SUCCESSIVE DIFFERENTIATION

SYNOPSIS

- If y = f(x) is differentiable at any point 'x' then its derivative is $f^1(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ and it is called the first derivative of f(x)
- If the derivative of y = f(x) is derivable at 'x', then the derivative of $f^1(x)$ denoted by $f^{11}(x)$ is called second order derivative of f(x) at 'x'.
  $f^{11}(x) = \lim_{h\to 0} \frac{f^1(x+h)-f^1(x)}{h}$
- If the (n-1)th derivative of y=f(x) is differentiable at x then its derivative is called the nth derivative of f(x) at x and it is denoted by
  $f^{(n)}(x)$ or $D^n y$ where $D \equiv \frac{d}{dx}$, or y_n or y^n.
- For $n \in N$ and n>1 then $\frac{d^n y}{dx^n} = \frac{d}{dx}\left(\frac{d^{n-1}y}{dx^{n-1}}\right)$
- If $y=(ax+b)^m$ and $m \in R - N$ then
  $y_n = a^n m(m-1)(m-2).......(m-n+1)\ (ax+b)^{m-n}$
  $m \in N$ and
  - n<m, then $y_n = \frac{m\,!}{(m-n)\,!} a^n (ax+b)^{m-n}$
  - n=m, then $y_n = a^n.n!$
  - n>m, then $y_n = 0$
- If $y = Tan^{-1}\left(\frac{x}{a}\right)$ then
  $y_n = (-1)^{n-1}(n-1)\,!\,a^{-m} \sin n\theta \sin^n \theta$
  where $\theta = Tan^{-1}\left(\frac{a}{x}\right)$
- $y = \frac{x}{x^2+a^2}$ then $y_n = \frac{(-1)^n n! Cos(n+1)\theta}{r^{n+1}}$
  where $r = \sqrt{x^2+a^2}; \theta = Tan^{-1}\left(\frac{a}{x}\right)$
- If $y = e^{ax}$ then $y_n = a^n e^{ax}$
- If $y = a^{mx}$ then $y_n = m^n a^{mx} (\log a)^n$
- If $y = \frac{1}{ax+b}$ then $y_n = \frac{(-1)^n a^n n!}{(ax+b)^{n+1}}$
- If $y = \log|ax+b|$ then $y_n = \frac{(-1)^{n-1} a^n (n-1)!}{(ax+b)^n}$
- If $y = \frac{1}{(ax+b)^m}$ then $y_n = \frac{(-1)^n a^n (m+n-1)\,!}{(m-1)\,!\,(ax+b)^{m+n}}$
- If $y = \frac{ax+b}{cx+d}$ then
  $y_n = \frac{(-1)^{n-1} c^{n-1} (ad-bc) n!}{(cx+d)^{n+1}}; (ad-bc \neq 0)$
- If y = Cos(ax+b) then $y_n = a^n. Cos\left(\frac{n\pi}{2} + ax + b\right)$
- If y = Sin(ax+b) then $y_n = a^n. Sin\left(\frac{n\pi}{2} + ax + b\right)$
- If $y = e^{ax}$ Sin(bx+c) then
  $y_n = (a^2+b^2)^{\frac{n}{2}} e^{ax}. Sin\,[bx+c+n\,Tan^{-1}(b/a)]$
- If $y = e^{ax}$ Cos(bx+c) then
  $y_n = (a^2+b^2)^{\frac{n}{2}} e^{ax}. Cos\,[bx+c+n\,Tan^{-1}(b/a)]$
- If $y = a^x$.Sin(bx+c) then
  $y_n = \left[(\log a)^2 + b^2\right]^{n/2} .a^x .Sin\left(bx+c+nTan^{-1}\left(\frac{b}{\log_e a}\right)\right)$
- If $y = a^x$.Cos(bx+c) then
  $y_n = \left[(\log a)^2 + b^2\right]^{n/2} .a^x .Cos\left(bx+c+nTan^{-1}\left(\frac{b}{\log_e a}\right)\right)$

- LEIBNITZ THEOREM: It helps to find the nth derivative of the product of two functions of x. If y = uv, where u and v are functions of x, having derivatives of nth order, then

$$y_n = {}^nC_0 u_n v + {}^nC_1 u_{n-1} v_1 + {}^nC_2 u_{n-2} v_2 + \dots\dots\dots$$
$$+ {}^nC_r u_{n-r} v_r + \dots + {}^nC_n u v_n$$

Where suffixes of u and v denote the number of times they are differentiated

- If x=f(t), y=g(t) then

$$\frac{dy}{dx} = \frac{g^1(t)}{f^1(t)}; \quad \frac{d^2y}{dx^2} = \frac{g^{11}(t)f^1(t) - g^1(t)f^{11}(t)}{\{f^1(t)\}^3}$$

- If f(x, y)=C then $\frac{dy}{dx} = -\frac{\left(\frac{\partial f}{\partial x}\right)}{\left(\frac{\partial f}{\partial y}\right)}$ and

$$\frac{d^2y}{dx^2} = -\frac{\left[f_{xx}.f_y^2 - 2f_{xy}f_x f_y + f_{yy}.f_x^2\right]}{f_y^3}$$

where $f_x, f_y, f_{xx}, f_{xy}, f_{yy}$ are partial derivatives.

MULTIPLE CHOICE QUESTIONS

LEVEL-I

1. $D^{10}(Cos2x)$
   1) 2^{10} Sin2x 2) -2^{10} Sin2x
   3) 2^{10} Cos2x 4) -2^{10} Cos2x

2. $D^{20}(Sin5x)$
   1) 5^{20} Sin5x 2) -5^{20} Cos5x
   3) -5^{20} Sin5x 4) 5^{20} Cos5x

3. $D^{20}[Sin(\frac{x}{2})] =$
   1) $2^{20} Sin(\frac{x}{2})$ 2) $\frac{1}{2^{20}} Sin(\frac{x}{2})$
   3) $-2^{20} Sin(\frac{x}{2})$ 4) $\frac{-1}{2^{20}} Sin(\frac{x}{2})$

4. $D^8(Sin^2 3x) =$
   1) $\frac{6^8}{2}$ Cos6x 2) $\frac{-6^8}{2}$ Cos6x
   3) $\frac{6^8}{2}$ Sin6x 4) $\frac{-6^8}{2}$ Sin6x

5. $D^8(Cos^2 7x) =$
   1) $2^8\, 2^8$ Cos14x 2) $2^7\, 7^8$ Cos14x
   3) $2^7\, 7^8$ Sin14x 4) $-2^7\, 7^8$ Sin14x

6. If S = a Cos 3t+b Sin3t then $\frac{d^2S}{dt^2} =$
   1) -9S 2) -S 3) 9S 4) S

7. $D^5(Cos3x) =$
   1) 3^5Cos3x 2) -3^5Sin3x
   3) 3^5Sin3x 4) -3^5Cos3x

8. $D^3(e^{5x}Cos4x)$
   1) $\frac{e^{5x}}{41}$ Cos4x 2) $41^{3/2}\, e^{5x}$ Cos4x
   3) $41^{3/2}\, e^{5x}\, Cos[4x+3Tan^{-1}(4/5)]$
   4) $41^{1/2}\, e^{5x}\, Cos[4x+Tan^{-1}(4/5)]$

9. $D^{16}(e^x \sin x)$
   1) $2^8\, e^x$ Sinx 2) $-2^8\, e^x$ Sinx
   3) $2^8\, e^x$ Cosx 4) $-2^8\, e^x$ Cosx

10. $D^{21}\left(e^{\sqrt{3}x} \cos x\right) =$
    1) $2^{21}\left(e^{\sqrt{3}x} \cos x\right)$ 2) $2^{21}\left(e^{\sqrt{3}x} \sin x\right)$
    3) $-2^{21}\left(e^{\sqrt{3}x} \cos x\right)$ 4) $-2^{21}\left(e^{\sqrt{3}x} \sin x\right)$

11. If $y = e^{3x}$ Sin(3x+4) then $y_n =$
    1) $(18)^n\, e^{3x}\, Sin[3x+4+\left(\frac{n\pi}{4}\right)]$
    2) $(3\sqrt{2})^n\, e^{3x}\, Sin[3x+4+\left(\frac{n\pi}{4}\right)]$
    3) $(3\sqrt{2})^n\, Sin[3x+4+\left(\frac{n\pi}{4}\right)]$
    4) $(18)^n\, Sin[3x+4+\left(\frac{n\pi}{4}\right)]$

12. If $y = e^{-12x}$ Cos(5x+2) then $y_n =$
    1) $(169)^n\, e^{-12x}\, Cos[5x + 2 - n\, Tan^{-1}\left(\frac{5}{12}\right)]$
    2) $(13)^n\, e^{-12x}\, Cos\,[5x + 2 - n\, Tan^{-1}\left(\frac{5}{12}\right)]$
    3) $(13)^n\, e^{12x}\, Cos\,[5x + 2 - n\, Tan^{-1}\left(\frac{5}{12}\right)]$
    4) $(13)^n\, e^{-12x}\, Sin\,[5x + 2 - n\, Tan^{-1}\left(\frac{5}{12}\right)]$

13. If $y=(7x+3)^{11}$ then $y_{11}=$

1) $7^{11}10!$ 2) $7^{11}11!$ 3) $11!$ 4) $7^{10}10!$

14. If $y=\frac{2x+3}{5x+4}$ then $y_n=$

1) $\frac{(-1)^n 5^n 7n!}{(5x+4)^{n+1}}$ 2) $\frac{(-1)^{n-1}5^{n-1}7n!}{(5x+4)^{n+1}}$

3) $\frac{(-1)^n 5^n 7n!}{(5x+4)^n(-1)^n}$ 4) $\frac{(-5)^{n-1}(-7)n!}{(5x+4)^{n+1}}$

15. If $y=\frac{1}{(ax+b)^2}$ then $y_n=$

1) $\frac{(-1)^n a^n n!}{(ax+b)^{n+2}}$ 2) $\frac{(-1)^n a^n (n+1)!}{(ax+b)^{n+2}}$

3) $\frac{(-1)^n a^n}{(ax+b)^{n+2}}$ 4) $\frac{a^n n!}{(ax+b)^{n+2}}$

16. If $y=(5x-7)^{-3}$ then $y_n=$

1) $\frac{(-1)^n(n+2)!5^n}{2!(5x-7)^{n+3}}$ 2) $\frac{(-1)^{n-1}5^{n-1}(n-1)!}{2!(5x-7)^{n+3}}$

3) $\frac{(-1)^n 5^n (n+2)!}{2!(5x-7)^{n+2}}$ 4) $\frac{(-1)^n 5^n (n)!}{2!(5x-7)^n}$

17. If $y=x^{n-1}\log x$ then $xy_1-(n-1)y=$

1) x^{n-2} 2) x^{n-1} 3) x^n 4) x^{n+1}

18. $y=\frac{1}{1+x+x^2+x^3}$ then $y_2(0)=$

1. 0 2. 1 3. -1 4. 1/2

19. If p(x) is a polynomial of r^{th} degree and $\frac{d^n}{dx^n}[p(x)]=0$ then

1. n=r 2. n>r 3. n<r 4. n=r/2

20. The n^{th} derivative of $\cos^4 x$ if n=2k+1 is

1. 1 2. 0

3. 4^n 4. $2^n \cos\frac{k\pi}{2}$

21. If $e^x+xy=e$ then $y_2(0)=$

1. $\frac{1}{e^3}$ 2. $\frac{1}{e^2}$ 3. $\frac{1}{e}$ 4. 1

22. If $y=\sin^{-1}x-\sin^{-1}\sqrt{1-x^2}$ then $\frac{d^2y}{dx^2}=$

1. $\frac{2}{\sqrt{1-x^2}}$ 2. $\frac{2x}{(1-x^2)^{3/2}}$

3. $\frac{2}{(1-x^2)^{3/2}}$ 4. $\frac{-2x}{(1-x^2)^{3/2}}$

23. If $x=\cos^n\theta, y=\sin^n\theta$ then $y_2=$

1. $\frac{n}{n-1}.\frac{\cos^{2n-1}\theta}{\sin^{n-3}\theta}$ 2. $\frac{n-1}{n}.\frac{\sin^{n-3}\theta}{\cos^{n-1}\theta}$

3. $n-1.\tan^{n-2}\theta.\sec^2\theta$

4. $n\frac{\sin^{n-1}\theta}{\cos^{n-2}\theta}$

24. If $y=xe^{1/x}$ then $x^3y_2+xy_1=$

1. 0 2. y 3. -y 4. 1/y

25. If $y=e^{-x}.\cos x$ then $y_4=$

1. 0 2. y 3. 4y 4. -4y

26. If $f(x)=x-x^2+x^3-x^4+.....for\ |x|<1$ then $f^n(0)=$

1. $\frac{-2}{(1+x)^2}$ 2. -2

3. 2 4. -1

27. If $f(x)=x^n+4$ then

$f(1)+\frac{f^1(1)}{1!}+\frac{f^{11}(1)}{2!}+\frac{f^{111}(1)}{3!}+.....=$

1. 2^{n-1} 2. 2^n+4

3. 2^n

4. $1+\frac{1}{1!}+\frac{1}{2!}+.......+\frac{1}{n!}$

KEY

1. 4	2. 1	3. 2	4. 2	5. 2
6. 1	7. 2	8. 3	9. 1	10. 2
11. 2	12. 2	13. 2	14. 4	15. 2
16. 1	17. 2	18. 1	19. 2	20. 1
21. 2	22. 2	23. 2	24. 2	25. 4
26. 2	27. 2			

LEVEL - II

1. If $y = \frac{c}{2}[e^{x/c} + e^{-x/c}]$ then $y^2 - c^2\left(\frac{dy}{dx}\right)^2 =$
   1) 0 2) c^2 3) $c^2/4$ 4) 1
2. If $\log y = \mathrm{Tan}^{-1}x$ then $(1+x^2)y_2 + (2x-1)y_1 =$
   1) 0 2) 1/2 3) -1 4) 1
3. If $y = a\sin(\log x)$ then $x^2y_2 + xy_1 =$
   1) y 2) ay 3) -y 4) a^2y
4. If $y = 2\,\mathrm{Sin}5x.\,\mathrm{Cos}x$ then $y_n =$
   1) $6^n \mathrm{Sin}\left(\frac{n\pi}{2}+6x\right) + 4^n \mathrm{Sin}\left(\frac{n\pi}{2}+4x\right)$
   2) $6^n \mathrm{Sin}\left(\frac{n\pi}{2}+6x\right) - 4^n \mathrm{Sin}\left(\frac{n\pi}{2}+4x\right)$
   3) $6^n \mathrm{Cos}\left(\frac{n\pi}{2}+6x\right) + 4^n \mathrm{Cos}\left(\frac{n\pi}{2}+4x\right)$
   4) $6^n \mathrm{Cos}\left(\frac{n\pi}{2}+6x\right) - 4^n \mathrm{Cos}\left(\frac{n\pi}{2}+4x\right)$
5. If $y = e^{a\mathrm{Cos}^{-1}x}$ then $(1-x^2)y_2 - xy_1 =$
   1) ay 2) $-a^2y$ 3) -ay 4) a^2y
6. $D^7(4\mathrm{Sin}x\,\mathrm{Cos}^2x) =$
   1) $3^7\mathrm{Cos}3x+\mathrm{Cos}x$ 2) $-(3^7\mathrm{Cos}3x+\mathrm{Cos}x)$
   3) $-3^7(\mathrm{Cos}3x+\mathrm{Cos}x)$ 4) $-3^7(\mathrm{Cos}3x+\mathrm{Cos}x)$
7. If $y = \mathrm{Tan}^{-1}\left(\frac{\log\left(\frac{e}{x^2}\right)}{\log(ex^2)}\right) + \mathrm{Tan}^{-1}\left(\frac{3+2\log x}{1-6\log x}\right)$ then $\frac{d^2y}{dx^2} =$
   1) 2 2) 1 3) 0 4) -1
8. If $y = \mathrm{Sin}(\mathrm{Sin}x)$ then $y_2 + \mathrm{Tan}x\, y_1 =$
   1) 0 2) y 3) $y\mathrm{Cos}^2x$ 4) $-y\mathrm{Cos}^2x$
9. If $y = e^{-x}\mathrm{Cos}x$ then $y_4 + 4y =$
   1) y 2) -y 3) y^2 4) 0
10. If $y = e^{mx}(ax+b)$ where a,b and m are constants, then $y_2 - 2my_1 =$
    1) m^2y 2) $-m^2y$ 3) m^2y^2 4) my^2
11. If $y = e^{3x}\mathrm{Sin}^2x$ then $y_n =$
    1) $\frac{3^n}{2}e^{3x} - \frac{13^{n/2}}{2}e^{3x}\mathrm{Cos}2x$
    2) $\frac{3^n}{2}e^{3x} - \frac{13^{n/2}}{2}e^{3x}\mathrm{Cos}[2x+n\,\mathrm{Tan}^{-1}(3/2)]$
    3) $\frac{3^n}{2} - \frac{13^{n/2}}{2}e^{3x}\mathrm{Cos}[2x+n\,\mathrm{Tan}^{-1}(2/3)]$
    4) $3^n e^{3x} - 13^{n/2}e^{3x}\mathrm{Cos}[x+n\,\mathrm{Tan}^{-1}(3/2)]$
12. If $y = e^{-2x}\mathrm{Cos}3x$ and $\frac{d^2y}{dx^2} + a\left(\frac{dy}{dx}\right) + by = 0$, then a =____, b=______
    1) 4, 13 2) 13, 4 3) -4, 13 4) -4, -13
13. $D^8(3^x\mathrm{Cos}x) =$
    1) $e^{x\log 3}\mathrm{Cos}[x+8\mathrm{Tan}^{-1}(\log 3^e)]$
    2) $[(\log 3)^2+1]^4 e^{x\log 3}\mathrm{Cos}[x+8\mathrm{Tan}^{-1}(\log_3 e)]$
    3) $[1+(\log 3)^2]^8 e^{8x\log 3}\mathrm{Cos}[x+4\pi]$
    4) $[1+(\log 3)^2]^8 e^{x\log 3}\mathrm{Cos}[x+8\mathrm{Tan}^{-1}(\log 3)]$
14. If $y = a\,x^{n+1} + b\,x^{-n}$ then $x^2y_2 =$
    1) ny 2) (n+1)y 3) -n(n+1)y 4) n(n+1)y
15. If $ax^2+2hxy+by^2=1$, then $y_2=$
    1) $\frac{h^2-ab}{hx+by}$ 2) $\frac{h^2-ab}{(hx+by)^3}$ 3) $\frac{h^2+ab}{(hx+by)^3}$ 4) $\frac{h^2+ab}{hx+by}$
16. If $y = \frac{\log(x+\sqrt{1+x^2})}{\sqrt{1+x^2}}$ then $(1+x^2)y_1 + xy =$
    1) y 2) -y 3) 0 4) 1
17. If $y = \log x^2$ then $y_n =$
    1) $\frac{(-1)^{n-1}(n-1)!}{x^n}$ 2) $\frac{(-1)^{n-1}(n-1)!}{2x^n}$
    3) $\frac{2(-1)^n(n-1)!}{x^n}$ 4) $\frac{2(-1)^{n-1}(n-1)!}{x^n}$
18. If $y = \frac{1}{x^2-a^2}$ then $y_n =$
    1) $\frac{(-1)^n n!}{2a}\left[\frac{1}{(x-a)^n} - \frac{1}{(x+a)^n}\right]$
    2) $\frac{(-1)^n n!}{2a}\left[\frac{1}{(x-a)^{n+1}} - \frac{1}{(x+a)^{n+1}}\right]$
    3) $\frac{(-1)^n n!}{2a}\left[\frac{1}{(x-a)^{n+1}} + \frac{1}{(x+a)^{n+1}}\right]$
    4) $\frac{(-1)^n n!}{2a}\left[\frac{1}{(x-a)^n} + \frac{1}{(x+a)^n}\right]$
19. If $y = \frac{x+1}{(x-1)(x-2)(x-3)}$ then $y_n =$
    1) $(-1)^n n!\left[\frac{1}{(x-1)^{n+1}} - \frac{3}{(x-2)^{n+1}} + \frac{2}{(x-3)^{n+1}}\right]$
    2) $(-1)^n n!\left[\frac{1}{(x-1)^{n+1}} + \frac{1}{(x-2)^{n+1}} + \frac{1}{(x+3)^{n+1}}\right]$
    3) $(-1)^n n!\left[\frac{1}{(x-1)^n} - \frac{1}{(x-2)^n} + \frac{1}{(x-3)^n}\right]$
    4) $(-1)^n\left[\frac{1}{(x-1)^{n+1}} + \frac{1}{(x-2)^{n+1}} + \frac{1}{(x+3)^{n+1}}\right]$

20. If $y = \log(x^3+6x^2+11x+6)$ then $y_n =$

1) $(-1)^n n!\left[\frac{1}{(x+1)^n}+\frac{1}{(x+2)^n}+\frac{1}{(x+3)^n}\right]$

2) $(-1)^{n-1}(n-1)!\left[\frac{1}{(x+1)^n}+\frac{1}{(x+2)^n}+\frac{1}{(x+3)^n}\right]$

3) $(-1)^{n-1}(n-1)!\left[\frac{1}{(x+1)^{n+1}}+\frac{1}{(x-2)^{n+1}}+\frac{1}{(x+3)^{n+1}}\right]$

4) $(-1)^{n-1}\left[\frac{1}{(x+1)^{n+1}}+\frac{1}{(x-2)^{n+1}}+\frac{1}{(x+3)^{n+1}}\right]$

21. $\frac{d^{31}}{dx^{31}}\left[\frac{x}{(x-1)^2}\right] =$

1) $-\frac{31!}{(x-1)^{32}}-\frac{32!}{(x-1)^{33}}$ 2) $-\frac{31!}{(x-1)^{32}}+\frac{32!}{(x-1)^{33}}$

3) $-\frac{31!}{(x-1)^{33}}-\frac{32!}{(x-1)^{32}}$ 4) $-\frac{31!}{(x-1)^{33}}+\frac{32!}{(x-1)^{32}}$

22. If $y = (1-x)^{-\alpha} e^{\alpha x}$ then $(1-x)y_1 =$

1) αy 2) αxy 3) $\frac{\alpha x}{y}$ 4) $-\alpha xy$

23. If $y = (A+Bx)e^{kx}$ then $y_2 - 2ky_1 + k^2y =$

1) k^2y 2) $-k^2y$ 3) 0 4) y

24. If $x^2+y^2 = 2$ and $y = Ay^{-3}$ then A =

1) -2 2) -1 3) 0 4) 1

25. If $(a+bx)e^{y/x} = x$ then $x^3y_2 =$

1) xy_1-y 2) $(y_1-xy)^2$ 3) $(xy_1-y)^2$ 4) $(y_1-xy)^3$

26. The function $y = e^{\sqrt{x}}+e^{-\sqrt{x}}$ then $xy^{11}+\frac{1}{2}y^1 =$

1) $\frac{1}{4}y$ 2) $-\frac{1}{4}y$ 3) 0 4) y

27. $D^{10}(x^2 \sin 3x) =$

1) $-3^{10}x^2\sin 3x + 20.\,3^9 x\cos 3x - 90.\,3^8\sin 3x$

2) $-3^{10}x^2\sin 3x$ 3) 0 4) 1

28. $D^{10}(x^3 \log x)$

1) $-9!\,x^{-7} + 30.\,8!\,x^{-7} + 270.\,7!\,x^{-7}$

2) $9!\,x^7 + 30.\,8!\,x^{-7} + 270.\,7!\,x^{-7}$

3) 0 4) 1

29. $D^n.(x.\cos 3x)$

1) $3^n\left\{x\cos\left(\frac{n\pi}{2}+3x\right)+n\cos\left(\frac{(n-1)\pi}{2}+3x\right)\right\}$

2) $3^{n-1}\left\{x\cos\left(\frac{n\pi}{2}+3x\right)+n\cos\left(\frac{n\pi}{2}+3x\right)\right\}$

3) $3^{n-1}\left\{x\cos\left(\frac{n\pi}{2}+3x\right)+n\cos\left(\frac{(n-1)\pi}{2}+3x\right)\right\}$

4) $3^{n-1}\left\{3x\cos\left(\frac{n\pi}{2}+3x\right)+n\cos\left(\frac{(n-1)\pi}{2}+3x\right)\right\}$

30. $D^n(x^3.e^{2x}) =$

1) $e^{2x}[8x^3+12nx^2+3n(n-1)x+n(n-1)(n-2)]$

2) $2^{n-2}e^{2x}[8x^3+12nx^2-3n(n-1)x+n(n-1)]$

3) $2^{n-3}e^{2x}[8x^3+12nx^2+6n(n-1)x+n(n-1)(n-2)]$

4) $2^ne^{2x}[8x^3+12nx^2+3n(n-1)x+n(n-1)(n-2)]$

31. Given $(1+x^2)y_2 + (2x-1)y_1 = 0$ then $(1+x^2)y_{n+2} + (2nx+2x-1)y_{n+1} =$

1) $-n^2y_n$ 2) $(n^2+n)y_n$

3) $-(n^2+n)y_n$ 4) $-(n^2+n)y_{n-1}$

32. If $x = \log t$ and $y = t^2-1$ then $y^{11}(1)$ at t=1 is equal to

1) 1 2) 2 3) 3 4) 4

33. If $y = 10^x - 10^{-x}$ then y_n = [When ‘n’ is even]

1) $y(\log 10)^n$ 2) 0

3) $(\log 10)^n(10^x - 10^{-x})$ 4) $(\log 10)^n$

34. If $y = \frac{ax+b}{cx+d}$ then $2y_1y_3 =$

1. $3y_2^2$ 2. y_2^2 3. $3y_1^2$ 4. $3y^2$

35. If $y^2 = 4ax$ then $\frac{\left(1+y_1^2\right)^{3/2}}{y_2}$ at x=a is

1. $-4\sqrt{2a}$ 2. $4\sqrt{2a}$

3. $\frac{4\sqrt{2}}{a}$ 4. $-4a$

36. If $p(x)$ is a polynomial of degree 3, p(0) =4, $p^1(0) = 3, p^{11}(0) = 6$ then $p^1(-1) =$

1. 10 2. 8 3. 6 4. 0

37. If $p(x)$ is a polynomial of degree n with $p(2) = -1, p^1(2) = 0, p^{11}(2) = -2,$ $p^{111}(2) = -12, p^{iv}(2) = 24$ then $p^{11}(1) =$

1. 26 2. 13 3. 29 4. 62

38. The least value of n so that $y_n = y_{n+1}$ where $y = x^2 + e^x$

1. 4 2. 3 3. 5 4. 2

39. If $f(x)=\sin(\sin x)$ and

$\cot x.f^{11}(x)+f^{1}(x)+g(x)\cot x=0$ then

$g(x)=$

1. $-\cos(\cos x)\sin^2 x$
2. $\cos(\cos x)\sin^2 x$
3. $-\sin(\sin x)\cos^2 x$
4. $\sin(\sin x)\cos^2 x$

KEY

1. 2	2. 1	3. 3	4. 1	5. 4
6. 2	7. 3	8. 4	9. 4	10. 1
11. 3	12. 1	13. 2	14. 4	15. 2
16. 4	17. 4	18. 2	19. 1	20. 2
21. 1	22. 2	23. 3	24. 1	25. 4
26. 1	27. 1	28. 1	29. 4	30. 3
31. 3	32. 4	33. 1	34. 1	35. 1
36. 4	37. 1	38. 2	39. 4	

NOTE: F_K Stands for K^{th} formula in the synopsis.

HINTS

1. Express the function as $c\cosh\frac{x}{c}$ and simplify.
4. Express 'y' as sin 6x +sin 4x and use F_{15}
7. Use the formula

$$Tan^{-1}\left(\frac{a+b}{1-ab}\right)=Tan^{-1}(a)+Tan^{-1}(b)$$

11. Express 'y' as $e^{3x}\left(\frac{1-\cos 2x}{2}\right)$ and use F_7,F_{17}
13. Use F_{19}
15. Use F_{22}
17. Express the function as 2 log x and use F_{10}
18. Express the funciton as $\frac{1}{2a}\left\{\frac{1}{x-a}-\frac{1}{x+a}\right\}$ and use F_9
19. Resolve into partial fractions and use F_9
20. Express the function as $\log(x+1)+\log(x+2)+\log(x+3)$ and use F_{10}
21. Express $\frac{x}{(x-1)^2}$ as $\frac{(x-1)+1}{(x-1)^2}=\frac{1}{x-1}+\frac{1}{(x-1)^2}$ and use F_9, F_{11}
28. Use F_{20}
29. Use F_{20}
31. Differentiate the given equation once and verify the options.
32. Use F_{21}
33. Use F_8

LEVEL - III

1. If $Cos^{-1}(y/b)=n\log(\frac{x}{n})$ then $x^2y_2+xy_1=$

   1) n^2y 2) n^2y^2 3) $-n^2y$ 4) ny

2. If y = aCos(logx) + b Sin(logx) then $x^2y_2+xy_1=$

   1) -y 2) 1/y 3) y 4) y^2

3. If $y=Cos(3Cos^{-1}x)$ then $y_3=$

   1) 12 2) -12 3) 24 4) 36

4. If $y=Cos^4x$ then $y_n=$

   1) $2^{n-1}Cos\left(\frac{n\pi}{2}+2x\right)+4^{n}Cos\left(\frac{n\pi}{2}+4x\right)$

   2) $2^{n-1}Cos\left(\frac{n\pi}{2}+2x\right)-2^{2n-3}Cos\left(\frac{n\pi}{2}+4x\right)$

   3) $2^{n-1}Cos\left(\frac{n\pi}{2}+2x\right)+4^{n-1}Cos\left(\frac{n\pi}{2}+4x\right)$

   4) $2^{n-1}Cos\left(\frac{n\pi}{2}+2x\right)+2^{2n-3}Cos\left(\frac{n\pi}{2}+4x\right)$

5. If $y=Sin^2x.Cos^2x$ then $y_n=$

   1) $2^{2n-3}Cos\left(\frac{n\pi}{2}+4x\right)$ 2) $2^{2n-3}Sin\left(\frac{n\pi}{2}+4x\right)$

   3) $-2^{2n-3}Cos\left(\frac{n\pi}{2}+4x\right)$ 4) $2^{2n+3}Sin\left(\frac{n\pi}{2}-4x\right)$

6. If y = 4Sin 3x. Sin 2x. Sin x then $y_n=$

   1) $2^nSin\left(\frac{n\pi}{2}+2x\right)+4^nSin\left(\frac{n\pi}{2}+4x\right)-6^nSin\left(\frac{n\pi}{2}+6x\right)$

   2) $2^n\ Cos\left(\frac{n\pi}{2}+2x\right)+3^n\ Cos\left(\frac{n\pi}{2}+4x\right)+2^n\ Cos\left(\frac{n\pi}{2}+6x\right)$

3) $2^n \operatorname{Sin}\left(\frac{n\pi}{2}+2x\right) + 2^n \operatorname{Sin}\left(\frac{n\pi}{2}+4x\right)$

$- 3^n \operatorname{Sin}\left(\frac{n\pi}{2}+6x\right)$

4) $-2^n \operatorname{Sin}\left(\frac{n\pi}{2}+2x\right) + 2^n \operatorname{Sin}\left(\frac{n\pi}{2}+4x\right)$

$- 3^n \operatorname{Sin}\left(\frac{n\pi}{2}+6x\right)$

7. If $f(x) = \operatorname{Sin}^6x+\operatorname{Cos}^6x$ then $f^{11}(x) =$
1) -6Sin4x 2) 6Sin4x
3) -6Cos4x 4) 6Cos4x

8. If $y = \frac{ax+b}{cx+d}$ then $2y_1.y_3=$
1) y_2^2 2) $-y_2^2$ 3) $3y_2^2$ 4) $4y_2^2$

9. If $y = \frac{5x^2-3x+4}{1-x}$ then $(1-x)y_3 =$
1) y_2 2) $2y_2$ 3) $3y_2$ 4) $4y_2$

10. If $y^{1/m}+y^{-1/m} = 2x$ then $(x^2-1)y_2 + xy_1 =$
1) my 2) -my 3) $-m^2y$ 4) m^2y

11. If $x^4-xy+y^4 = 1$ then y_2 at (0,1) is
1) 1/8 2) -1/8 3) 1/16 4) -1/16

12. The function $y = (x^2+1)^{50}$ should be differentiated n times to result in polynomial of the 30th degree, then n is equal to
1) 20 2) 30 3) 50 4) 70

13. If $y = \sqrt{2x-x^2}$ then $y^3.y_2 =$
1) -1 2) 0 3) 1 4) 2

14. If $f(x) = \frac{x^2}{2}$, If $0\le x\le 1$, $f(x) = 2x^2-3x+(3/2)$, If $1\le x\le 2$ then the function f^{11} is.....
1) Continuous 2) Discontinuous
3) Differentiable 4) Not differentiable

15. If $f(x) = \frac{x^3}{x^2-1}$ then $f^{111}(0) =$
1) - 3! 2) 0 3) 1 4) 3!

16. $D^4\left\{\frac{x^4}{(x-1)(x-2)}\right\} =$

1) $4!\left\{\frac{16}{(x-2)^5}-\frac{1}{(x-1)^5}\right\}$

2) $-24\left\{\frac{16}{(x-2)^5}-\frac{1}{(x-1)^5}\right\}$

3) $4!\left\{\frac{16}{(x-2)^4}-\frac{1}{(x-1)^4}\right\}$

4) $4!\left\{\frac{16}{(x-2)^4}+\frac{1}{(x-1)^4}\right\}$

17. $D^n(x^n.\log x) =$
1) n! [1+logx+(1/2)+(1/3)+....(1/n)]
2) n [1+logx + (1/2) + (1/3) + ... (1/n)]
3) 1+logx+(1/2)+(1/3)+..... + (1/n)]
4) n! [logx+ (1/2) + (1/3) +.....(1/n)]

18. If $y = x^4\log x$ then $y_4 =$
1) 74 2) 0 3) 30 4) 24logx+50

19. If $x = \operatorname{Sin}\theta$, $y = \operatorname{Cos}3\theta$ then $(1-x^2)y_2 - xy_1 =$
1) y 2) -3y 3) 3y 4) -9y

20. If $x = a\operatorname{Cos}^3\theta$, $y = b\operatorname{Sin}^3\theta$ then the value of $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{6}$
1) $\frac{32b}{27a^2}$ 2) $\frac{27b}{32a^2}$ 3) $\frac{32a^2}{27b}$ 4) $\frac{24a^2}{27b}$

21. If $x = e^t\operatorname{Cos}t$, $y = e^t\operatorname{Sin}t$ then $\frac{d^2y}{dx^2}$ at t=0 is
1) 0 2) 2 3) 2/e 4) 1/2

22. If x = a(t-sin t) and y = a(1+ Cos t) then the value of y_2 at $t = \frac{\pi}{2}$ is
1) a 2) a^2 3) 1/a 4) $1/a^2$

23. If x = 2 Cos t - Cos 2t, y = 2Sin t - Sin 2t then the value of y_2 at $t = \pi/2$ is
1) 1/2 2) 3/2 3) -1/2 4) -3/2

24. If x = Cos t and y = Sin 4t then $(1-x^2)y_2-xy_1 =$
1) 4y 2) -4y 3) 16y 4) -16y

25. If x = Sint, y = Sinkt then $(1-x^2)y_2-xy_1 =$
1) ky 2) -ky 3) k^2y 4) $-k^2y$

26. If $x = \operatorname{Sin}^{-1}t$ and $y = \log(1-t^2)$ then $\frac{d^2y}{dx^2}$ at t = 1/2 is
1) -8/3 2) 8/3 3) 3/4 4) -3/4

27. Let f be a twice differentiable function such that $f^{11}(x) = -f(x)$ and $f^1(x) = g(x)$. If $h^1(x) = [f(x)]^2 + [g(x)]^2$, h(1) = 8 and h(0) = 2 then h(2) =
1) 14 2) 12 3) 2 4) 3

28. If $y^2 = p(x)$ is a polynomial of degree 3, then $2\frac{d}{dx}\left[y^3\frac{d^2y}{dx^2}\right] =$

1) $p(x) + p^1(x)$ 2) $p(x).\ p^1(x)$
3) $p(x).\ p^{111}(x)$ 4) A constant

29. If $f(x) = (1+x)^n$ then the value of

$f(0) + f^1(0) + \frac{f^{11}(0)}{2!} + ...+ \frac{f^n(0)}{n!}$ is

1) n 2) 2^n 3) 2^{n-1} 4) 2^{n+1}

30. If $f(x) = \begin{vmatrix} x^n & \text{Sinx} & -\text{Cosx} \\ n! & \text{Sin}\left(\frac{n\pi}{2}\right) & \text{Cos}\left(\frac{n\pi}{2}\right) \\ a & a^2 & a^3 \end{vmatrix}$ then the value of $\frac{d^nf}{dx^n}$ at x = 0, for n = 2m+1 is equal to

1) -1 2) 0 3) 1 4) 2

31. Let f(x) = Sinx, $g(x) = x^2$, h(x) = logx, if f(x) = h {f(g(x))} then $\frac{d^2f}{dx^2} =$

1) $2\ \text{Cos}^3x$ 2) $2\text{Cot}\ x^2 - 4x^2\text{Cos}^2x^2$
3) $2x\text{Cot}x^2$ 4) -2Cosec^2x

32. The third derivative of a function f(x) vanishes for all x. If f(0) = 1, $f^1(1) = 2$ and $f^1(2) = -1$ then f(x) =

1) x^3+3x+2 2) $\frac{-x^2-3x+1}{2}$

3) $\frac{x^2+3x-1}{2}$ 4) $\frac{-3x^2+10x+2}{2}$

33. Let P(x) be a polynomial of degree 4, with P(2) = -1, $P^1(2) = 0$, $P^{11}(2) = 2$, $P^{111}(2) = -12$, $P^{iv}(2) = 24$ then the value of $P^{11}(1) =$

1) -12 2) 2 3) 24 4) 26

34. If f: R $\to$ R is a function such that $f(x) = x^3+x^2f^1(1)+xf^{11}(2)+f^{111}(3)$ for x $\in$ R then f(2) =

1) 2 2) -2 3) f(1) 4) f(0)

35. If $x^2+y^2 = R^2$ and K = 1/R then K=

1) $\frac{y_2}{\sqrt{1+y_1^2}}$ 2) $\frac{|y_2|}{\sqrt{(1+y_1^2)^3}}$

3) $\frac{2|y_2|}{\sqrt{1+y_1^2}}$ 4) $\frac{3|y_2|}{\sqrt{(1+y_1^2)^3}}$

36. If $f(x) = x^n$ then

$f(1) - \frac{f^1(1)}{1!} + \frac{f^{11}(1)}{2!} - \frac{f^{111}(1)}{3!} + ... + \frac{(-1)^n.f^n(1)}{n!} =$

(AIEEE-2003)

1. 2^{n-1} 2. 0 3. 1 4. 2^n

37. If $y = \sin\left(3\sin^{-1}x\right)$ then $\underset{x\to 0}{Lt}\left(\frac{\frac{d^2y}{dx^2}}{\log(1+x)}\right) =$

1. 24 2. -24 3. -6 4. 0

38. If $f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$ where p is constant then $f^{111}(x)$ at x=0 is

1. p 2. $p+p^2$
3. $p+p^3$ 4. independent of p

39. If $y = \sin px$ then $\begin{vmatrix} y & y_1 & y_2 \\ y_3 & y_4 & y_5 \\ y_6 & y_7 & y_8 \end{vmatrix} =$

1. 0 2. 1 3. y 4. -y

KEY

1. 3	2. 3	3. 3	4. 4	5. 3
6. 1	7. 3	8. 3	9. 3	10. 4
11. 4	12. 4	13. 1	14. 2	15. 1
16. 1	17. 1	18. 4	19. 4	20. 1
21. 2	22. 3	23. 4	24. 4	25. 4
26. 1	27. 1	28. 3	29. 2	30. 2
31. 2	32. 4	33. 4	34. 2	35. 2
36. 2	37. 2	38. 4	39. 1	

HINTS

3. Express y as $4x^3-3x$

4. Express Cos^4x as $\frac{3}{8} + \frac{\cos 2x}{2} + \frac{\cos 4x}{8}$ and use F_{14}

5. Express the function as $\frac{1-\cos 4x}{8}$ and use F_{14}

6. Express the function as sin2x+sin4x-sin6x and use F_{15}
8. Use F_{12}
9. Cross multiply and use F_{20}
11. use F_{22}
12. Here 'y' is a 100th degree polynomial
13. Square the function and differentiate.
14. Evaluate $f''(1) = \underset{x\to 1}{Lt}\dfrac{f'(x)-f'(1)}{x-1}$
15. Express the function $\dfrac{x^3}{x^2-1}$ as

$$\frac{(x^2-1+1)x}{x^2-1} = x+\frac{1}{2}\left\{\frac{1}{x-1}+\frac{1}{x+1}\right\}$$

and use F_9
17. By verifying the options
18. Use F_{20}
19. Use F_{21}
20. Use F_{21}
21. Use F_{21}
27. Differentiate $h^1(x) = \{f(x)\}^2 + \{g(x)\}^2$ and use the hypothesis. That gives $h^{11}(x)=0 \Rightarrow h(x) = C_1 x + C_2$
use conditions to get the result.
28. Differentiate $y^2=p(x)$ three times.
and obtain,
$2(3y_1y_2+yy_3) = p^{111}(x)$ --> (1) say
since $2\dfrac{d}{dx}\left(y^3y_2\right) = 2\left\{3y^2y_1y_2+y^3y_3\right\}$
multiplying equation (1) by 'y^2' the result follows.
29. Differentiate f(x) 'n' times successively and use $^nc_0+{}^nc_1+.....+{}^nc_n=2^n$.
32. Verifying the given conditions from the options.
35. Differentiate the function two times and obtain the result.

LEVEL - IV

NEW PATTERN QUESTIONS

1. I: If $y=\cos\left(3\cos^{-1}x\right)$ then $\dfrac{d^3y}{dx^3}=24$

II: If $y=\sin\left(7\sin^{-1}x\right)$ then

$\left(1-x^2\right)y_2 - xy_1 + 49y = 0$

1) only I is true 2) only II is true
3) both I and II are true
4) neither I nor II true

2. I: If $f(x)=(1+x)^n$ then the value of

$f(0)+f^{|}(0)+\dfrac{1}{2!}f^{||}(0)+......+\dfrac{1}{n!}f^{n}(0)$ is 2^n

II: $f(x)=x^n+4$ then the value of

$f(1)+f^{|}(1)+\dfrac{1}{2!}f^{||}(1)+......+\dfrac{1}{n!}f^{n}(1)$ is

2^n+4

1) only I is true 2) only II is true
3) both I and II are true
4) neither I nor II true

3. I: If $y=\sin x\sin 2x\sin 3x$ then

$$y_n=\frac{1}{4}\left[4^n\sin\left(\frac{n\pi}{2}+4x\right)-6^n\sin\left(\frac{n\pi}{2}+6x\right)+2^n\sin\left(\frac{n\pi}{2}+2x\right)\right]$$

II: If $y=\sin 2x\sin 3x\sin 4x$ then

$$y_n=\frac{1}{4}\left[\begin{array}{l}-9^n\sin\left(\frac{n\pi}{2}+9x\right)+5^n\sin\left(\frac{n\pi}{2}+5x\right)\\+3^n\sin\left(\frac{n\pi}{2}+3x\right)+\sin\left(\frac{n\pi}{2}+x\right)\end{array}\right]$$

1) only I is true 2) only II is true
3) both I and II are true
4) neither I nor II true

4. I: If $y=\cos\left(m\sin^{-1}x\right)$ then

$\left(1-x^2\right)y_2-xy_1+m^2y=0$

II: If $\log y=\tan^{-1}x$ then

$\left(1+x^2\right)\dfrac{d^2y}{dx^2}+(2x-1)\dfrac{dy}{dx}=0$

1) only I is true 2) only II is true
3) both I and II are true
4) neither I nor II true

5. I: If $a=y_2(1)$ where $y=\cos(\log x)$, $b=y_2(0)$ where $y=\tan^{-1}x$ and $c=y_2(1)$ where $y=(\log x)/x$ then the ascending order of a, b, c is

1) a,b,c 2) b,c,a 3) c,a,b 4) a,c,b

6. Match the following

I. If $y = a + be^{-4x}$ then $y_2 + 4y_1 =$ a) 0

II. If $y = ax + \frac{b}{x}$ then $x^2 y_2 + xy_1 =$ b) y

III. If $xy = ae^x + be^{-x}$

then $xy_2 + 2y_1 =$ c)xy

IV. If $y = xe^{-\frac{1}{x}}$ then $x^3 y_2 + xy_1 =$ d) $-y$

1) a,b,c,d 2) b,d,a,c 3)a,d,b,c 4) d,c,b,a

7. A : The n^{th} derivative of $\log(3x+4)$ is

$$\frac{3^n (-1)^{n-1} (n-1)!}{(3x+4)^n}$$

R: If $y = \log|ax+b|$ then

$$y_n = \frac{(-1)^{n-1} (n-1)! a^n}{(ax+b)^n}$$

1) Both A and R are true and R is the correct explanation of A
2) Both A and R are true but R is not correct explanation of A
3) A is true but R is false
4) A is false but R is true

8. A : The n^{th} derivative of $\sin 5x \cos 3x$ is

$$\frac{1}{2}\left[8^n \sin\left(\frac{n\pi}{2} + 8x\right) + 2^n \sin\left(\frac{n\pi}{2} + 2x\right)\right]$$

R : If $y = \sin(ax+b)$ then

$$y_n = a^n \sin\left(ax + b + \frac{n\pi}{2}\right)$$

1) Both A and R are true and R is the correct explanation of A
2) Both A and R are true but R is not correct explanation of A
3) A is true but R is false
4) A is false but R is true

KEY

1.3	2.3	3.3	4.3	5.3
6.1	7.1	8.1		

LEVEL - V

1. If $f(x) = y = \frac{1}{ax+b}$ then $y_n = \frac{(-1)^n a^n n!}{(ax+b)^{n+1}}$

i) *If* $y = \frac{2x+3}{5x+4}$ *then yn* =

1) $\frac{(-1)^n 5^{n+1} (7) n!}{(5x+4)^{n+1}}$ 2) $\frac{(-1)^{n-1} 5^{n-1} 7n!}{(5x+4)^{n+1}}$

3) $\frac{(-1)^n 5^n 7n!}{(5x+4)^{n+1} (-1)^n}$ 4) $\frac{(-5)^{n-1} (-7) n!}{(5x+4)^{n+1}}$

ii) $D^4\left[\frac{x^4}{(x-1)(x-2)}\right] =$

1) $4!\left[\frac{16}{(x-2)^5} - \frac{1}{(x-1)^5}\right]$

2) $-24\left[\frac{16}{(x-2)^5} - \frac{1}{(x-1)^5}\right]$

3) $4!\left[\frac{16}{(x-2)^4} - \frac{1}{(x-1)^4}\right]$

4) $4!\left[\frac{16}{(x-2)^4} + \frac{1}{(x-1)^4}\right]$

PREVIOUS EAMCET QUESTIONS

2004

1. $y = \sin^{-1} x \Rightarrow (1-x^2)\frac{d^2y}{dx^2} =$

1) $-x\frac{dy}{dx}$ 2) 0

3) $x\frac{dy}{dx}$ 4) $x\left(\frac{dy}{dx}\right)^2$

2. If $f : R \to R$ is an even function having derivatives of all orders then an add function among the following is

1) $f^{||}$ 2) $f^{|||}$ 3) $f^{|} + f^{||}$ 4) $f^{||} + f^{|||}$

2003

3. $f(x) = x^2 e^{-x/a}$ then $f^n(0) =$

1) 0 2) 1

3) $\frac{{}^nC_2}{a^n}$ 4) $\frac{n(n-1)(-1)^n}{a^{n-2}}$

2002

4. If $y=ae^{x}+be^{-x}+c$, where a, b, c are parameters then $y^{111}=$

1) y 2) y^{1} 3) 0 4) y^{11}

5. If y=aCos(logx)+bSin(logx) where a, b are parameters, then $x^2y^{11}+xy^{1}=$

1) y 2) -y 3) 3y 4) -2y

2001

6. If y_k is the k^{th} derivative of y w.r.t. x and y=Cos(Sinx) then y=Cos(Sinx) then y_1Sinx + y_2Cosx=

1) $ySin^3x$ 2) $-ySin^3x$ 3) $yCos^3x$ 4) $-yCos^3x$

7. If $f(x)=\frac{x^2}{x+a}$ then $f^{11}(a)=$

1) 4a 2) $\frac{1}{8a}$ 3) $\frac{1}{4a}$ 4) 8a

1999

8. If y = Sin(7Sin^{-1}x) then $(1-x^2)y_2-xy_1=$

1) -49y 2) -7y 3) 49y 4) 7y

1998

9. If $\log y = \tan^{-1}x$ then

$$\left(1+x^2\right)\frac{d^2y}{dx^2}+(2x-1)\frac{dy}{dx}+4=$$

1) 0 2) 2logy 3) 4 4) 1

10. If y=sin $\left(7\sin^{-1}x\right)$ then $\left(1-x^2\right)y_2-xy_1=$

1) -49y 2) -7y 3) 49y 4) 7y

11. $\frac{d^n\left(e^x Sin\ x\right)}{dx^n}=$

1) $2^{\frac{n}{2}}e^x Cos\left(x+\frac{n\pi}{4}\right)$ 2) $2^{\frac{n}{2}}e^x Cos\left(x-\frac{n\pi}{4}\right)$

3) $2^{\frac{n}{2}}e^x \sin\left(x+\frac{n\pi}{4}\right)$ 4) $2^{\frac{n}{2}}e^x \sin\left(x-\frac{n\pi}{4}\right)$

1997

12. If x=Sin t, y=Sin pt then

$$\left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+p^2y=$$

1) 0 2) 1 3) -1 4) $\frac{1}{\sqrt{2}}$

13. If $y=\left(x^2-1\right)^n$ then $\left(x^2-1\right)y_{n+2}+2x\ y_{n+1}=$

1) $\left(n^2+1\right)y_n$ 2) $\left(n^2-1\right)y_n$

3) $n\left(n^2+1\right)y_n$ 4) $n(n+1)y_n$

14. If $y=\sqrt{cox2x}$ then $y\frac{d^2y}{dx^2}+2y^2=$

1) 0 2) $-\left(\frac{dy}{dx}\right)^2$ 3) $\left(\frac{dy}{dx}\right)^2$ 4) $y\frac{dy}{dx}$

1995

15. If $y=\cos\left(m\sin^{-1}x\right)$, the value of $\left(1-x^2\right)y_2-xy_1$ is

1) $-m^2y$ 2) $\frac{y}{m^2}$ 3) m^2y 4) $\frac{m^2}{y}$

16. If $y=e^{m\sin^{-1}x}$ then $\left(1-x^2\right)y_3-3xy_2$ is

1) my^2 2) $\left(1+m^2\right)y$ 3) m^2y 4) $\left(1+m^2\right)y_1$

1994

17. If $x=t^2, y=t^3$ then $y_2=$

1) $\frac{3}{2}$ 2) $\frac{3}{4t}$ 3) $\frac{3}{2t}$ 4) $\frac{3t}{2}$

18. $\frac{d^{20}(2\cos x\cos 3x)}{dx^{20}}=$

1) $2^{20}\left(Cos2x-2^{20}\cos 4x\right)$

2) $2^{20}\left(Cos2x+2^{20}\cos 4x\right)$

3) $2^{20}\left(\sin 2x+2^{20}\sin 4x\right)$

4) $2^{20}\left(\sin 2x-2^{20}\sin 4x\right)$

19. In $y=a+bx^2$; a,b arbitrary constants; then

1) $\frac{d^2y}{dx^2}=2xy$ 2) $x\frac{d^2y}{dx^2}=y_1$

3) $x\frac{d^2y}{dx^2}-\frac{dy}{dx}+y=0$ 4) $x\frac{d^2y}{dx^2}=2xy$

1992

20. If $y = Cos\left(3Cos^{-1}x\right)$ then $y_3 =$

1) 0 2) 24 3)24x 4) $24x^2$

21. $D^4\left(Cos^4 x\right) =$

1)8Cos2x-32Cos4x 2)8Cos2x+32Cos4x

3) $4^4 Cos^4(4\pi + x)$ 4) None

1990

22. n^{th} derivative of $\log(ax+b)$

1) $\frac{(-1)^n(n-1)!a^{n-1}}{(ax+b)^{n-1}}$ 2) $\frac{(-1)^{n-1}(n-1)!a^{n-1}}{(ax+b)^{n-1}}$

3) $\frac{(-1)^{n-1}(n-1)!a^n}{(ax+b)^n}$ 4) $\frac{(-1)^n(n)!a^n}{(ax+b)^{n+1}}$

23. If $x = Cos\theta + \theta Sin\theta, y = Sin\theta - \theta Cos\theta$ then $y_2 =$

1) $\frac{Sec^3\theta}{\theta}$ 2) $\frac{\theta}{Sec^3\theta}$ 3) $\frac{\theta}{COS^3\theta}$ 4) $Sec^2\theta$

1989

24. If $y = Cos^4 x$ then $y_n =$

1) $2^{n-1}Cos\left(\frac{n\pi}{2} + 2x\right) + 4^n Cos\left(\frac{n\pi}{2} + 4x\right)$

2) $2^{n-1}Cos\left(\frac{n\pi}{2} + 2x\right) - 2^{2n-3}Cos\left(\frac{n\pi}{2} + 4x\right)$

3) $2^{n-1}Cos\left(\frac{n\pi}{2} + 2x\right) + 4^{n-1}Cos\left(\frac{n\pi}{2} + 4x\right)$

4) $2^{n-1}Cos\left(\frac{n\pi}{2} + 2x\right) + 2^{2n-3}Cos\left(\frac{n\pi}{2} + 4x\right)$

KEY

1) 3	2) 2	3) 4	4) 2	5) 2
6) 4	7) 3	8) 1	9) 3	10) 1
11) 3	12) 1	13) 4	14) 2	15) 1
16) 4	17) 2	18) 2	19) 2	20) 2
21) 2	22) 3	23) 1	24) 4	