30. Bayes's Theorem and its Applications

Exercise 30

1. Question

In a bulb factory, three machines, A, B, C, manufacture 60%, 25% and 15% of the total production respectively. Of their respective outputs, 1%, 2% and 1% are defective. A bulb is drawn at random from the total product, and it is found to be defective. Find the probability that it was manufactured by machine C.

Answer

Let

D : Bulb is defective

We want to find P(C|D), i.e. probability that the selected defective bulb is manufactured by C

$$P(C|D) = \frac{P(C).P(D|C)}{P(A).P(D|A) + P(B).P(D|B) + P(C).P(D|C)}$$

Where, P(A) = probability that bulb is made by machine A $= \frac{60}{100}$

P(B) = probability that bulb is made by machine B $= \frac{25}{100}$

P(C) = probability that bulb is made by machine C $= \frac{15}{100}$

P(D|A) = probability of defective bulb from machine A $= \frac{1}{100}$

P(D|B) = probability of defective bulb from machine B $= \frac{2}{100}$

P(D|C) = probability of defective bulb from machine C $= \frac{1}{100}$

$$P(C|D) = \frac{15}{60 + 50 + 15}$$

$$= \frac{15}{125}$$

$$= \frac{3}{25}$$

Conclusion: Therefore, the probability of selected defective bulb is from machine C is $\frac{3}{25}$

2. Question

A company manufactures scooters at two plants, A and B. plant A produces 80% and plant B produces 20% of the total product. 85% of the scooters produced at pant A and 65% of the scooters produced at plant B are of standard quality. A scooter produced by the company is selected at random, and it is found to be of standard quality. What is the probability that it was manufactured at

plant A?

Answer

Let S : Standard quality

We want to find P(A|S), i.e. probability that selected standard scooter is from

plant A

$$P(A|S) = \frac{P(A).P(A|S)}{P(A).P(S|A) + P(B).P(S|B)}$$

Where, P(A) = probability that scooter is from A $= \frac{80}{100}$

P(B) = probability that scooter is from B = $\frac{20}{100}$

P(S|A) = probability that standard scooter from A = $\frac{85}{100}$

P(S|B) = probability that standard scooter from B = $\frac{65}{100}$

$$P(A|S) = \frac{(80)(85)}{(80)(85) + (20)(65)}$$

$$= \frac{6800}{6800 + 1300} = \frac{68}{81}$$

Conclusion: Therefore, the probability of selected standard scooter is from plant A is $\frac{68}{81}$

3. Question

In a certain college, 4% of boys and 1% of girls are taller than 1.75 meters. Furthermore, 60% of the students are girls. If a student is selected at random and is taller than 1.75 meters, what is the probability that the selected student is a girl?

Answer

Let, T :students taller than 1.75

B : Boys in class

G : Girls in class

We want to find P(G|T), i.e. probability that selected taller is a girl

$$P(G|T) = \frac{P(G).P(T|G)}{P(G).P(T|G) + P(B).P(T|B)}$$

$$= \frac{\left(\frac{60}{100}\right)\left(\frac{1}{100}\right)}{\left(\frac{60}{100}\right)\left(\frac{1}{100}\right) + \left(\frac{40}{100}\right)\left(\frac{4}{100}\right)}$$

$$= \frac{60}{220} = \frac{3}{11}$$

Conclusion : Therefore, the probability of selected taller student is a girl is $\frac{3}{11}$

4. Question

In a class, 5% of the boys and 10% of the girls have an IQ of more than 150. In this class, 60% of the students are boys. If a student is selected at random and found to have and IQ of more than 150, find the probability that the student is a boy.

Answer

Let, I : students having IQ more than 150

B : Boys in the class

G : Girls in the class

We want to find P(B|I) i.e. probability that selected student having IQ greater than 150 is a boy

$$P(B|I) = \frac{P(B).P(I|B)}{P(G).P(I|G) + P(B).P(I|B)}$$

$$= \frac{\left(\frac{60}{100}\right)\left(\frac{5}{100}\right)}{\left(\frac{60}{100}\right)\left(\frac{5}{100}\right) + \left(\frac{40}{100}\right)\left(\frac{10}{100}\right)}$$

$$= \frac{300}{300 + 400} = \frac{3}{7}$$

Conclusion: Therefore, the probability that selected student having IQ greater than 150 is a boy is $\frac{3}{7}$

5. Question

Suppose 5% of men and 0.25% of women have grey hair. A grey-haired person is selected at random. What is the probability of this person being male? Assume that there is an equal number of males and females.

Answer

Let MG : Men having grey hair

WG: Women having grey hair

G : Having grey hair

Given an equal number of males and females. So let's assume both the probability be $\frac{1}{2}$

We want to find P(MG|G), i.e. probability of a randomly selected grey person to be male

$$P(MG|G) = \frac{P(MG).P(G|MG)}{P(MG).P(G|MG) + P(WG).P(G|WG)}$$

$$= \frac{\left(\frac{1}{2}\right)\left(\frac{5}{100}\right)}{\left(\frac{1}{2}\right)\left(\frac{5}{100}\right) + \left(\frac{1}{2}\right)\left(\frac{0.25}{100}\right)}$$

$$= \frac{5}{5.25}$$

$$= \frac{20}{21}$$

Conclusion: Therefore, the probability of a randomly selected grey person to be male is $\frac{20}{21}$

6. Question

Two groups are competing for the positions on the board of directors of a corporation. The probabilities that the first and the second groups will win are 0.6 and 0.4, respectively. Further, if the first group wins, the probability of introducing a new product is 0.7, and when the second groups win, the corresponding probability is 0.3. Find the probability that the new product introduced was by the second group.

Answer

Let F : First group

S : Second group

N : Introducing a new product

We want to find P(S|N), i.e. new product introduced by the second group

$$P(S|N) = \frac{P(S).P(N|S)}{P(S).P(N|S) + P(F).P(N|F)}$$

$$= \frac{(0.4)(0.3)}{(0.6)(0.7) + (0.4)(0.3)}$$

$$= \frac{0.12}{0.54}$$

$$= \frac{2}{9}$$

Conclusion: Therefore, the probability of the second group introduced a new product is $\frac{2}{9}$

7. Question

A bag A contains 1 white and 6 red balls. Another bag contains 4 white and 3 red balls. One of the bags is selected at random, and a ball is drawn from it, which is found to be white. Find the probability that the ball is drawn is from bag A.

Answer

Let R : Red ball

W : White ball

A : Bag A

B : Bag B

Assuming, selecting bags is of equal probability i.e. $\frac{1}{2}$

We want to find P(A|W), i.e. the selected white ball is from bag A

$$P(A|W) = \frac{P(A).P(W|A)}{P(A).P(W|A) + P(B).P(W|B)}$$

$$= \frac{\left(\frac{1}{2}\right)\left(\frac{1}{7}\right)}{\left(\frac{1}{2}\right)\left(\frac{1}{7}\right) + \left(\frac{1}{2}\right)\left(\frac{4}{7}\right)}$$

$$= \frac{1}{5}$$

Conclusion: Therefore, the probability of selected white ball is from

bag A is $\frac{1}{5}$

8. Question

There are two I and II. The bag I contains 3 white and 4 black balls, and bag II contains 5 white and 6 black balls. One ball is drawn at random from one of the bags and is found to be white. Find the probability that it was drawn from the bag I.

Answer

Let W : White ball

B : Black ball

X : 1^{st} bag

Y : 2^{nd} bag

Assuming, selecting bags is of equal probability i.e. $\frac{1}{2}$

We want to find P(X|W), i.e. probability of selected white ball is from the 1^{st} bag

$$P(X|W) = \frac{P(X).P(W|X)}{P(X).P(W|X) + P(Y).P(W|Y)}$$

$$= \frac{\left(\frac{1}{2}\right)\left(\frac{3}{7}\right)}{\left(\frac{1}{2}\right)\left(\frac{3}{7}\right) + \left(\frac{1}{2}\right)\left(\frac{5}{11}\right)}$$

$$= \frac{\frac{3}{7}}{\frac{3}{7} + \frac{5}{11}}$$

$$= \frac{33}{68}$$

Conclusion: Therefore, the probability of selected white ball is from the 1^{st} bag is $\frac{33}{68}$

9. Question

A box contains 2 gold and 3 silver coins. Another box contains 3 gold and 3 silver coins. A box is chosen at random, and a coin is drawn from it. If the selected coin is a gold coin, find the probability that it was drawn from the second box.

Answer

Let G : Gold coins

S : Siler coins

A : 1^{st} box

B : 2^{nd} box

Assuming, selecting bags is of equal probability i.e. $\frac{1}{2}$

We want to find P(B|G), i.e. probability of selected gold coin is from the 2^{nd} box

$$P(B|G) = \frac{P(B).P(G|B)}{P(A).P(G|A) + P(B).P(G|B)}$$

$$= \frac{\left(\frac{3}{6}\right)\left(\frac{1}{2}\right)}{\left(\frac{2}{5}\right)\left(\frac{1}{2}\right) + \left(\frac{3}{6}\right)\left(\frac{1}{2}\right)}$$

$$= \frac{5}{9}$$

Conclusion: Therefore, the probability of selected gold coin is from the 2^{nd} box is $\frac{5}{9}$

10. Question

Three urns A, B and C contains 6 red and 4 white; 2 red and 6 white; and 1 red and 5 white balls respectively. An urn is chosen at random, and a ball is drawn. If the ball drawn is found to be red, find the probability that the balls was drawn from the first urn A.

Answer

let A : Ball drawn from bag A

B : Ball is drawn from bag B

C : Ball is drawn from bag C

R : Red ball

W : White ball

Assuming, selecting bags is of equal probability i.e. $\frac{1}{3}$

We want to find P(A|R), i.e. probability of selected red ball is from bag A

$$P(A|R) = \frac{P(A).P(R|A)}{P(A).P(R|A) + P(B).P(R|B) + P(C).P(R|C)}$$

$$= \frac{\left(\frac{1}{3}\right)\left(\frac{6}{10}\right)}{\left(\frac{1}{3}\right)\left(\frac{6}{10}\right) + \left(\frac{1}{3}\right)\left(\frac{2}{8}\right) + \left(\frac{1}{3}\right)\left(\frac{1}{6}\right)}$$

$$= \frac{\left(\frac{3}{5}\right)}{\left(\frac{3}{5}\right) + \left(\frac{1}{4}\right) + \left(\frac{1}{6}\right)} = \frac{36}{61}$$

Conclusion: Therefore, the probability of selected red ball is from bag A is $\frac{36}{61}$

11. Question

Three urns contain 2 white and 3 black balls; 3 white and 2 black balls, and 4 white and 1 black ball respectively. One ball is drawn from an urn chosen at random, and it was found to be white. Find the probability that it was drawn from the first urn.

Answer

let A : Ball drawn from bag A

B : Ball is drawn from bag B

C : Ball is drawn from bag C

BB : Black ball

WB : White ball

Assuming, selecting bags is of equal probability i.e. $\frac{1}{3}$

We want to find P(A|W), i.e. probability of selected White ball is from bag A

$$P(A|W) = \frac{P(A).P(W|A)}{P(A).P(W|A) + P(B).P(W|B) + P(C).P(W|C)}$$

$$= \frac{\left(\frac{1}{3}\right)\left(\frac{2}{5}\right)}{\left(\frac{1}{3}\right)\left(\frac{2}{5}\right) + \left(\frac{1}{3}\right)\left(\frac{3}{5}\right) + \left(\frac{1}{3}\right)\left(\frac{4}{5}\right)}$$

$$= \frac{2}{9}$$

Conclusion: Therefore, the probability of selected white ball is from bag A is $\frac{2}{9}$

12. Question

There are three boxes, the first one containing 1 white, 2 red and 3 black balls; the second one containing 2 white, 3 red and 1 black ball and the third one containing 3 white, 1 red and 2 black balls. A box is chosen at random, and from it, two balls are drawn at random. One ball is red and the other, white. What is the probability that they come from the second box?

Answer

let A : Ball drawn from bag A

B : Ball is drawn from bag B

C : Ball is drawn from bag C

BB : Black ball

WB : White ball

RB : Red ball

Assuming, selecting bags is of equal probability i.e. $\frac{1}{3}$

We want to find P(B|WR) i.e. probability of selected White and red ball is from bag B

$$= \frac{\left(\frac{1}{3}\right)\left(\frac{3}{5}\right)}{\left(\frac{1}{3}\right)\left(\frac{2}{5}\right) + \left(\frac{1}{3}\right)\left(\frac{3}{5}\right) + \left(\frac{1}{3}\right)\left(\frac{1}{6}\right)} = \frac{6}{11}$$

Conclusion: Therefore, the probability of selected white and red ball from bag B is $\frac{6}{11}$

13. Question

Urn A contains 7 white and 3 black balls; urn B contains 4 white and 6 black balls; urn C contains 2 white and

8 black balls. One of these urns is chosen at random with probabilities 0.2, 0.6 and 0.2 respectively. From the chosen urn, two balls are drawn at random without replacement. Both the balls happen to be white. Find the probability that the balls are drawn are from urn C.

Answer

Let A : Ball is drawn from bag A

B : Ball is drawn from bag B

C : Ball is drawn from bag C

BB : Black ball

WB : White ball

RB : Red ball

Probability of picking 2 white balls fro urn A $= \frac{7c2}{10c2} = \frac{21}{45}$

Probability of picking 2 white balls fro urn B $= \frac{4c2}{10c2} = \frac{6}{45}$

Probability of picking 2 white balls fro urn C $= \frac{2c2}{10c2} = \frac{1}{45}$

We want to find the probability of 2 white balls picked from urn C

$$= \frac{(0.2)\left(\frac{1}{45}\right)}{(0.2)\left(\frac{21}{45}\right) + (0.6)\left(\frac{6}{45}\right) + (0.2)\left(\frac{1}{45}\right)}$$

$$= \frac{1}{40}$$

Conclusion: Therefore, the probability of both selected white balls are from urn C is $\frac{1}{40}$

14. Question

There are 3 bags, each containing 5 white and 3 black balls. Also, there are 2 bags, each containing 2 white and 4 black balls. A white ball is drawn at random. Find the probability that this ball is from a bag of the first group.

Answer

Let A : the set of first 3 bags

B : a set of next 2 bags

WB : White ball

BB : Black ball

Now we can change the problem to two bags, i.e. bag A containing 15 white and 9 black balls(5 white and 3 black in each bag) and bag B containing 4 white and 8 black balls(2 white and 4 black balls in each bag)

Probability of selecting bag A is $\frac{3}{5}$ (3 bags are in A) and selecting B is $\frac{2}{5}$ (2 bags are in B)

We want to find the probability of selected white ball is from bag A

$$P(A|WB) = \frac{P(A).P(WB|A)}{P(A).P(WB|A) + P(B).P(WB|B)}$$

$$= \frac{\left(\frac{3}{5}\right)\left(\frac{15}{24}\right)}{\left(\frac{3}{5}\right)\left(\frac{15}{24}\right) + \left(\frac{2}{5}\right)\left(\frac{4}{12}\right)}$$

$$= \frac{45}{61}$$

Conclusion: Therefore, the probability of selected white ball is from the first group is $\frac{45}{61}$

15. Question

There are four boxes, A,B,C and D, containing marbles. A contains 1 red, 6 white and 3 black marbles; B contains 6 red, 2 white and 2 black marbles; C contains 8 red, 1 white and 1 black marbles; and D contains 6 white and 4 black marbles. One of the boxes is selected at random and a single marble is drawn from it. If the marble is red, what is the probability that it was drawn from the box A?

Answer

Let A : Ball drawn from bag A

B : Ball is drawn from bag B

C : Ball is drawn from bag C

D : Ball is drawn from bag D

BB : Black ball

WB : White ball

RB : Red ball

Assuming all boxes have an equal probability for picking i.e. $\frac{1}{4}$

We want to find P(A|RB), i.e. probability of selected red ball is from box A

$$P(A|RB) = \frac{P(A).P(RB|A)}{P(A).P(RB|A) + P(B).P(RB|B) + P(C).P(RB|C) + P(D).P(RB|D)}$$

$$= \frac{\left(\frac{1}{4}\right)\left(\frac{1}{10}\right)}{\left(\frac{1}{4}\right)\left(\frac{1}{10}\right) + \left(\frac{1}{4}\right)\left(\frac{6}{10}\right) + \left(\frac{1}{4}\right)\left(\frac{8}{10}\right) + \left(\frac{1}{4}\right)\left(\frac{0}{10}\right)}$$

$$= \frac{1}{15}$$

Conclusion: Therefore, the probability of selected red ball is from box A is $\frac{1}{15}$

16. Question

A car manufacturing factory has two plants X and Y. Plant X manufactures 70% of the cars, and plant Y manufactures 30%. At pant X, 80% of the cars are rated of standard quality, and at plant Y, 90% are rated of standard quality. A car is picked up at random and is found to be of standard quality. A car is picked up at random and is found to be of standard quality. Find the probability that it has come from plant X.

Answer

Let X : Car produced from plant X

Y : Car produced from plant Y

S : Car rated as standard quality

We want to find P(X|S), i.e. selected standard quality car is from plant X

$$P(X|S) = \frac{P(X).P(S|X)}{P(X).P(S|X) + P(Y).P(S|Y)}$$

$$= \frac{\left(\frac{70}{100}\right)\left(\frac{80}{100}\right)}{\left(\frac{70}{100}\right)\left(\frac{80}{100}\right) + \left(\frac{30}{100}\right)\left(\frac{90}{100}\right)}$$

$$= \frac{56}{83}$$

Conclusion: Therefore, the probability of selected standard quality car is from plant X is $\frac{56}{83}$

17. Question

An insurance company insured 2000 scooters and 3000 motorcycles. The probability of an accident involving a scooter is 0.01, and that of motorcycles is 0.02. An insured vehicle met with an accident. Find the probability that the accidented vehicle was a motorcycle.

Answer

Let M : Motorcycle

S : Scooter

A : Accident vechicle

We want to find P(M|A), i.e. probability of accident vehicle was a motorcycle

$$P(M|A) = \frac{P(M).P(A|M)}{P(M).P(A|M) + P(S).P(A|S)}$$

$$= \frac{\left(\frac{3000}{5000}\right)(0.02)}{\left(\frac{3000}{5000}\right)(0.02) + \left(\frac{2000}{5000}\right)(0.01)}$$

$$= \frac{6}{8}$$

$$= \frac{3}{4}$$

Conclusion: Therefore, the probability of accident vechile was motorcycle is $\frac{3}{4}$

18. Question

In a bulb factory, machines A, B and C manufactures 60%, 30% and 10% bulbs respectively. Out of these bulbs 1%, 2% and 3% of the bulbs produced respectively by A, B and C are found to be defective. A bulb is picked up at random from the total production and found to be defective. Find the probability that this bulb was produced by machine A.

Answer

Let A : Manufactured from machine A

B : Manufactured from machine B

C : Manufactured from machine C

D : Defective bulb

We want to find P(A|D), i.e. probability of selected defective bulb is from

machine A

$$P(A|D) = \frac{P(A).P(D|A)}{P(A).P(D|A) + P(B).P(D|B) + P(C).P(D|C)}$$

$$= \frac{\left(\frac{60}{100}\right)\left(\frac{1}{100}\right)}{\left(\frac{60}{100}\right)\left(\frac{1}{100}\right) + \left(\frac{30}{100}\right)\left(\frac{2}{100}\right) + \left(\frac{10}{100}\right)\left(\frac{3}{100}\right)}$$

$$= \frac{6}{15}$$

$$= \frac{2}{5}$$

Conclusion: Therefore, the probability of selected defective bulb is from machine A is $\frac{2}{5}$