## **BOARD QUESTION PAPER: JULY 2018**

## MATHEMATICS AND STATISTICS

Time: 3 Hours Total Marks: 80

### Note:

- i. All questions are compulsory.
- Figures to the right indicate full marks. ii.
- Graph of L.P.P. should be drawn on graph paper only. iii.
- Answer to every new question must be written on a new page. iv.
- Answers to both sections should be written in the same answer book. v.
- Use of logarithmic table is allowed. vi.

# SECTION-I

#### Select and write the appropriate answer from the given alternatives in each of the Q.1. (A) following sub-questions: (6)[12]

- If the sum of the slopes of the lines represented by  $x^2 + kxy 3y^2 = 0$  is twice their product, i. then the value of 'k' is \_\_\_\_\_.

(A) 2 (C) -1

- If the vectors  $\vec{i} = 2\vec{j} + \vec{k}$ ,  $a\vec{i} = 5\vec{j} + 3\vec{k}$  and  $5\vec{i} = 9\vec{j} + 4\vec{k}$  are coplanar, then the value of a is II.
  - (A) 3

(C) 2

- The acute angle between the line  $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$  and the plane 10x + 2y 11z = 8 is
  - (A)  $\sin^{-1}\left(\frac{8}{21}\right)$

(B)  $\cos^{-1}\left(\frac{8}{21}\right)$ 

(D)  $\cos^{-1}\left(\frac{1}{e}\right)$ 

#### Attempt any THREE of the following: (B)

(6)

- Write the dual of each of the following statements: 1.
  - $-p \wedge (q \vee c)$
  - "Shweta is a doctor or Seema is a teacher."
- In  $\triangle$  ABC, prove that ac cos B bc cos A =  $a^2 b^2$ . ii.
- Show that the equation  $2x^2 + xy y^2 + x + 4y 3 = 0$  represents a pair of straight lines. iii.
- If  $\bar{a}$ ,  $\bar{b}$ ,  $\bar{c}$  are the position vectors of the points A, B, C respectively such that  $3\bar{a} + 5\bar{b} = 8\bar{c}$ , then find the ratio in which C divides AB.
- If points A (5, 5, λ), B (-1, 3, 2) and C (-4, 2, -2) are collinear, then find the value of λ. V.

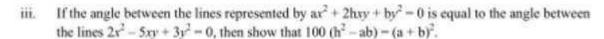
#### Q.2. (A) Attempt any TWO of the following:

(6)[14]

Using truth table, examine whether the following statement pattern is a tautology, a contradiction or a contingency:

$$(p \vee q) \vee r \leftrightarrow p \vee (q \vee r)$$

Find the inverse of the matrix A where  $A = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$  by using adjoint method. ii.



#### (B) Attempt any TWO of the following:

(8)

- Prove that three vectors a, b and c are coplanar if and only if there exists non-zero linear î. combination  $x \overline{a} + y \overline{b} + z \overline{c} = \overline{0}$ .
- Maximize z = 6x + 4y subject to constraints,  $x \le 2$ ,  $x + y \le 3$ ,  $-2x + y \le 1$ ,  $x, y \ge 0$ . Also find the ii. maximum value of 'z'.
- iii. Find the general solution of  $\sin 2x + \sin 4x + \sin 6x = 0$ .

#### Q.3. (A) Attempt any TWO of the following:

(6)[14]

- Write the negations of the following statements:
  - If diagonals of a parallelogram are perpendicular, then it is a rhombus.
  - Mangoes are delicious, but expensive.
  - A person is rich if and only if he is a software engineer.
- 11. Express the following equations in matrix form and solve them by the method of reduction: x+y+z=6, 3x-y+3z=6 and 5x+5y-4z=3
- Find the vector equation of the line passing through the point (-1, -1, 2) and parallel to the line iii. 2x-2=3y+1=6z-2.

#### (B) Attempt any TWO of the following:

- A plane meets the coordinate axes in A, B, C such that the centroid of the triangle ABC is the point (p, q, r). Show that the equation of the plane is  $\frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 3$ .
- If  $\alpha$ ,  $\beta$ ,  $\gamma$  are direction angles of the line l, then prove that  $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$ . ii. Hence deduce that  $\sin^2 \alpha + \sin^2 \beta + \sin^2 \psi = 2$
- Using the Sine rule, prove the Cosine rule. iii.

# SECTION - II

#### Select and write the appropriate answer from the given alternatives in each of the Q.4. (A) following sub-questions: (6)[12]

- Equation of the tangent to the curve  $2x^2 + 3y^2 5 = 0$  at (1, 1) is 1. (A) 2x-3y-5=0

(C) 2x + 3y + 5 = 0

- (B) 2x + 3y 5 = 0(D) 3x + 2y + 5 = 0
- The order and the degree of the differential equation  $\left(\frac{d^3y}{dx^3}\right)^{\frac{1}{6}} \left(\frac{dy}{dx}\right)^{\frac{1}{3}} = 0$  are respectively
  - (A) 3, 2

- Given  $X \sim B(n, p)$ . If p = 0. 6, E(X) = 6, then the value of Var(X) is \_\_\_\_\_. iii.
  - (A) 2.4

(B) 2.6

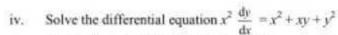
(C) 2.5

(D) 2.3

#### (B) Attempt any THREE of the following:

(6)

- The displacement s of a particle at time t is given by  $s = t^3 4t^2 5t$ . Find its velocity and acceleration at t - 2.
- If  $y = \cos^{-1}(1 2\sin^2 x)$ , find  $\frac{dy}{dx}$ . ii.
- Evaluate:  $\int \frac{1}{\sin x \cdot \cos^2 x} dx$ iii.



v. Obtain the probability distribution of the number of sixes in two tosses of a fair die.

## Q.5. (A) Attempt any TWO of the following:

(6)[14]

i. If  $f(x) = \frac{1 - \sqrt{3} \tan x}{\pi - 6x}$ , for  $x \ne \frac{\pi}{6}$  is continuous at  $x = \frac{\pi}{6}$ , find  $f\left(\frac{\pi}{6}\right)$ .

ii. If 
$$\sec^{-1}\left(\frac{x+y}{x-y}\right) = a^2$$
, show that  $\frac{dy}{dx} = \frac{y}{x}$ .

iii. Evaluate: 
$$\int \frac{e^{r}}{(1+e^{r})(2+e^{r})} dr$$

## (B) Attempt any TWO of the following:

(8

 A stone is dropped into a pond. Waves in the form of circles are generated and the radius of the outermost ripple increases at the rate of 2 inch/sec. How fast will the area of the wave increase

a. when the radius is 5 inch?

b. after 5 seconds?

ii. Evaluate: 
$$\int_{1}^{x} \frac{1}{x + \sqrt{a^2 - x^2}} dx$$

 The rate of growth of bacteria is proportional to the number present. If initially, there were 1000 bacteria and the number doubles in 1 hour, find the number of bacteria after 2½ hours.
[Take √2 = 1, 414]

## Q.6. (A) Attempt any TWO of the following:

(6)[14]

i. Discuss the continuity of the following function in its domain, where

$$f'(x) = x^2 - 4$$
, for  $0 \le x \le 2$   
=  $2x + 3$ , for  $2 < x \le 4$   
=  $x^2 - 5$ , for  $4 < x \le 6$ 

 $= x^2 - 5$ , for  $4 < x \le 6$ ii. If y = f(u) is differentiable function of u, and u = g(x) is a differentiable function of x, then prove that y = f[g(x)] is a differentiable function of x and  $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ 

 Suppose that 80% of all families own a television set. If 10 families are interviewed at random, find the probability that at most three families own a television set.

## (B) Attempt any TWO of the following:

(8)

i. If u and v are integral functions of x, then show that  $\int u \cdot v \, dx = u \int v \, dx - \int \left[ \frac{du}{dx} \int v \, dx \right] dx$ 

Hence evaluate  $\int \log x \, dx$ .

ii. Prove that :

$$\int_{a}^{2a} f(x) dx = \int_{a}^{\infty} f(x) dx + \int_{a}^{\infty} f(2a - x) dx$$

iii. Find k if the function f(x) is defined by

$$f(x) = kx(1-x)$$
, for  $0 < x < 1$   
= 0 , otherwise,

is the probability density function (p.d.f.) of a random variable (r.v.) X. Also find  $P\left(X < \frac{1}{2}\right)$