# Manipal – 2024

# **Solved Paper**

# Physics

**1.** If E = energy, G = gravitational constant, I = impulse and M = mass, then dimensions of  $\frac{GIM^2}{E^2}$  are same as that of

A. time

B. mass

C. length

D. force

**2.** Two points move in the same straight line starting at the same moment from the same point in it. The first moves with constant velocity u and the second with constant acceleration *f*. During the time elapses before the second catches, the first greatest distance between the particle is

A. u/f

B. u<sup>2</sup>/2f

C. f/2u<sup>2</sup>

D.  $f/u^2$ 

**3.** A car turns on a banked road of radius 300 m. If the coefficient of friction between the tyres and the road is 0.3, then maximum speed of the car on the turn is

[ Take , g = 10 m/s<sup>2</sup> ] A. 10 m/s

B. 30 m/s

C. 40 m/s

D. 50 m/s

**4.** A particle is projected from the surface of the earth with an initial speed of 4.0kms<sup>-1</sup>. The maximum height (in km) attained by the particle is \_\_\_\_\_

[Radius of earth = 6400 km and  $g = 9.8 \text{ ms}^{-2}$ ]

Numerical

**5.** The stress versus strain graphs for wires of two materials A and B are as shown in the figure. If  $Y_A$  and  $Y_B$  are the Young's modulii of the materials, then



**6.** Two pendulums of time periods 3 s and 7 s , respectively start oscillating simultaneously from two opposite extreme positions. After how much time they will be in same phase?

A. 21/8 s

B. 21/4 s

C. 21/2 s

D. 21/10 s

**7.** Fundamental frequency of a sonometer wire is n, if the tension is made 3 times and length and diameter are also increased 3 times, what is the new frequency?

A.  $\frac{n}{3\sqrt{3}}$ 

B. 3n

C.√3n

D. n/√3

8. An electric dipole shown in the figure. Work done to move a charge particle of 1µC from point Q to P is x  $\times$  10<sup>-7</sup> J, then the value of x is .....



Numerical

**9.** In the following circuit diagram, potential difference across  $4\mu$  F capacitor is



D. 8 V

**10.** An electric kettle has two coils. When one coil is switched on, it takes 10 min to boil water and when the second coil is switched on it takes 20 min to boil same amount of water. The time taken by electric kettle to boil water when-both coil are used in parallel, is found to be n second. Find the value of n.

Numerical

**11.** When 100 V DC is applied across a solenoid, a current of 1 A flows in it. When 100 VAC is applied across the same coil, the current drop is 0.5 A. If the frequency of the AC source is 50 Hz, then inductance of solenoid is found to be xmH, then find the value of x.

Numerical

**12.** When a lens is cut into two halves along XOX', then focal length of each half lens



A. increases

B. decreases

C. remains same

D. None of the above

**13.** If the frequency of incident photon on metal surface is doubled, then stopping potential will become

A. doubled

B. less than double

C. more than double

D. less than existing value

**14.** If an electron in n = 4 orbit of hydrogen atom jumps down to n = 3 orbit, the amount of energy released and the wavelength of radiation emitted are

#### A. 0.66eV ,1.88 imes 10<sup>-6</sup> m

B. 1.89eV ,1.98 imes 10<sup>-7</sup> m

C. 0.29eV ,1.78  $\times$   $10^{-5}\,\mathrm{m}$ 

D. 0.98eV ,0.93 imes 10<sup>-6</sup> m

15. In the circuit shown below, the diode has  $20\Omega$  in forward bias resistance. When V<sub>i</sub> increases from 8 V to 12 V, then change in the current is found to be x mA. Find the value of x.



Numerical

# Chemistry

**1.** The wave number of the shortest wavelength of absorption spectrum of hydrogen atom is \_\_\_\_\_ (Rydberg constant =  $109700 \text{ cm}^{-1}$ ).

Numerical

2. Electronegativity of the following elements increases in the order

A. C, N, Si, P

B. N, Si, C, P

C. Si, P, C, N

D. P, Si, N, C

3. Match List-I with List-II.

	List-I		List-II
	(Compound)		(Hybridisation)
А.	${ m CuCl}_5^{3-}$	I.	$sp^3d^2$
В.	${ m MnCl}_5^{3-}$	II.	$d^2sp^3$
C.	${\rm XeOF_4}$	III.	$dsp^3$
D.	$Fe(CO)_5$	IV.	$sp^{3}d$

Choose the correct match from the options given below

A. A-IV, B-III, C-I, D-II

B. A-IV, B-III, C-II, D-I

C. A-IV, B-I, C-III, D-II

D. A-IV, B-II, C-III, D-I

**4.** The spin only magnetic moment of  $[NiCl_4]^{2-}$  is \_\_\_\_\_ (Nearest integer)

Numerical

5. The maximum work obtained from a reversible process is given as

Α. –ΔΑ

Β. ΔΑ

 $C. -\Delta G$ 

D. ∆G

**6.** If  $K_{\rho}$  for a reaction,

 $A(g) + 2B(g) \rightleftharpoons 3C(g) + D(g)$  is 0.05 atm at 1000 K. Its K<sub>c</sub> in term of R is  $\frac{x \times 10^{-5}}{R}$ . The value of x is \_\_\_\_\_

Numerical

**7.** Boiling point of water at 750 mm Hg is 99.63°C. The amount of sucrose that should be added to 500 g of water such that it boil at 100°C is \_\_\_\_\_ g. (nearest integer)

(Molar elevation constant for water = 0.5 K kg/mol)

Numerical

8. For the cell reaction,

 $Cu(s) + 2Ag^+(aq) \rightarrow Cu^{2+}(aq) + 2Ag(s) \xrightarrow{E_{cell}^{\circ}}$  is 0.46 V. The equilibrium constant of the reaction is

A.  $3.92 \times 10^{12}$ 

B.  $3.92 \times 10^{15}$ 

 $C. 8.92 \times 10^{17}$ 

 $D. 8.92 \times 10^{10}$ 

**9.** For a first order reaction, time required for 99% completion of a reaction is x times the time required for completion of 90%. The value of x is ......

Numerical

10.  $F_2$  is formed by reacting  $K_2MnF_6$  with

A. SbF<sub>5</sub>

B. MnF<sub>3</sub>

C. KSbF<sub>6</sub>

D. MnF<sub>4</sub>

**11.** Which of the following ion is colourless inspite of the presence of unpaired electrons?

A. La<sup>3+</sup>

B. Eu<sup>3+</sup>

C. Gd<sup>3+</sup>

D. Lu<sup>3+</sup>

**12.** The oxidation state of Cr in  $[Cr(H_2O)_6]$  Cl<sub>3</sub>,  $[Cr(C_6H_6)_2]$ ,  $K_2[Cr(CN)_2(O)_2(O_2)(NH_3)]$  respectively are

- A. +3, +4, +6
- B. +3, +2, +4
- C. +3, 0, + 6
- D. +3, 0, + 4
- 13. In the following sequence of reactions,

$$\operatorname{CH}_3 - \operatorname{Br} \xrightarrow{\operatorname{KCN}} X \xrightarrow{\operatorname{H}_3\operatorname{O}^+} Y \xrightarrow[\operatorname{Ether}]{\operatorname{LiAlH}_4} Z$$

The final product Z is

- A. acetone
- B. methane
- C. acetaldehyde
- D. ethyl alcohol

**14.** The major product Y in the following reaction is

$$\overbrace{O}^{\text{ACH}_2\text{OH}} Y$$

- A. hemiacetal
- B. acetal
- C. an ether
- D. an ester
- **15.** Consider the following amino acids.
- (i) Lysine
- (ii) Glutamine

(iii) Arginine

(iv) Leucine

(v) Serine

(vi) Proline

(vii) Valine

Which of the given amino acids are basic in nature?

- A. (i) and (iii)
- B. (i), (ii) and (iv)
- C. (iii) and (vii)
- D. (iii), (v) and (vi)

# Mathematics

1. The solution of the equation  $\log(\log_4(\sqrt{x+4} + \sqrt{x})) = 0$  is A. 2 B. 4 C. 9/4 D. 8 2.  $\frac{a}{b} = \frac{1}{3}$  and  $\frac{b}{c} = \frac{3}{4}$ , then the value of  $\frac{a+2b}{b+2c}$  is A. 28/33 B. 7/11 C. 1/2 D. None of these 3. Total number of even divisions of 2079000 which are divisible by 15 are A. 54 B. 128

- C. 108
- D. 72

**4.** If N denotes number of 8 digit numbers that contains exactly four nines, then unit digit of N is

Numerical

**5.** If expression  $x + 1/x^2$ , |x > 0| attains is minimum value at  $x = \alpha$ , then  $\alpha^3$  is

Numerical

**6.** If the number of terms in the expansion of  $(x\sqrt{180} + \sqrt[3]{432})^{200}$  having integral coefficients is n, then the vaule of [n/6] is (where [.] denotes greater integer function)

A. 4

B. 5

С. 6

D. 7

7. If the coefficient of  $x^m$ ,  $m \in N$  in the expansion of  $\left(\sqrt{2x} + \sqrt[3]{\frac{3}{x^2}}\right)^9$  is equal to  $k(k \neq 0)$ , the k is equal to

A. 1008

B. 2016

C. 3024

D. 1016

**8.** If the angle between the pair of straight lines formed by joining the points of intersection of  $x^2 + y^2 = 4$  and y = 3x + c to the origin is a right angle, then  $c^2$  is

A. 20

B. 13

C. 1/5

D. 5

**9.** The equation of mirror image of the circle  $x^2 + y^2 - 6x - 10y + 33 = 0$  about the line mirror y = x is .....

A.  $x^{2} + y^{2} - 10x + 6y + 33 = 0$ B.  $x^{2} + y^{2} + 10x - 6y + 33 = 0$ C.  $x^{2} + y^{2} - 10x - 6y + 33 = 0$ D.  $x^{2} + y^{2} + 10x + 6y + 33 = 0$ 

**10.** If two tangents drawn from the point (h, k) to the parabola  $y^2 = 64x$  be such that the slope of one tangent is 8 times of the other, then the value of  $k^2/2h$  is

A. 9

B. 27

C. 81

D. 162

**11.** Let  $f(x) = \left[\frac{\sin x}{x}\right] + \left[\frac{2\sin x}{x}\right] + \ldots + \left[\frac{10\sin x}{x}\right]$  (where [.] is the largest integer). then, find the value of  $\lim_{x\to 0} f(x)$ .

Numerical

**12.** If in a  $\triangle$ ABC, sin<sup>2</sup> A + sin<sup>2</sup> B + sin<sup>2</sup> C = 2, then the triangle is always:

A. isosceles triangle

- B. right angled
- C. acute angled
- D. obtuse angled

**13.** triangle ABC, sin A, sin B and sin C are in AP ( $C > 90^{\circ}$ ), then cos A is

A.  $\frac{3c-4b}{2b}$ B.  $\frac{3c-4b}{2c}$ C.  $\frac{4c-3b}{2b}$ D.  $\frac{4c-3b}{2c}$ Let  $D = \begin{vmatrix} n & n^2 & n^3 \\ n^2 & n^3 & n^5 \\ 1 & 2 & 3 \end{vmatrix}$ , then  $\lim_{n \to \infty} \frac{M_{11} + C_{33}}{(M_{13})^2}$  is equal to (where  $M_{ij}$  is the minor and  $C_{ij}$  is the 14. cofactor of  $i^{\text{th}}$  row and  $j^{\text{th}}$  column's element) A. 0 B. -1 C. -2 D. 3 15. If  $x = \sin \left( 2 \tan^{-1} 2 \right), y = \sin \left( \frac{1}{2} \tan^{-1} \frac{4}{3} \right)$ , then A. x = 1 - yB.  $x^2 = 1 - y$ C.  $x^2 = 1 + y$ D.  $y^2 = 1 - x$ 

Let: f:N
ightarrow N be defined as

$$f(n)=\left\{egin{array}{cc} rac{n+1}{2}, & ext{if is $n$ odd} \ rac{n}{2}, & ext{if $n$ is even} \end{array}
ight.$$
 for all  $n\in N.$  Then  $f$  is 16.

A. injective but not surjective

B. surjective but not injective

C. both injective and surjective

D. neither injective nor surjective

**17.** Let f(x) is a polynomial function such that  $f(x) + f(1/x) = f(x) \cdot f(1/x)$ , x > 0 and f(3) = 10, if  $\int f(x)dx = g(x) + c$  and g(1) = 4/3, then the value of g(3) is

A. 10

B. 9

C. 8

D. 12

A real valued differentiable function f satisfies  $f(x) + f(y) + 2xy = f(x+y) \forall x, y \in R$ . Given 18.  $f^n(0) = 0$ , then value of  $\int_0^{\frac{\pi}{2}} f(\sin x) dx$  will be

A. 0

B. π/4

C. π/2

D. π

**19.** Given  $dy/dx + 2y \tan x = \sin x$  and y = 0 for  $x = \pi/3$ . If maximum value of y is 1/k, then ' k ' is equal to

Numerical

**20.** Given three vectors a, b and c each two of which are non collinear. Further if (a + b) is collinear with c, (b + c) is collinear with a and  $|a|=|b|=|c|=\sqrt{2}$ . Then, the value of  $|a \cdot b + b \cdot c + c \cdot a|$  is

Numerical

# Solution

# **Physics**

Ans 1

### **Correct Option.** A

### Solution:

The SI unit in which *G* is measured = N - m<sup>2</sup>/kg<sup>2</sup>  $\therefore \text{ Its dimensions will be } \frac{[\text{MLT}^{-2}][\text{L}^2]}{[\text{M}^2]} = [\text{M}^{-1} \text{ L}^3 \text{ T}^{-2}]$   $\therefore \text{ Impulse} = \text{Force} \times \text{Time} = [\text{MLT}^{-2}][\text{T}] = [\text{MLT}^{-1}]$   $\therefore \text{ Dimensions for energy} = [\text{ML}^2 \text{ T}^{-2}]$ Now put the dimensions of *G*, *I*, *M* and *E* in  $\frac{GIM^2}{E^2}$   $= \frac{[\text{M}^{-1} \text{ L}^3 \text{ T}^{-2}][\text{MLT}^{-1}][\text{M}^2]}{[\text{ML}^2 \text{ T}^{-2}]^2}$   $= \frac{[\text{M}^{-1} \text{ L}^3 \text{ T}^{-2}][MLT^{-1}][\text{M}^2]}{[\text{M}^2 \text{ L}^4 \text{ T}^{-4}]} = \frac{[\text{M}^2 \text{ L}^4 \text{ T}^{-3}]}{[\text{M}^2 \text{ L}^4 \text{ T}^{-4}]}$ 

The equation  $\frac{GIM^2}{E^2}$  has dimension of time.

# Ans 2

# **Correct Option. B**

### Solution:

The greatest distance between the particles will be at the moment when velocity of both particles gets equal

 $\therefore$  Using v = u + at, for acceleratod particle

$$u=0+(F)(t) \ t=rac{u}{(F)}$$

Distance travelled by accelerated in time t,

$$egin{aligned} D_1 &= O + rac{1}{2}(F) \Big(rac{u}{F}\Big)^2 \ D_1 &= rac{u^2}{2F} \end{aligned}$$

Distance travelled by constantly moving particle,

$$egin{aligned} D_2 &= (u)\left(rac{u}{F}
ight) + rac{1}{2}(0)\left(rac{u}{F}
ight)^2 \ D_2 &= rac{u^2}{F} \end{aligned}$$

Greatest distance between two particles

$$D=rac{u^2}{f}-rac{u^2}{2f}$$
 $D=rac{u^2}{2f}$ 

#### Ans 3

# **Correct Option. B**

#### Solution:

Given,  $r=300~\mathrm{m}, \mu=0.3$ ,

 $g=10~{\rm m/s^2}$ 

If v is the velocity of the vehicle while turning on the road, then

$$\frac{mv^2}{r} \leq \mu mg$$

 $\Rightarrow v \leq \sqrt{\mu r g}$ 

 $\Rightarrow v_{
m max} = \sqrt{\mu r g}$ 

 $=\sqrt{0.3\times300\times10}=30~m/s$ 

#### Ans 4

# **Correct Option. 935**

Solution:

Using conservation of mechanical energy.

Decrease in kinetic energy = Increase in gravitational potential energy

$$egin{array}{lll} \Rightarrow & rac{1}{2}mv^2 = \Delta U \ \Rightarrow & rac{1}{2}mv^2 = rac{mgh}{1+rac{h}{R}} \ \Rightarrow & h = rac{v^2}{2g-rac{v^2}{R}} \end{array}$$

Substituting the values, we get

$$h = rac{ig(4.0 imes 10^3ig)^2}{2 imes 9.8 - rac{ig(4 imes 10^3ig)^2}{6.4 imes 10^6}} = 9.35 imes 10^5 ext{ m} = 935 ext{ km}$$

#### Ans 5

# **Correct Option.** C

#### Solution:

We know that the slope of the line in a stress-strain curve represents the young's modulus for a wire. We also know that the slope of a line can be calculated as the tangent of the angle the line makes with the positive x -axis of the graph.

So, for wire A, the stress-strain line of the material is at an angle of  $60^{\circ}$  from the positive x-axis. So the slope of the line (m<sub>A</sub>) will be

 $m_A = an 60^\circ \ \Rightarrow m_A = \sqrt{3}$ 

Hence the young's modulus of wire A will also be  $\sqrt{3}$ .

Similarly, for wire B , the stress-strain line of the material is at an angle of  $30^\circ$  from the positive x axis. So, the slope of the line  $(m_B)$  will be

 $m_B = an 30^\circ \ \Rightarrow m_B = rac{1}{\sqrt{3}}$ 

Hence the young's modulus of wire B will also be  $1/\sqrt{3}$ .

Then taking the ratio of the young's modulus for wire A and B , we get

$$rac{Y_A}{Y_B} = rac{\sqrt{3}}{1/\sqrt{3}} \ dots rac{Y_A}{Y_B} = 3$$

# Correct Option. A

#### Solution:

General equation of oscillation are

$$y_{1} = A \sin\left(\omega_{1}t + \frac{\pi}{2}\right)$$

$$y_{2} = A \sin\left(\omega_{2}t + \frac{\pi}{2}\right)$$
Now,  $\omega_{1}t + \frac{\pi}{2} = \omega_{2}t - \frac{\pi}{2}$ 
or
$$t = \frac{\pi}{\omega_{2} - \omega_{1}}$$

$$= \frac{\pi}{(2\pi/T_{2}) - (2\pi/T_{1})}$$

$$= \frac{T_{1}T_{2}}{2(T_{1} - T_{2})}$$

$$= \frac{3 \times 7}{2(7 - 3)} = \frac{21}{8} \text{ s}$$

### Ans 7

Correct Option. A

### Solution:

Fundamental frequency,

$$n = \frac{v}{2l} = \frac{\sqrt{T/\mu}}{2l} = \frac{\sqrt{T/\rho S}}{2l} = \frac{\sqrt{T/\rho \pi r^2}}{2l}$$
$$\therefore n \propto \frac{\sqrt{T}}{2l}$$

Given that tension, diameter and length all are made 3 times.

$$\therefore n$$
 will become  $\frac{1}{3\sqrt{3}}$  times

### Ans 8

# **Correct Option. 1.8**

# Solution:

From figure, electric dipole moment,

$$p=2ql=q(2l)$$
  
 $=2 imes 10^{-6} imes \left(1 imes 10^{-9}
ight)$   
 $\Rightarrow p=2 imes 10^{-15} 
m cm$ 

Electric potential at point p due to dipole

$$\begin{split} V_p &= \frac{1}{4\pi\varepsilon_0} \cdot \frac{p\cos\theta}{r^2} \\ V_p &= 9 \times 10^9 \times \frac{2 \times 10^{-15} \times \cos 60^\circ}{\left(1 \times 10^{-2}\right)^2} \\ &= \frac{18 \times 10^{-6} \times \frac{1}{2}}{10^{-4}} = 9 \times 10^{-2} = 0.09 \text{ V} \end{split}$$

Similarly, electric potential at point  ${\boldsymbol{Q}}$  due to electric dipole

$$V_Q = rac{9 imes 10^9 imes \left(2 imes 10^{-15}
ight) imes \cos 120^\circ}{\left(1 imes 10^{-2}
ight)^2} = -0.09 \, {
m V}$$

 $\therefore$  Potential difference between point Q and P.

$$V_{QP} = V_P - V_Q = 0.09 - (-0.09) = 0.18 \text{ V}$$

 $\therefore$  Work done to move the charge of  $1\mu C$  from point Q to P.

$$W = V_{QP} \cdot q = 0.18 imes 1 imes 10^{-6} \ = 1.8 imes 10^{-7} \ {
m J}$$

Ans 9

#### **Correct Option. D**

#### Solution:

1. Parallel Combination:

The  $2\mu F$  and  $6\mu F$  capacitors are in parallel, so their equivalent capacitance:

$$C_{eq} = 2 + 6 = 8 \mu F$$

2. Series Combination:

The  $4\mu F'$  capacitor is in series with the  $8\mu F$  equivalent capacitor. The total capacitance is:

$$rac{1}{C_{total}} = rac{1}{4} + rac{1}{8} = rac{3}{8} \Longrightarrow C_{total} = rac{8}{3} \mu F$$

3. Charge Calculation:

The total charge in the circuit is:

$$Q = C_{total} \cdot V = \frac{8}{3} \cdot 12 = 32 \mu C$$

4. Voltage Across 4µF Capacitor:

Since the charge is the same in series, the voltage across the  $4\mu F$  capacitor is:

$$V = rac{Q}{C} = rac{32}{4} = 8 \ {
m V}$$

Final answer: 8V.

#### Ans 10

#### **Correct Option. 400**

#### Solution:

Heat produced in a resistance R in time t is given by

$$H = \frac{V^2}{R}t$$
 [ where ,  $V =$  source voltage ]

For first coil,

$$H_1 = rac{V^2}{R_{
m i}} t_1 = rac{V^2}{R_{
m i}} imes 10 imes 60 \, [::t_1 = 10 imes 60 \, {
m s}$$

For second coil,  $H_2 = rac{V^2}{R_2} t_2 \left[\because t_2 = 20 imes 60 ext{ s}
ight]$ 

$$=rac{V^2}{R_2} imes 20 imes 60$$

Given that,  $H_1=H_2$ 

$$egin{array}{lll} \Rightarrow & rac{V^2}{R_1} imes 10 imes 60 = rac{V^2}{R_2} imes 20 imes 60 \ \Rightarrow & R_2 = 2R_1 . . (i) \end{array}$$

Let t be the time taken by electric kettle to boil water when both coil are used in parallel, then

$$\begin{split} H_p = & \left(\frac{V^2}{R_1} + \frac{V^2}{R_2}\right) \times t = V^2 t \left(\frac{1}{R_1} + \frac{1}{2R_1}\right) [\text{ from Eq. (i) }] \\ H_p = & \frac{3V^2 t}{2R_1} \end{split}$$

But,  $H_P = H_1$ 

 $\Rightarrow t = 400 \text{ s}$ 

$$\therefore \frac{3V^2t}{2R_1} = \frac{V^2}{R_1} \times 10 \times 60$$

Ans 11

# Correct Option. 550

#### Solution:

When DC is applied across a solenoid, then resistance, i.e.  $R=rac{V}{T}$ 

 $R=rac{100}{1}=100\Omega$ 

When AC is applied, then

 $Z=rac{100}{0.5}=200\Omega$ 

Net impedance,

$$Z = \sqrt{R^2 + X_L^2}$$

$$200 = \sqrt{(100)^2 + X_L^2}$$

$$\Rightarrow (200)^2 = (100)^2 + X_L^2$$

$$\Rightarrow 30000 = X_L^2 \Rightarrow X_L = 173.205\Omega$$
Now,  $X_L = \omega L$ 

$$\Rightarrow 173.205 = 2\pi fL \Rightarrow \frac{173.205}{2\pi \times f} = L$$

$$\Rightarrow \frac{173.205}{2 \times 3.14 \times 50} = L \Rightarrow 0.55\Omega = L$$

$$L = 0.55H = 550mH$$

# Ans 12

# Correct Option. C

#### Solution:

If R be the magnitude of radius of curvature of equiconvex lens, then its focal length *f* is given by

$$egin{aligned} &rac{1}{f} = (\mu-1)\left(rac{1}{R_1} - rac{1}{R_2}
ight) \ &= (\mu-1)\left(rac{1}{R} - rac{1}{-R}
ight) = rac{2(\mu-1)}{R} \ &\Rightarrow rac{1}{f} = rac{2(\mu-1)}{R} \dots (i) \end{aligned}$$

When equiconvex lens is cut into two halves along XOX', then

 $R_1=R, R_2=-R$ 

∴ Focal length f' is given as

$$\frac{1}{f'} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) = \frac{2(\mu - 1)}{R} = \frac{1}{f} \quad [\text{ from (i) }]$$
$$\Rightarrow f' = f$$

### Ans 13

# Correct Option. C

#### Solution:

According to Einstein's photoelectric equation,

$$hf = W_0 + eV_0 \dots (i)$$

If frequency is doubled, then

$$egin{aligned} h(2f) &= W_0 + eV_0' \ 2hf &= W_0 + eV_0' \dots (ii) \end{aligned}$$

Substituting hf from Eq. (i) in Eq. (ii), we have,

or 
$$V_0'=2V_0+rac{W_0}{e}$$
 or  $V_0'>2V_0$ 

Ans 14

**Correct Option.** A

Solution:

$$E = rac{-13.6}{n^2} \mathrm{eV}$$
  
 $E_4 - E_3 = 13.6 \left[rac{1}{(3)^2} - rac{1}{(4)^2}
ight]$   
 $= 13.6 \left[rac{7}{144}
ight] \mathrm{eV} = 0.66\mathrm{eV}$   
 $\lambda = rac{1}{R\left[rac{1}{(3)^2} - rac{1}{(4)^2}
ight]} = rac{144}{1.09 imes 10^7 imes 7} \mathrm{m}$   
 $= 1.88 imes 10^{-6} \mathrm{m}$ 

#### **Correct Option. 9**

#### Solution:

Diode will be in forward bias when  $V_i$  is greater than 10 V and it conducts till  $V_i$  is 12 V .

Total circuit resistance

 $=200+20=220\Omega$ 

When  $V_i$  is less than 10 V , the diode will not conduct due to its reverse biasing. Thus, initial current,  $i_1 = 0$ .

When  $V_i=12~{
m V}$  , then diode will be in forward bias with effective voltage,  $V'=12-10=2~{
m V}$ 

 $\therefore$  Maximum current,  $I_2 = rac{V'}{R} = rac{2}{220} = 0.009 \ \mathrm{A} = 9 \ \mathrm{mA}$ 

 $\therefore$  Change in current,  $\Delta I = I_2 - I_1 = 9~\mathrm{mA}$ 

# Chemistry

Ans 1

### Correct Option. 109700

#### Solution:

Given,  $R=109700~{
m cm}^{-1}$ 

Shortest wavelength mean transition is from infinity to n = 1.

Using the formula,

$$\begin{split} \bar{v} &= R \left[ \frac{1}{n_f^2} - \frac{1}{n_i^2} \right] \\ n_t &= \infty, n_i = n = 1 \\ \bar{v} &= 109700 \left[ \frac{1}{1} - \frac{1}{\infty} \right] \\ \bar{v} &= 109700 \ \mathrm{cm}^{-1} \end{split}$$

### Ans 2

# Correct Option. C

### Solution:

As electronegativity increases across the period, therefore, electronegativity of nitrogen ( N ) is more than carbon (C) and electronegativity of P is more than Si .

Also electronegativity decreases down the group, therefore electronegativity of carbon is more than Si and electronegativity of N is more than P. Hence, the correct increasing order will be Si, P, C, N.

Ans 3

**Correct Option.** A

Solution:

The correct match is A-IV, B-III, C-I, D-II.

Hybridisation involved in the given complexes are

 $egin{aligned} {
m CuCl}_5^{3-}, {
m Cu(II)} &= sp^3d(4s,4p,4d) \ {
m MnCl}_5^{3-}, {
m Mn(II)} &= dsp^3(3d,4s,4p) \ {
m XeOF}, {
m Xe(VI)} &= sp^3d^2(5s,5p,5d) \ {
m Fe(CO)}_5, {
m Fe}(0) &= d^2sp^3 \end{aligned}$ 

#### **Correct Option.** 3

#### Solution:

 $Ni: 3d^84s^2$ 

for the compound  $[NiCl_4]^{2-}$ , Ni is +2 oxidation state. Hence, the electronic configuration of  $Ni^{2+}$  is given by  $Ni^{2+}:3d^8$ 

Since,  $Cl^{-1}$  is a weak field ligand, so it will not cause pairing of electrons. So, for  $Ni^{2+}$  the configuration is



Hence, n = 2

Spin only magnetic moment is given by

$$\mu_s = \sqrt{n(n+2)} = \sqrt{2(2+2)}$$
BM =  $\sqrt{8} = 2.82$ BM  
 $\mu_s \simeq 3$  BM

#### Ans 5

**Correct Option.** A

#### Solution:

Decrease in the value of Gibbs energy  $(-\Delta G)$  gives the maximum useful work or net work done by the system for the given change whereas decrease in Helmholtz free energy or work function  $(-\Delta A)$  gives the maximum work done for the given change.

Ans 6

**Correct Option. 5** 

#### Solution:

Given reaction,

$$egin{aligned} A(g)+2B(g)&\rightleftharpoons 3C(g)+D(g)\ \Delta n_g&=4-3=1 \end{aligned}$$
 Formula for  $K_p, \ K_p&=K_c(RT)^{\Delta n}\ 0.05&=K_c(R imes1000)^1\ K_c&=rac{5 imes10^{-5}}{R} \end{aligned}$ 

#### **Correct Option. 127**

#### Solution:

Elevation in boiling point of water required  $(\Delta T_b)$ .

 $= 100 - 99.63 = 0.37^{\circ}\mathrm{C}$ 

Mass of solvent (water),  $W_1=500~{
m g}$ 

Molar mass of solvent,  $M_1 = 18~{
m g/mol}$ 

Molar mass of solute (sucrose)

 $(C_{12}H_{22}O_{11}) = 342 \text{ g/mol}$  $M_2 = rac{1000 imes K_b imes w_2}{w_1 \Delta T_b}$ or  $w_2 = rac{M_2 imes W_1 imes \Delta T_b}{1000 imes K_b} = rac{342 imes 500 imes 0.37}{1000 imes 0.5}$  $w_2 = 126.54 pprox 127$ 

#### Ans 8

#### **Correct Option. B**

#### Solution:

$$\operatorname{Cu}(s) + 2\operatorname{Ag}^+(aq) \longrightarrow \operatorname{Cu}^{2+}(aq) + 2\operatorname{Ag}(s)$$
  
 $n = 2, E^{\circ}_{\operatorname{cell}} = 0.46 \operatorname{V}$   
Now using the equation for  $E^{\circ}_{\operatorname{cell}}$   
 $E^{\circ}_{\operatorname{cell}} = 0.0591 \log K$ 

$$E_{
m coll}^{\circ} = rac{n}{n} \log K_c \ 0.46 = rac{0.0591}{n} \log K_c \ K_c = 
m Antilog 15.5668 \ K_c = 3.92 imes 10^{15}$$

#### **Correct Option. 2**

#### Solution:

$$t_{99\%} = rac{2.303}{k} \log rac{(a)}{a - 0.99a} \ = rac{2.303}{k} \log 10^2 \Rightarrow 2 imes rac{2.303}{k} \dots (i) \ t_{90\%} = rac{2.303}{k} \log rac{a}{a - 0.90a} \ = rac{2.303}{k} \log 10 = rac{2.303}{k} \dots (ii)$$

Take the ratio of (i) and (ii)

 $rac{t_{99\%}}{t_{90\%}}=2 \ t_{99\%}=2 imes t_{90\%}$ 

Hence, 2 times is the final answer.

#### Ans 10

#### **Correct Option.** A

#### Solution:

 $K_2MnF_6$  react with  $SbF_5$  to form  $F_2.$  In this reaction stronger Lewis acid  $SbF_5$  displace the weaker one  $(MnF_4)$  from its salt.  $MnF_4$  being unstable, readily decompose to give  $MnF_3$  and fluorine.

The complete reaction is as follows,  $K_2MnF_6+2SbF_5\longrightarrow 2KSbF_6+MnF_4$  (unstable)

 $2MnF_4 \longrightarrow 2MnF_3 + F_2$ 

Overall reaction,

 $2 \ K_2 MnF_6 + 4SbF_5 \longrightarrow 4KSbF_6 + 2MnF_3 + F_2$ 

#### Ans 11

#### **Correct Option.** C

#### Solution:

Gadolinium does not absorb light in the visible region of spectrum, hence it is colourless inspite of the presence of seven unpaired electrons.

#### Ans 12

### **Correct Option.** C

### Solution:

Oxidation state of Cr,

$$\begin{split} &In \left[ \mathrm{Cr}(\mathrm{H}_2O)_6 \right] \mathrm{Cl}_3 \\ &x + 0 \times 6 - 1 \times 3 = 0 \\ &x = 3...(i) \\ &\ln \left[ \mathrm{Cr}(\mathrm{C}_6\mathrm{H}_6)_2 \right] \\ &x + 0 \times 2 = 0 \\ &x = 0..(ii) \\ &\mathrm{In} \mathrm{K}_2 \left[ \mathrm{Cr}(\mathrm{CN})_2(\mathrm{O})_2 \left( \mathrm{O}_2 \right) (\mathrm{NH}_3) \right] \\ &(1 \times 2) + x - (1 \times 2) - (2 \times 2) - (1 \times 2) - (0 \times 1) = 0 \\ &x - 6 = 0 \\ &x = +6...(iii) \\ &\mathrm{Therefore, oxidation states are } +3, 0, +6. \end{split}$$

# Ans 13

**Correct Option. D** 

# Solution:

The complete reaction is as follows,

$$\begin{array}{c} \mathsf{CH}_{3} \longrightarrow \mathsf{Br} & \xrightarrow{\mathsf{KCN}} \mathsf{CH}_{3} \longrightarrow \mathsf{CH}_{3} \longrightarrow \mathsf{CH}_{3} \mathsf{O}^{+} \\ \xrightarrow{(X)} & \mathsf{Hydrotysis} & \mathsf{CH}_{3} \mathsf{COOH} \\ \xrightarrow{(Y)} & \mathsf{Acetontrile} & \mathsf{CH}_{3} \mathsf{COOH} \\ \xrightarrow{(Y)} & \mathsf{Acetocacd} \end{array}$$

$$\begin{array}{c} \mathsf{Feduction} \\ & \mathsf{CH}_{3} \mathsf{CH}_{2} \mathsf{OH} \\ \xrightarrow{(Z)} \\ \mathsf{Ethyl} \ \mathsf{alcohol} \end{array}$$

### Ans 14

**Correct Option. B** 

### Solution:

Dihydropyran reacts with alcohols in presence of anhydrous H<sup>+</sup> to form acetal.

The complete reaction is as follows





# Correct Option. A

# Solution:

Among the given amino acids, only two i.e. lysine and arginine are basic amino acid. While all other are neutral amino acid.

# **Mathematics**

Ans 1

### **Correct Option.** C

### Solution:

 $\begin{array}{l} \because \log(\log_4(\sqrt{x+4}+\sqrt{x})) = 0 \\ \Rightarrow \log_4(\sqrt{x+4}+\sqrt{x}) = 1 \\ \Rightarrow \sqrt{x+4}+\sqrt{x} = 4^1 = 4 \end{array}$ From the given options,  $x = \frac{9}{4}$  satisfy the equation.

### Ans 2

# **Correct Option. B**

### Solution:

 $\therefore \frac{a}{b} = \frac{1}{3} \Rightarrow b = 3a \text{ and } \frac{b}{c} = \frac{3}{4} \Rightarrow \frac{3a}{c} = \frac{3}{4}$  $\Rightarrow \frac{a}{c} = \frac{1}{4} \Rightarrow c = 4a$  $\text{Then, } \frac{a+2b}{b+2c} = \frac{a+2(3a)}{(3a)+2(4a)}$  $= \frac{a+6a}{3a+8a} = \frac{7a}{11a} = \frac{7}{11}$ 

### Ans 3

### **Correct Option.** C

### Solution:

 $: 2079000 = 2^3 \times 3^3 \times 5^3 \times 7 \times 11$ 

for the divisor to be even and divisible by 15;2,3 and 5 must be occur once.

There, the total number of required divisors are

 $3 \times 3 \times 3 \times 2 \times 2 = 108$ 

### Ans 4

# **Correct Option. 5**

### Solution:

N = Number of 8 digit number with exactly four 9.

$$\begin{array}{l} \Rightarrow N = \text{ Number with } \left[ \left\{ 1^{\text{st}} \text{ digit is } 9 \right\} + \left\{ 1^{\text{st}} \text{ digit is not } 9 \right\} \right] \\ = {}^7C_3 \cdot 9^4 + 8 \cdot {}^7C_4 \cdot 9^3 \\ = 9^3 \left( 9 \cdot \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} + 8 \cdot \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} \right) \\ = 729 \times (315 + 280) = 729 \times 595 \end{array}$$

Hence, unit digit of N is 5 .

#### Ans 5

### **Correct Option. 2**

#### Solution:

 $\begin{array}{l} \because x + \frac{1}{x^2} = \frac{x}{2} + \frac{x}{2} + \frac{1}{x^2} \\ \text{and } AM \ge GM \\ \\ \Rightarrow \frac{x}{2} + \frac{x}{2} + \frac{1}{x^2} \ge \left(\frac{x}{2} \times \frac{x}{2} \times \frac{1}{x^2}\right)^{\frac{1}{3}} \\ \\ \Rightarrow \frac{x}{2} + \frac{x}{2} + \frac{1}{x^2} \ge \left(\frac{1}{4}\right)^{\frac{1}{3}} \\ \\ \\ \text{Minimum value of } x + \frac{1}{x^2} = \sqrt[3]{\frac{1}{4}} \end{array}$ 

and for minimum value

$$rac{x}{2}=rac{1}{x^2}\Rightarrow x^3=2$$
Hence,  $lpha^3=2$ 

#### Ans 6

# **Correct Option. B**

#### Solution:

$$\begin{aligned} \mathbf{T}_{r+1} &= {}^{200}C_r(x\cdot\sqrt{180}){}^{200-r}\cdot(\sqrt[3]{432})^r \\ &= {}^{200}C_r 2^{200-r}\cdot 3{}^{200-r}\cdot 5\frac{{}^{200-r}}{2}\cdot x{}^{200-r} 2^r 3^r 2^{\frac{r}{3}} \\ &\therefore \text{ Coefficient } = {}^{200}C_r 2^{200}\cdot 3{}^{200}\cdot 5{}^{100}\cdot \frac{2^{\frac{r}{3}}}{5^{\frac{r}{2}}} \end{aligned}$$

r must be multiple of 6 and takes values between 0 to 200.

$$\therefore$$
 34 terms =  $n$ 

 $\left[\frac{n}{6}\right] = \left[\frac{34}{6}\right] = 5$ 

Ans 7

# Correct Option. B

#### Solution:

$$:: T_{r+1} = {}^{9}C_{r}(2x)^{\frac{9-r}{2}} \left(\frac{3}{x^{2}}\right)^{\frac{3}{2}}$$

$$= {}^{9}C_{r}2^{\frac{9-r}{2}} \cdot 3^{\frac{r}{3}} \cdot x^{\left(\frac{9-r}{2}-\frac{2}{3}\right)}$$

$$= {}^{9}C_{r}2^{\frac{9-r}{2}} \cdot 3^{\frac{r}{3}} \cdot x^{\frac{27-r}{6}}$$
So,  $r = 3$ 
Then, coefficient  $= {}^{9}C_{3} \cdot 2^{\frac{9-3}{2}} \cdot 3^{1}$ 

$$= \frac{9 \cdot 8 \cdot 7}{3 \cdot 2} \times 2^{3} \times 3$$

$$= 2016$$

# Ans 8

# Correct Option. A

# Solution:

$$egin{array}{lll} & \therefore x^2+y^2=4 \Rightarrow x^2+y^2-4(1)^2=0\ldots(i) \ & ext{and} \; y=3x+c \Rightarrow rac{y-3x}{c}=1 \end{array}$$

Put the value of '1' in Eq. (i)

$$\Rightarrow x^2 + y^2 - 4\left(\frac{y - 3x}{c}\right)^2 = 0$$
$$\Rightarrow x^2 + y^2 - 4\left(\left(\frac{y}{c}\right)^2 + \left(\frac{3x}{c}\right)^2 - \frac{6xy}{c^2}\right) = 0$$
$$\Rightarrow x^2\left(1 - \frac{36}{c^2}\right) + y^2\left(1 - \frac{4}{c^2}\right) + \frac{24xy}{c^2} = 0$$

According to the question,

$$egin{pmatrix} 1-rac{36}{c^2} \end{pmatrix} + egin{pmatrix} 1-rac{4}{c^2} \end{pmatrix} = 0 \Rightarrow 2 = rac{40}{c^2} \ \Rightarrow \ c^2 = 20 \end{split}$$

# Ans 9

Correct Option. C

Solution:

Given circle is  $x^2+y^2-6x-10y+33=0$ 

Centre of the circle = (3,5)

Radius of required circle is same as of given circle  $=\sqrt{(3)^2+(5)^2-33}=1$ 

Mirror image of centre about y = x is (5,3)

Hence, Required equation  $(x-5)^2+(y-3)^2=1$ 

$$\Rightarrow x^2+y^2-10x-6y+33=0$$

### Ans 10

#### **Correct Option.** C

#### Solution:

Let the equation of tangent is

$$y = mx + \frac{16}{m} [:: a = 16]$$

It passes through (h,k)

Then,  $k=mh+rac{16}{m}\Rightarrow m^2h+16-mk=0$ 

According to the question, roots of the equation will be  $m_1$  and  $8m_1$ .

So, 
$$m_1+8m_1=rac{k}{h}\Rightarrow 9m_1=rac{k}{h}\dots(i)$$
  
and  $m_1\cdot 8m_1=rac{16}{h}\Rightarrow 8m_1^2=rac{16}{h}\dots(ii)$ 

By solving these equations, we get

$$\frac{k^2}{2h} = 81$$

Ans 11

**Correct Option. 375** 

#### Solution:

As we know,

 $\lim_{x 
ightarrow 0} rac{\sin x}{x} \simeq 1$ 

but it is actually less than 1.

So,  $\lim_{x\to 0} \left[\frac{\sin x}{x}\right] = 0 = 1^2 - 1$ Similarly,  $\lim_{x\to 0} \left[2\frac{\sin 2x}{x}\right] = \lim_{x\to 0} \left[4\frac{\sin 2x}{2x}\right]$  $= 3 = 2^2 - 1$ 

Hence,  $\lim_{x
ightarrow 0}\left[nrac{\sin nx}{x}
ight]=n^2-1$ 

Therefore,

$$egin{aligned} &\lim_{x o 0} f(x) = \left(1^2-1
ight) + \left(2^2-1
ight) + \ldots + \left(10^2-(1)
ight) \ &= \left(1^2+2^2+\ldots+10^2
ight) - (1+1+\ldots+1 ext{ times }) \ &= rac{10 imes 11 imes 21}{6} - 10 \ &= 385 - 10 = 375 \end{aligned}$$

### Ans 12

Correct Option. B

So, exactly one angle is 90°.

#### Solution:

$$\sin^2 A + \sin^2 B + \sin^2 C = 2$$

$$= \frac{1 - \cos 2A}{2} + \frac{1 - \cos 2B}{2} + \frac{1 - \cos 2C}{2} = 2$$

$$\Rightarrow \cos 2A + \cos 2B + \cos 2C = -1$$

$$\Rightarrow 2\cos(A + B)\cos(A - B) = -2\cos^2 C$$

$$\Rightarrow 2\cos(\pi - C)\cos(A - B) = -2\cos^2 C [\because A + B + C = \pi]$$

$$\Rightarrow 2\cos C[\cos C - \cos(A - B)] = 0$$

$$\Rightarrow -2\cos C[\cos(A + B) + \cos(A - B)] = 0$$

$$\Rightarrow -\cos A \cdot \cos B \cdot \cos C = 0$$

$$\Rightarrow \cos A \cdot \cos B \cdot \cos C = 0$$
Therefore, one of them should be equal to zero.

# Correct Option. D

### Solution:

As we know,

 $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ 

and  $\sin A, \sin B$  and  $\sin C$  are in AP.

So, a, b, c will also be in AP

$$egin{aligned} &\therefore 2b = a+c \ ext{and } \cos A &= rac{b^2+c^2-a^2}{2bc} = rac{b^2+c^2-(2b-c)^2}{2bc} \ &= rac{b^2+c^2-(4b^2+c^2-4bc)}{2bc} \ &= rac{-3b^2+4bc}{2bc} \ &= rac{b(4c-3b)}{2bc} = rac{4c-3b}{2c} \end{aligned}$$

# Ans 14

# **Correct Option.** A

#### Solution:

$$\therefore D = \begin{vmatrix} n & n^2 & n^3 \\ n^2 & n^3 & n^5 \\ 1 & 2 & 3 \end{vmatrix}$$
  
Then,  $M_{11} = \begin{vmatrix} n^3 & n^5 \\ 2 & 3 \end{vmatrix} = 3n^3 - 2n^5$   
and  $C_{33} = \begin{vmatrix} n & n^2 \\ n^2 & n^3 \end{vmatrix} = n^4 - n^4 = 0$   
 $M_{13} = \begin{vmatrix} n^2 & n^3 \\ 1 & 2 \end{vmatrix} = 2n^2 - n^3$   
 $\therefore \lim_{n \to \infty} = \frac{M_{11} + C_{33}}{(M_{13})^2} = \lim_{n \to \infty} \frac{n^3 (3 - 2n^2) + 0}{n^4 (2 - n)^2}$   
 $= \lim_{n \to \infty} \frac{3 - 2n^2}{n [4 + n^2 - 4n]} = \lim_{n \to \infty} \frac{n^2 (\frac{3}{n^2} - 2)}{n^3 (\frac{4}{n^2} + 1 - \frac{4}{n^2})}$   
put  $n = \infty$ 

)

$$=rac{\left(rac{3}{\varpi^2}-2
ight)}{\infty\left(rac{4}{\varpi^2}+1-rac{4}{\varpi}
ight)}=rac{-2}{1 imes\infty}=0$$

# Correct Option. D

# Solution:

we have 
$$x = \sin(2 \tan^{-1} 2)$$
,  
 $\Rightarrow x = \sin(2\theta), \tan^{-1} 2 = \theta$   
 $\Rightarrow x = \frac{2 \tan \theta}{1 + \tan^2 \theta}, \tan \theta = 2$   
 $\Rightarrow x = \frac{4}{5}$   
Also,  $y = \sin(\frac{1}{2}\tan^{-1}(\frac{4}{3}))$   
 $\Rightarrow y = \sin(\theta), \frac{1}{2}\tan^{-1}(\frac{4}{3}) = \theta$   
 $\Rightarrow y = \sin(\theta), \tan(2\theta) = \frac{4}{3}$   
 $\Rightarrow y = \sin(\theta), \tan(\theta) = \frac{1}{2}$   
 $\Rightarrow y = \frac{1}{\sqrt{5}}$   
Hence,  $y^2 = \frac{1}{5} = 1 - \frac{4}{5} = 1 - x$ 

### Ans 16

# Correct Option. B

# Solution:

$$\therefore f(n) = \begin{cases} \frac{n+1}{2}, & n \text{ is odd} \\ \frac{n}{2}, & n \text{ is even} \end{cases}$$
$$\therefore f(1) = \frac{1+1}{2} = 1$$
and  $f(2) = \frac{2}{2} = 1$ 
$$\therefore f(1) = f(2) = 1$$
So, f is not injective  
and Range of  $f(n)$  is all natural numbers.  
So, it is surjective

Hence, *f* is surjective but not injective.

# Ans 17

#### **Correct Option. D**

#### Solution:

Let,  $f(x) = 1 + x^n$  or  $f(x) = 1 - x^n$   $\therefore f(3) = 10$ So,  $f(x) = 1 + x^2$ Then,  $\int f(x)dx = \int (1 + x^2) dx = x + \frac{x^3}{3} + c$   $\therefore \quad g(x) = x + \frac{x^3}{3} + c$   $\therefore \quad g(1) = \frac{4}{3}$   $\Rightarrow \quad \frac{4}{3} = 1 + \frac{1}{3} + c$   $\Rightarrow \quad C = 0$ Therefore,  $g(x) = x + \frac{x^3}{3}$ and  $g(3) = 3 + \frac{3^3}{3} = 12$ 

#### Ans 18

#### **Correct Option. B**

#### Solution:

f(x) + f(y) + 2xy = f(x + y)...(i)differentiating the equation,  $\Rightarrow f'(y) + 2x = f'(x + y)$ at y = 0f'(0) + 2x = f'(x) $\Rightarrow f'(x) = 2x$ Integrating both sides.  $\int f'(x) = \int 2x \cdot dx$  $\Rightarrow f(x) = x^{2}$  $\therefore I = \int_{0}^{\frac{\pi}{2}} f(\sin x) dx$ 

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \sin^2 x \cdot dx \dots (ii)$$
  

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \sin^2 \left(\frac{\pi}{2} - x\right) dx = \int_0^{\frac{\pi}{2}} \cos^2 x \cdot dx \dots (iii)$$
  
Adding Eqs. (ii) and (iii)  

$$2l = \int_0^{\frac{\pi}{2}} \left(\sin^2 x + \cos^2 x\right) dx = \int_0^{\frac{\pi}{2}} 1 \cdot dx$$
  

$$\Rightarrow 2l = \frac{\pi}{2}$$
  

$$\Rightarrow l = \frac{\pi}{4}$$

# **Correct Option. 8**

#### Solution:

$$\frac{dy}{dx} + 2y\tan x = \sin x$$

The given differential equation is of the form

$$\frac{dy}{dx} + Py = Q$$

By comparing, we get

$$P=2 an x$$
 and  $Q=\sin x$ 

 $\therefore$  I.F.  $= e^{\int \cdot 2 \tan \cdot dx} = e^{2 \log \sec x} = e^{\log \sec^2 x} = \sec^2 x$ 

Then, solution of the differential equation

$$y \sec^{2} x = \int \sin x \cdot \sec^{2} x \cdot dx + c$$
  

$$\Rightarrow y \sec^{2} x = \int \frac{\sin x}{\cos^{2} x} dx + c$$
  

$$\Rightarrow y \sec^{2} x = \int \tan x \cdot \sec x dx + c$$
  

$$\Rightarrow y \sec^{2} x = \sec x + c$$
  

$$\Rightarrow y = \frac{1}{\sec x} + \frac{c}{\sec^{2} x} = \cos x + (\cos^{2} x) \cdot c$$
  
at  $x = \frac{\pi}{3}, y = 0$   

$$\therefore 0 = \cos \frac{\pi}{3} + (\cos^{2} \frac{\pi}{3} \cdot c)$$
  

$$\Rightarrow 0 = \frac{1}{2} + \frac{c}{4} \Rightarrow c = -2$$
  

$$\therefore y = \cos x - 2 \cos^{2} x$$

$$\Rightarrow \frac{y}{2} = -\cos^2 x + \frac{1}{2}\cos x = -\left(\cos^2 x - \frac{1}{2}\cos x\right)$$
$$\Rightarrow \frac{y}{2} = -\left[\left(\cos x - \frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2\right] = -\left[\left(\cos x - \frac{1}{4}\right)^2\right] + \frac{1}{16}$$
So, maximum value of  $\frac{y}{2} = \frac{1}{16}$ 
$$\Rightarrow \text{Maximum value of } y = \frac{1}{8} \Rightarrow k = 8$$

### **Correct Option. 3**

#### Solution:

Let,  $\mathbf{a} + \mathbf{b} = \lambda \mathbf{c} \dots (i)$ and  $\mathbf{b} + \mathbf{c} = \mu \mathbf{a} \dots (ii)$ Subtracting Eq. (ii) from Eq. (i),  $\mathbf{a} - \mathbf{c} = \lambda \mathbf{c} - \mu \mathbf{a}$   $\Rightarrow (1 + \mu)\mathbf{a} = (1 + \lambda)\mathbf{c}$   $\therefore$  a and c are non-collinear. So,  $1 + \mu = 0$  and  $1 + \lambda = 0$   $\Rightarrow \lambda = \mu = -1$ Therefore,  $\mathbf{a} + \mathbf{b} = -\mathbf{c}$   $\Rightarrow \mathbf{a} + \mathbf{b} + \mathbf{c} = 0 \Rightarrow |\mathbf{a} + \mathbf{b} + \mathbf{c}|^2 = 0$   $\Rightarrow |\mathbf{a}|^2 + |\mathbf{b}|^2 + |\mathbf{c}|^2 + 2(\mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a}) = 0$   $\Rightarrow 2 + 2 + 2 + 2(\mathbf{b} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a}) = 0$  $\Rightarrow \mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a} = -3$ 

Hence,  $|\mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a}| = 3$