 4) = 0$

$\Rightarrow -3 + 9a - 6 = 0 \Rightarrow a = 1$

29. We have, $f(x) = \dfrac{x-1}{x-2}$

For one-one : Let $x, y \in A$ and consider $f(x) = f(y)$

$\Rightarrow \dfrac{x-1}{x-2} = \dfrac{y-1}{y-2}$

$\Rightarrow (x - 1)(y - 2) = (x - 2)(y - 1)$

$\Rightarrow xy - y - 2x + 2 = xy - x - 2y + 2 \Rightarrow x = y$

Thus, $f(x) = f(y) \Rightarrow x = y$ for all $x, y \in A$

So, f is one-one.

For onto : Let y be an arbitrary element of B. Then,

$f(x) = y \Rightarrow \dfrac{x-1}{x-2} = y \Rightarrow (x - 1) = y(x - 2) \Rightarrow x = \dfrac{1-2y}{1-y}$

Clearly, $x = \dfrac{1-2y}{1-y}$ is a real number for all $y \neq 1$.

Also, $\dfrac{1-2y}{1-y} \neq 2$ for any y, for, if we take $\dfrac{1-2y}{1-y} = 2$, then we get $1 = 2$, which is wrong.

So, f is onto. Hence, f is a bijective.

30. $f(0) = k$ (Given) ...(i)

Since, $f(x)$ is continuous at $x = 0$.

$\therefore\ f(0) = \lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^+} f(x)$

Now, $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} \dfrac{\cos^2 x - \sin^2 x - 1}{\sqrt{x^2+1} - 1}$

$= \lim\limits_{x\to 0} \dfrac{1 - \sin^2 x - \sin^2 x - 1}{\sqrt{x^2+1} - 1} \times \dfrac{\sqrt{x^2+1}+1}{\sqrt{x^2+1}+1}$

$= \lim\limits_{x\to 0} \dfrac{-2\sin^2 x(\sqrt{x^2+1}+1)}{x^2+1-1}$

$= \lim\limits_{x\to 0} -2\dfrac{\sin^2 x}{x^2}.(\sqrt{x^2+1}+1)$

$= -2\left(\lim\limits_{x\to 0}\dfrac{\sin^2 x}{x^2}\right) \times \lim\limits_{x\to 0}(\sqrt{x^2+1}+1)$

$= -2(1)^2(1 + 1) = -4$ ...(ii)

From (i) and (ii), we get $k = -4$.

31. Given, $f(x) = (x(x - 2))^2 = x^2(x - 2)^2$, $D_f = R$.

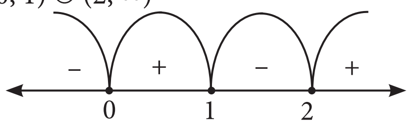
Differentiating w.r.t. x, we get

$f'(x) = x^2 \cdot 2(x - 2) + (x - 2)^2 \cdot 2x$

$= 2x(x-2)(x+x-2) = 2x(x-2)(2x-2) = 4x(x-1)(x-2)$

Now, the given function f is (strictly) increasing iff $f'(x) > 0$

$\Rightarrow \; x \in (0, 1) \cup (2, \infty)$



Further, the tangents will be parallel to x-axis iff $f'(x) = 0$

$\Rightarrow \; x = 0, 1, 2$

The given curve is $y = x^2(x-2)^2$

When $x = 0, y = 0$;

When $x = 1, y = 1^2(1-2)^2 = 1 \times (-1)^2 = 1 \times 1 = 1$;

When $x = 2, y = 2^2(2-2)^2 = 4 \times 0 = 0$.

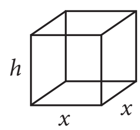
$\therefore$ The points on the given curve, where the tangents are parallel to x-axis are (0, 0), (1, 1) and (2, 0).

OR

Let h be height and x be the side of the square base of the open box.

Then its area $= x \times x + 4\,h \times x = c^2$ (given)

$$\Rightarrow \; h = \frac{c^2 - x^2}{4x}$$



Now V = volume of the box

$$= x^2 h = x^2 \cdot \frac{c^2 - x^2}{4x} = \frac{1}{4}(c^2 x - x^3)$$

$$\Rightarrow \; \frac{dV}{dx} = \frac{1}{4}(c^2 - 3x^2) \text{ and } \frac{d^2V}{dx^2} = \frac{1}{4}(-6x) = \frac{-3}{2}x$$

For maxima or minima $\dfrac{dV}{dx} = 0 \Rightarrow x^2 = \dfrac{c^2}{3}$

$$\Rightarrow \; x = \frac{c}{\sqrt{3}} \quad (\because x \nless 0)$$

For this value of x, $\dfrac{d^2V}{dx^2} < 0$

$\Rightarrow V$ is maximum at $x = \dfrac{c}{\sqrt{3}}$ and its maximum volume is,

$$V = \frac{1}{4}x(c^2 - x^2) = \frac{1}{4} \cdot \frac{c}{\sqrt{3}}\left(c^2 - \frac{c^2}{3}\right) = \frac{c^3}{6\sqrt{3}} \text{ cubic units.}$$

32. Consider, $\displaystyle\int_0^1 \{\tan^{-1}x + \tan^{-1}(1-x)\}\,dx$

$$= \int_0^1 \tan^{-1}x\,dx + \int_0^1 \tan^{-1}(1-x)dx$$

$$= \int_0^1 \tan^{-1}x\,dx + \int_0^1 \tan^{-1}\{1-(1-x)\}dx$$

$$\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

$$= \int_0^1 \tan^{-1}x\,dx + \int_0^1 \tan^{-1}x\,dx = 2\int_0^1 \tan^{-1}x\,dx$$

$$= 2[(\tan^{-1}x)\cdot x]_0^1 - 2\int_0^1 \frac{x}{(1+x^2)}dx$$

$$= 2[(\tan^{-1}1)\cdot 1 - 0\} - [\log(1+x^2)]_0^1$$

$$= \left(2 \times \frac{\pi}{4}\right) - (\log 2 - \log 1) = \left(\frac{\pi}{2} - \log 2\right)$$

33. Here, $y = x\log\left(\dfrac{x}{a+bx}\right)$ ... (i)

$\Rightarrow \; y = x[\log x - \log(a+bx)] = x\log x - x\log(a+bx)$

$$\Rightarrow \; \frac{dy}{dx} = x \cdot \frac{1}{x} + 1 \cdot \log x - \left[1 \cdot \log(a+bx) + x \cdot \frac{1}{a+bx} \cdot b\right]$$

$$= 1 - \frac{bx}{a+bx} + \log x - \log(a+bx)$$

$$\Rightarrow \; \frac{dy}{dx} = \frac{a}{a+bx} + \log\left(\frac{x}{a+bx}\right)$$

$$\Rightarrow \; \frac{dy}{dx} = \frac{a}{a+bx} + \frac{y}{x} \qquad \text{[Using (i)]} \qquad \text{...(ii)}$$

Again differentiating (ii) w.r.t. x, we get

$$\frac{d^2y}{dx^2} = a \cdot (-1)(a+bx)^{-2} \cdot b + \frac{x\dfrac{dy}{dx} - y}{x^2}$$

$$= \frac{-ab}{(a+bx)^2} + \frac{a}{x(a+bx)} = \frac{-abx + a(a+bx)}{x(a+bx)^2} = \frac{a^2}{x(a+bx)^2}$$

Now, $\text{R.H.S.} = \left(x\dfrac{dy}{dx} - y\right)^2$

$$= \left\{x \cdot \left[\frac{a}{a+bx} + \frac{y}{x}\right] - y\right\}^2 = \left(\frac{ax}{a+bx}\right)^2$$

and $\text{L.H.S.} = x^3 \dfrac{d^2y}{dx^2} = \dfrac{a^2x^2}{(a+bx)^2} = \left[\dfrac{ax}{a+bx}\right]^2 = \text{R.H.S.}$

34. We have, $\dfrac{dy}{dx} = \dfrac{e^x(\sin^2 x + \sin 2x)}{y(2\log y + 1)}$

$$\Rightarrow \; \int y(2\log y + 1)\,dy = \int e^x(\sin^2 x + \sin 2x)\,dx$$

$$\Rightarrow \; 2\int y\log y\,dy + \int y\,dy = \int e^x(\sin^2 x + \sin 2x)dx$$

$$\Rightarrow \; 2\left[\log|y| \cdot \frac{y^2}{2} - \int \frac{1}{y} \times \frac{y^2}{2}dy\right] + \frac{y^2}{2} = e^x \sin^2 x + C$$

$$\left[\because \int e^x(f(x) + f'(x))dx = e^x f(x) + C\right]$$

$$\Rightarrow \; y^2\log|y| - \frac{y^2}{2} + \frac{y^2}{2} = e^x \sin^2 x + C$$

$\Rightarrow \; y^2\log|y| = e^x\sin^2 x + C$, which is required solution.

OR

We have $\dfrac{dy}{y^2 - y - 2} = \dfrac{dx}{x^2 + 2x - 3}$

Integrating both sides, we get

$$\int \frac{dy}{y^2-y-2} = \int \frac{dx}{x^2+2x-3}$$

$$\Rightarrow \int \frac{dy}{\left(y-\frac{1}{2}\right)^2-\left(\frac{3}{2}\right)^2} = \int \frac{dx}{(x+1)^2-2^2}+c$$

$$\Rightarrow \frac{1}{2\cdot\frac{3}{2}}\log\left|\frac{y-\frac{1}{2}-\frac{3}{2}}{y-\frac{1}{2}+\frac{3}{2}}\right| = \frac{1}{2\cdot 2}\log\left|\frac{x+1-2}{x+1+2}\right|+c$$

$$\Rightarrow \frac{1}{3}\log\left|\frac{y-2}{y+1}\right| = \frac{1}{4}\log\left|\frac{x-1}{x+3}\right|+c$$

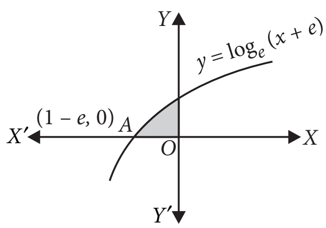
35. The bounded area is as shown in figure.

Curve is $y = \log_e (x + e)$

If $y = 0$, then $x = 1 - e$

$A \equiv (1 - e, 0)$

Required area is



$$A = \int_{1-e}^{0} \log_e (x+e)\, dx$$

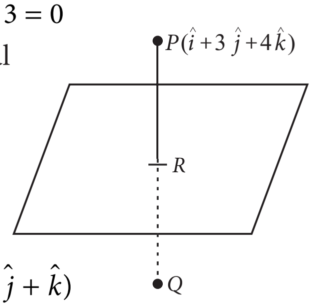
Put $x + e = t \Rightarrow dx = dt$ and $x = 1 - e \Rightarrow t = 1$ and $x = 0 \Rightarrow t = e$

$$A = \int_1^e \log t\, dt = \left[t\log t - t\right]_1^e = e\log e - e - 0 + 1$$

$= 1$ sq. unit

36. Let Q be the image of the point $P(\hat{i} + 3\hat{j} + 4\hat{k})$ in the plane $\vec{r}\cdot(2\hat{i} - \hat{j} + \hat{k}) + 3 = 0$

Then, PQ is normal to the plane. Since PQ passes through P and is normal to the given plane, therefore equation of line PQ is



$\vec{r} = (\hat{i} + 3\hat{j} + 4\hat{k}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$

Since Q lies on line PQ, so let the position vector of Q be $(\hat{i} + 3\hat{j} + 4\hat{k}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$

$= (1 + 2\lambda)\hat{i} + (3 - \lambda)\hat{j} + (4 + \lambda)\hat{k}.$

Since, R is the mid-point of PQ. Therefore, position vector of R is

$$\frac{[(1+2\lambda)\hat{i}+(3-\lambda)\hat{j}+(4+\lambda)\hat{k}]+[\hat{i}+3\hat{j}+4\hat{k}]}{2}$$

$$= (\lambda+1)\hat{i} + \left(3-\frac{\lambda}{2}\right)\hat{j} + \left(4+\frac{\lambda}{2}\right)\hat{k}$$

Since R lies on the plane $\vec{r}\cdot(2\hat{i} - \hat{j} + \hat{k}) + 3 = 0$

$$\Rightarrow \left\{(\lambda+1)\hat{i}+\left(3-\frac{\lambda}{2}\right)\hat{j}+\left(4+\frac{\lambda}{2}\right)\hat{k}\right\}\cdot(2\hat{i}-\hat{j}+\hat{k})+3=0$$

$$\Rightarrow 2\lambda + 2 - 3 + \frac{\lambda}{2} + 4 + \frac{\lambda}{2} + 3 = 0 \Rightarrow \lambda = -2$$

Thus, the position vector of Q is

$(\hat{i} + 3\hat{j} + 4\hat{k}) - 2(2\hat{i} - \hat{j} + \hat{k}) = -3\hat{i} + 5\hat{j} + 2\hat{k}.$

OR

The given line is $\frac{x+2}{1} = \frac{y+1}{2} = \frac{z-3}{2}$ ...(i)

Let $P(-2, -1, 3)$ lies on the line.

The direction ratios of line (i) are 1, 2, 2

$\therefore$ The direction cosines of line are $\frac{1}{3}, \frac{2}{3}, \frac{2}{3}$

Equation (i) may be written as

$$\frac{x+2}{\frac{1}{3}} = \frac{y+1}{\frac{2}{3}} = \frac{z-3}{\frac{2}{3}} \quad \text{...(ii)}$$

Coordinates of any point on the line (ii) may be taken as $\left(\frac{1}{3}r-2, \frac{2}{3}r-1, \frac{2}{3}r+3\right)$

Let $Q \equiv \left(\frac{1}{3}r-2, \frac{2}{3}r-1, \frac{2}{3}r+3\right)$

Given $|r| = 2$, $\therefore$ $r = \pm 2$

Putting the values of r, we have

$Q \equiv \left(-\frac{4}{3}, \frac{1}{3}, \frac{13}{3}\right)$ or $Q \equiv \left(\frac{-8}{3}, \frac{-7}{3}, \frac{5}{3}\right)$

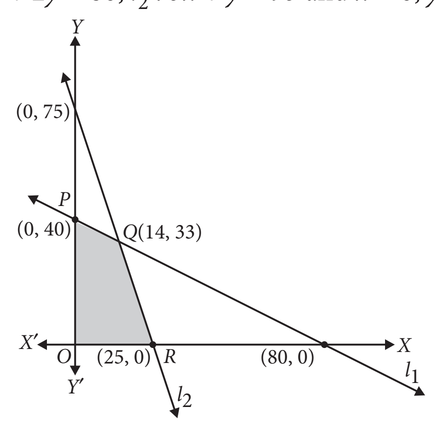
37. We have maximize $Z = 4x + 6y$.

Subject to constraints :

$x + 2y \le 80$, $3x + y \le 75$ and $x \ge 0$, $y \ge 0$

Now we draw the graphs of the lines

$l_1 : x + 2y = 80$, $l_2 : 3x + y = 75$ and $x = 0$, $y = 0$.



We obtain shaded region as the feasible region.

The lines l_1 and l_2 intersect at $Q(14, 33)$.

Thus, the vertices of the feasible region are $P(0, 40)$, $Q(14, 33)$, $R(25, 0)$ and $O(0, 0)$.

Corner Points	Value of $Z = 4x + 6y$
$P(0, 40)$	240
$Q(14, 33)$	254 (Maximum)
$R(25, 0)$	100
$O(0, 0)$	0

Thus, Z has maximum value 254 at $Q(14, 33)$.

OR

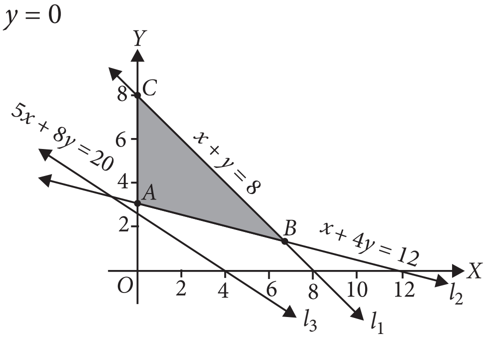
We have minimize $Z = 30x + 20y$.

Subject to constraints :

$$x + y \le 8,\ x + 4y \ge 12,\ 5x + 8y \ge 20,\ x, y \ge 0$$

Now, we draw the graphs of

$l_1 : x + y = 8$, $l_2 : x + 4y = 12$, $l_3 : 5x + 8y = 20$ and $x = 0$, $y = 0$



Shaded region ABC is the required feasible region. $B\left(\frac{20}{3}, \frac{4}{3}\right)$ is the point of intersection of the lines l_1 and l_2.

Thus, the vertices of the feasible region are

$$A(0, 3),\ B\left(\frac{20}{3}, \frac{4}{3}\right) \text{ and } C(0, 8).$$

Corner Points	Value of $Z = 30x + 20y$
$A(0, 3)$	60 (Minimum)
$B(20/3, 4/3)$	226.6
$C(0, 8)$	160

$\therefore$ Z has minimum value 60 at $A(0, 3)$.

38. $$AC = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix}\begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0\cdot 2 + 6\cdot(-2) + 7\cdot 3 \\ (-6)\cdot 2 + 0\cdot(-2) + 8\cdot 3 \\ 7\cdot 2 + (-8)\cdot(-2) + 0\cdot 3 \end{bmatrix} = \begin{bmatrix} 9 \\ 12 \\ 30 \end{bmatrix}$$

$$BC = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}\begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0\cdot 2 + 1\cdot(-2) + 1\cdot 3 \\ 1\cdot 2 + 0\cdot(-2) + 2\cdot 3 \\ 1\cdot 2 + 2\cdot(-2) + 0\cdot 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \\ -2 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0 & 6+1 & 7+1 \\ -6+1 & 0+0 & 8+2 \\ 7+1 & -8+2 & 0+0 \end{bmatrix} = \begin{bmatrix} 0 & 7 & 8 \\ -5 & 0 & 10 \\ 8 & -6 & 0 \end{bmatrix}$$

$$\therefore\ (A + B)C = \begin{bmatrix} 0 & 7 & 8 \\ -5 & 0 & 10 \\ 8 & -6 & 0 \end{bmatrix}\begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0\cdot 2 + 7\cdot(-2) + 8\cdot 3 \\ (-5)\cdot 2 + 0\cdot(-2) + 10\cdot 3 \\ 8\cdot 2 + (-6)(-2) + 0\cdot 3 \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \\ 28 \end{bmatrix} \quad \text{...(i)}$$

$$\text{Now, } AC + BC = \begin{bmatrix} 9 \\ 12 \\ 30 \end{bmatrix} + \begin{bmatrix} 1 \\ 8 \\ -2 \end{bmatrix} = \begin{bmatrix} 9+1 \\ 12+8 \\ 30-2 \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \\ 28 \end{bmatrix} \quad \text{...(ii)}$$

From (i) and (ii), we get

$(A + B)C = AC + BC$

OR

Let $B = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 7 \\ 3 & 5 \end{bmatrix}$

Now, $|B| = 3 - 4 = -1 \ne 0$

$|C| = 20 - 21 = -1 \ne 0$

Hence B^{-1} and C^{-1} exist.

$\therefore$ The given matrix equation becomes $BAC = I$

$\Rightarrow B^{-1}(BAC)\,C^{-1} = B^{-1}I\,C^{-1} \Rightarrow IAI = B^{-1}C^{-1}$

$\Rightarrow A = B^{-1}C^{-1}$ ...(i)

Now, adj $B = \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix}$

$$\therefore\ B^{-1} = \frac{1}{|B|}(\text{adj } B) = \frac{1}{-1}\begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$$

$$\text{Also, adj } C = \begin{bmatrix} 5 & -3 \\ -7 & 4 \end{bmatrix}' = \begin{bmatrix} 5 & -7 \\ -3 & 4 \end{bmatrix}$$

$$\therefore\ C^{-1} = \frac{1}{|C|}(\text{adj } C) = \frac{1}{-1}\begin{bmatrix} 5 & -7 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} -5 & 7 \\ 3 & -4 \end{bmatrix}$$

$$\text{Now, from (i), } A = B^{-1}C^{-1} = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} -5 & 7 \\ 3 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 15+6 & -21-8 \\ -10-3 & 14+4 \end{bmatrix} = \begin{bmatrix} 21 & -29 \\ -13 & 18 \end{bmatrix}$$

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