



Ratio and Proportion

Ratio

The ratio of two quantities of the same kind and in the same unit is the fraction that one quantity is of the other.

The ratio a is to b is the fraction a/b and it is written as $a : b$,

where a is called the first term or **antecedent** and b is called second term or **consequent**.

Simplest or Standard Form

The ratio $(a : b)$ are in the simplest form, if HCF of a and b is 1, i.e. there is no common factor between a and b other than 1. It is also called the lowest terms.

Properties of Ratios

Some properties of ratios are as follows:

1. The value of a ratio remains unchanged, if each one of its terms is multiplied or divided by a same non-zero number.
2. $a^2 : b^2$ is the duplicate ratio of $a : b$.
3. $a^3 : b^3$ is the triplicate ratio of $a : b$.
4. $\sqrt{a} : \sqrt{b}$ is the sub-duplicate ratio of $a : b$.
5. $\sqrt[3]{a} : \sqrt[3]{b}$ is the sub-triplicate ratio of $a : b$.
6. If $a : b$ and $c : d$ are two ratios, then the compounded ratio is $ac : bd$.

7. If $a : b$ is ratio, then $\frac{1}{a} : \frac{1}{b}$ i.e. $b : a$ is the reciprocal ratio.

Example 1 If $a : b = 3 : 4$, then $(6a + b) : (4a + 5b)$ is

- (a) 1 : 2 (b) 3 : 5 (c) 7 : 9 (d) 11 : 16

Sol. (d) We have,
$$\frac{6a + b}{4a + 5b} = \frac{6\left(\frac{a}{b}\right) + 1}{4\left(\frac{a}{b}\right) + 5}$$
$$= \frac{6 \times \frac{3}{4} + 1}{4 \times \frac{3}{4} + 5} = \frac{11}{16}$$

Proportion

An equality of two ratios is called the proportion.

If $\frac{a}{b} = \frac{c}{d}$ or $a : b = c : d$, then we can say that a, b, c and d are in proportion and can be written as $a : b :: c : d$. Where symbol ' $::$ ' represents proportion and it is read as a is to b as c is to d .

Here, a and d are called '**extremes**' and b and c are called '**means**'.

(i) If $a : b :: c : d$

(a) $(a \times d) = (b \times c)$, i.e. Product of extremes = Product of means

(b) d is called the **fourth proportional** of a, b and c .

(ii) If $a : b :: b : c$, then

- (a) a, b, c are said to be in **continued proportion**.
 (b) c is called **third proportional** to a, b and **fourth proportional** to a, b, b .
 (c) $\frac{a}{b} = \frac{b}{c} \Leftrightarrow b^2 = ac \Leftrightarrow b = \sqrt{ac}$

Where, b is called the **mean proportional** or **geometric mean** between a and c .

(iii) If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$ and $\frac{a-b}{a+b} = \frac{c-d}{c+d}$

Which is known as Componendo and Dividendo.

Example 2 The mean proportion between 4 and 64 is.

- (a) 64 (b) 45 (c) 16 (d) 24

Sol. (c) Let the mean proportion between 4 and 64 be x .

$$\text{So, } 4 : x :: x : 64 \Rightarrow \frac{4}{x} = \frac{x}{64}$$

$$\Rightarrow x \times x = 64 \times 4 \Rightarrow x^2 = 256$$

$$\Rightarrow x^2 = 16 \times 16$$

[\because product of means = product of extremes]

$$x = 16$$

Hence, the mean proportion between 4 and 64 is 16.

Alternate Method

Here, $a = 4$, $c = 64$ and b (mean proportion)

$$\therefore b = \sqrt{ac} = \sqrt{4 \times 64} = 2 \times 8 = 16$$

Hence, the mean proportion is 16.

Example 3 The fourth proportional to 3, 7, 15 is

- (a) 35 (b) 37 (c) 30 (d) 21

Sol. (a) Let the fourth proportional be x .

$$\therefore 3 : 7 :: 15 : x$$

$$\Rightarrow x = \frac{7 \times 15}{3} = 35$$

Unitary Method

A method in which the value of unit quantity is first obtained to find the value of any required quantity is called unitary method.

In general problems based on unitary method, we face two types of variations.

- (i) **Direct Variation** Two quantities a and b vary directly, if the ratio a/b remains constant.

e.g.

- (a) The cost of articles varies directly as the number of articles. If we buy more article, then we have to pay more money and *vice-versa*.
 (b) The work done varies directly as the number of men doing work. If work is done by more workers, then work will be much more than as previous work done by less workers.

- (ii) **Inverse Variation** Two quantities a and b vary inversely, if the product ab remains constant.

e.g. Suppose a vehicle covers a certain distance at a uniform speed. Then, we can say for certain that, more is the speed of the vehicle, less is the time taken to cover the distance.

Thus, speed varies inversely as the time taken to cover a certain distance.

Example 4 A worker is paid ₹ 1815 for 15 days. The amount, he receives if he works for 8 days is

- (a) ₹ 968 (b) ₹ 955 (c) ₹ 950 (d) ₹ 940

Sol. (a)

Days	Payment
15	1815
8	x

$$\text{By direct variation, } \frac{1815}{15} = \frac{x}{8}$$

$$\Rightarrow x = \frac{1815 \times 8}{15} = 968 \Rightarrow x = 968$$

Hence, he will receive ₹ 968 for 8 days.

Per cent

Per cent means parts per hundred.

Fraction as a per cent To convert a fraction into a per cent, we multiply the fraction by 100 and put the per cent sign '%'.
 e.g. $\frac{1}{2} = \left(\frac{1}{2} \times 100 \right) \% = 50\%$

To convert a per cent into a fraction, we divide it by 100 and remove the per cent sign '%'.
e.g. $10\% = \frac{10}{100} = \frac{1}{10}$

Ratio as per cent To convert a ratio into a per cent, convert the ratio into the fraction and then multiply the fraction by 100 and put the per cent sign '%'.
e.g. $4 : 5 = \frac{4}{5} = \left(\frac{4}{5} \times 100\right)\% = 80\%$

To convert a per cent into a ratio, convert the per cent into a fraction and then express it as a ratio in lowest terms.
e.g. $50\% = \frac{50}{100} = 1 : 2$

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To convert a decimal into a per cent, move the decimal point on the right side by two digits and put the per cent sign '%'.
e.g. $0.135 = 13.5\%$

Per cent as decimal To convert a per cent in decimal form, express it as a fraction with denominator as 100 and then write it in decimal form. e.g. $15\% = \frac{15}{100} = 0.15$

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Example 5 25% of 32 is

- (a) 8 (b) 7 (c) 6 (d) 5

Sol. (a) We have, 25% of 32

$$= 32 \times \frac{25}{100} = \frac{32}{4} = 8$$

Example 6 A picnic is being planned in a school for 20 Class VIII girls which are 40% of the total number of students. Find the ratio of girls to the number of boys in the class.

- (a) 2 : 3 (b) 2 : 4 (c) 1 : 2 (d) 3 : 2

Sol. (a) Let the total number of students be x . Then, 40% of x are girls.

Therefore, 40% of $x = 20$

$$\Rightarrow \frac{40}{100} \times x = 20$$

$$\Rightarrow x = \frac{20 \times 100}{40} = 50$$

So, the number of boys $= 50 - 20 = 30$

Ratio of number of girls to the number of boys

$$= 20 : 30 = \frac{20}{30} = \frac{2}{3}$$

Hence, $\frac{2}{3}$ is written as 2 : 3 and read as 2 is to 3.

Practice Exercise

1. Which of the following is in standard form ?

- (a) $\frac{49}{71}$ (b) $\frac{28}{106}$ (c) $\frac{27}{99}$ (d) $\frac{13}{117}$

2. Which of the following is not in standard form

- (a) $\frac{71}{65}$ (b) $\frac{28}{106}$ (c) $\frac{13}{59}$ (d) $\frac{49}{17}$

3. Third proportional to 9 and 12 is

- (a) 16 (b) 10.5
(c) $6\sqrt{3}$ (d) None of these

4. The fourth proportional to 3, 5 and 21 is

- (a) 35 (b) $\frac{5}{7}$
(c) $\frac{7}{5}$ (d) None of these

5. If $\frac{a}{3} = \frac{b}{4} = \frac{c}{7}$, then $\frac{a+b+c}{c}$ is equal to

- (a) $\frac{1}{7}$ (b) $\frac{1}{2}$
(c) 7 (d) 2

6. If $\frac{a}{b} = \frac{b}{c} = \frac{c}{d}$, then $\frac{b^3 + c^3 + d^3}{a^3 + b^3 + c^3}$ will be equal to
 (a) $\frac{d}{a}$ (b) $\frac{c}{d}$ (c) $\frac{b}{c}$ (d) $\frac{a}{b}$
7. If $yz : zx : xy = 1 : 2 : 3$, then $\frac{x}{yz} \div \frac{y}{zx}$ is
 (a) 4 : 1 (b) 3 : 2
 (c) 1 : 2 (d) 1 : 4
8. Out of the ratios 7 : 20; 13 : 25; 17 : 30 and 11 : 15, the smallest one is
 (a) 10 : 7 (b) 7 : 20
 (c) 17 : 20 (d) None of these
9. The mean proportional between 45 and a certain number is three times the mean proportional between 5 and 22. The number is
 (a) 24 (b) 49 (c) 22 (d) 9
10. If the mean proportional between X and Y is n times the third proportional, then the ratio X : Y will be
 (a) $n^2 : 1$ (b) $1 : n^2$
 (c) $n^{2/3} : 1$ (d) $1 : n^{2/3}$
11. What must be added to each term of the ratio 49 : 68 so that it becomes 3 : 4 ?
 (a) 11 (b) 10 (c) 7 (d) 8
12. The sum of two numbers be 40 and their difference is 4. What is the ratio of the two numbers?
 (a) 9 : 11 (b) 9 : 4
 (c) 11 : 9 (d) 4 : 9
13. The ratio of zinc and copper in a brass piece is 13 : 7. How much zinc will be there in 100 kg of such a piece?
 (a) 65 kg (b) 40 kg
 (c) 45 kg (d) 50 kg
14. The incomes of A and B are in the ratio 3 : 2 and their expenditures in the ratio 5 : 3. If each saves ₹ 1500, then B's income is
 (a) ₹ 6000 (b) ₹ 4700
 (c) ₹ 3000 (d) ₹ 7500
15. The ratio of A's and B's income last year was 3 : 4. The ratio of their own incomes of last year and this year is 4 : 5 and 2 : 3, respectively. If the total sum of their present incomes is ₹ 4160, then find the present income of A.
 (a) ₹ 1600 (b) ₹ 6500
 (c) ₹ 2500 (d) ₹ 41600
16. The ratio of ages of two boys is 5 : 6. After 2 yr the ratio will be 7 : 8. The ratio of their ages after 12 yr will be
 (a) 15 : 6 (b) 17 : 18
 (c) 11 : 12 (d) 22 : 24
17. At present, the ratio of the ages of Maya and Chhaya is 6 : 5 and fifteen years from now, the ratio will get changed to 9 : 8. Maya's present age is?
 (a) 15 yr (b) 20 yr
 (c) 30 yr (d) 10 yr
18. If x varies as y and $x = 8$, when $y = 15$, then the value of x when $y = 10$ is
 (a) 5 (b) $1\frac{7}{8}$
 (c) $\frac{8}{15}$ (d) $5\frac{1}{3}$
19. In a race, Ravi covers 5 km in 20 min. How much distance will he cover in 100 min?
 (a) 40 km (b) 35 km
 (c) 26 km (d) 25 km
20. If 45 m of a uniform rod weighs 171 kg, what will be the weight of 12 m of the same rod?
 (a) 49 kg (b) 42.5 kg
 (c) 55 kg (d) 45.6 kg
21. If the price of m articles is n , what is the price of 5 articles?
 (a) $\frac{5n}{m}$ (b) $\frac{5m}{n}$ (c) $\frac{m}{n}$ (d) $\frac{5m}{n}$
22. If the cost of 24 oranges is ₹ 72, then the cost of 120 oranges is
 (a) ₹ 180 (b) ₹ 360
 (c) ₹ 172 (d) ₹ 500

- 23.** How many sacks are required for filling 1026 kg of rice, if each sack is filled with 114 kg of rice?
 (a) 19 (b) 15
 (c) 7 (d) 9
- 24.** 6 men can complete a piece of work in 20 days. In how many days will 8 men complete the same piece of work?
 (a) 12 days (b) 14 days
 (c) 15 days (d) 16 days
- 25.** If 30% of x is 450, then the value of x is
 (a) 1500 (b) 3000
 (c) 2000 (d) 2500
- 26.** What per cent of 500 is 35
 (a) 7% (b) 5%
 (c) 8% (d) 6%
- 27.** After spending 20% of the salary, Aakash has ₹ 20000 in his hand. Find the amount of his salary.
 (a) ₹ 35000 (b) ₹ 30000
 (c) ₹ 25000 (d) ₹ 50000
- 28.** The price of an article was ₹ 68000. If its price is increased by 20%, then find the new price of that article.
 (a) ₹ 45200 (b) ₹ 71200 (c) ₹ 81600 (d) ₹ 2600
- 29.** 72% of 25 students are good in Mathematics. How many students are not good in Mathematics?
 (a) 7 (b) 6 (c) 5 (d) 8
- 30.** If Chameli had ₹ 600 left after spending 75% of her money, how much did she have in the beginning?
 (a) ₹ 2200 (b) ₹ 2400 (c) ₹ 2100 (d) ₹ 2600
- 31.** A football team won 10 matches out of the total number of matches they played. If their win percentage was 40, then how many matches did they play in all?
 (a) 35 (b) 30 (c) 25 (d) 50
- 32.** If 40% of the students are girls and the number of boy students is 600, then find the number of girls students
 (a) 500 (b) 300 (c) 450 (d) 400

Answers

1	(a)	2	(b)	3	(a)	4	(a)	5	(d)	6	(a)	7	(a)	8	(b)	9	(c)	10	(c)
11	(d)	12	(c)	13	(a)	14	(a)	15	(a)	16	(b)	17	(c)	18	(d)	19	(d)	20	(d)
21	(a)	22	(b)	23	(d)	24	(c)	25	(a)	26	(a)	27	(c)	28	(c)	29	(a)	30	(b)
31	(c)	32	(d)																

Hints & Solutions

- 1.** (a) Except $\frac{49}{71}$, all are not in standard form.
 Because all fractions have common factor.
- 2.** (b) Except $\frac{28}{106}$, all are in standard form.
 Because it have common factor. $\frac{28}{106} = \frac{14}{53}$.
 $\frac{14}{53}$ is an standard form.
- 3.** (a) Let third proportion be x.
 $\therefore 9:12::12:x$
 $\Rightarrow 9x = 12 \times 12 \Rightarrow x = 16$
- 4.** (a) Let the fourth proportional be x.
 $\therefore 3:5::21:x$
 $\Rightarrow x = \frac{5 \times 21}{3} = 35$
- 5.** (d) Let $\frac{a}{3} = \frac{b}{4} = \frac{c}{7} = k$.
 Then, $a = 3k$, $b = 4k$
 and $c = 7k$.
 $\therefore \frac{a+b+c}{c} = \frac{3k+4k+7k}{7k}$
 $= 2$

6. (a) Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = k$

$$\Rightarrow a = bk = ck^2 = dk^3$$

$$b = dk^2 \text{ and } c = dk$$

$$\begin{aligned} \therefore \frac{b^3 + c^3 + d^3}{a^3 + b^3 + c^3} &= \frac{(dk^2)^3 + (dk)^3 + d^3}{(dk^3)^3 + (dk^2)^3 + (dk)^3} \\ &= \frac{(k^6 + k^3 + 1)}{(k^9 + k^6 + k^3)} = \frac{1}{k^3} = \frac{d}{dk^3} = \frac{d}{a} \end{aligned}$$

7. (a) Let $yz = k$, $zx = 2k$ and $xy = 3k$

$$\begin{aligned} \therefore \frac{x}{yz} + \frac{y}{xz} &= \frac{x}{yz} \times \frac{zx}{y} \\ &= \frac{x^2}{y^2} = \left(\frac{xz}{yz}\right)^2 = \left(\frac{2k}{k}\right)^2 = 4 : 1 \end{aligned}$$

8. (b) Now, $\frac{7}{20} = 0.35$, $\frac{13}{25} = 0.52$, $\frac{17}{30} = 0.56$ and $\frac{11}{15} = 0.73$.

\therefore The smallest value is 0.35 i.e. ratio 7 : 20.

9. (c) Mean proportional between 5 and 22

$$= \sqrt{22 \times 5} = \sqrt{110}$$

Mean proportional between 45 and $x = \sqrt{45x}$

$$\therefore 3\sqrt{110} = \sqrt{45x}$$

On squaring both sides, we get

$$9(110) = 45x \Rightarrow x = 22$$

10. (c) Mean proportional of X and Y = \sqrt{XY}

Third proportional of X and Y = $\frac{Y^2}{X}$

$$\text{Now, } \sqrt{XY} = n \cdot \frac{Y^2}{X} \Rightarrow XY = n^2 \frac{Y^4}{X^2} \Rightarrow X^3 = n^2 Y^3$$

$$\Rightarrow \frac{X}{Y} = \frac{n^{2/3}}{1}$$

11. (d) Let the number be x added in the given ratio.

$$\therefore \frac{49 + x}{68 + x} = \frac{3}{4}$$

$$\Rightarrow 4(49 + x) = 3(68 + x) \Rightarrow x = 8$$

12. (c) Required ratio = $40 + 4 : 40 - 4$

$$= 44 : 36 = 11 : 9$$

13. (a) Amount of zinc = $\left(100 \times \frac{13}{20}\right) = 65 \text{ kg}$

14. (a) Let their incomes be ₹ 3x and ₹ 2x and their corresponding expenditure be ₹ 5y and ₹ 3y.

According to the given conditions,

$$3x - 5y = 1500 \text{ and } 2x - 3y = 1500$$

On solving, we get $x = 3000$ and $y = 1500$

Hence, B's income = $2x = ₹ 6000$

15. (a) The ratio of present incomes = $3 \times \frac{5}{4} : 4 \times \frac{3}{2}$

$$= \frac{15}{4} : \frac{12}{2} = 30 : 48 = 5 : 8$$

$$\therefore \text{A's present income} = \frac{4160}{5 + 8} \times 5 = ₹ 1600$$

16. (b) Let the ages of two boys be 5x and 6x, respectively.

According to the question, $\frac{5x + 2}{6x + 2} = \frac{7}{8}$

$$\Rightarrow 40x + 16 = 42x + 14 \Rightarrow 2x = 2 \Rightarrow x = 1$$

Now, ratio of their ages after 12 yr

$$= \frac{5x + 12}{6x + 12} = 17 : 18$$

17. (c) Let the present ages of Maya and Chhaya are 6x and 5x yr, respectively

$$\therefore \frac{6x + 15}{5x + 15} = \frac{9}{8}$$

$$\Rightarrow 48x + 120 = 45x + 135 \Rightarrow 3x = 15$$

$$\Rightarrow x = 5$$

Hence, present age of Maya = $5 \times 6 = 30$ yr

18. (d) Since, $x \propto y \Rightarrow x = ky$... (i)

$$\Rightarrow 8 = 15k \Rightarrow k = \frac{8}{15}$$

When $y = 10$

$$x = \frac{8}{15} \times 10 = \frac{16}{3} \quad [\text{from Eq. (i)}]$$

$$= 5 \frac{1}{3}$$

19. (d) \therefore Distance covered in 20 min = 5 km

$$\therefore \text{Distance covered in 1 min} = \frac{5}{20} \text{ km}$$

$$\therefore \text{Distance covered in 100 min} = \frac{5}{20} \times 100$$

$$= 5 \times 5 = 25 \text{ km}$$

20. (d) \therefore Weight of 45 m rod = 171 kg

$$\therefore \text{Weight of 1 m rod} = \frac{171}{45} \text{ kg}$$

$$\therefore \text{Weight of 12 m rod} = \frac{171}{45} \times 12 = 45.6 \text{ kg}$$

21. (a) \therefore Price of m articles = n

$$\therefore \text{Price of 1 article} = \frac{n}{m}$$

$$\therefore \text{Price of 5 articles} = \frac{5n}{m}$$

22. (b)

Orange	Cost
24 ↓	72 ↓
120 ↓	x ↓

$$\text{i.e. } 24 : 120 :: 72 : x$$

$$\Rightarrow 24 \times x = \frac{120 \times 72}{24} = ₹ 360$$

23. (d) Required number of sacks

$$= \frac{\text{Total quantity}}{\text{Quantity of each sack}} = \frac{1026}{114} = 9$$

24. (c)

Men	Days
$6 \uparrow$	$20 \downarrow$
$8 \uparrow$	$x \downarrow$

$$8 : 6 :: 20 : x$$

$$\therefore x = \frac{6 \times 20}{8} = 15$$

25. (a) 30% of $x = 450$

$$\Rightarrow x \times \frac{30}{100} = 450$$

$$\Rightarrow x = \frac{450 \times 100}{30} = 1500$$

26. (a) Let $500 \times x\% = 35$

$$\Rightarrow 500 \times \frac{x}{100} = 35$$

$$x = 7\%$$

27. (c) Let salary be ₹ x .

$$\text{Then, } x \times 20\% = x \times \frac{20}{100} = \frac{x}{5}$$

$$\therefore x - \frac{x}{5} = 20000$$

$$\Rightarrow \frac{4x}{5} = 20000$$

$$\therefore x = 25000$$

28. (c) We have, price of the article = ₹ 68000

\therefore New price of the article

$$= \text{Old price} + \text{Increased price}$$

$$= 68000 + 68000 \times \frac{20}{100}$$

$$= 68000 + 13600 = ₹ 81600$$

29. (a) Total number of students = 25

Student good in Mathematics = 72%

Students, who are not good in Mathematics
(100 - 72)% = 28%.

Now, number of student who are not good In
Mathematics = 28% of 25

$$= \frac{28}{100} \times 25 = 7$$

Hence, 7 students are not good in Mathematics.

30. (b) Let Chameli had total money be ₹ x .

Percentage of money spent by Chameli = 75%

Chameli had left money after spending

$$= (100 - 75)\% = 25\%$$

But money left = ₹ 600

[given]

$$\therefore 25\% \text{ of } x = 600$$

$$\Rightarrow \frac{25}{100} \times x = 600 \Rightarrow x = \frac{600 \times 100}{25} = 2400$$

Hence, Chameli had ₹ 2400 in the beginning.

31. (c) Number of matches won by the football
team = 10.

Let x matches be played by the team.

$$\therefore 40\% \text{ of } x = 10$$

$$\Rightarrow \frac{40}{100} \times x = 10$$

$$\Rightarrow x = \frac{10 \times 100}{40} = 25$$

Hence, a football team played 25 matches in all.

32. (d) \therefore 40% of students are girls.

\therefore 60% of students are boys

$$\therefore x \times 60\% = 600$$

$$\Rightarrow x \times \frac{60}{100} = 600$$

$$\Rightarrow x = \frac{600 \times 100}{60} = 1000$$

\therefore 40% of girls,

$$\text{i.e. } 1000 \times \frac{40}{100} = 400$$