Chapter 12. Rational Expressions and Equations

Answer 1PT3.

Match the algebraic expression:

1. $\dfrac{\frac{a}{b}}{\frac{x}{y}}$ a. Complex fraction.

2. $3-\dfrac{a+1}{a-1}$ c. Mixed fraction.

3. $\dfrac{2}{x^2+2x-4}$ b. rational fraction.

Answer 1STP.

A cylindrical container is 8 inches in height and has a radius of 2.5 inches.

Use the formula $V=\pi r^2 h$ to find the volume of the container:

$$V=\pi r^2 h \qquad \text{Formula.}$$
$$=3.14(2.5)^2 8 \qquad \text{Substitute.}$$
$$=157 \qquad \text{Simplify.}$$

Thus, the volume of the container is option $\boxed{D.\ 157\ \text{in}^2}$.

Answer 1VC.

The given statement is false.

The true statement is:

A rational expression is a fraction whose numerator and denominator are polynomials.

The given statement "The complex fraction $\frac{\frac{4}{5}}{\frac{2}{3}}$ can be simplified as $\frac{6}{5}$." is true.

$$\frac{\frac{4}{5}}{\frac{2}{3}} = \frac{4}{5} \div \frac{2}{3} \quad \text{Rewrite as a division sentence.}$$

$$= \frac{4}{5} \cdot \frac{3}{2} \quad \text{Rewrite as multiplucation by the reciprocal.}$$

$$= \frac{\cancel{2} \cdot 2 \cdot 3}{5 \cdot \cancel{2}} \quad \text{Divide by common factors.}$$

$$= \frac{6}{5} \quad \text{Simplify.}$$

Answer 3VC.

The given statement "The equation $\frac{x}{x-1} + \frac{2x-3}{x-1} = 2$ has an extraneous solution of 1." is true.

For this equation, exclude the values for which $x - 1 = 0$.

$$x - 1 = 0 \quad \text{The denominator cannot equalzero.}$$

$$x = 1$$

Answer 4PT.

First write an inverse variation equation that relates *x* and *y*.

$$xy = k \quad \text{Inverse variation equation}$$

$$(40)(21) = k \quad \text{Substitue } x = 40, \text{and } y = 21$$

$$840 = k \quad \text{Multiply}$$

Therefore, an inverse variation equation that relates *x* and *y* is $\boxed{xy = 840}$.

Now solve for *y*:

If $y = 21$ when $x = 40$, find *y* when $x = 84$

Let $x_1 = 40, y_1 = 21, \text{and } x_2 = 84$. Solve for y_2.

Use product property.

$$x_1 y_1 = x_2 y_2 \quad \text{Product rule for inverse variation.}$$

$$(40)(21) = (84) y_2 \quad \text{Substitue } x_1 = 40, y_1 = 21, \text{and } x_2 = 84.$$

$$\frac{840}{84} = y_2 \quad \text{Divide each side by 84.}$$

$$y_2 = 10 \quad \text{Simplify.}$$

Thus, $\boxed{y = 10}$ when $x = 84$.

Answer 4VC.

The given statement "The mixed expression $6-\frac{a-2}{a+3}$ can be written as $\frac{5a+16}{a+3}$." is false.

$$6-\frac{a-2}{a+3}=\frac{6(a+3)}{a+3}-\frac{a-2}{a+3} \quad \text{The LCD is } (x+3).$$

$$=\frac{6a+18-a+2}{a+3} \quad \text{Distributive property.}$$

$$=\frac{5a+20}{a+3} \quad \text{Simplify,}$$

The true statement is:

The mixed expression $6-\frac{a-2}{a+3}$ can be written as $\frac{5a+20}{a+3}$.

Answer 5PT.

First write an inverse variation equation that relates *x* and *y*.

$$xy=k \quad \text{Inverse variation equation.}$$

$$(4)(22)=k \quad \text{Substitue } x=4, \text{and } y=22.$$

$$88=k \quad \text{Multiply.}$$

Therefore, an inverse variation equation that relates *x* and *y* is $\boxed{xy=88}$.

Now solve for *x*:

If $y=22$ when $x=4$, find *x* when $y=16$

Let $x_1=4, y_1=22, \text{and } y_2=16$. Solve for x_2.

Use product property.

$$x_1y_1=x_2y_2 \quad \text{Product rule for inverse variation.}$$

$$(4)(22)=x_2(16) \quad \text{Substitue } x_1=4, y_1=22, \text{and } y_2=16.$$

$$\frac{88}{16}=x_2 \quad \text{Divide each side by 3.}$$

$$x_2=5.5 \quad \text{Simplify.}$$

Thus, $\boxed{x=5.5}$ when $y=16$.

Answer 5VC.

The given statement "The least common multiple for (x^2-144) and $(x+12)$ is $\underline{x+12}$." is <u>false</u>.

Express each polynomial in factored form.

$$(x^2-144)=(x+12)(x-12)$$

$$(x+12) \text{ is Prime factor.}$$

Use each factor the greatest number of times it appears.

Thus, the least common multiple is $(x+12)(x-12)=(x^2-144)$

The true statement is:

The least common multiple for (x^2-144) and $(x+12)$ is $\underline{x^2-144}$.

Answer 6PT.

Consider the following expression:

$$\frac{5-2m}{6m-15}$$

$$=\frac{5-2m}{-3(5-2m)} \qquad \text{Factor}$$

$$=\frac{\overset{1}{\cancel{(5-2m)}}}{-3\underset{1}{\cancel{(5-2m)}}} \qquad \text{Divide the numerator and denominator by the GCF, } 5-2m.$$

$$=-\frac{1}{3} \qquad \text{Simplify}$$

Therefore, the simplify form of the expression is $\boxed{-\frac{1}{3}}$.

To find the excluded values for the rational expression, factor the denominator, set each factor equal to 0, and solve for *m*.

Exclude the value for which $6m-15=0$

$$6m-15=0 \qquad \text{The denominator cannot be equal to 0}$$

$$m=\frac{5}{2}$$

Therefore, *m* cannot be equal to $\boxed{\frac{5}{2}}$.

Answer 6VC.

The given statement "The excluded values for $\frac{4x}{x^2-x-12}$ are -3 and 4." is <u>true</u>.

Exclude the values for which $x^2-x-12=0$.

$$\begin{aligned} x^2-x-12&=0 && \text{The denominator cannot equalzero.}\\ (x-4)(x+3)&=0 && \text{Factor.}\\ x-4=0 \quad &\text{or} \quad x+3=0 && \text{Zero factor property.}\\ x=4 \quad &\text{or} \quad x=-3 && \text{Solve for } x. \end{aligned}$$

Therefore, *x* cannot equal -3 and 4.

Answer 7E.

First write an inverse variation equation that relates *x* and *y*.

$$\begin{aligned} xy&=k && \text{Inverse variation equation}\\ (42)(28)&=k && \text{Substitue } x=42 \text{ and } y=28\\ 1176&=k && \text{Multiply} \end{aligned}$$

Therefore, an inverse variation equation that relates *x* and *y* is $\boxed{xy=1176}$.

Now solve for *y*:

If $y=28$ when $x=42$, find *y* when $x=56$

Let $x_1=42, y_1=28, \text{and } x_2=56$. Solve for y_2.

Use product property.

$$\begin{aligned} x_1y_1&=x_2y_2 && \text{Product rule for inverse variation.}\\ (42)(28)&=(56)y_2 && \text{Substitue } x_1=42, y_1=28, \text{and } x_2=56.\\ \frac{1176}{56}&=y_2 && \text{Divide each side by 56.}\\ y_2&=21 && \text{Simplify.} \end{aligned}$$

Thus, $\boxed{y=21}$ when $x=56$.

Answer 7PT.

Consider the following expression:

$$\begin{aligned} &\frac{3+x}{2x^2+5x-3}\\ &=\frac{x+3}{(2x-1)(x+3)} && \text{Factor}\\ &=\frac{\cancel{(x+3)}^{1}}{(2x-1)\cancel{(x+3)}_{1}} && \text{Divide the numerator and denominator by the GCF.}\\ &=\frac{1}{2x-1} && \text{Simplify} \end{aligned}$$

Therefore, the simplify form of the expression is $\boxed{\frac{1}{2x-1}}$.

To find the excluded values for the rational expression, factor the denominator, set each factor equal to 0, and solve for *x*.

Exclude the value for which $2x^2+5x-3=0$

$$2x^2+5x-3=0 \quad \text{The denominator cannot be equal to 0}$$
$$(2x-1)(x+3)=0 \quad \text{Factor}$$
$$2x-1=0 \quad \text{or} \quad x+3=0 \quad \text{Use zero factor property to solve for } x.$$
$$x=\frac{1}{2} \quad \text{or} \quad x=-3$$

Therefore, *x* cannot be equal to $\boxed{-3,\frac{1}{2}}$.

The length of a rectangular door is 2.5 times its width. The area of the door is 9750 square inches.

Since, the width of the door is *w* then length of the door is $2.5w$.

Hence

$$A=lb \quad \text{Formula.}$$
$$9750=2.5w\cdot w \quad \text{Substitute.}$$
$$9750=2.5w^2 \quad \text{Simplify.}$$

Therefore, the area of the door is option $\boxed{B.\ 2.5w^2=9750}$.

Answer 8E.

First write an inverse variation equation that relates *x* and *y*.

$$xy=k \quad \text{Inverse variation equation}$$
$$(5)(15)=k \quad \text{Substitue } x=5, \text{and } y=15$$
$$75=k \quad \text{Multiply}$$

Therefore, an inverse variation equation that relates *x* and *y* is $\boxed{xy=75}$.

Now solve for *y*:

If $y=15$ when $x=5$, find *y* when $x=3$

Let $x_1=5, y_1=15, \text{and } x_2=3$. Solve for y_2.

Use product property.

$$x_1y_1=x_2y_2 \quad \text{Product rule for inverse variation.}$$
$$(5)(15)=(3)y_2 \quad \text{Substitue } x_1=5, y_1=15, \text{and } x_2=3.$$
$$\frac{75}{3}=y_2 \quad \text{Divide each side by 3.}$$
$$y_2=25 \quad \text{Simplify.}$$

Thus, $\boxed{y=25}$ when $x=3$.

Answer 8PT.

Consider the following expression:

$$\frac{4c^2+12c+9}{2c^2-11c-21}$$

$$=\frac{(2c+3)(2c+3)}{(2c+3)(c-7)} \quad \text{Factor}$$

$$=\frac{\overset{1}{\cancel{(3c+3)}}(2c+3)}{\underset{1}{\cancel{(2c+3)}}(c-7)} \quad \text{Divide the numerator and denominator by the GCF.}$$

$$=\frac{2c+3}{c-7} \quad \text{Simplify}$$

Therefore, the simplify form of the expression is $\boxed{\frac{2c+3}{c-7}}$.

To find the excluded values for the rational expression, factor the denominator, set each factor equal to 0, and solve for *c*.

Exclude the value for which $2c^2-11c-21=0$

$$2c^2-11c-21=0 \quad \text{The denominator cannot be equal to 0.}$$

$$(2c+3)(c-7)=0 \quad \text{Factor.}$$

$$2c+3=0 \quad \text{or} \quad c-7=0 \quad \text{Use zero factor property to solve for } c.$$

$$c=-\frac{3}{2} \quad \text{or} \quad c=7$$

Therefore, *x* cannot be equal to $\boxed{7,-\frac{3}{2}}$.

Answer 9E.

First write an inverse variation equation that relates *x* and *y*.

$$xy=k \quad \text{Inverse variation equation.}$$

$$(8)(18)=k \quad \text{Substitue } x=8, \text{and } y=18.$$

$$144=k \quad \text{Multiply.}$$

Therefore, an inverse variation equation that relates *x* and *y* is $\boxed{xy=144}$.

Now solve for *x*:

If $y=18$ when $x=8$, find *x* when $y=3$

Let $x_1=8, y_1=18, \text{and } y_2=3$. Solve for x_2.

Use product property.

$x_1y_1=x_2y_2$	Product rule for inverse variation.
$(8)(18)=x_2(3)$	Substitue $x_1=8, y_1=18, \text{and } y_2=3$.
$\frac{144}{3}=x_2$	Divide each side by 3.
$x_2=48$	Simplify.

Thus, $\boxed{x=48}$ when $y=3$.

Answer 9PT.

Consider the following expression:

$$\frac{1-\frac{9}{t}}{1-\frac{81}{t^2}}$$

$$=\frac{\frac{t}{t}-\frac{9}{t}}{\frac{t^2}{t^2}-\frac{81}{t^2}} \quad \left[\begin{array}{l}\text{The LCD of the fractions in the numerator} \\ \text{is } t \text{ and denominator is } t^2.\end{array}\right]$$

$$=\frac{\frac{t-9}{t}}{\frac{t^2-81}{t^2}} \quad \text{Simplify.}$$

$$=\frac{t-9}{t}\div\frac{t^2-81}{t^2} \quad \text{Rewrite as a division sentence.}$$

$$=\frac{t-9}{t}\cdot\frac{t^2}{t^2-81} \quad \text{Rewrite as multiplication by the reciprocal.}$$

$$=\frac{\cancel{t-9}}{\cancel{t}}\cdot\frac{t\cdot\cancel{t}}{\cancel{(t-9)}(t+9)} \quad \text{Factor.}$$

$$=\frac{t}{t+9} \quad \text{Simplify.}$$

Therefore, the simplify form of the expression is $\boxed{\frac{t}{t+9}}$.

To find the excluded values for the rational expression, factor the denominator, set each factor equal to 0, and solve for *t*.

Exclude the value for which $t = 0, t^2 - 81 = 0$ and $t^2 = 0$.

$t = 0$ or $t^2 = 0$

$t = 0$

$$t^2 - 81 = 0 \qquad \text{The denominator cannot be equal to 0.}$$

$$(t+9)(t-9) = 0 \qquad \text{Factor.}$$

$$t+9 = 0 \quad \text{or} \quad t-9 = 0 \qquad \text{Use zero factor property to solve for } t.$$

$$t = -9 \quad \text{or} \quad t = 9$$

Therefore, *x* cannot be equal to $\boxed{0, 0, \pm 9}$.

Answer 10E.

First write an inverse variation equation that relates *x* and *y*.

$$xy = k \qquad \text{Inverse variation equation}$$

$$(175)(35) = k \qquad \text{Substitue } x = 175, \text{and } y = 35$$

$$6{,}125 = k \qquad \text{Multiply}$$

Therefore, an inverse variation equation that relates *x* and *y* is $\boxed{xy = 6{,}125}$.

Now solve for *y*:

If $y = 35$ when $x = 175$, find *y* when $x = 75$

Let $x_1 = 175, y_1 = 35, \text{and } x_2 = 75$. Solve for y_2.

Use product property.

$$x_1 y_1 = x_2 y_2 \qquad \text{Product rule for inverse variation.}$$

$$(175)(35) = (75) y_2 \qquad \text{Substitue } x_1 = 175, y_1 = 35, \text{and } x_2 = 75.$$

$$\frac{6{,}125}{75} = y_2 \qquad \text{Divide each side by 75.}$$

$$y_2 = 81.7 \qquad \text{Simplify.}$$

Thus, $\boxed{y = 81.7}$ when $x = 75$.

Answer 10PT.

Consider the following expression.

$$\frac{\frac{5}{6}+\frac{u}{t}}{\frac{2u}{t}-3}$$

$$=\frac{\frac{5t}{6t}+\frac{6u}{6t}}{\frac{2u}{t}-\frac{3t}{t}}$$ [The LCD of the fractions in the numerator is $6t$ and denominator is t.]

$$=\frac{\frac{5t+6u}{6t}}{\frac{2u-3t}{t}}$$ Simplify.

$$=\frac{5t+6u}{6t}\div\frac{2u-3t}{t}$$ Rewrite as a division sentence.

$$=\frac{5t+6u}{6t}\cdot\frac{t}{2u-3t}$$ Rewrite as multiplication by the reciprocal.

$$=\frac{t(5t+6u)}{6t(2u-3t)}$$ Multiply.

$$=\frac{\cancel{t}(5t+6u)}{6\cancel{t}(2u-3t)}$$ Divide by the common factor.

$$=\frac{5t+6u}{6(2u-3t)}$$ Simplify.

Therefore, the answer is $\boxed{\frac{5t+6u}{6(2u-3t)}}$.

To find the excluded values for the rational expression, factor the denominator, set each factor equal to 0, and solve for variables.

Exclude the value for which $t=0$, and $2u=0$.

$t=0$ or $2u=0$
$u=0$

Therefore, t cannot be equal to $\boxed{0}$ and u cannot be equal to $\boxed{0}$.

Answer 11E.

Consider the following expression:

$$\frac{3x^2y}{12xy^3z}$$

$$=\frac{(3xy)x}{(3xy)4y^2z}$$ The GCF of the numerator and denominator is $3xy$.

$$=\frac{\overset{1}{\cancel{(3xy)}}x}{\underset{1}{\cancel{(3xy)}}4y^2z}$$ Divide the numerator and denominator by the GCF $3xy$.

$$=\frac{x}{4y^2z}$$ Simplify.

Therefore, the simplify form of the expression is $\boxed{\frac{x}{4y^2z}}$

Answer 11PT.

Consider the following expression.

$$\frac{x+4+\frac{5}{x-2}}{x+6+\frac{15}{x-2}}$$

$$=\frac{\frac{(x+4)(x-2)}{x-2}+\frac{5}{x-2}}{\frac{(x+6)(x-2)}{x-2}+\frac{15}{x-2}}$$ [The LCD of the fractions in the numerator is $x-2$ and denominator is $x-2$.]

$$=\frac{\frac{x^2+2x-8}{x-2}+\frac{5}{x-2}}{\frac{x^2+4x-12}{x-2}+\frac{15}{x-2}}$$ Distributive property.

$$=\frac{\frac{x^2+2x-3}{x-2}}{\frac{x^2+4x+3}{x-2}}$$ Simplify.

$$=\frac{x^2+2x-3}{x-2}\div\frac{x^2+4x+3}{x-2}$$ Rewrite as a division sentence.

$$=\frac{x^2+2x-3}{x-2}\cdot\frac{x-2}{x^2+4x+3}$$ Rewrite as multiplication by the reciprocal.

$$=\frac{(x+3)(x-1)}{x-2}\cdot\frac{x-2}{(x+3)(x+1)}$$ Factor numerator and denominator.

$$=\frac{\overset{1}{\cancel{(x+3)}}(x-1)}{\underset{1}{\cancel{(x-2)}}}\cdot\frac{\overset{1}{\cancel{(x-2)}}}{\underset{1}{\cancel{(x+3)}}(x+1)}$$ Divide by the common factor.

$$=\frac{x-1}{x+1}$$ Simplify.

Therefore, the answer is $\boxed{\frac{x-1}{x+1}}$.

To find the excluded values for the rational expression, factor the denominator, set each factor equal to 0, and solve for *t*.

Exclude the value for which $x-2=0$.

$$x-2=0$$
$$x=2$$

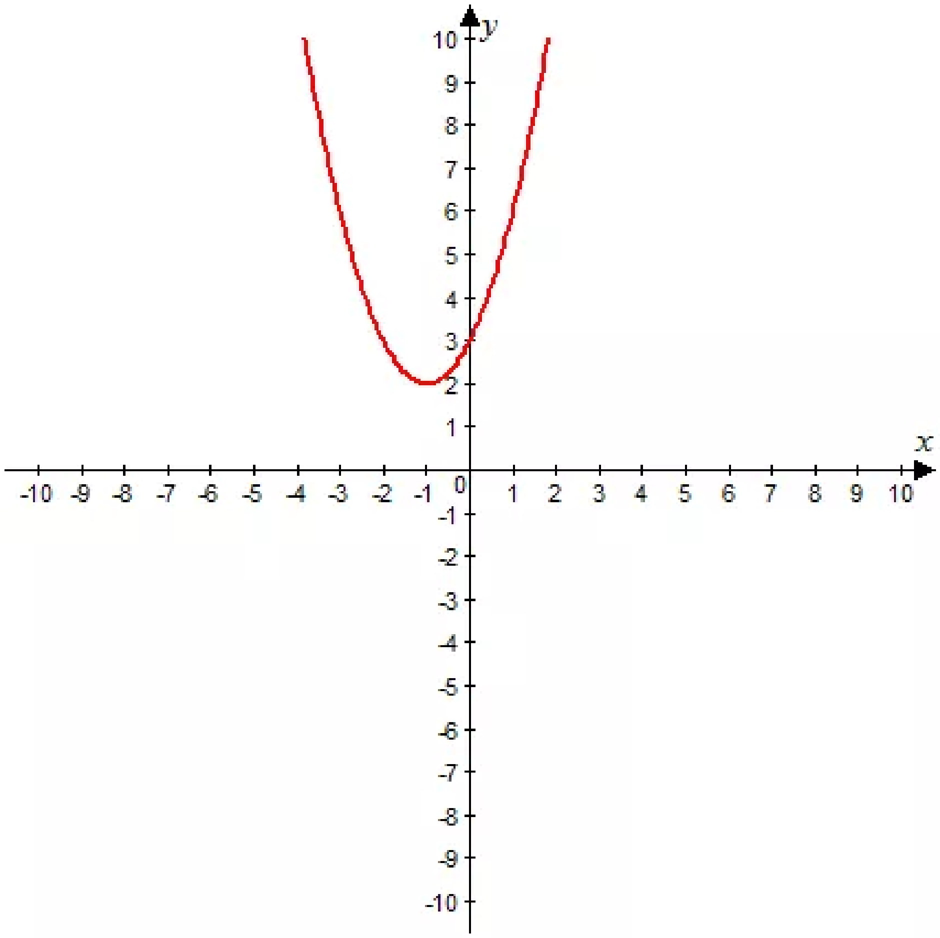
Therefore, *x* cannot be equal to $\boxed{2}$.

Answer 11STP.

Consider the following function:

$$y=x^2+2x+3$$

Graph of the function is as shown below:



The graph has only one root $\boxed{-3}$.

Answer 12E.

Consider the following expression:

$$\frac{n^2-3n}{n-3}=\frac{n(n-3)}{n-3} \quad \text{Factor.}$$

$$=\frac{n(\cancel{n-3})^{1}}{\cancel{n-3}_{1}} \quad \text{Divide the numerator and denominator by the GCF, } n-3.$$

$$=n \quad \text{Simplify.}$$

Therefore, the simplify form of the expression is $\boxed{n}$

Answer 12PT.

Consider the following subtraction.

$$\frac{2x}{x-7}-\frac{14}{x-7}=\frac{2x-14}{x-7} \quad \text{The common denominator is } (x-7).$$

$$=\frac{2(x-7)}{x-7} \quad \text{Factor.}$$

$$=\frac{2(\cancel{x-7})^{1}}{\cancel{x-7}_{1}} \quad \text{Divide by the common factors, } (x-7).$$

$$=2 \quad \text{Simplify.}$$

Thus, difference is $\boxed{2}$.

Answer 13E.

Consider the following expression:

$$\frac{a^2-25}{a^2+3a-10}$$

$$=\frac{(a+5)(a-5)}{(a+5)(a-2)} \quad \text{Factor.}$$

$$=\frac{(\cancel{a+5})^{1}(a-5)}{(\cancel{a+5})_{1}(a-2)} \quad \text{Divide the numerator and denominator by the GCF, } a+5.$$

$$=\frac{a-5}{a-2} \quad \text{Simplify.}$$

Therefore, the simplify form of the expression is $\boxed{\frac{a-5}{a-2}}$

Answer 13PT.

Consider the following rational expression.

$$\frac{n+3}{2n-8}\cdot\frac{6n-24}{2n+1}$$

$$=\frac{n+3}{2(n-4)}\cdot\frac{6(n-4)}{2n+1}$$ Factor the numerators and denominators.

$$=\frac{6(n+3)(n-4)}{2(n-4)(2n+1)}$$ Multiply the numerators and denominators.

$$=\frac{\overset{3}{\cancel{6}}(n+3)\overset{1}{\cancel{(n-4)}}}{\underset{1}{\cancel{2}}\underset{1}{\cancel{(n-4)}}(2n+1)}$$ Factor out the GCF.

$$=\frac{3(n+3)}{2n+1}$$ Simplify.

Thus, the product is $\boxed{\frac{3(n+3)}{2n+1}}$.

Answer 14E.

Consider the following expression:

$$\frac{x^2+10x+21}{x^3+x^2-42x}$$

$$=\frac{(x+7)(x+3)}{x(x+7)(x-6)}$$ Factor.

$$=\frac{\overset{1}{\cancel{(x+7)}}(x+3)}{x\underset{1}{\cancel{(x+7)}}(x-6)}$$ Divide the numerator and denominator by the GCF, $x+7$.

$$=\frac{x+3}{x(x-6)}$$ Simplify.

Therefore, the simplify form of the expression is $\boxed{\frac{x+3}{x(x-6)}}$

Answer 14PT.

Consider the following division:

$$(10m^2+9m-36)\div(2m-3)$$

Use long division process to divide a polynomial by a binomial.

$$\begin{array}{r} 5m+12 \\ 2m-3\overline{)\;10m^2+9m-36} \\ \underline{(-)10m^2-15m} \\ 24m-36 \\ \underline{(-)24m-36} \\ 0 \end{array}$$

Thus, the quotient $(10m^2+9m-36)\div(2m-3)$ is $\boxed{5m+12}$.

Answer 14STP.

Consider the following expression:

$$\frac{x^2-7x+6}{x^2+6x-7}$$

$$=\frac{(x-6)(x-1)}{(x+7)(x-1)}$$ The GCF of the numerator and denominator is $x-1$.

$$=\frac{(x-6)\cancel{(x-1)}^{1}}{(x+7)\cancel{(x-1)}_{1}}$$ Divide the numerator and denominator by the GCF, $x-1$.

$$=\frac{x-6}{x+7}$$ Simplify.

Therefore, the simplify form of the expression is $\boxed{\frac{x-6}{x+7}}$.

To find the excluded values for the rational expression, factor the denominator, set each factor equal to 0, and solve for *x*.

Exclude the value for which $x^2+6x-7=0$

$x^2+6x-7=0$ The denominator cannot be equal to 0

$(x+7)(x-1)=0$ Factor

$x+7=0$ or $x-1=0$ Use zero factor property to solve for x.

$x=-7$ or $x=1$

Therefore, *x* cannot be equal to $\boxed{-7,1}$.

Answer 15E.

Consider the following rational expression.

$$\frac{7b^2}{9}\cdot\frac{6a^2}{b}=\frac{42a^2b^2}{9b}$$ Multiply the numerators and denominators.

$$=\frac{\cancel{3b}^{1}(14a^2b)}{\cancel{3b}_{1}(3)}$$ The GCF is $3b$.

$$=\frac{14a^2b}{3}$$ Simplify.

Thus, the product is $\boxed{\frac{14a^2b}{3}}$.

Answer 15PT.

Consider the following rational expression.

$$\frac{x^2+4x-32}{x+5}\cdot\frac{x-3}{x^2-7x+12}$$

$$=\frac{(x+8)(x-4)}{x+5}\cdot\frac{x-3}{(x-3)(x-4)} \quad \text{Factor the numerators and denominators.}$$

$$\frac{(x+8)(x-4)(x-3)}{(x+5)(x-3)(x-4)} \quad \text{Multiply the numerators and denominators.}$$

$$=\frac{(x+8)\cancel{(x-4)}^{1}\cancel{(x-3)}^{1}}{(x+5)\cancel{(x-3)}_{1}\cancel{(x-4)}_{1}} \quad \text{The GCF is } (x-3)(x-4).$$

$$=\frac{x+8}{x+5} \quad \text{Simplify.}$$

Thus, the product is $\boxed{\frac{x+8}{x+5}}$.

Answer 15STP.

Consider the following rational expression.

$$\frac{1030 \text{ kilometers}}{1 \text{ hour}}\cdot\frac{1000 \text{ meters}}{1 \text{ kilometer}}\cdot\frac{1 \text{ hour}}{60 \text{ minutes}}\cdot\frac{1 \text{ minutes}}{60 \text{ seconds}}$$

$$=\frac{1030 \;\cancel{\text{kilometers}}}{1 \;\cancel{\text{hour}}}\cdot\frac{1000 \text{ meters}}{1 \;\cancel{\text{kilometer}}}\cdot\frac{1 \;\cancel{\text{hour}}}{60 \;\cancel{\text{minutes}}}\cdot\frac{1 \;\cancel{\text{minutes}}}{60 \text{ seconds}}$$

$$=\frac{1030\cdot1000\cdot1\cdot1 \text{ meters}}{1\cdot1\cdot60\cdot60 \text{seconds}}$$

$$=\frac{1{,}030{,}000 \text{ meters}}{3{,}600 \text{seconds}} \quad \text{Multiply.}$$

$$\approx\frac{16.67 \text{ meters}}{1 \text{seconds}} \quad \text{Simplify.}$$

$$\approx 286.11 \text{ meters/seconds}$$

Thus, the answer is $\boxed{286.11 \text{ m/s}}$.

Answer 16E.

Consider the following rational expression.

$$\frac{5x^2y}{8ab}\cdot\frac{12a^2b}{25x}=\frac{60a^2bx^2y}{200abx}$$ Multiply the numerators and denominators.

$$=\frac{\cancel{20xab}(3axy)}{\cancel{20xab}(10)}$$ The GCF is $20xab$.

$$=\frac{3axy}{10}$$ Simplify.

Thus, the product is $\boxed{\frac{3axy}{10}}$.

Answer 16PT.

Consider the following polynomials.

$$\frac{z^2+2z-15}{z^2+9z+20}\div(z-3)$$

$$=\frac{z^2+2z-15}{z^2+9z+20}\cdot\frac{1}{z-3}$$ Multiply by the reciprocal of $(z-3)$.

$$=\frac{(z+5)(z-3)}{(z+5)(z+4)}\cdot\frac{1}{z-3}$$ Factor.

$$=\frac{\cancel{(z+5)}\cancel{(z-3)}}{\cancel{(z+5)}(z+4)}\cdot\frac{1}{\cancel{(z-3)}}$$ Divide by common factors.

$$=\frac{1}{z+4}$$ Simplify.

Thus, the quotient is $\boxed{\frac{1}{z+4}}$.

Answer 16STP.

Consider the following rational expression.

$$\frac{x^2-9}{x^3+x}\cdot\frac{3x}{x-3}$$

$$=\frac{(x-3)(x+3)}{x(x^2+1)}\cdot\frac{3x}{x-3}$$ Factor the numerators.

$$=\frac{3x(x-3)(x+3)}{x(x-3)(x^2+1)}$$ Multiply the numerators and denominators.

$$=\frac{3\cancel{x}\cancel{(x-3)}(x+3)}{\cancel{x}\cancel{(x-3)}(x^2+1)}$$ The GCF is $x(x-3)$.

$$=\frac{3(x+3)}{x^2+1}$$ Simplify.

Thus, the product is $\boxed{\frac{3(x+3)}{x^2+1}}$.

Answer 17E.

Consider the following rational expression.

$$(3x+30)\cdot\frac{10}{x^2-100}$$

$$=\frac{10(3x+30)}{x^2-100}$$ Multiply the numerators and denominators.

$$=\frac{30(x+10)}{(x+10)(x-10)}$$ Factor the numerators.

$$=\frac{30\,\cancel{(x+10)}^{1}}{\cancel{(x+10)}_{1}\,(x-10)}$$ The GCF is $(x+10)$.

$$=\frac{30}{x-10}$$ Simplify.

Thus, the product is $\boxed{\frac{30}{x-10}}$.

Answer 17PT.

Consider the following polynomials.

$$\frac{4x^2+11x+6}{x^2-x-6}\div\frac{x^2+8x+16}{x^2+x-12}$$

$$=\frac{4x^2+11x+6}{x^2-x-6}\cdot\frac{x^2+x-12}{x^2+8x+16}$$ Multiply by the reciprocal of $\frac{x^2+8x+16}{x^2+x-12}$.

$$=\frac{(x+2)(4x+3)}{(x+2)(x-3)}\cdot\frac{(x+4)(x-3)}{(x+4)(x+4)}$$ Factor.

$$=\frac{\cancel{(x+2)}^{1}(4x+3)}{\cancel{(x+2)}_{1}\,\cancel{(x-3)}_{1}}\cdot\frac{\cancel{(x+4)}^{1}\,\cancel{(x-3)}^{1}}{(x+4)\,\cancel{(x+4)}_{1}}$$ Divide by common factors.

$$=\frac{4x+3}{x+4}$$ Simplify.

Thus, the quotient is $\boxed{\frac{4x+3}{x+4}}$.

Answer 17STP.

Consider the following binomials.

$$\frac{a+3}{2a+6} \div \frac{6a-24}{4a+12}$$

$$= \frac{a+3}{2a+6} \cdot \frac{4a+12}{6a-24}$$ Multiply by $\frac{4a+12}{6a-24}$, the reciprocal of $\frac{6a-24}{4a+12}$.

$$= \frac{a+3}{2(a+3)} \cdot \frac{4(a+3)}{6(a-4)}$$ Factor.

$$= \frac{\overset{1}{\cancel{4}}(a+3)\overset{1}{\cancel{(a+3)}}}{\underset{3}{\cancel{12}}\underset{1}{\cancel{(a+3)}}(a-4)}$$ Divide by common factors.

$$= \frac{a+3}{3(a-4)}$$ Simplify.

Thus, the quotient is $\boxed{\frac{a+3}{3(a-4)}}$.

Answer 18E.

Consider the following rational expression.

$$\frac{3a-6}{a^2-9} \cdot \frac{a+3}{a^2-2a}$$

$$= \frac{3(a-2)}{(a-3)(a+3)} \cdot \frac{a+3}{a(a-2)}$$ Factor the numerators and denominators.

$$= \frac{3(a-2)(a+3)}{a(a-2)(a+3)(a-3)}$$ Multiply the numerators and denominators.

$$= \frac{3\overset{1}{\cancel{(a-2)}}\overset{1}{\cancel{(a+3)}}}{a\underset{1}{\cancel{(a-2)}}\underset{1}{\cancel{(a+3)}}(a-3)}$$ The GCF is $(a-2)(a+3)$.

$$= \frac{3}{a(a-3)}$$ Simplify.

Thus, the product is $\boxed{\frac{3}{a(a-3)}}$.

Answer 18PT.

To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

Consider the following division:

$$\left(10z^4+5z^3-z^2\right)\div 5z^3=\frac{10z^4+5z^3-z^2}{5z^3} \quad \text{Write as a rational expression.}$$

$$=\frac{10z^4}{5z^3}+\frac{5z^3}{5z^3}-\frac{z^2}{5z^3} \quad \text{Divide each term by } 5z^3.$$

$$=\frac{\overset{2z}{\cancel{10z^4}}}{\underset{1}{\cancel{5z^3}}}+\frac{\overset{1}{\cancel{5z^3}}}{\underset{1}{\cancel{5z^3}}}-\frac{\overset{1}{\cancel{z^2}}}{\underset{5z}{\cancel{5z^3}}} \quad \text{Simplify each term.}$$

$$=2z+1-\frac{1}{5z} \quad \text{Simplify.}$$

Thus, the quotient is $\boxed{2z+1-\frac{1}{5z}}$.

Answer 18STP.

Consider the following binomials.

$$\frac{x}{x+4}\div\frac{4x}{x^2-16}$$

$$=\frac{x}{x+4}\cdot\frac{x^2-16}{4x} \quad \text{Multiply by } \frac{x^2-16}{4x}\text{, the reciprocal of } \frac{4x}{x^2-16}.$$

$$=\frac{x}{x+4}\cdot\frac{(x-4)(x+4)}{4x} \quad \text{Factor.}$$

$$=\frac{\overset{1}{\cancel{x}}(x-4)\overset{1}{\cancel{(x+4)}}}{4\underset{1}{\cancel{x}}\underset{1}{\cancel{(x+4)}}} \quad \text{Divide by common factors.}$$

$$=\frac{x-4}{4} \quad \text{Simplify.}$$

Thus, the quotient is $\boxed{\frac{x-4}{4}}$.

Answer 19E.

Consider the following rational expression.

$$\frac{x^2+x-12}{x+2}\cdot\frac{x+4}{x^2-x-6}$$

$$=\frac{(x+4)(x-3)}{x+2}\cdot\frac{x+4}{(x-3)(x+2)} \quad \text{Factor the numerators and denominators.}$$

$$=\frac{(x+4)(x-3)(x+4)}{(x+2)(x-3)(x+2)} \quad \text{Multiply the numerators and denominators.}$$

$$=\frac{(x+4)\overset{1}{\cancel{(x-3)}}(x+4)}{(x+2)\underset{1}{\cancel{(x-3)}}(x+2)} \quad \text{The GCF is } (x-3).$$

$$=\frac{(x+4)^2}{(x+2)^2} \quad \text{Simplify.}$$

Thus, the product is $\boxed{\frac{(x+4)^2}{(x+2)^2}}$.

Answer 19PT.

Consider the following addition.

$$\frac{y}{7y+14}+\frac{6}{6-3y}$$

$$=\frac{y}{7(y+2)}+\frac{6}{-3(y-2)} \quad \text{Factor the denominators.}$$

$$=\frac{y}{7(y+2)}+\frac{-2}{y-2} \quad \text{Simplify.}$$

$$=\frac{y}{7(y+2)}\cdot\frac{y-2}{y-2}+\frac{-2}{y-2}\cdot\frac{7(y+2)}{7(y+2)} \quad \text{The LCD is } 7(y+2)(y-2).$$

$$=\frac{y(y-2)}{7(y+2)(y-2)}+\frac{-14(y+2)}{7(y+2)(y-2)} \quad \text{Multiply.}$$

$$=\frac{y^2-2y}{7(y+2)(y-2)}+\frac{-14y-28}{7(y+2)(y-2)} \quad \text{Distributive property.}$$

$$=\frac{y^2-2y-14y-28}{7(y+2)(y-2)} \quad \text{Add the numerators.}$$

$$=\frac{y^2-16y-28}{7(y+2)(y-2)} \quad \text{Combine like terms.}$$

Therefore, the sum is $\boxed{\frac{y^2-16y-28}{7(y+2)(y-2)}}$.

Answer 19STP.

Consider the following figure:

To find the perimeter of the triangle add sides of the triangle:

$$
\begin{aligned}
P &= \frac{5t+3}{t-10}+\frac{7-t}{t-10}+\frac{-4t-1}{t-10} \\
&= \frac{5t+3+7-t-4t-1}{t-10} && \text{The common denominator is } t-10. \\
&= \frac{9}{t-10} && \text{Combine like terms.}
\end{aligned}
$$

Thus, the perimeter of the triangle is $\boxed{\frac{9}{t-10}}$.

Answer 20E.

Consider the following rational expression.

$$
\begin{aligned}
&\frac{b^2+19b+84}{b-3}\cdot\frac{b^2-9}{b^2+15b+36} \\
&= \frac{(b+12)(b+7)}{b-3}\cdot\frac{(b-3)(b+3)}{(b+12)(b+3)} && \text{Factor the numerators and denominators.} \\
&= \frac{(b+12)(b+7)(b-3)(b+3)}{(b-3)(b+12)(b+3)} && \text{Multiply the numerators and denominators.} \\
&= \frac{\overset{1}{\cancel{(b+12)}}(b+7)\overset{1}{\cancel{(b-3)}}\overset{1}{\cancel{(b+3)}}}{\underset{1}{\cancel{(b-3)}}\underset{1}{\cancel{(b+12)}}\underset{1}{\cancel{(b+3)}}} && \text{The GCF is } (b-3)(b+12)(b+3). \\
&= b+7 && \text{Simplify.}
\end{aligned}
$$

Thus, the product is $\boxed{b+7}$.

Answer 20PT.

Consider the following mixed expression.

$$
\begin{aligned}
\frac{x+5}{x+2}+6 &= \frac{x+5}{x+2}+6\cdot\frac{x+2}{x+2} && \text{The LCD is } x+2. \\
&= \frac{x+5}{x+2}+\frac{6x+12}{x+2} && \text{Multiply.} \\
&= \frac{x+5+6x+12}{x+2} && \text{Add the numerators.} \\
&= \frac{7x+17}{x+2} && \text{Simplify.}
\end{aligned}
$$

Therefore, the answer is $\boxed{\frac{7x+17}{x+2}}$.

Answer 21E.

Consider the following monomials.

$$\frac{p^3}{2q} \div \frac{p^2}{4q} = \frac{p^3}{2q} \cdot \frac{4q}{p^2} \quad \text{Multiply by } \frac{4q}{p^2}\text{, the reciprocal of } \frac{p^2}{4q}.$$

$$= \frac{p \cdot p^2}{2q} \cdot \frac{2 \cdot 2q}{p^2} \quad \text{Factor.}$$

$$= \frac{p \cdot \cancel{p^2}^{1}}{\cancel{2q}_{1}} \cdot \frac{2 \cdot \cancel{2q}^{1}}{\cancel{p^2}_{1}} \quad \text{Divide by common factors.}$$

$$= 2p \quad \text{Simplify.}$$

Thus, the quotient is $\boxed{2p}$.

Answer 21PT.

Consider the following subtraction.

$$\frac{x^2 - 1}{x + 1} - \frac{x^2 + 1}{x - 1}$$

Change each rational expression into an equivalent expression with the LCD. Then subtract.

$$\frac{x^2-1}{x+1} - \frac{x^2+1}{x-1} = \frac{x^2-1}{x+1} \cdot \frac{x-1}{x-1} - \frac{x^2+1}{x-1} \cdot \frac{x+1}{x+1} \quad \text{The LCD is } (x-1)(x+1).$$

$$= \frac{x^3 - x^2 - x + 1}{(x-1)(x+1)} - \frac{x^3 + x^2 + x + 1}{(x-1)(x+1)} \quad \text{Multiply.}$$

$$= \frac{x^3 - x^2 - x + 1 - (x^3 + x^2 + x + 1)}{(x-1)(x+1)} \quad \text{Subtract the numerators.}$$

$$= \frac{x^3 - x^2 - x + 1 - x^3 - x^2 - x - 1}{(x-1)(x+1)} \quad \text{Distribute the negative.}$$

$$= \frac{-2x^2 - 2x}{(x-1)(x+1)} \quad \text{Combine like terms.}$$

$$= \frac{-2x(x+1)}{(x-1)(x+1)} \quad \text{Factor.}$$

$$= \frac{-2x\,\cancel{(x+1)}^{1}}{(x-1)\,\cancel{(x+1)}_{1}} \quad \text{The GCF is } (x+1).$$

$$= \frac{-2x}{x-1} \quad \text{Simplify.}$$

Therefore, the difference is $\boxed{\frac{-2x}{x-1}}$.

Answer 22E.

Consider the following binomials.

$$\frac{y^2}{y+4} \div \frac{3y}{y^2-16}$$

$$= \frac{y^2}{y+4} \cdot \frac{y^2-16}{3y} \quad \text{Multiply by } \frac{y^2-16}{3y}\text{, the reciprocal of } \frac{3y}{y^2-16}.$$

$$= \frac{y \cdot y}{y+4} \cdot \frac{(y-4)(y+4)}{3 \cdot y} \quad \text{Factor.}$$

$$= \frac{y \cdot \cancel{y}^{1}}{\cancel{y+4}_{1}} \cdot \frac{(y-4)\cancel{(y+4)}^{1}}{3 \cdot \cancel{y}_{1}} \quad \text{Divide by common factors.}$$

$$= \frac{y(y-4)}{3} \quad \text{Simplify.}$$

Thus, the quotient is $\boxed{\frac{y(y-4)}{3}}$.

Answer 22PT.

In order to find the extraneous solution the both sides of the equation are multiplied by LCD of the two ration expression. This can give solutions that are not solutions to the original equation. This type of solution is called extraneous solution.

Consider the following equation:

$$\frac{2n}{n-4} - 2 = \frac{4}{n+5} \quad \text{Original equation}$$

$$(n-4)(n+5)\left(\frac{2n}{n-4} - 2\right) = \frac{4}{n+5}(n-4)(n+5) \quad \text{The LCD is } (n-4)(n+5)$$

$$\left[\cancel{(n-4)}(n+5) \cdot \frac{2n}{\cancel{n-4}} - 2(n-4)(n+5)\right] = \frac{4}{\cancel{n+5}}(n-4)\cancel{(n+5)} \quad \text{Distributive property.}$$

$$2n(n+5) - 2(n-4)(n+5) = 4(n-4)$$

$$2n^2 + 10 - 2n^2 - 2n + 40 = 4n - 16 \quad \text{Distributive property.}$$

$$-2n + 50 = 4n - 16 \quad \text{Combine like terms.}$$

$$66 = 6n \quad \text{Simplify}$$

$$11 = n \quad \text{Divide both sides.}$$

Thus the solution of the equation is $\boxed{11}$.

Answer 23E.

Consider the following polynomials.

$$\frac{3y-12}{y+4}\div\left(y^2-6y+8\right)$$

$$=\frac{3y-12}{y+4}\cdot\frac{1}{y^2-6y+8} \quad \text{Multiply by the reciprocal of } \left(y^2-6y+8\right).$$

$$=\frac{3(y-4)}{y+4}\cdot\frac{1}{(y-4)(y-2)} \quad \text{Factor.}$$

$$=\frac{3\cancel{(y-4)}}{(y+4)}\cdot\frac{1}{\cancel{(y-4)}(y-2)} \quad \text{Divide by common factors.}$$

$$=\frac{3}{(y+4)(y-2)} \quad \text{Simplify.}$$

Thus, the quotient is $\boxed{\frac{3}{(y+4)(y-2)}}$.

Answer 23PT.

In order to find the extraneous solution the both sides of the equation are multiplied by LCD of the two ration expression. This can give solutions that are not solutions to the original equation. This type of solution is called extraneous solution.

Consider the following equation:

$$\frac{3}{x^2+5x+6}-\frac{7}{x+3}=-\frac{x-1}{x+2} \quad \text{Original equation.}$$

$$\frac{3}{(x+3)(x+2)}-\frac{7}{x+3}=-\frac{x-1}{x+2} \quad \text{Factor.}$$

$$(x+3)(x+2)\left(\frac{3}{(x+3)(x+2)}-\frac{7}{x+3}\right)=\left(-\frac{x-1}{x+2}\right)(x+3)(x+2)$$

$$\text{The LCD is } (x+3)(x+2).$$

$$\frac{3\cancel{(x+3)}\cancel{(x+2)}}{\cancel{(x+3)}\cancel{(x+2)}}-\frac{7\cancel{(x+3)}(x+2)}{\cancel{x+3}}=-\frac{(x-1)(x+3)\cancel{(x+2)}}{\cancel{x+2}}$$

$$\text{Distributive property.}$$

$$3-7(x+2)=-(x-1)(x+3) \quad \text{Simplify}$$

$$3-7x-14=-x^2-2x+3 \quad \text{Distributive property.}$$

$$x^2-5x-14=0 \quad \text{Set equal to 0.}$$

$$(x-7)(x+2)=0 \quad \text{Factor.}$$

$x-7=0$ or $x+2=0$ Zero-product property and solve for x

$x=7$ $\quad$ $x=-2$

Since, -2 is an excluded value of x, the number -2 is an extraneous solution.

Thus, the solution of the equation is $\boxed{7}$.

Answer 24E.

Consider the following polynomials.

$$\frac{2m^2+7m-15}{m+5} \div \frac{9m^2-4}{3m+2}$$

$$= \frac{2m^2+7m-15}{m+5} \cdot \frac{3m+2}{9m^2-4} \quad \text{Multiply by the reciprocal of } \frac{9m^2-4}{3m+2}.$$

$$= \frac{(2m-3)(m+5)}{m+5} \cdot \frac{3m+2}{(3m-2)(3m+2)} \quad \text{Factor.}$$

$$= \frac{(2m-3)\cancel{(m+5)}^{1}}{\cancel{m+5}_{1}} \cdot \frac{\cancel{(3m+2)}^{1}}{(3m-2)\cancel{(3m+2)}_{1}} \quad \text{Divide by common factors.}$$

$$= \frac{2m-3}{3m-2} \quad \text{Simplify.}$$

Thus, the quotient is $\boxed{\frac{2m-3}{3m-2}}$.

Answer 25E.

To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

Consider the following division:

$$\left(4a^2b^2c^2-8a^3b^2c+6abc^2\right) \div 2ab^2$$

$$= \frac{4a^2b^2c^2-8a^3b^2c+6abc^2}{2ab^2}$$

$$= \frac{4a^2b^2c^2}{2ab^2} - \frac{8a^3b^2c}{2ab^2} + \frac{6abc^2}{2ab^2} \quad \text{Divide each term by } 2ab^2.$$

$$= \frac{\cancel{4a^2b^2c^2}^{2ac^2}}{\cancel{2ab^2}_{1}} - \frac{\cancel{8a^3b^2c}^{4a^2c}}{\cancel{2ab^2}_{1}} + \frac{\cancel{6abc^2}^{3c^2}}{\cancel{2ab^2}_{b}} \quad \text{Simplify each term.}$$

$$= 2ac^2 - 4a^2c + \frac{3c^2}{b} \quad \text{Simplify.}$$

Thus, the quotient is $\boxed{2ac^2 - 4a^2c + \frac{3c^2}{b}}$.

Answer 25PT.

Consider the following figure:

Use formula to find the area of the triangle:

$$A=\frac{1}{2}(\text{base})(\text{height}) \qquad \text{Formula.}$$

$$=\frac{1}{2}\cdot\frac{36}{x+y}\cdot\frac{x^2-y^2}{12} \qquad \text{Substitute.}$$

$$=\frac{1}{2}\cdot\frac{36}{x+y}\cdot\frac{(x+y)(x-y)}{12} \qquad \text{Factor } x^2-y^2.$$

$$=\frac{1}{2}\cdot\frac{\overset{3}{\cancel{36}}}{\underset{1}{\cancel{x+y}}}\cdot\frac{\overset{1}{\cancel{(x+y)}}(x-y)}{\underset{1}{\cancel{12}}} \qquad \text{Factor out the common factor.}$$

$$=\frac{3}{2}(x-y) \qquad \text{Simplify.}$$

Thus, the correct option is $\boxed{B.\frac{3}{2}(x-y)}$.

Answer 26E.

Consider the following division:

$$(x^3+7x^2+10x-6)\div(x+3)$$

Use long division process to divide a polynomial by a binomial.

$$\begin{array}{r} x^2+4x-2 \\ x+3\overline{\big)\; x^3+7x^2+10x-6} \\ \underline{(-)x^3+3x^2} \\ 4x^2+10x \\ \underline{(-)4x^2+12x} \\ -2x-6 \\ \underline{(-)-2x-6} \\ 0 \end{array}$$

The quotient of $(x^3+7x^2+10x-6)\div(x+3)$ is $\boxed{x^2+4x-2}$.

Answer 27E.

Consider the following division:

$$\frac{x^3-7x+6}{x-2}$$

Rename the x^2 term using a coefficient of 0.

$$\frac{x^3+0x^2-7x+6}{x-2}$$

Use long division process to divide a polynomial by a binomial.

$$\begin{array}{r}
x^2+2x-3 \\
x-2\overline{\smash{)}\;x^3-0x^2-7x+6} \\
\underline{(-)x^3-2x^2} \\
2x^2-7x \\
\underline{(-)2x^2-4x} \\
-3x+6 \\
\underline{(-)-3x+6} \\
0
\end{array}$$

The quotient of $(x^3+7x^2+10x-6)\div(x+3)$ is $\boxed{x^2+4x-2}$.

Answer 28E.

Consider the following division:

$$(48b^2+8b+7)\div(2b-1)$$

Use long division process to divide a polynomial by a binomial.

$$\begin{array}{r}
24b+16 \\
2b-1\overline{\smash{)}\;48b^2+\;8b+7} \\
\underline{(-)48b^2-24b} \\
32b+7 \\
\underline{(-)32b-16} \\
23
\end{array}$$

The quotient of $(48b^2+8b+7)\div(2b-1)$ is $24b+16$ with a remainder 23, which can be written as $\boxed{24b+16+\frac{23}{2b-1}}$.

Answer 29E.

Consider the following addition.

$$\frac{m+4}{5}+\frac{m-1}{5}$$

$$=\frac{m+4+m-1}{5}$$ The common denominator is 5.

$$=\frac{2m+3}{5}$$ Simplify.

Thus, sum is $\boxed{\frac{2m+3}{5}}$.

Answer 30E.

Consider the following addition.

$$\frac{-5}{2n-5}+\frac{2n}{2n-5}=\frac{-5+2n}{2n-5}$$ The common denominator is $(2n-5)$.

$$=\frac{2n-5}{2n-5}$$ Simplify the numerator.

$$=\frac{\cancel{2n-5}^{1}}{\cancel{2n-5}_{1}}$$ Divide by the common factors, $(2n-5)$.

$$=1$$ Simplify.

Thus, sum is $\boxed{1}$.

Answer 31E.

Consider the following addition.

$$\frac{a^2}{a-b}+\frac{-b^2}{a-b}=\frac{a^2-b^2}{a-b}$$ The common denominator is $(a-b)$.

$$=\frac{(a+b)(a-b)}{a-b}$$ Factor.

$$=\frac{(a+b)\cancel{(a-b)}^{1}}{\cancel{a-b}_{1}}$$ Divide by the common factors, $(a-b)$.

$$=a+b$$ Simplify.

Thus, sum is $\boxed{a+b}$.

Answer 32E.

Consider the following subtraction.

$$\frac{7a}{b^2} - \frac{5a}{b^2} = \frac{7a-5a}{b^2}$$ The common denominator is b^2.

$$= \frac{2a}{b^2}$$ Subtract the numerator.

Thus, difference is $\boxed{\frac{2a}{b^2}}$.

Answer 33E.

Consider the following subtraction.

$$\frac{2x}{x-3} - \frac{6}{x-3} = \frac{2x-6}{x-3}$$ The common denominator is $(x-3)$.

$$= \frac{2(x-3)}{x-3}$$ Factor.

$$= \frac{2\,\cancel{(x-3)}^{1}}{\cancel{x-3}_{1}}$$ Divide by the common factors, $(x-3)$.

$$= 2$$ Simplify.

Thus, difference is $\boxed{2}$.

Answer 34E.

Consider the following subtraction.

$$\frac{m^2}{m-n} - \frac{2mn-n^2}{m-n} = \frac{m^2-2mn+n^2}{m-n}$$ The common denominator is $(m-n)$.

$$= \frac{(m-n)(m-n)}{m-n}$$ Factor.

$$= \frac{\cancel{(m-n)}^{1}(m-n)}{\cancel{m-n}_{1}}$$ Divide by the common factors, $(m-n)$.

$$= m-n$$ Subtract the numerator.

Thus, difference is $\boxed{m-n}$.

Answer 35E.

Consider the following addition.

$$\frac{2c}{3d^2}+\frac{3}{2cd}$$

Change each rational expression into an equivalent expression with the LCD, $6cd^2$.

Then add.

$$\frac{2c}{3d^2}+\frac{3}{2cd}=\frac{2c}{3d^2}\cdot\frac{2c}{2c}+\frac{3}{2cd}\cdot\frac{3d}{3d} \qquad \text{The LCD is } 6cd^2.$$

$$=\frac{4c^2}{6cd^2}+\frac{9d}{6cd^2} \qquad \text{Multiply.}$$

$$=\frac{4c^2+9d}{6cd^2} \qquad \text{Add the numerators.}$$

Therefore, the sum is $\boxed{\frac{4c^2+9d}{6cd^2}}$.

Answer 36E.

Consider the following addition.

$$\frac{r^2+21r}{r^2-9}+\frac{3r}{r+3}$$

Factor each denominator and find the LCD.

$$r^2-9=(r+3)(r-3)$$
$$r+3=(r+3)$$
$$\text{LCD}=(r+3)(r-3).$$

Since the denominator of $\frac{r^2+21r}{r^2-9}=\frac{r^2+21r}{(r+3)(r-3)}$ is already $(r+3)(r-3)$, only $\frac{3r}{r+3}$ needs to be renamed.

$$\frac{r^2+21r}{r^2-9}+\frac{3r}{r+3}$$

$$=\frac{r^2+21r}{(r+3)(r-3)}+\frac{3r(r-3)}{(r+3)(r-3)} \qquad \text{The LCD is } (r+3)(r-3).$$

$$=\frac{r^2+21r}{(r+3)(r-3)}+\frac{3r^2-9r}{(r+3)(r-3)} \qquad 3r(r-3)=3r^2-9r.$$

$$=\frac{r^2+21r+3r^2-9r}{(r+3)(r-3)} \qquad \text{Add the numerators.}$$

$$=\frac{4r^2+12r}{(r+3)(r-3)} \qquad \text{Combine like terms.}$$

$$=\frac{4r\cancel{(r+3)}^{1}}{\cancel{(r+3)}_{1}(r-3)} \qquad \text{Factor.}$$

$$=\frac{4r}{r-3} \qquad \text{Simplify.}$$

Therefore, the sum is $\boxed{\frac{4r}{r-3}}$.

Answer 37E.

Consider the following addition.

$$\frac{3a}{a-2}+\frac{5a}{a+1}$$

The denominators $a-2$ and $a+1$ are already completely factor.

Thus, LCD is $(a-2)(a+1)$.

$$\begin{aligned}\frac{3a}{a-2}+\frac{5a}{a+1}&=\frac{3a(a+1)}{(a-2)(a+1)}+\frac{5a(a-2)}{(a-2)(a+1)} && \text{The LCD is } (a-2)(a+1).\\ &=\frac{3a^2+3a}{(a-2)(a+1)}+\frac{5a^2-10a}{(a-2)(a+1)} && \text{Distributive property.}\\ &=\frac{3a^2+3a+5a^2-10a}{(a-2)(a+1)} && \text{Add the numerators.}\\ &=\frac{8a^2-7a}{(a-2)(a+1)} && \text{Combine like terms.}\end{aligned}$$

Therefore, the sum is $\boxed{\frac{8a^2-7a}{(a-2)(a+1)}}$.

Answer 38E.

Consider the following subtraction.

$$\frac{7n}{3}-\frac{9n}{7}$$

Since LDC of 3 and 7 is 21.

$$\begin{aligned}\frac{7n}{3}-\frac{9n}{7}&=\frac{7n\cdot 7}{3\cdot 7}-\frac{9n\cdot 3}{7\cdot 3} && \text{The LCD is 21.}\\ &=\frac{49n}{21}-\frac{27n}{21} && \text{Multiply.}\\ &=\frac{49n-27n}{21} && \text{Subtract the numerators.}\\ &=\frac{22n}{21} && \text{Simplify.}\end{aligned}$$

Therefore, the difference is $\boxed{\frac{22n}{21}}$.

Answer 39E.

Consider the following subtraction.

$$\frac{7}{3a}-\frac{3}{6a^2}$$

Factor each denominator and find the LCD.

$$6a^2 = 2\cdot 3\cdot a\cdot a$$
$$3a = 3\cdot a$$
$$\text{LCD} = 2\cdot 3\cdot a\cdot a = 6a^2.$$

Since the denominator of $\frac{3}{6a^2}$ is already $6a^2$, only $\frac{7}{3a}$ needs to be renamed.

$$\frac{7}{3a}-\frac{3}{6a^2}=\frac{7}{3a}\cdot\frac{2a}{2a}-\frac{3}{6a^2} \qquad \text{Multiply } \frac{7}{3a} \text{ by } \frac{2a}{2a}.$$
$$=\frac{14a}{6a^2}-\frac{3}{6a^2} \qquad \text{Multiply.}$$
$$=\frac{14a-3}{6a^2} \qquad \text{Subtract the numerators.}$$

Therefore, the difference is $\boxed{\frac{14a-3}{6a^2}}$.

Answer 40E.

Consider the following subtraction.

$$\frac{2x}{2x+8}-\frac{4}{5x+20}$$

Factor each denominator and find the LCD.

$$2x+8=2(x+4)$$
$$5x+20=5(x+4)$$
$$\text{LCD}=10(x+4).$$

Change each rational expression into an equivalent expression with the LCD.

$$\frac{2x}{2x+8}-\frac{4}{5x+20}=\frac{2x}{2(x+4)}-\frac{4}{5(x+4)}$$
$$=\frac{2x}{2(x+4)}\cdot\frac{5}{5}-\frac{4}{5(x+4)}\cdot\frac{2}{2} \qquad \text{The LCD is } 10(x+4).$$
$$=\frac{10x}{10(x+4)}-\frac{8}{10(x+4)} \qquad \text{Multiply.}$$
$$=\frac{10x-8}{10(x+4)} \qquad \text{Subtract the numerators.}$$
$$=\frac{2(5x-4)}{10(x+4)} \qquad \text{Factor.}$$
$$=\frac{5x-4}{5(x+4)} \qquad \text{Simplify.}$$

Therefore, the difference is $\boxed{\frac{5x-4}{5(x+4)}}$.

Answer 41E.

Consider the following mixed expression.

$$
\begin{aligned}
4+\frac{x}{x-2} &= 4\cdot\frac{x-2}{x-2}+\frac{x}{x-2} && \text{The LCD is } x-2.\\
&= \frac{4x-8}{x-2}+\frac{x}{x-2} && \text{Multiply.}\\
&= \frac{4x-8+x}{x-2} && \text{Add the numerators.}\\
&= \frac{5x-8}{x-2} && \text{Simplify.}
\end{aligned}
$$

Therefore, the answer is $\boxed{\frac{5x-8}{x-2}}$.

Answer 42E.

Consider the following mixed expression.

$$
\begin{aligned}
&2-\frac{x+2}{x^2-4}\\
&= 2-\frac{x+2}{(x+2)(x-2)} && \text{Factor the denominators.}\\
&= 2-\frac{1}{x-2} && \text{Divide by the common factor, } x+2.\\
&= 2\cdot\frac{x-2}{x-2}-\frac{1}{x-2} && \text{The LCD is } (x-2).\\
&= \frac{2x^2-4}{x-2}-\frac{1}{x-2} && \text{Distributive property.}\\
&= \frac{2x^2-4-1}{x-2} && \text{Subtract the numerators.}\\
&= \frac{2x^2-5}{x-2} && \text{Simplify.}
\end{aligned}
$$

Therefore, the answer is $\boxed{\frac{2x^2-5}{x-2}}$.

Answer 43E.

Consider the following mixed expression.

$$3+\frac{x^2+y^2}{x^2-y^2}$$

$$=3\cdot\frac{x^2-y^2}{x^2-y^2}+\frac{x^2+y^2}{x^2-y^2}$$ The LCD is $\left(x^2-y^2\right)$.

$$=\frac{3x^2-3y^2}{x^2-y^2}+\frac{x^2+y^2}{x^2-y^2}$$ Distributive property.

$$=\frac{3x^2-3y^2+x^2+y^2}{x^2-y^2}$$ Add the numerators.

$$=\frac{4x^2-2y^2}{x^2+y^2}$$ Simplify.

Therefore, the answer is $\boxed{\frac{4x^2-2y^2}{x^2+y^2}}$.

Answer 44E.

Consider the following expression.

$$\frac{\frac{x^2}{y^3}}{\frac{3x}{9y^2}}=\frac{x^2}{y^3}\div\frac{3x}{9y^2}$$ Rewrite as a division sentence.

$$=\frac{x^2}{y^3}\cdot\frac{9y^2}{3x}$$ Rewrite as multiplication by the reciprocal.

$$=\frac{\cancel{x^2}^{\,x}}{\cancel{y^3}_{\,y}}\cdot\frac{\cancel{9}^{\,3}\,\cancel{y^2}^{\,1}}{\cancel{3}_{\,1}\,\cancel{x}_{\,1}}$$ Divide common factor.

$$=\frac{3x}{y}$$ Simplify

Therefore, the answer is $\boxed{\frac{3x}{y}}$.

Answer 45E.

Consider the following expression.

$$\frac{5+\frac{4}{a}}{\frac{a}{2}-\frac{3}{4}}$$

The numerator and denominator contain mixed expression. Rewrite it as rational expression first.

$$=\frac{\frac{5a}{a}+\frac{4}{a}}{\frac{2a}{4}-\frac{3}{4}} \quad \left[\begin{array}{l}\text{The LCD of the fractions in the numerator} \\ \text{is } a \text{ and denominator is 4.}\end{array}\right]$$

$$=\frac{\frac{5a+4}{a}}{\frac{2a-3}{4}} \quad \text{Simplify.}$$

$$=\frac{5a+4}{a}\div\frac{2a-3}{4} \quad \text{Rewrite as a division sentence.}$$

$$=\frac{5a+4}{a}\cdot\frac{4}{2a-3} \quad \text{Rewrite as multiplication by the reciprocal.}$$

$$=\frac{20a+16}{2a^2-3a} \quad \text{Simplify.}$$

Therefore, the answer is $\boxed{\frac{20a+16}{2a^2-3a}}$.

Answer 46E.

Consider the following expression.

$$\frac{y+9-\frac{6}{y+4}}{y+4+\frac{2}{y+1}}$$

The numerator and denominator contain mixed expression. Rewrite it as rational expression first.

$$=\frac{\frac{(y+9)(y+4)}{y+4}-\frac{6}{y+4}}{\frac{(y+4)(y+1)}{y+1}+\frac{2}{y+1}}$$ [The LCD of the fractions in the numerator is $y+4$ and denominator is $y+1$.]

$$=\frac{\frac{y^2+13y+36}{y+4}-\frac{6}{y+4}}{\frac{y^2+5y+4}{y+1}+\frac{2}{y+1}}$$ Distributive property.

$$=\frac{\frac{y^2+13y+36-6}{y+4}}{\frac{y^2+5y+4+2}{y+1}}$$ Simplify the numerator and denominator.

$$=\frac{\frac{y^2+13y+30}{y+4}}{\frac{y^2+5y+6}{y+1}}$$ Simplify.

$$=\frac{y^2+13y+30}{y+4}\div\frac{y^2+5y+6}{y+1}$$ Rewrite as a division sentence.

$$=\frac{y^2+13y+30}{y+4}\cdot\frac{y+1}{y^2+5y+6}$$ Rewrite as multiplication by the reciprocal.

$$=\frac{(y+3)(y+10)}{y+4}\cdot\frac{y+1}{(y+3)(y+2)}$$ Factor numerator and denominator.

$$=\frac{\cancel{(y+3)}^{1}(y+10)}{y+4}\cdot\frac{y+1}{\cancel{(y+3)}_{1}(y+2)}$$ Divide by the common factor.

$$=\frac{(y+10)(y+1)}{(y+4)(y+2)}$$ Simplify.

Therefore, the answer is $\boxed{\frac{(y+10)(y+1)}{(y+4)(y+2)}}$.

Answer 47E.

In order to find the extraneous solution the both sides of the equation are multiplied by LCD of the two ration expression. This can give solutions that are not solutions to the original equation. This type of solution is called extraneous solution.

Consider the following equation:

$$\frac{4x}{3}+\frac{7}{2}=\frac{7x}{12}-\frac{1}{4} \quad \text{Original equation.}$$

$$12\left(\frac{4x}{3}+\frac{7}{2}\right)=12\left(\frac{7x}{12}-\frac{1}{4}\right) \quad \text{The LCD is 12.}$$

$$\frac{\overset{4}{\cancel{12}}\cdot 4x}{\underset{1}{\cancel{3}}}+\frac{\overset{6}{\cancel{12}}\cdot 7}{\underset{1}{\cancel{2}}}=\frac{\overset{1}{\cancel{12}}\cdot 7x}{\underset{1}{\cancel{12}}}-\frac{\overset{3}{\cancel{12}}\cdot 1}{\underset{1}{\cancel{4}}} \quad \text{Use distributive property.}$$

$$16x+42=7x-3 \quad \text{Simplify.}$$

$$9x=-45$$

$$x=-5 \quad \text{Divide both sides by 9.}$$

Therefore, the solution is $\boxed{-5}$.

Answer 48E.

In order to find the extraneous solution the both sides of the equation are multiplied by LCD of the two ration expression. This can give solutions that are not solutions to the original equation. This type of solution is called extraneous solution.

Consider the following equation:

$$\frac{11}{2x}-\frac{2}{3x}=\frac{1}{6} \quad \text{Original equation.}$$

$$6x\left(\frac{11}{2x}-\frac{2}{3x}\right)=\left(\frac{1}{6}\right)6x \quad \text{The LCD is } 6x.$$

$$\frac{\overset{3}{\cancel{6x}}\cdot 11}{\underset{1}{\cancel{2x}}}-\frac{\overset{2}{\cancel{6x}}\cdot 2}{\underset{1}{\cancel{3x}}}=\frac{1\cdot \overset{x}{\cancel{6x}}}{\underset{1}{\cancel{6}}} \quad \text{Use distributive property}$$

$$33-4=x \quad \text{Simplify.}$$

$$29=x \quad \text{Subtract.}$$

Therefore, the solution is $\boxed{29}$.

Answer 49E.

In order to find the extraneous solution the both sides of the equation are multiplied by LCD of the two ration expression. This can give solutions that are not solutions to the original equation. This type of solution is called extraneous solution.

Consider the following equation:

$$\frac{2}{3r}-\frac{3r}{r-2}=-3 \quad \text{Original equation.}$$

$$3r(r-2)\left(\frac{2}{3r}-\frac{3r}{r-2}\right)=-3\cdot 3r(r-2) \quad \text{The LCD is } 3r(r-2).$$

$$\frac{\cancel{3r}(r-2)2}{\cancel{3r}}-\frac{3r\cdot 3r\cancel{(r-2)}}{\cancel{r-2}}=-3\cdot 3r(r-2) \quad \text{Use distributive property.}$$

$$2r-4-9r^2=-9r^2+18r \quad \text{Simplify.}$$

$$-4=16r \quad \text{Simplify}$$

$$-\frac{1}{4}=r \quad \text{Divide both sides by 16.}$$

Therefore, the solution is $\boxed{-\frac{1}{4}}$.

Answer 50E.

In order to find the extraneous solution the both sides of the equation are multiplied by LCD of the two ration expression. This can give solutions that are not solutions to the original equation. This type of solution is called extraneous solution.

Consider the following equation:

$$\frac{x-2}{x}-\frac{x-3}{x-6}=\frac{1}{x} \quad \text{Original equation.}$$

$$x(x-6)\left(\frac{x-2}{x}-\frac{x-3}{x-6}\right)=\left(\frac{1}{x}\right)x(x-6) \quad \text{The LCD is } x(x-6).$$

$$\frac{\cancel{x}(x-6)(x-2)}{\cancel{x}}-\frac{x\cancel{(x-6)}(x-3)}{\cancel{(x-6)}}=\frac{1}{\cancel{x}}\cancel{x}(x-6) \quad \text{Use distributive property.}$$

$$(x-6)(x-2)-x(x-3)=x-6 \quad \text{Simplify.}$$

$$x^2-8x+12-x^2+3x=x-6 \quad \text{Multiply.}$$

$$-5x+12=x-6 \quad \text{Combine like terms.}$$

$$-6x=-18 \quad \text{Simplify.}$$

$$x=3 \quad \text{Multiply both sides by} -6.$$

Therefore, the solution is $\boxed{3}$.

Answer 51E.

In order to find the extraneous solution the both sides of the equation are multiplied by LCD of the two ration expression. This can give solutions that are not solutions to the original equation. This type of solution is called extraneous solution.

Consider the following equation:

$$\frac{3}{x^2+3x}+\frac{x+2}{x+3}=\frac{1}{x} \qquad \text{Original equation}$$

$$\frac{3}{x(x+3)}+\frac{x+2}{x+3}=\frac{1}{x} \qquad \text{Factor.}$$

$$x(x+3)\left(\frac{3}{x(x+3)}+\frac{x+2}{x+3}\right)=\frac{1}{x}x(x+3) \qquad \text{The LCD is } x(x+3).$$

$$\frac{x(x+3)3}{x(x+3)}+\frac{x(x+3)(x+2)}{(x+3)}=\frac{x(x+3)}{x} \qquad \text{Use distributive property.}$$

$$3+x(x+2)=x+3 \qquad \text{Simplify.}$$

$$3+x^2+2x=x+3 \qquad \text{Use distributive property.}$$

$$x^2+x=0 \qquad \text{Set equal to 0.}$$

$$x(x+1)=0 \qquad \text{Factor.}$$

$x=0$ or $x+1=0$, $x=-1$ Zero-product property and solve for x

Since, 0 is an excluded value of x, the number 0 is an extraneous solution.

Thus, the solution of the equation is $\boxed{-1}$.

Answer 52E.

In order to find the extraneous solution the both sides of the equation are multiplied by LCD of the two ration expression. This can give solutions that are not solutions to the original equation. This type of solution is called extraneous solution.

Consider the following equation:

$$\frac{1}{n+4}-\frac{1}{n-1}=\frac{2}{n^2+3n-4} \qquad \text{Original equation.}$$

$$\frac{1}{n+4}-\frac{1}{n-1}=\frac{2}{(n+4)(n-1)} \qquad \text{Factor.}$$

$$(n+4)(n-1)\left(\frac{1}{n+4}-\frac{1}{n-1}\right)=\left(\frac{2}{(n+4)(n-1)}\right)(n+4)(n-1)$$

The LCD is $(n+4)(n-1)$

$$\frac{(n+4)(n-1)}{n+4}-\frac{(n+4)(n-1)}{n-1}=\frac{2(n+4)(n-1)}{(n+4)(n-1)}$$

Distributive property.

$$n-1-n-4=2$$

$$-5=2 \qquad \text{Combine like terms.}$$

Thus, the equation has no solutions.