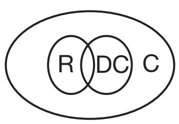
Theory of Computation Test I

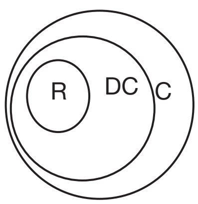
Number of Questions: 25 **Section Marks: 30**

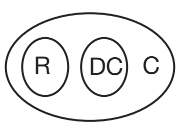
Directions for questions 1 to 25: Select the correct alternative from the given choices.

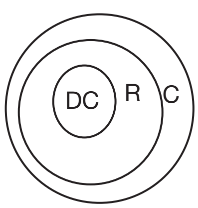
1. Which of the following is FALSE?
I: Any regular language is context free language.
II: There exists a DPDA (Deterministic Push Down Automata) for every CFL.
(A) I only (B) II only
(C) Both I and II (D) Neither I nor II

2. Which of the following figure correctly specifies the relation between regular languages (R), deterministic CFL's (DC) and CFL's (C)?

(A)


(B)


(C)


(D)


3. Consider the regular expression, $RE = ab^* + ba^*$. Then the reversal of RE, given by RE^R is equal to:
(A) $ab^* + ba^*$ (B) a^*b
(C) $b^*a + a^*b$ (D) $(a^*b)(b^*a)$

4. The statement: 'For every regular language L, every subset of L is regular as well' is:
(A) TRUE
(B) False
(C) TRUE, only if L is CFG
(D) False, only if L is CFG.

5. Which of the following strings are not accepted by the language L $(((010 \cup 10)^*1)^*)$?
(A) 1 (B) 0101
(C) 10101110 (D) 01001011

6. Which of the following is FALSE?
(A) $(L_1^*)^* = L_1^*$
(B) $L_1^* = (L_1L_1)^*$
(C) If $L_1 \subset L_2$ then $L_1^* \subset L_2^*$
(D) If L_1 is finite then it is regular.

7. Which of the following strings are generated by the regular expression,

$$R = (ab\varepsilon)^*(a + b)ba?$$

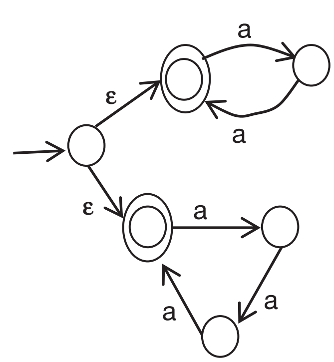
(i) ε (ii) aba
(iii) $ababba$ (iv) $abababa$
(A) (i), (iii) (B) (ii), (iii)
(C) (ii), (iv) (D) (i), (iv)

8. The regular expression $R = (ab\varepsilon)^* ((a + b) \cdot \phi)\ ba$ is equivalent to
(A) $(ab\varepsilon)^*(a + b)ba$ (B) $(ab\varepsilon)^*ba$
(C) ba (D) ϕ

9. Which of the following identity is FALSE for regular expressions?
(i) $R + R = R$ (ii) $RR^* = R^*R$
(iii) $\varepsilon R = R\varepsilon = R$
(A) (i), (ii), (iii) (B) (i) only
(C) (ii), (iii) (D) None of these

10. Which of the following language is Regular?
I: $\{ww | w \in \{0, 1\}^*\}$
II: $\{w/w = w^R, w \in \{0, 1\}^*\}$
III: $\{ww^R/w \in \{0, 1^*\}$
IV: Set of all strings with un-equal number of 0's and 1's.
(A) I, III only (B) II, IV only
(C) IV only (D) None of the above

11. Consider the following NFA:



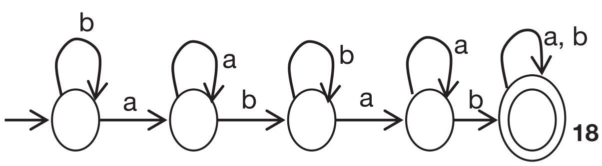
Which of the following gives the language accepted by given NFA?
(A) All strings of the form a^k, $k \geq 0$.
(B) All strings of the form $\{a^m a^n | m \geq 0, n \geq 0\}$
(C) All strings of the form $\{a^m a^n \mid m$ is a multiple of 2 and n is multiple of 3$\}$
(D) All strings of the form $\{a^k \mid k$ is a multiple of 2 or 3$\}$

12. Which of the following is not context-free language?
(i) $\{a^k \mid k$ is a perfect square$\}$

(ii) $\{a^i b^j c^i d^j \mid i, j \geq 0\}$
(iii) $\{a^i b^{2i} a^i \mid i \geq 0\}$

(A) (i), (ii) (B) (ii), (iii)
(C) (i), (iii) (D) (i), (ii), (iii)

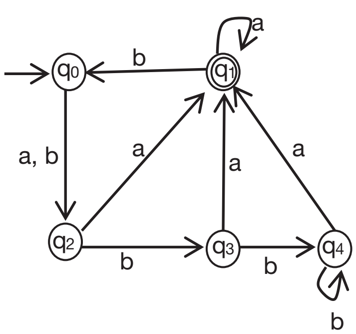
13. Consider the following DFA D:



Which of the following is TRUE?
(i) D accepts all strings which contain the sub-word 'ab' two times only.
(ii) D accepts all strings which terminate with 'b'.
(iii) The strings *baabb*, *abbba* are not in the language.

(A) (i), (ii) only (B) (iii) only
(C) (ii), (iii) (D) (i), (iii)

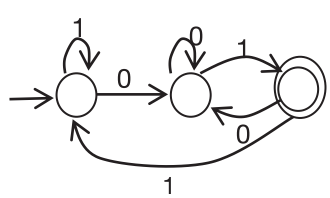
14. Consider the following FA:



Which of the following states are equivalent?
(i) q_0, q_1 (ii) q_2, q_3
(iii) q_3, q_4 (iv) q_0, q_4

(A) (i), (ii) (B) (ii), (iv)
(C) (iii) only (D) (ii), (iii)

15. Consider the following FA:

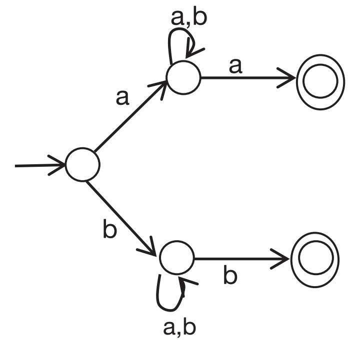


The number of states in the minimized FA is
(A) 3 (B) 2
(C) 1 (D) None of the above

16. Which of the following represents a language in automata theory?
(i) Σ^* (ii) ε
(iii) ϕ (iv) $\{\phi\}$
(v) $\{\varepsilon\}$

(A) (i), (iii) only (B) (i), (iii), (v) only
(C) (ii), (iv) only (D) (iv), (v) only

17. Consider the following NFA:



The number of states in its equivalent DFA is
(A) 4 (B) 5
(C) 6 (D) 7

18. Which of the following language is regular?
(i) $\{x = y + z \mid x, y, z$ are binary integers and x is the sum of y and $z\}$
(ii) $\{w \mid w$ is a binary representation of a number greater than $3\}$

(A) (i) only (B) (ii) only
(C) Both (i) and (ii) (D) Neither (i) nor (ii)

19. Consider the following grammar:
S → PaP
P → ε|PaPbP|PbPaP|Pa|aP
What is the language generated by this grammar over $\{a, b\}$?
(A) Set of all strings with more a's than b's
(B) Set of all strings with more b's than a's
(C) Set of all strings with twice a's than b's
(D) Set of all strings with equal number of a's and b's.

20. Consider a regular language L. A new language DELchar(L) = {W|W is some string from L with exactly one character deleted} is defined. Then DELchar(L) is
(A) a regular language
(B) a CFG but not regular
(C) neither CFG nor regular
(D) not accepted by a PDA

21. Consider the following grammar:
S → a Sb | P
P → bP | Pa | ε
Which of the following language represents the grammar?
(A) $\{a^n b^n \mid n \in N\}$
(B) $\{a^n b^m b^n \mid m, n \in N\}$
(C) $\{a^n b^m a^p b^n \mid n, m, p \in N\}$
(D) $\{a^n b^m \mid n > m\}$

22. Let M_1 and M_2 are two DFA's with 5-tuple format as given below:

$$M_1 = \{Q_1, \Sigma, \delta_1, S_1, F_1)$$
$$M_2 = (Q_2, \Sigma, \delta_2, S_2, F_2)$$

where Q_1, Q_2 are set of states; Σ is the alphabet set; δ_1, δ_2 are transition functions; S_1, S_2 are start states;

F_1, F_2 are final states. Then which of the following are necessary for $L(M_1) = L(M_2)$?

(i) $Q_1 = Q_2$ (ii) $F_1 = F_2$

(iii) $S_1 = S_2$ (iv) $\delta_1 = \delta_2$

(A) (i), (ii) (B) (i), (ii), (iii)

(C) (i), (ii), (iii), (iv) (D) (iii), (iv)

23. Which of the following language is both regular and context free?

(i) $\{a^n (bc)^n : n \geq 0\}$ (ii) $\{a^n\, a^n\, a^n : n \geq 0\}$

(A) (i) only (B) (ii) only

(C) Both (i) and (ii) (D) Neither (i) nor (ii)

24. Match the following:

List I		List II	
1	$r^*s + s$	A	r^*
2	ϕ^*	B	ϕ
3	ε^*	C	ε
4	$(\varepsilon + r)^*$	D	$r^*\, s$

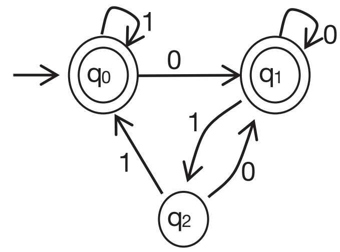
(A) 1–a, 2–b, 3–c, 4–d

(B) 1–d, 2–c, 3–c, 4–a

(C) 1–a, 2–c, 3–c, 4–d

(D) 1–d, 2–c, 3–b, 4–a

25. Consider the following FA:



What is the language accepted by above FA?

(A) $\{w|w \in \{0, 1\}^*$ and w do not end with 1$\}$

(B) $\{w|w \in \{0, 1\}^*$ and w contains more zeros than 1's$\}$

(C) $\{w|w \in \{0, 1\}^*$ and w do not end with 01$\}$

(D) $\{w|w \in \{0, 1\}^*$ and w do not have consequent 0's and 1's$\}$

Answer Keys

1. B	**2.** B	**3.** C	**4.** B	**5.** C	**6.** B	**7.** C	**8.** D	**9.** D	**10.** D
11. D	**12.** D	**13.** B	**14.** D	**15.** A	**16.** B	**17.** B	**18.** B	**19.** A	**20.** A
21. C	**22.** C	**23.** B	**24.** B	**25.** C					

Hints and Explanations

1. Every regular language is a CFL but not vice versa. Every CFL has a PDA but that PDA need not be a deterministic PDA. Choice (B)

2. Regular $\subset$ DCFL $\subset$ CFL Choice (B)

3. Given regular expression,

$$RE = ab^* + ba^*$$
$$RE^R = (ab^* + ba^*)^R$$
$$= (ab^*)^R + (ba^*)^R$$
$$= (b^*)^R\, a^R + (a^*)^R\, b^R$$
$$= (b^R)^*a + (a^R)^*b$$
$$= b^*a + a^*b$$

Choice (C)

4. Given statement is false.
Ex: $L = \{a, b\}^*$; subset of L is $\{a^n\, b^n \mid n \in N\}$, which is not regular. Choice (B)

5. Given language,

$$L(((010 \cup 10)^*1)^*)$$

The regular expressions which are accepted by 'L' are of the form $((m)^*1)^*$, where $m = 010 \cup 10$.
Any string in L is either ε or ends with '1'. Choice (C)

6. $L_1^* = (L_1\, L_1)^*$ is false.
Let $L_1 = \{a\}$ then L_1^* will have 'a' but $(L_1\, L_1)^*$ do not have 'a'.
Choice (B)

7. Given regular expression,

$$R = (abe)^*\,(a + b)\, ba$$

'ε' is not accepted.
'aba' is accepted.
'$ababba$' is not accepted.
'$abababa$' is accepted. Choice (C)

8. $\phi R = R\phi = \phi$, so $(a + b)\phi = \phi$
$\phi \cdot ba = \phi$, $(ab\varepsilon)^* \,.\, \phi = \phi$ Choice (D)

9. All the three are valid identities of regular expressions. Choice (D)

10. None of the four languages is regular. Finite automata cannot check the equality of two substrings of a string. Choice (D)

11. Given NFA accepts number of a's which is a multiple of 2 or 3. Choice (D)

12. $\{a^k \mid k$ is a perfect square$\}$
This is not context-free. The PDA can't check whether a number is perfect square or not.
$\{a^i\, b^j\, c^i\, d^j \mid i, j \geq 0\}$
Not context free. PDA can check the equality of a, b and c, d or a, d and b, c but not a, c and b, d.
$\{a^i\, b^{2i}\, a^i \mid i \geq 0\}$ is also not recognized by a PDA.
Choice (D)

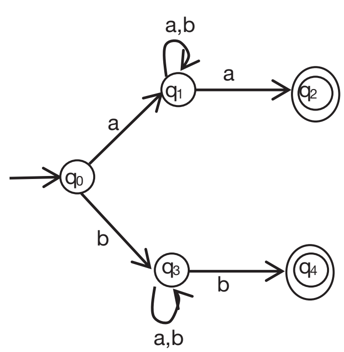
13. Given DFA accepts all strings which contain at least two sub-words '*ab*'. It do not accept *baabb*, *abbba*. Choice (B)

14. In given FA, non-final states are $\{q_0, q_2, q_3, q_4\}$ and final state is $\{q_1\}$.
A state is equivalent to another state if both are either non-final or final states.
Also each transition from those states leads to either final state or non final state only.
$\therefore$ q_0, q_1 are not equivalent.
q_0, q_4, are not equivalent.
q_2, q_3 are equivalent as both are non-final states and q_2, q_3 with '*a*' reaches a final state and q_2, q_3 with '*b*' reaches a non-final state.
Similarly, q_3, q_4 are also equivalent. Choice (D)

15. Given FA is the minimal FA. No minimization is possible. ($\because$ With transition '1', both the non-final states are reaching a final and non-final state). Choice (A)

16. Σ^*, ϕ(empty), $\{\varepsilon\}$ are languages. Choice (B)

17. Given NFA,



The DFA equivalent to given NFA is given as,

	a	b
$\rightarrow [q_0]$	$[q_1]$	$[q_3]$
$[q_1]$	$[q_1, q_2]$	$[q_1]$
$[q_3]$	$[q_3]$	$[q_3, q_4]$
$[q_1, q_2]$	$[q_1, q_2]$	$[q_1]$
$[q_3, q_4]$	$[q_3]$	$[q_3, q_4]$

$\therefore$ Number of states in the equivalent DFA is 5. Choice (B)

18. (i) is not regular. (This will be shown using pumping Lemma). (ii) is regular and the expression is given as $0^*1\ (0 + 1)\ (0 + 1)\ (0 + 1)^*$ Choice (B)

19. Given grammar is

$$S \rightarrow PaP$$
$$P \rightarrow e|PaPbP|PbPaP|Pa|aP$$

Consider some derivations:

1. $S \rightarrow PaP$
 $\rightarrow a$
2. $S \rightarrow PaP$
 $\rightarrow PaaP$
 $\rightarrow aa$
3. $S \rightarrow PaP$
 $\rightarrow PaPbPaP$
 $\rightarrow aba$
4. $S \rightarrow PaP$
 $\rightarrow baa$

$\therefore$ More *a*'s than *b*'s. Choice (A)

20. DELChar (L) is a regular language. Choice (A)

21. Given grammar,

$$S \rightarrow aSb|P$$
$$P \rightarrow bP|Pa|e$$

The strings generated by given grammar are
$S \rightarrow P$
$\rightarrow \varepsilon$
$S \rightarrow aSb$
$\rightarrow aPb$
$\rightarrow ab$
$S \rightarrow a\ Sb$
$\rightarrow aPb$
$\rightarrow aPab$
$\rightarrow aab$
$S \rightarrow aSb$
$\rightarrow aPb$
$\rightarrow abPb$
$\rightarrow abb$
$S \rightarrow aSb$
$\rightarrow aaSbb$
$\rightarrow aabbb$
In the starting and end of the string we need to have equal number of *a*'s and *b*'s. in between there will be any number of *b*'s and *a*'s.
$\therefore$ The language accepted by given grammar is $\{a^n\, b^m\, a^p\, b^n \mid n, m, p \in N\}$. Choice (C)

22. $L(M_1) = L(M_2)$ if $Q_1 = Q_2$; $F_1 = F_2$; $S_1 = S_2$; $\delta_1 = \delta_2$. Choice (C)

23. (i) is CFL but not regular.
(Checking the equality of *a*'s and (*bc*) is not done using FA)
(ii) is regular and CFL.
$\{a^{3n} \mid n \geq 0\}$ is regular.
Every regular language is CFL. Choice (B)

24.

$$r^*\,s + s = r^*s$$
$$\phi^* = \varepsilon^* = \varepsilon$$
$$(\varepsilon + r)^* = r^*$$

Choice (B)

25. Given FA do not accept the strings which will terminate with '01'. Choice (C)